Checking and improving the geometric accuracy of non-interpolating curved high-order meshes

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Abstract *We define a new disparity measure between curved high-order meshes and parameterized manifolds in terms of a differentiable norm. The main application of the proposed definition is to measure and minimize the distance between a curved high-order mesh and a target parameterized curve or surface. We obtain exponential convergence rates of geometric accuracy even when non-uniform parameterizations of the target manifold are prescribed. Thus, we can generate coarse curved high-order meshes significantly more accurate than finer low-order meshes for the same resolution. The approach deals with nodes on top of the curve or surface (interpolative), or floating freely in the physical space (non-interpolative).*

Introduction

The ability to **measure and enhance the geometric accuracy** of a curved high-order mesh is essential to perform **solution convergence studies for unstructured high-order methods**. Note that geometrically inaccurate meshes can pollute the approximated solution and therefore, impede to obtain the **exponential rates of convergence** predicted by the theory. There are several ways to define the distance between two discretized manifolds [1-4].

Methodology

Given two m-dimensional manifolds in $\mathbb{R}^n$, Σ_1 and Σ_2, the new $\mathcal{L}_2$-disparity of Σ_1 and Σ_2 is defined as

$$d\left(\Sigma_1, \Sigma_2\right) = \inf_{\boldsymbol{\phi}^U} \left\|\boldsymbol{\varphi}_1 - \boldsymbol{\varphi}_2 \circ \boldsymbol{\phi}^U\right\| = \inf_{\boldsymbol{\phi}^U} \sqrt{\int_{U_1} \left\|\boldsymbol{\varphi}_1 - \boldsymbol{\varphi}_2 \circ \boldsymbol{\phi}^U\right\|^2 \,\mathrm{d}\Omega},$$

where $\|\cdot\|$ is the Euclidean norm, and $\boldsymbol{\phi}^U$ are all the possible orientation-preserving diffeomorphisms between $\mathcal{U}_2$ and $\mathcal{U}_1$.

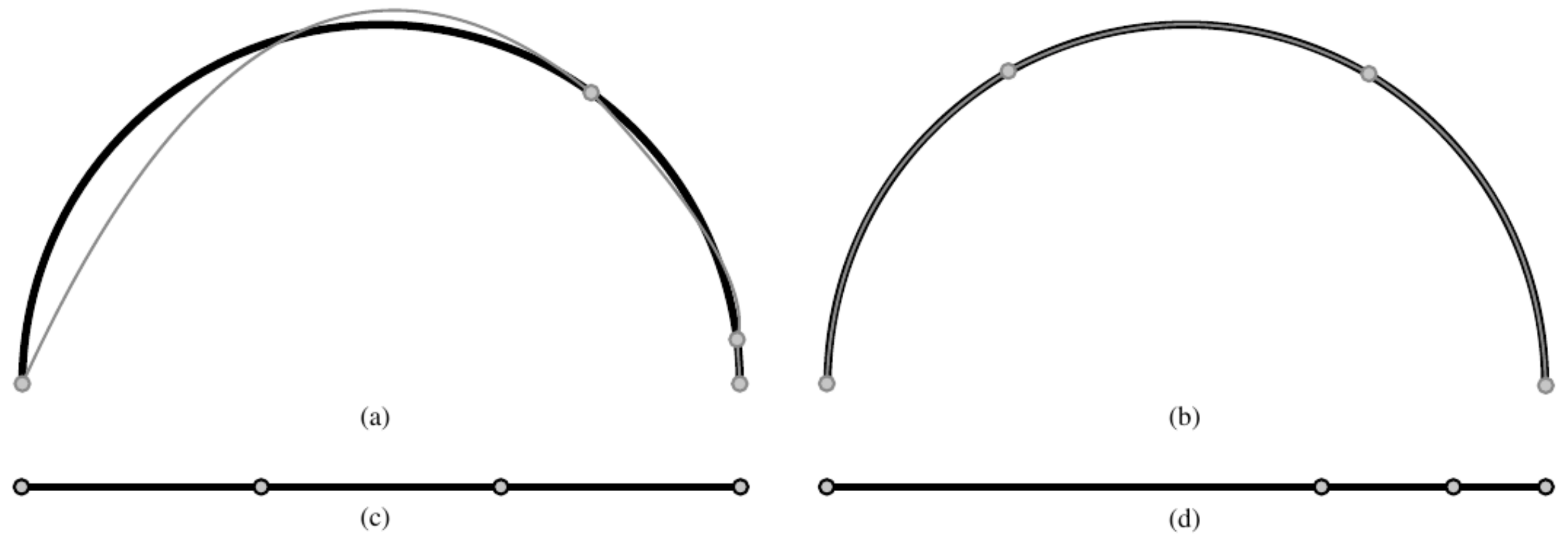
Non-interpolative but more accurate

By minimizing the square of the disparity measure, the disparity between the circular arc and a non-interpolative linear mesh (b) is smaller (d = 0.15) than against a standard interpolative mesh (a) (d = 0.065). **Non-interpolative meshes have more freedom to accurately approximate target geometries**.

