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LMI-Based Design of Distributed Energy-Dissipation Systems for Vibration Control of Large Multi-story Structures

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Abstract

In this paper, we present an advanced computational procedure that allows obtaining distributed energy-dissipation systems for large multi-story structures. The proposed methodology is based on a decentralized velocity-feedback energy-to-componentwise-peak (ECWP) controller design approach and can be formulated as a linear matrix inequality (LMI) optimization problem with structure constraints. To demonstrate the effectiveness of the proposed design methodology, a passive damping system is computed for the seismic protection of a 20-story building equipped with a complete set of interstory viscous dampers. The high-performance characteristics of the obtained passive ECWP control system are clearly evidenced by the numerical simulation results. Also, the computational effectiveness of the proposed design procedure is confirmed by the low computation time of the associated LMI optimization problem.

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Keywords: vibration control; large scale control; energy-dissipation systems; passive control; seismic protection

1. Introduction

Designing distributed energy-dissipation systems for vibration control of large structures is a complex and challenging problem [1]. When viscous dampers are implemented as interstory actuation devices at different levels of the structure, the resulting damping forces are proportional to the corresponding interstory velocities. From a control design perspective, this kind of passive control configuration is equivalent to a static fully decentralized velocity-feedback active controller. By considering this relation and exploiting recent results in static output-feedback control, linear matrix inequality (LMI) formulations can be used to design distributed energy-dissipation systems [2]. The energy-to-componentwise-peak (ECWP) norm is a kind of generalized H_2 norm that can produce very positive results in designing state-feedback controllers for the seismic protection of medium-size multi-story buildings. However, the high computational cost of the associated LMI optimization problem makes this approach unsuitable for large-dimension problems [3]. In this paper, we present an advanced computational procedure that allows obtaining distributed energy-dissipation systems with high-performance characteristics. The proposed methodology is based on

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a decentralized velocity-feedback ECWP approach and has a reduced computational cost, which makes it appropriate for large-scale systems. Following this ECWP design strategy, the damping constants of a complete system of inter-story viscous dampers are computed for the seismic protection of a 20-story building. For the same building, an active state-feedback H_∞ controller and an active state-feedback ECWP controller are also designed. To assess the performance of the obtained energy-dissipation system, the building seismic response is computed for the different control configurations using the North-South Kobe seismic record as ground acceleration disturbance. The computation times corresponding to the associated LMI optimization problems are also compared to evaluate the computational effectiveness. The obtained results clearly show the high-performance characteristics of the ECWP damping system and the low computational cost of the proposed design procedure. The rest of the paper is organized as follows: In Section 2, the state-feedback and output-feedback ECWP controller design strategies are presented. In Section 3, the computational procedure to design passive ECWP damping systems is discussed. In Section 4, the numerical results corresponding to the 20-story building are provided, and the effectiveness of the different controllers are compared. Additionally, a mathematical model for the lateral displacement of an n-story building is provided in Appendix A, and a brief summary of the state-feedback H_∞ controller design methodology is included in Appendix B.

2. ECWP controller design

2.1. State-feedback ECWP controllers

In this section, we provide an LMI formulation for the design of state-feedback ECWP controllers. We firstly consider a linear system in the following form:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{w}(t) \\ \mathbf{z}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^{n_x}$ is the state, $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ is the control input, $\mathbf{w}(t) \in \mathbb{R}^{n_w}$ is the disturbance input, $\mathbf{z}(t) \in \mathbb{R}^{n_z}$ is the controlled output and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{C}$, $\mathbf{D}$ are constant real matrices with appropriate dimensions. A state-feedback controller $\mathbf{u}(t) = \mathbf{G}\mathbf{x}(t)$ with gain matrix $\mathbf{G} \in \mathbb{R}^{n_u \times n_x}$ defines the closed-loop system

