APPLICATION OF THE MEASURED EQUATION OF INVARIANCE TO TRANSMISSION LINES AND DISCONTINUITIES

Mark D. Prouty*, Rafael Pous, Kenneth K. Mei, and Steven E. Schwarz
Department of Electrical Engineering and Computer Science
Electronics Research Laboratory*
University of California
Berkeley, CA 94720

1. Introduction

The Measured Equation of Invariance (MEI) method is a recently developed method for terminating finite-difference meshes extremely closely to structures of interest [1]. The method has been applied previously to scattering [2] and antenna problems [3]. In this work, we will show how the method may be applied to transmission structures, such as waveguides or microstrip lines, and discontinuities in such structures.

A brief explanation of the MEI method will be given here to review some of the terminology. In the MEI approach, one generates measuring solutions to calculate finite-difference-type equations which are applicable at the mesh boundaries, where the usual finite difference equations fail. These generalized equations are called equations of invariance. In order to generate the measuring solutions, a set of assumed charge or current distributions are integrated against the Green's function for the geometry. These assumed distributions are called metrons. The Green's function need not satisfy boundary conditions on the structures of interest, which are covered by the finite-difference mesh. It must, however, satisfy all boundary conditions which are outside of the mesh.

2. Uniform Transmission Lines

The MEI method may easily be applied to static analyses of uniform transmission lines, such as single and coupled microstrip lines. In this case, Laplace's equation is solved for the static electric potential, and the total charges found on the metal structures. Solving the problem with the correct permitivity values for the dielectrics yields the capacitance of the structures, while solving the problem with all permitivities equal to the free-space value yields the inductances. The quasi-static impedance values may then be obtained.

Consider first the simple case of a single microstrip structure. This example illustrates the capability of the MEI method. The geometry and mesh for a typical problem are shown in Figure 1 (ignore for the moment the second, coupled microstrip line). At interior points of the mesh, a standard finite difference approximation to Laplace's equation is used. The potential of points on the metal strip is fixed at 1 volt. At the mesh boundaries measured equations of the form

$$\Phi_0 = \sum_{i=1}^{N} a_i \cdot \Phi_i$$

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are written, where Φ_0 is the voltage at an edge node, and the other Φ_i are the voltages of its nearest neighbors. These equations of invariance are obtained by generating a set of measuring solutions and solving for the coefficients, a_i, using a singular value decomposition method. The measuring solutions are obtained by

$$\Phi^i(\bar{r}) = \int_S \sigma^i(\bar{r}') \cdot G(\bar{r}|\bar{r}') \cdot dS'$$

In this case, the metrons, σ^i, are given by

$$\sigma^i = \cos((i-1)\pi x / W) \qquad \text{(x origin is at center of mesh)}$$

while the Green's function, $G(\bar{r}|\bar{r}')$, is the usual static Green's function, usually obtained from images [4]. Results for the single microstrip case agree well with established results, but are not plotted here to save space.

Coupled microstrip lines may be analyzed by simply changing the Green's functions used to generate the measuring solutions. For symmetric coupled lines, where the normal modes are either even (same voltage on each conductor) or odd (opposite voltage on each conductor), the geometry may be modeled as a single conductor next to a magnetic or electric wall. The Green's function used satisfies this boundary condition through the use of images. Results for the even and odd mode impedances of coupled structures are shown in Figure 1. Note that the mesh is held fixed for all ranges of S and H. Only the Green's function is changed to reflect the different dimensions.

3. Scattering by Waveguide Discontinuities

The MEI method may be easily applied to more general transmission line problems. Consider the case of scattering by an inductive post in a rectangular waveguide. For the lowest order TE_{01} mode, this is a two dimensional problem, as the incident E field has no variation in the direction of the post. This problem may be solved in a manner similar to the preceding static problem. In this case, the Helmholtz equation is solved, and the Green's function is that of a rectangular waveguide. The finite difference mesh covers only the post, to a depth of 3 layers. The boundary condition on the scatterer is no longer a simple constant, but that the sum of the scattered and incident E field has no tangential component. Results are shown in Figure 2.

4. Microstrip Discontinuities

Next, we will consider planar microstrip-type structures, where currents are confined to two dimensions. In this paper, we will show results for cases where no dielectric is present. This simplifies the Green's function calculation for our present purposes, but the method may be applied to more general cases.

For full-wave solutions of Maxwell's equations, one is faced with the choice of what field components or vector or scalar potential components to calculate. For the case of currents on only two dimensions, the fields may be specified in terms of two components of the magnetic vector potential. The two components taken are those spanning the plane containing the current. See reference [3] for more details on the potential formulation.