Linear meshes for a circular arc: (a) an interpolative mesh (nodes on top) and (b) a non-interpolative mesh (nodes floating around).

Independence of parameterization

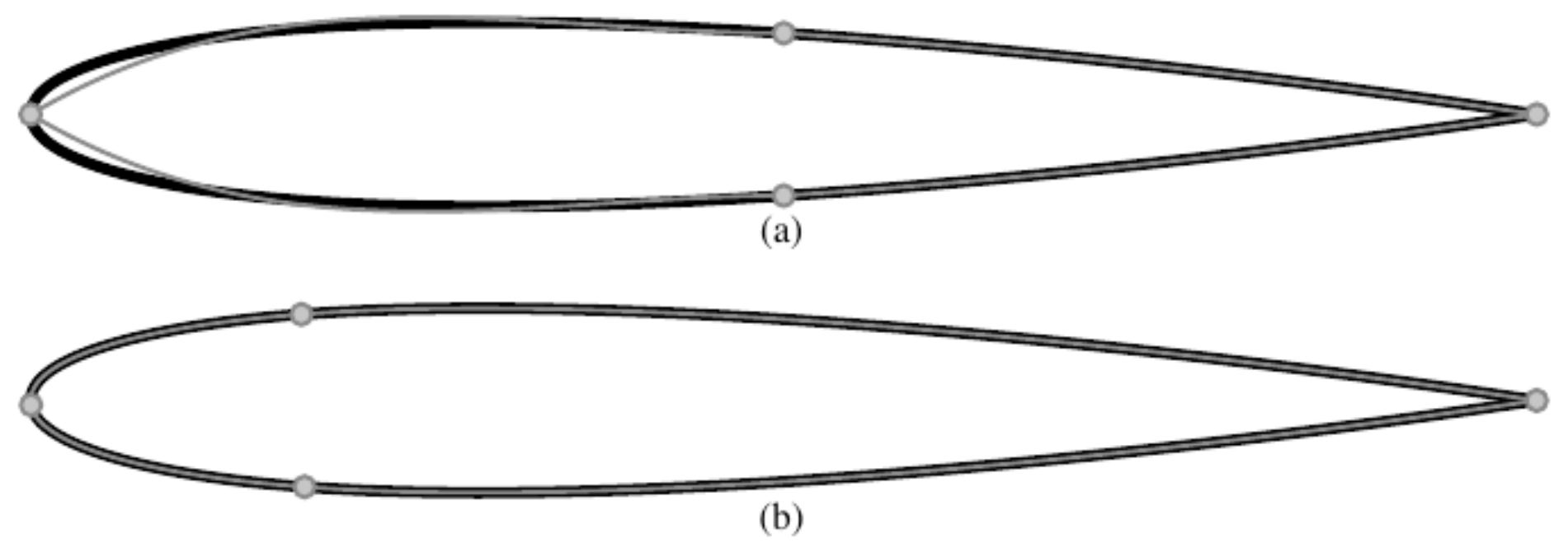
The proposed formulation is independent of the parameterization of the target geometry. Accordingly, is able to relocate the nodes in the parametric space in such a manner that they lead to non-interpolative meshes of optimal disparity in the physical space.