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_G\mathbf{x}(t) + \mathbf{E}\mathbf{w}(t) \\ \mathbf{z}(t) = \mathbf{C}_G\mathbf{x}(t) \end{cases} \tag{2}$$

with $\mathbf{A}_G = \mathbf{A} + \mathbf{B}\mathbf{G}$ and $\mathbf{C}_G = \mathbf{C} + \mathbf{D}\mathbf{G}$. In the ECWP controller design, we consider the system norm

$$\gamma(\mathbf{G}) = \sup_{\|\mathbf{w}\|_2 \neq 0} \frac{\|\mathbf{z}\|_\infty}{\|\mathbf{w}\|_2}, \tag{3}$$

where $\|\mathbf{z}\|_\infty = \sup_{0 \le t < +\infty} \max_i |z_i(t)|$ is the continuous componentwise-peak-norm, and $\|\mathbf{w}\|_2 = \left[\int_0^\infty \mathbf{w}^T(t)\mathbf{w}(t)dt\right]^{1/2}$ is the usual continuous energy-norm. The design strategy consists in obtaining an optimal control gain matrix $\widetilde{\mathbf{G}}$ that produces an asymptotically stable closed-loop matrix $\mathbf{A}_{\widetilde{G}}$ and, simultaneously, minimizes the worst-case gain $\gamma(\mathbf{G})$. Using an LMI approach, this optimization problem can be formulated as follows [4]:

$$\mathcal{P}: \begin{cases} \text{minimize} \quad \eta \\ \text{subject to:} \quad \mathbf{X} > 0,\ \eta > 0, \\ \qquad \mathbf{A}\mathbf{X} + \mathbf{X}\mathbf{A}^T + \mathbf{B}\mathbf{Y} + \mathbf{Y}^T\mathbf{B}^T + \mathbf{E}\mathbf{E}^T < 0, \\ \qquad \begin{bmatrix} \mathbf{X} & \mathbf{X}\mathbf{C}_i^T + \mathbf{Y}^T\mathbf{D}_i^T \\ \mathbf{C}_i\mathbf{X} + \mathbf{D}_i\mathbf{Y} & \eta \end{bmatrix} > 0,\ 1 \le i \le n_z, \end{cases} \tag{4}$$

where $\mathbf{C}_i$ and $\mathbf{D}_i$ denote the ith row of the matrices $\mathbf{C}$ and $\mathbf{D}$, respectively, and $\mathbf{X} = \mathbf{X}^T \in \mathbb{R}^{n_x \times n_x}$, $\mathbf{Y} \in \mathbb{R}^{n_u \times n_x}$ are the LMI variables. If the problem $\mathcal{P}$ attains an optimal value $\widetilde{\eta}$ for the LMI matrices $(\widetilde{\mathbf{X}}, \widetilde{\mathbf{Y}})$, then the state-feedback gain matrix $\widetilde{\mathbf{G}} = \widetilde{\mathbf{Y}}\widetilde{\mathbf{X}}^{-1}$ defines an optimal ECWP controller with an associated γ-value $\gamma\left(\widetilde{\mathbf{G}}\right) = \widetilde{\eta}^{1/2}$.

Table 1. Mass and stiffness characteristics of the 20-story building model.

Story	1–5	6–11	12–14	15–17	18–19	20
mass ($\times 10^6$ Kg)	1.10	1.10	1.10	1.10	1.10	1.10
stiffness ($\times 10^8$ N/m)	8.62	5.54	4.54	2.91	2.56	1.72

2.2. Static output-feedback ECWP controllers

Let us now consider the linear system in Eq. (1) and a vector of measured outputs $\mathbf{y}(t) \in \mathbb{R}^{n_y}$, which can be written as a linear combination of the states in the form $\mathbf{y}(t) = \mathbf{C}_\mathrm{y}\mathbf{x}(t)$. We are interested in designing a static output-feedback controller $\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t)$, which allows computing the control actions $\mathbf{u}(t)$ from the measured-output information $\mathbf{y}(t)$ through a simple matrix product. A static output-feedback controller with gain matrix $\mathbf{K} \in \mathbb{R}^{n_u \times n_y}$ defines the closed-loop system

