Application of Reduced order Modelling in Geophysics

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ABSTRACT

Simulations in science and engineering, and more specifically geophysics require huge resources, both in terms of processors and time. To tackle the problem of dealing with a large complex model reduced order methods have been devised to solve the model in less number of state variables. Proper Generalized Decomposition or PGD in short, is one such *apriori* technique to solve a high dimensional model in a reduced basis of low dimensions. PGD works by way of separated formulation of the original wave field. In this work, a detailed description of PGD is shown. PGD is applied to wave and heat models to successfully reduce higher dimensional problems and generate solution over the complete parameter range of interest in the offline phase. In the online phase accessing of solution inside the parameter range for any given value is efficient and fast.

A. *Objectives*

The main objective of this work is to effectively design an *apriori* Reduce Order Method solver to solve parametric high dimensional geophysical problems in a cost effective and fast manner.

In the field of geophysics, seismic inversion is an important process to identify geological parameters. Since, the geophysical data obtained (ground/sea) surveys does not produce the information of model of earth directly, inversion methods are applied, example being reverse time migration, Kirchhoff migration etc.[1, 3] The general technique for that is to find a representative smooth model, which in fact is a forward problem using the transient wave equation. Once, the forward problem is generated for all the parameters sufficiently the inversion process is performed by minimizing a suitable function characteristic of representative parameters [2]. In this case of geophysics with standard evaluation of forward models, this a cumbersome and time taking process considering the number of parameters involved (anisotropy, density etc.). To elaborate, each new parametric value requires complete calculation of overall 3D spatial model over the frequencies involved. The aim of this research is to minimize the time taken during such process effectively using Reduced Order Method (ROM) techniques especially *a - priori* method and hence aid in inversion processes.

B. *Methodology*

ROM modeling or Model Order Reduction (MOR) works by way of looking for a generalized solution characteristic of the complete model [4]. It is classified into two types depending on whether it is an *a posteriori* method like, POD/SVD (Proper Orthogonal Decomposition/Singular Value Decomposition) or an *a priori* method like PGD (Proper Generalized Decomposition).

In the context of the problem statement being investigated, PGD is used.

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C. *Research Problem*

Two problems are being investigated to find the effectiveness of PGD algorithm. The first is the heat equation to check the accuracy of the PGD and second is the simulation of a Helmholtz problem which requires more work because of its inherent difficulty in finding a good solution.

- The generic heat equation model is defined as follows;

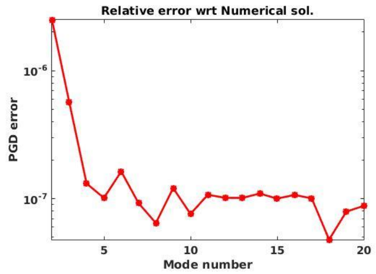
$\frac{\partial u}{\partial t} - k.\Delta u = f.$ Where, k = conductivity.

And, $u = u(x, k, t)$, $f = f(x, k, t)$, $x \in [0,1]$, $t \in [0,0.1]$ and $k \in [1, 5]$.

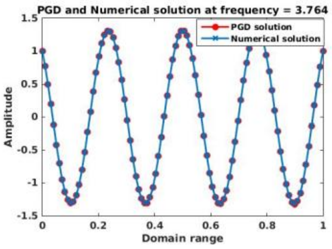
- The Helmholtz equation with 1D in space and 1D in frequency;

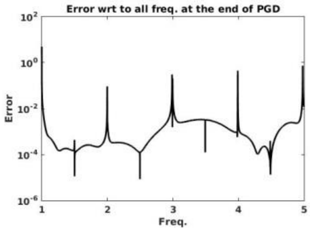
$\nabla \cdot \nabla u + k^2 u = f.$ Where, k = wave number, $u = u(x, \omega)$ and, $f = f(x, \omega)$

D. *Results*



The problem of heat equation is solved through finite difference scheme. Mode number represents the number of mode required to reach the final solution (here, final solution is the solution generated through direct solution of heat equation). It perfectly shows the error by the PGD formulation has reached until 10^{-7} after 10 nodes.





Similarly for the Helmholtz problem with a frequency range of 1-5Hz, the error corresponding to the direct finite difference formulation is shown. It has taken 50 modes to reach the desired level of error.

E. *Conclusion and Future Enhancement*

The problems solved within the purview of the PGD shows it is efficient and is suitable for solving further higher dimensional Helmholtz problem in geophysics. Currently, PGD application in 2D spatial Helmholtz equation is being worked upon with few successes. Also, implementation of perfect absorbing layers, variable source location and different velocity regions is being processed.

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Author biography

Prattya Datta was born in Kolkata, India, in 1989. He received the B.E. degree in Chemical engineering from the Jadavpur University, Kolkata, India, in 2011, and the M.Tech. degree in Petroleum engineering from the Indian Institute of Technology (IIT) Madras, Chennai, India, in 2014. He has also obtained degree of M.S. in Geophysics from Institut de Physics du Globe de Paris (IPGP) in 2013.

Since October 2015, he has been with the Department of CASE, BSC-CNS, where he is registered as Pre-Doctoral student. He is associated with La-caixa fellowship. Prior to joining he has also worked as business analyst in India for a period of 1 year. His current research interests include geophysics, reduced order modeling, data science and machine learning.