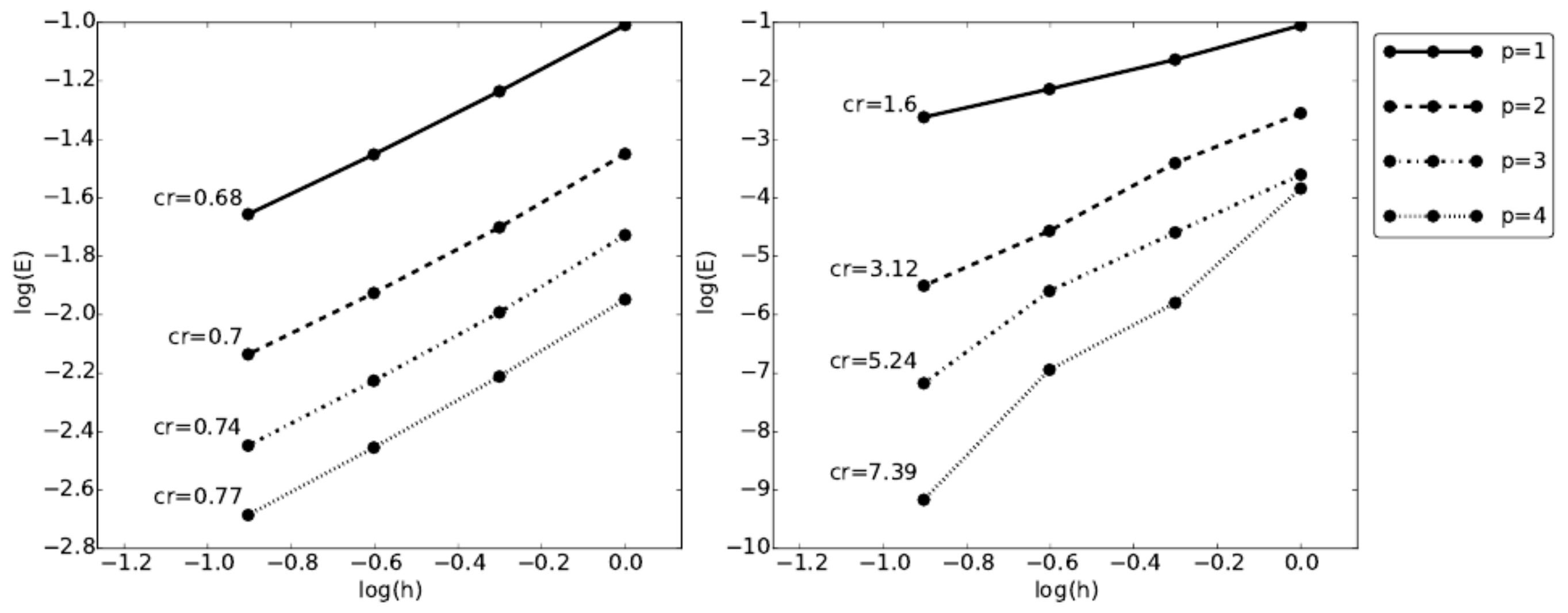
Quadratic meshes for a non-uniformly parameterized circular arc: (a) physical mesh for (c) equi-sized elements on the parametric space, and (b) a non-interpolative mesh (nodes floating around) with optimal disparity with (d) non-uniformly distributed elements on the parametric space.

Exponential convergence rates

For non-straight sided meshes our formulation provides **exponential convergence rates for geometric accuracy**, even when non-uniform parameterizations of the target manifold are used.