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_K\mathbf{x}(t) + \mathbf{E}\mathbf{w}(t) \\ \mathbf{z}(t) = \mathbf{C}_K\mathbf{x}(t) \end{cases} \tag{5}$$

with $\mathbf{A}_K = \mathbf{A} + \mathbf{B}\mathbf{K}\mathbf{C}_y$ and $\mathbf{C}_K = \mathbf{C} + \mathbf{D}\mathbf{K}\mathbf{C}_y$. In this case, the design goal is to obtain an optimal output-feedback gain matrix $\widetilde{\mathbf{K}}$ that produces an asymptotically stable closed-loop matrix $\mathbf{A}_{\widetilde{K}}$ and, simultaneously, minimizes the worst-case gain $\gamma(\mathbf{K})$. Using the results presented in [5], if the measured-output matrix $\mathbf{C}_y$ is full row-rank, a suboptimal static output-feedback ECWP controller can be designed by solving the following LMI optimization problem:

$$\mathcal{P}_{\mathrm{out}}: \begin{cases} \text{minimize} \quad \eta \\ \text{subject to:} \quad \mathbf{X}_Q > 0,\ \mathbf{X}_R > 0,\ \eta > 0, \\ \qquad \mathbf{A}\mathbf{Q}\mathbf{X}_Q\mathbf{Q}^T + \mathbf{Q}\mathbf{X}_Q\mathbf{Q}^T\mathbf{A}^T + \mathbf{A}\mathbf{R}\mathbf{X}_R\mathbf{R}^T + \mathbf{R}\mathbf{X}_R\mathbf{R}^T\mathbf{A}^T + \mathbf{B}\mathbf{Y}_R\mathbf{R}^T + \mathbf{R}\mathbf{Y}_R^T\mathbf{B}^T + \mathbf{E}\mathbf{E}^T < 0, \\ \qquad \begin{bmatrix} \mathbf{Q}\mathbf{X}_Q\mathbf{Q}^T + \mathbf{R}\mathbf{X}_R\mathbf{R}^T & \mathbf{Q}\mathbf{X}_Q\mathbf{Q}^T\mathbf{C}_i^T + \mathbf{R}\mathbf{X}_R\mathbf{R}^T\mathbf{C}_i^T + \mathbf{R}\mathbf{Y}_R^T\mathbf{D}_i^T \\ \mathbf{C}_i\mathbf{Q}\mathbf{X}_Q\mathbf{Q}^T + \mathbf{C}_i\mathbf{R}\mathbf{X}_R\mathbf{R}^T + \mathbf{D}_i\mathbf{Y}_R\mathbf{R}^T & \eta \end{bmatrix} > 0,\ 1 \le i \le n_z. \end{cases} \tag{6}$$

This optimization problem includes three variable matrices: $\mathbf{X}_Q$, $\mathbf{X}_R$ and $\mathbf{Y}_R$, and two constant matrices: $\mathbf{Q}$ and $\mathbf{R}$. The columns of the matrix $\mathbf{Q}$ contain a basis of Ker($\mathbf{C}_\mathrm{y}$), and the matrix $\mathbf{R}$ has the form $\mathbf{R} = \mathbf{C}_\mathrm{y}^T(\mathbf{C}_\mathrm{y}\mathbf{C}_\mathrm{y}^T)^{-1} + \mathbf{Q}\mathbf{L}$, where $\mathbf{L}$ is a constant matrix that acts as a free parameter in the controller design. If an optimal value $\widetilde{\eta}_{\mathrm{out}}$ is attained in $\mathcal{P}_{\mathrm{out}}$ for the triplet $(\widetilde{\mathbf{X}}_Q, \widetilde{\mathbf{X}}_R, \widetilde{\mathbf{Y}}_R)$, then the corresponding output-feedback gain matrix can be written in the form $\widetilde{\mathbf{K}} = \widetilde{\mathbf{Y}}_R\widetilde{\mathbf{X}}_R^{-1}$, and the value $\widetilde{\gamma}_{_K} = (\widetilde{\eta}_{\mathrm{out}})^{1/2}$ provides an upper bound of the ECWP controller norm $\gamma(\widetilde{\mathbf{K}})$.

Remark 1. Structured static output-feedback controllers can be designed by setting a suitable sparsity pattern on the LMI variable matrices. In particular, a diagonal output-feedback gain matrix can be obtaining by constraining the matrices $\mathbf{X}_R$ and $\mathbf{Y}_R$ to diagonal form.