For these examples, care must be taken in specifying the equations on the conductor boundaries. At interior points of the conductor surface, two components of the electric field vanish, yielding two equations for the two vector potential components. At conductor edges only one component of the electric field vanishes and another equation is necessary as well. Integrating the continuity equation, $\nabla \cdot J = -j\omega\rho$, over a pillbox at the metal edge, and using some additional Maxwell's equations, one obtains the condition we use at the edge, $J_z = j\omega D_z$, where the direction z points off the edge of the metal.

The problem of a microstrip bend is shown in Figure 3. An incident wave is applied to the bend from the z-direction, and is partially reflected. The standing wave can be seen in the figure, as well as the current continuing along the other arm of the bend. Figure 4 shows the currents for a line which has a step in width, and a gap at the step. The incident wave approaches from the left in the diagram.

References

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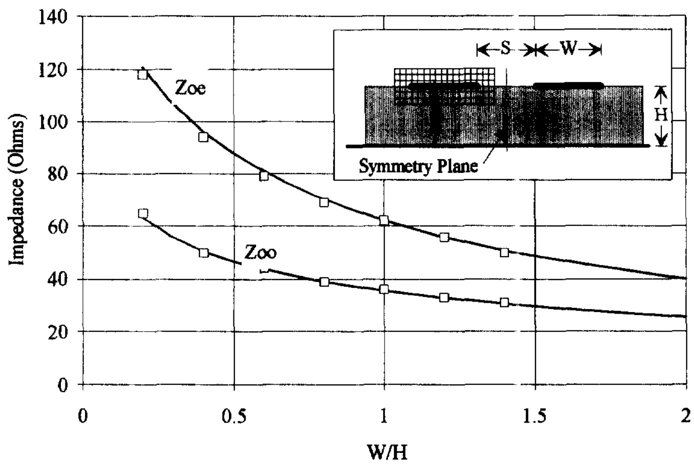

Figure 1. Calculated even (Zoe) and odd (Zoo) mode characteristic impedances for coplanar coupled lines. The solid lines are this work, and the points are from [4]. For the case plotted, S/H = 0.4. Substrate permitivity is 9.6.

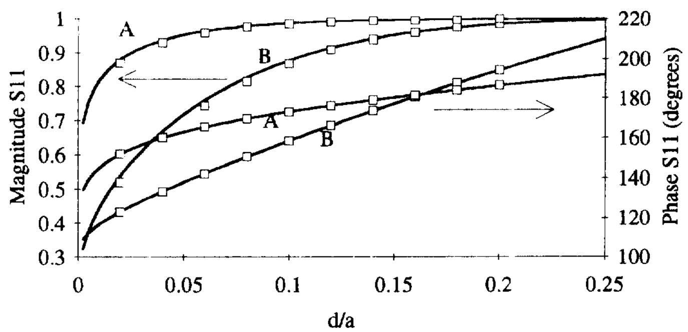

Figure 2. Reflection coefficient for a circular post of diameter d, placed at the center of a rectangular waveguide (h=a/2), for $\lambda = 1.8a$ (A), and $\lambda = 1.2a$ (B). Solid lines are from reference [5], points are this work.

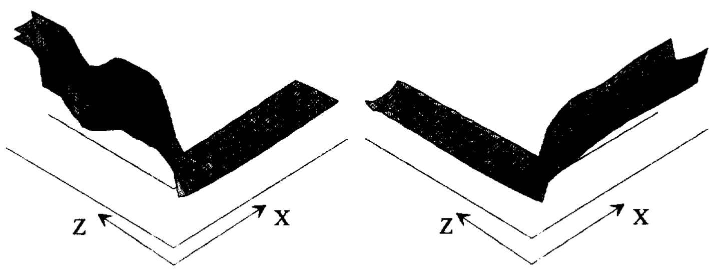

Figure 3. Currents on a bend in a microstrip. Z-directed currents are shown on the left, and X-directed currents are shown on the right. Wavelength is 7.5 W, H = 2.5 W.

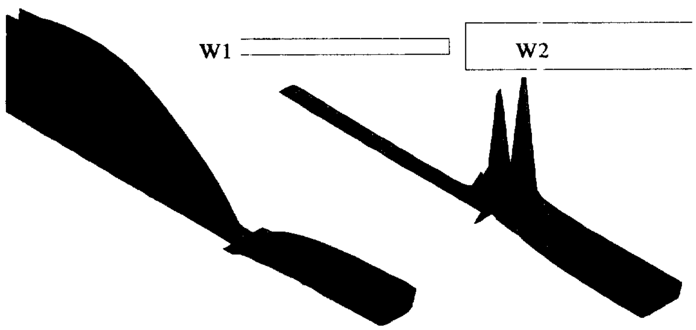

Figure 4. Currents on a step gap structure. The magnitude of longitudinal component is shown on the left, and the transverse component (magnified 20 times) is shown on the right. W1 = 3 W2, gap width = W1, wavelength = 15 W1, H = 2.5 W1.