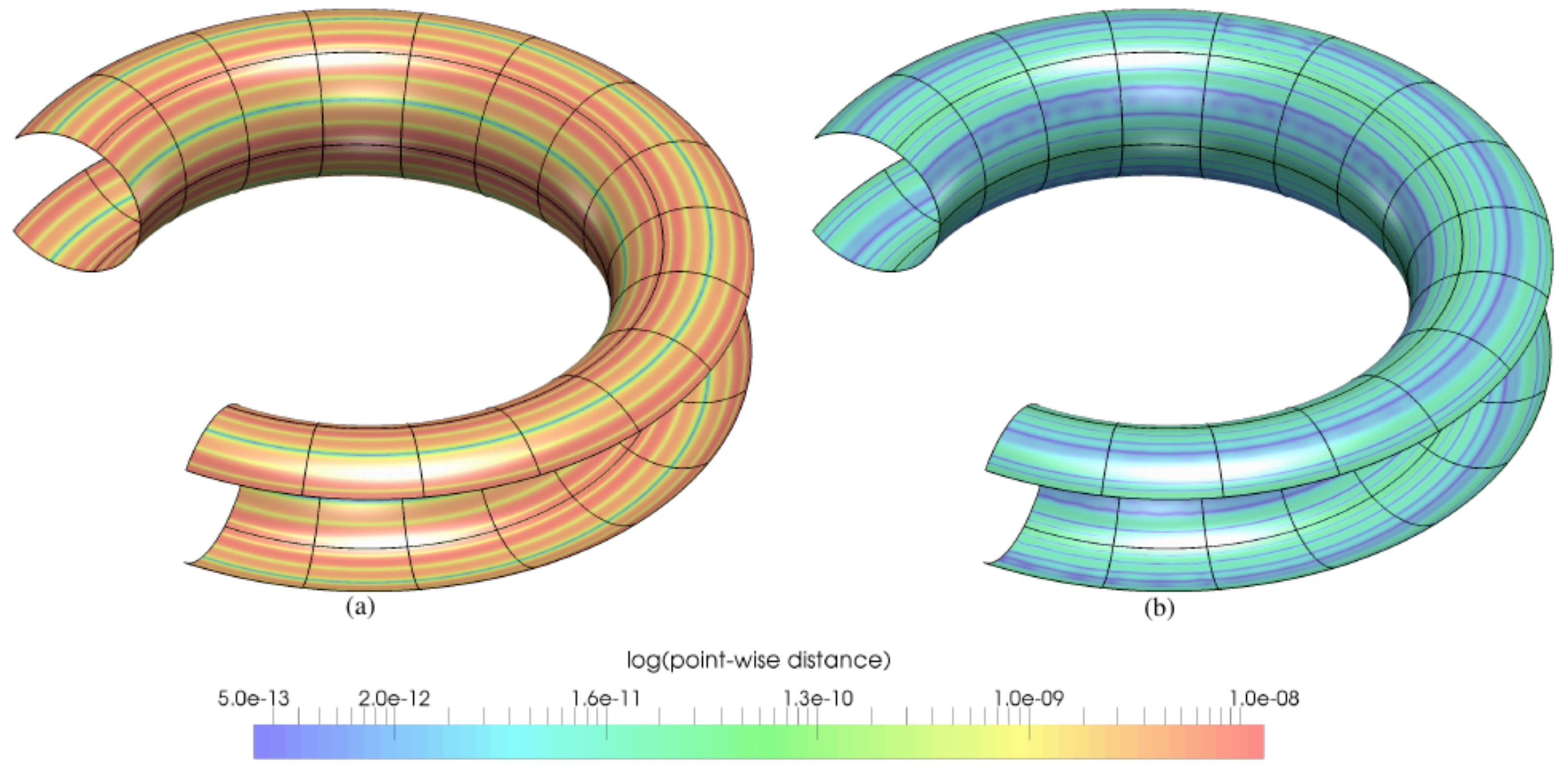
Cubic meshes (gray line) for a NACA0012 (black): (a) an interpolative mesh and (b) a non-interpolative mesh with optimal disparity.



Convergence rate (cr) of the disparity of the meshes generated for the: (a) the NACA0012 using the initial interpolating meshes, and (right) the non-interpolative meshes with optimal disparity (right).