Remark 2. The LMI constant matrix $\mathbf{L}$ is an important element that can contribute to improve the effectiveness of the proposed design methodology. From a computational point of view, using a null $\mathbf{L}$ matrix is a particularly convenient option in large-dimension problems. However, the choice $\mathbf{L} = \mathbf{0}$ can produce feasibility issues in the LMI optimization problem. After extensive numerical simulations, it has been observed that, frequently, these feasibility issues can be overcome by using a slightly perturbed state matrix of the form $\mathbf{A}_p = \mathbf{A} - \epsilon\mathbf{I}_{n_x}$ [2].

Remark 3. An advanced choice of the matrix $\mathbf{L}$ that can help to avoid feasibility issues in the optimization problem $\mathcal{P}_{\mathrm{out}}$ has been proposed in [6]. However, due to the additional cost of computing the matrix $\mathbf{L}$, this design strategy is only effective for problems with low or medium-size dimension.

3. Design of distributed passive damping systems

Let us consider an n-story building equipped with a system of force-actuation devices a_j, $j = 1, \ldots, n$, implemented between each pair of consecutive stories. If these devices are assumed to be ideal viscous dampers, then the actuation

Table 2. Damping constants corresponding to the ECWP damping system ($\times 10^7$Ns/m).

d_1 = 3.8639	d_2 = 3.5995	d_3 = 3.3819	d_4 = 3.1996	d_5 = 3.0477	d_6 = 1.8784	d_7 = 1.8286	d_8 = 1.8101
d_9 = 1.8254	d_{10} = 1.8788	d_{11} = 1.9774	d_{12} = 1.7474	d_{13} = 1.9400	d_{14} = 2.2250	d_{15} = 1.6975	d_{16} = 1.5868
d_{17} = 1.4600	d_{18} = 1.1647	d_{19} = 1.0347	d_{20} = 0.5953				

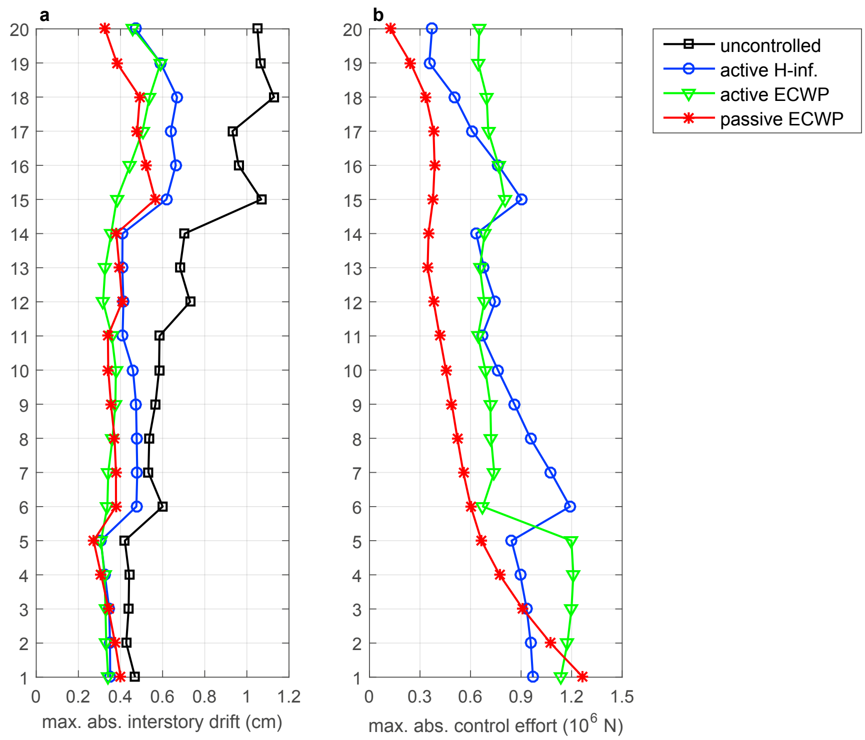

Fig. 1. Seismic response of the twenty-story building model corresponding to the uncontrolled configuration (black line with squares), the active state-feedback H_∞ controller (blue line with circles), the active state-feedback ECWP controller (green line with triangles) and the passive ECWP controller (red line with asterisks). (a) Maximum absolute interstory drifts. (b) Maximum absolute control efforts. The North-South Kobe seismic record scaled to a peak-value of 1 m/s^2 has been used as ground acceleration disturbance.

force u_j produced by the interstory actuator a_j can be modeled as $u_j = -d_j\,\dot{r}_j$, where $\dot{r}_j$ is the j-th interstory velocity defined in Appendix A, and d_j is the corresponding damping constant. Following the ideas presented in [2], we can consider the vector of measured outputs $\mathbf{y}(t) = [\dot{r}_1, \ldots, \dot{r}_n]^T$ and design a fully decentralized velocity-feedback controller $\mathbf{u}(t) = \widehat{\mathbf{K}}\,\mathbf{y}(t)$, with a diagonal gain matrix $\widehat{\mathbf{K}} = \mathrm{diag}\,(\hat{k}_1, \ldots, \hat{k}_n)$. If the diagonal elements $\hat{k}_j$ are all negative, then the obtained velocity-feedback controller defines a set of damping constants $d_j = -\hat{k}_j,\ j = 1, \ldots, n$. This approach allows designing distributed damping systems with high-performance characteristics by taking advantage of advanced design methodologies for decentralized output-feedback active control. Specifically, in order to design a passive damping system following the static output-feedback ECWP design methodology presented in Section 2.2, we consider the state-space building model $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}w(t)$ presented in Appendix A, and the vector of measured outputs $\mathbf{y}(t) = \mathbf{C}_y\mathbf{x}(t)$ with $\mathbf{C}_y = [[\mathbf{0}]_{n\times n}\ \mathbf{I}_n]$. Next, assuming that the control objective is to reduce the interstory drift values in the seismically excited building by means of moderate control actions, we select the controlled-output vector $\mathbf{z}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$ defined by the matrices

$$\mathbf{C} = \begin{bmatrix} \mathbf{I}_n & [\mathbf{0}]_{n\times n} \\ [\mathbf{0}]_{n\times n} & [\mathbf{0}]_{n\times n} \end{bmatrix}, \quad \mathbf{D} = \alpha \begin{bmatrix} [\mathbf{0}]_{n\times n} \\ \mathbf{I}_n \end{bmatrix}, \tag{7}$$

where α is a scaling factor that compensates the different magnitude of interstory drifts and control forces. Finally, to compute a diagonal velocity-feedback gain matrix $\widehat{\mathbf{K}}$, we solve the LMI optimization problem $\mathcal{P}_{\text{out}}$ in Eq. (6), constraining the structure of the LMI variables $\mathbf{X}_\text{R}$ and $\mathbf{Y}_\text{R}$ to diagonal form.

4. Numerical results

By applying the proposed design procedure to a 20-story building model with the mass and stiffness characteristics presented in Table 1, a 5% of relative damping, and the scaling coefficient $\alpha = 10^{-8.2}$, we obtain the set of damping constants collected in Table 2. For the same building model, we also design an optimal state-feedback ECWP controller and an optimal state-feedback H_∞ controller by solving, respectively, the LMI optimization problem $\mathcal{P}$ in Eq. (4) with the scaling coefficient $\alpha = 10^{-8.2}$, and the LMI optimization problem $\mathcal{P}_\infty$ in Eq. (B.2) with the scaling coefficient $\alpha = 10^{-8.0}$. The computation times required to solve the different LMI optimization problems are the following: 33.18 seconds for ECWP passive controller, 355.16 seconds for the state-feedback ECWP controller, and 102.45 seconds for the state-feedback H_∞ controller. To demonstrate the performance of the computed damping system, a proper set of numerical simulations has been carried out using the North-South Kobe seismic record scaled to an acceleration peak of 1 m/s^2. The plots in Fig. 1(a) show the maximum absolute interstory drifts obtained for the uncontrolled building (black line with squares), the state-feedback H_∞ controller (blue line with circles), the state-feedback ECWP controller (green line with triangles), and the passive control system defined by the damping constants in Table 2 (red line with asterisks). The corresponding maximum absolute control efforts are displayed in Fig. 1(b) using the same colors and symbols. These plots provide a good demonstration of the high-performance characteristics of the designed passive control system when compared with the optimal state-feedback active controllers. Additionally, the obtained computation times clearly confirm the effectiveness of the proposed methodology in large-dimension problems. Considering the positive results indicated by the numerical simulations, an experimental validation should be carried out to explore the practical applicability of the proposed energy-dissipation systems.