Dealing with different types of curvatures

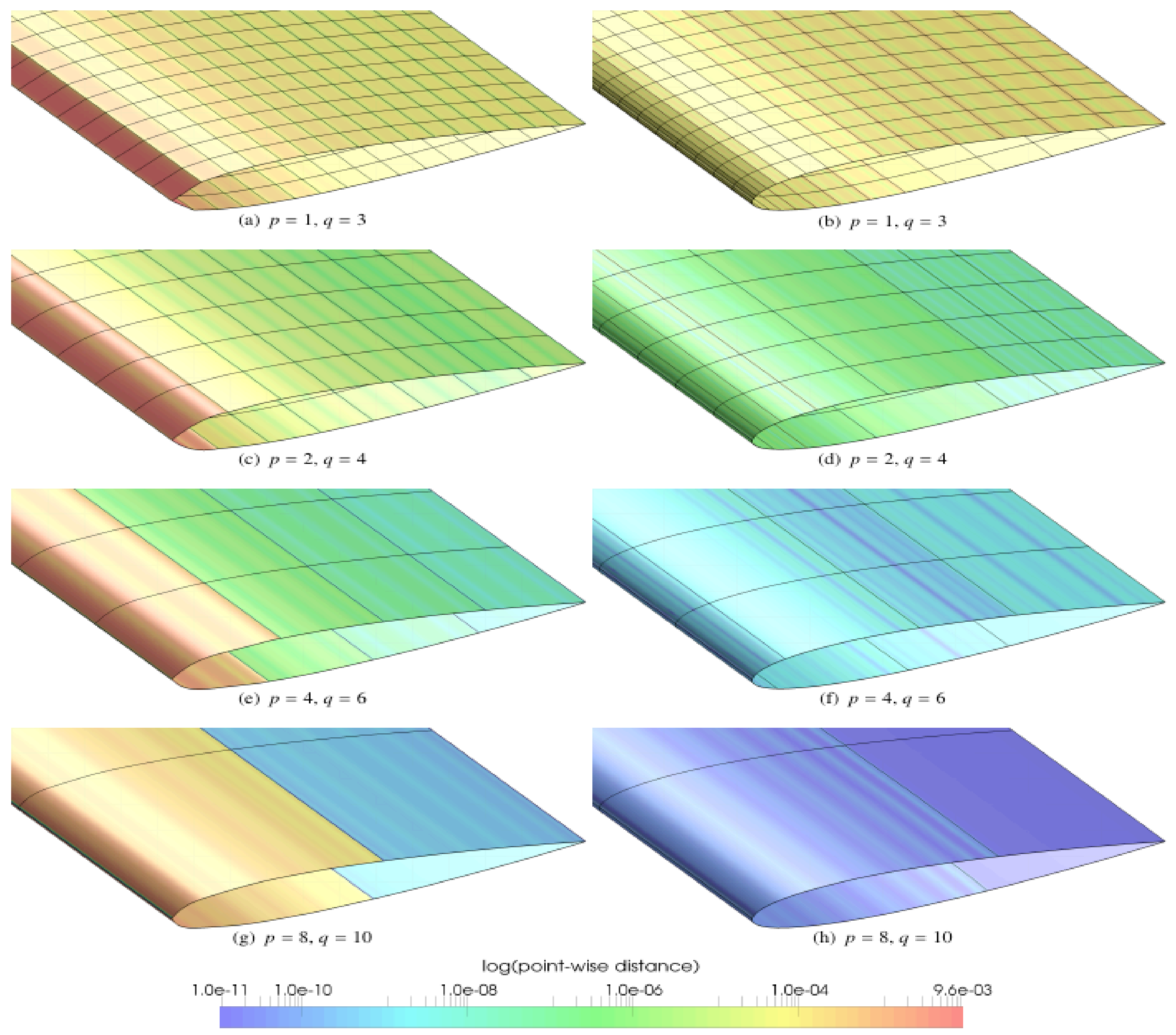
Our method is independent of the manifold dimension. Specifically, it **deals with different types of surface curvatures: elliptic, hyperbolic, parabolic, planar**.



Sixtic meshes for a piece of a torus (logarithm of the point-wise distance): (a) an interpolative mesh and (b) a non-interpolative mesh with optimal disparity measure.

Higher accuracy for the same resolution

Using **high polynomial degrees** we can generate **coarse curved meshes significantly more geometrically accurate than finer linear meshes**.



Meshes generated for the NACA0012 airfoil surface. In columns, initial interpolating meshes (a), (c), (e) and (g); and optimal disparity meshes (b), (d) , (f) and (h). In rows, the polynomial degrees of the physical and parametric meshes.

Concluding remarks

Our method to enhance the geometric accuracy of a curved high-order mesh: is non-interpolative and independent of parameterization, features exponential convergence rates, is independent of manifold dimension, and deals with high polynomial degrees. Therefore, it is really well suited to generate **accurate high-order approximations of non-uniformly parameterized CAD entities that may arise in practical applications**.

In the near future we would like to: apply our method to complex assembly models, improve the implementation performance, and combine it with our smoothing and untangling methods for curved high-order meshes [5-7].

References

[1] P Cignoni, C Rocchini, and R Scopigno. Metro: measuring error on simplified surfaces. Computer Graphics Forum, 17, 167-174, 1998.
[2] H Alt and M Buchin. Can we compute the similarity between surfaces? Disc. & Comp. Geom., 43(1):78–99, 2010.
[3] H Alt and M Godau. Computing the Fréchet distance between two polygonal curves. Int. J. Comp. Geom. & App., 5(1):75–91, 1995.
[4] JF Remacle, J Lambrechts, C Geuzaine, and T Toulorge. Optimizing the geometrical accuracy of 2D curvilinear meshes. Procedia Engineering,82:228–239, 2014.
[5] A Gargallo-Peirró, X Roca, and J Sarrate. A surface mesh smoothing and untangling method independent of the CAD parameterization. Comput. Mech., 53(4):587–609, 2014.
[6] A Gargallo-Peiró, X Roca, J Peraire, and J Sarrate. Optimization of a regularized distortion measure to generate curved high-order unstructured tetrahedral meshes. Int. J. Numer. Meth. Engng, Online, 2015.
[7] E Ruiz-Gironés, X Roca, and J Sarrate. High-order mesh curving by distortion minimization with boundary nodes free to slide on a 3D CAD representation. CAD, Online, 2015.