Remark 4. In order to reduce the computation time, the matrix $\mathbf{R}$ in the LMI optimization problem $\mathcal{P}_{\text{out}}$ has been computed using a null $\mathbf{L}$ matrix. Also, a perturbed system matrix $\mathbf{A}_p = \mathbf{A} - 0.02\,\mathbf{I}_{40}$ has been used to avoid the encountered feasibility issues.

Remark 5. All the computations in this paper have been carried out using Matlab© R2015b on a regular laptop with an Intel© Core™ i7-2640M processor at 2.80 GHz. The LMI optimization problems corresponding to the different controller designs have been solved with the function `mincx()` included in the Robust Control Toolbox™.

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Appendix A. Building Model

In this Appendix we briefly present a state-space model for the lateral displacement of an n-story building structure equipped with a complete set of interstory actuation devices. An illustrative figure and a more detailed derivation can be found in [3]. Let us consider the state vector $\mathbf{x}_I(t) = [q_1(t), \ldots, q_n(t), \dot{q}_1(t), \ldots, \dot{q}_n(t)]^T$, where $q_j(t)$ is the relative displacement of the j-th story with respect to the ground. The building's lateral motion can be described by the state-space model $\dot{\mathbf{x}}_I(t) = \mathbf{A}_I\mathbf{x}_I(t) + \mathbf{B}_I\mathbf{u}(t) + \mathbf{E}_I w(t)$, where $\mathbf{u}(t) = [u_1(t), \ldots, u_n(t)]^T$ is the vector of control actions and $w(t)$ is the ground acceleration disturbance. The model matrices have the following structure:

$$\mathbf{A}_I = \begin{bmatrix} [\mathbf{0}]_{n\times n} & \mathbf{I}_n \\ -\mathbf{M}^{-1}\mathbf{K}_s & -\mathbf{M}^{-1}\mathbf{C}_d \end{bmatrix}, \quad \mathbf{B}_I = \begin{bmatrix} [\mathbf{0}]_{n\times n} \\ \mathbf{M}^{-1}\mathbf{T}_u \end{bmatrix}, \quad \mathbf{E}_I = \begin{bmatrix} [\mathbf{0}]_{n\times 1} \\ -[\mathbf{1}]_{n\times 1} \end{bmatrix}, \tag{A.1}$$

where $[\mathbf{0}]_{n\times m}$ is a zero matrix of dimension $n \times m$, $\mathbf{I}_n$ is the identity matrix of dimension n and $[\mathbf{1}]_{n\times 1}$ denotes a vector of dimension n with all its entries equal to 1. $\mathbf{M}$ and $\mathbf{K}_s$ are the mass and stiffness matrices, respectively, which have the following diagonal and tridiagonal form:

$$\mathbf{M} = \begin{bmatrix} m_1 & & & \\ & \cdots & & \\ & & \cdots & \\ & & & m_n \end{bmatrix}, \quad \mathbf{K}_s = \begin{bmatrix} k_1 + k_2 & -k_2 & & & & \\ -k_2 & k_2 + k_3 & -k_3 & & & \\ & \cdots & \cdots & \cdots & & \\ & & \cdots & \cdots & \cdots & \\ & & & -k_{n-1} & k_{n-1} + k_n & -k_n \\ & & & & -k_n & k_n \end{bmatrix}, \tag{A.2}$$

where m_j and k_j denote the mass and stiffness coefficients of the j-th story, respectively. $\mathbf{C}_\mathrm{d}$ is the damping matrix, which can be computed from $\mathbf{M}$ and $\mathbf{K}_\mathrm{s}$ by setting a proper damping ratio on the building modes [7]. Finally, $\mathbf{T}_\mathrm{u}$ is the control location matrix, which models the action of the control forces. For a complete system of interstory actuation devices, $\mathbf{T}_\mathrm{u}$ is a square matrix of size n with the following upper-diagonal band structure:

$$\mathbf{T}_u = \begin{bmatrix} 1 & -1 & & & & \\ & 1 & -1 & & & \\ & & \cdots & \cdots & & \\ & & & \cdots & \cdots & \\ & & & & 1 & -1 \\ & & & & & 1 \end{bmatrix}. \tag{A.3}$$

Frequently, it can be more convenient to use a state vector formed by the interstory drifts and the interstory velocities $\mathbf{x}(t) = [r_1(t), \ldots, r_n(t), \dot{r}_1(t), \ldots, \dot{r}_n(t)]^T$, where $r_1(t) = q_1(t)$, and $r_j(t) = q_j(t) - q_{j-1}(t)$, $j = 2, \ldots, n$. The new state-space model $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}w(t)$ can be obtained with the matrices $\mathbf{A} = \mathbf{P}\mathbf{A}_I\mathbf{P}^{-1}$, $\mathbf{B} = \mathbf{P}\mathbf{B}_I$, $\mathbf{E} = \mathbf{P}\mathbf{E}_I$, where $\mathbf{P}$ is the change-of-basis matrix corresponding to the state transformation $\mathbf{x}(t) = \mathbf{P}\mathbf{x}_I(t)$, which in this particular case can be written in the form $\mathbf{P} = \text{blockdiag}\left(\mathbf{T}_\mathrm{u}^T, \mathbf{T}_\mathrm{u}^T\right)$.

Appendix B. H_∞ controller design

In this Appendix, we provide a concise summary of the state-feedback H_∞ controller design approach. For the linear system given in Eq. (1) and the state-feedback controller $\mathbf{u}(t) = \mathbf{G}\mathbf{x}(t)$, we consider the H_∞ system norm

$$\gamma_\infty(\mathbf{G}) = \sup_{||\mathbf{w}||_2 \neq 0} \frac{||\mathbf{z}||_2}{||\mathbf{w}||_2}, \tag{B.1}$$

where $||\cdot||_2$ is the usual continuous 2-norm. In this case, the objective is to obtain an optimal state-feedback control gain matrix $\hat{\mathbf{G}}$ that produces an asymptotically stable closed-loop matrix $\mathbf{A}_{\hat{G}}$ and, simultaneously, minimizes the worst-case gain $\gamma_\infty(\mathbf{G})$. This matrix can be computed by solving the following LMI optimization problem [8]:

$$\mathcal{P}_\infty : \begin{cases} \text{maximize} \quad \eta \\ \text{subject to: } \mathbf{X} > 0,\ \eta > 0, \\ \qquad \begin{bmatrix} \mathbf{AX} + \mathbf{XA}^T + \mathbf{BY} + \mathbf{Y}^T\mathbf{B}^T + \eta\mathbf{EE}^T & \mathbf{XC}^T + \mathbf{Y}^T\mathbf{D}^T \\ \mathbf{CX} + \mathbf{DY} & -\mathbf{I}_{n_z} \end{bmatrix} < 0, \end{cases} \tag{B.2}$$

where $\mathbf{X} = \mathbf{X}^T \in \mathbb{R}^{n_x \times n_x}$ and $\mathbf{Y} \in \mathbb{R}^{n_u \times n_x}$ are the optimization variables. If an optimal value $\hat{\eta}_\infty$ is attained in $\mathcal{P}_\infty$ for the pair $(\hat{\mathbf{X}}, \hat{\mathbf{Y}})$, then the state-feedback gain matrix $\hat{\mathbf{G}} = \hat{\mathbf{Y}}\hat{\mathbf{X}}^{-1}$ defines an optimal H_∞ controller and the corresponding γ-value can be computed as $\gamma_\infty(\hat{\mathbf{G}}) = (\hat{\eta}_\infty)^{-1/2}$.

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