VII International Conference on Textile Composites and Inflatable Structures
STRUCTURAL MEMBRANES 2015
E. Oñate, K.-U.Bletzinger and B. Kröplin (Eds)

Attractive Planar Panelization using Dynamic Relaxation Principles

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Key words: Dynamic Relaxation, Planar Quad Mesh Approximation, Planar Relaxation

Summary. In the presented paper a new method is proposed to approximate a given NURBS surface with a PQ (Planar Quad) mesh. The desired mesh layout will be generated in plan and then attracted to the target surface. The process iteratively pulls the mesh vertices towards the target surface and then planarizes the faces thereafter using dynamic relaxation methods.

1 INTRODUCTION

Free form architecture building envelopes need to be broken down into elements that not only size-wise but also with respect to their geometrical description can be manufactured and assembled. This process always requires the faceting of surfaces. Triangulation of a surface is a well-documented process however from an architectural and economical point of view the planar quadrilateral (PQ) faceting of a surface offers several advantages.

In the previous published paper [1] the Planar Relaxation (PR) method was proposed to approximate doubly curved surfaces with quad dominated meshes. The process started with quad meshes generated on the target surface consisting of non - planar quads. The optimization then moves the vertices into a planar state using a modified dynamic relaxation algorithm. The mesh moves away from the surfaces to reach the desired PQ mesh.

In the current paper the vice versa approach is presented. The procedure starts with a mesh consisting of planar quads only, covering the footprint of the target surface. This could be either the projection of a mesh generate on the target surface or any other 2D mesh. Attracting forces then pulling the vertices towards the target surface and hence applying an imperfection. In order to establish the planar state of the mesh faces back again the Planar Relaxation method according to paper [1] is performed. The attraction and Planar Relaxation are applied iteratively until the mesh is close enough to the target surface or reaches the defined residual.

In the following chapters we will recall the Planar Relaxation method and introduce the attracting forces. The proposes procedure is then tested, in order to produce a planar quad mesh solution, for a realized grid shell roof using triangular faces only.

2 PLANAR RELAXATION ALGORITHM

In order to achieve the desired planarity for all faces of the approximation mesh we perform a non-linear optimization. Here we adopt a modified dynamic relaxation algorithm which is suitable to solve approximation problems of discrete systems.

2.1. Dynamic Relaxation (DR)

The dynamic relaxation method is widely used for form finding and nonlinear structural analyses of fabric or cable structures. The method uses the dynamic equation of a damped system with or without externally applied loads to calculate the static behavior of structures. To reduce the computation storage requirements the DR method is formulated as direct vector method which only considers geometric nonlinearity.

The motion of mesh nodes representing a 2D structure is traced over time until the sum of residual forces in the nodes reach the convergence criteria. This indicates the state of equilibrium of the structural system. The first DR algorithm for structures was proposed in 1965 by A. S. Day [4] and J.R.H. Otter [5].

2.2. The Dynamic Relaxation Algorithm

In this chapter we briefly summarize the dynamic relaxation algorithm. The DR method has been implemented into a 3D-CAD environment (Rhinoceros 4.0 ®) as a 'PlugIn` application. The algorithm consists of the iterative computation of a series of vector operations applied to the vertices of a given mesh.

The equation for a viscously damped system is given in formula (1) an needs to be executed for all mesh vertices at each iterative step until the weighted nodal residual r' falls below the specified target value.

$$r'_{[i]} = (f_{[i]} + acc_{[i-1]} * M + vel_{[i-1]} * C) - f_{[i-1]} \tag{1}$$

r'...Weighted residual vector
f...Internal force vector
acc...Acceleration vector
M...Fictitious mass
vel...Velocity vector
C...Fictitious damping
i...Iteration

2.3. Planar Relaxation (PR)

In order to modify the DR algorithm to planarize the mesh faces we only need to define a new set of pulling forces which are acting towards the desired condition of the mesh. Let us introduce the new planar pulling forces (fp):

$$\mathrm{fp}_{[i]} = \Sigma(vp_{[1]} \leftarrow \ldots vp_{[j]} \leftarrow) \quad (2)$$

$vp_{[j]}\leftarrow$...out of planarity vector adjacent face
j...number of adjacent faces

For every vertex we compute the out of plane vectors (vp) of the adjacent faces. The plane of the adjacent face is defined by the vertices of this face, other than the current one. The out of planarity condition can be expressed with the distance (d) of the forth vertex to the three point plane defined by the first three vertices of the mesh face. Finally we replace the internal forces (f[i]) in Equation 1 with the new defined planar pulling forces (fp[i]) as defined in equation 2.

3 ATTRACTIVE PLANAR RELAXATION ALGORITHM

With the planarization algorithm establish we can now define the attracting forces which move the mesh nodes towards the target surface.

3.1 Attractive Forces

The attraction forces (fa[i]) are defined as the vector connecting the individual mesh vertices (p[i]) with its closest point (cp[i]) on the surface:

$$\mathrm{fa}_{[i]} = cp_{[i]} - p_{[i]} \quad (3)$$

$p_{[j]}$...vertex point
$cp_{[i]}$...closest point on target surface

We obviously need to introduce a force scale as the full force would pull the mesh strait away on the target surface. The scale will be applied as a constant modifier to all pulling forces.

$$\mathrm{fa\ mod}_{[i]} = \mathrm{fa}_{[i]} s \quad (4)$$

s…modifying scale

The magnitudes of the attracting forces are in direct relationship of the individual node distance to the surface. Hence closer to surface mesh nodes generate smaller forces and therefore less imperfections then further away mesh nodes.

3.2 Attractive Planar Relaxation

The procedure starts with the computation of the attracting forces. Depended on the user defined force scale the mesh vertices will be moved into a now non-planar state. A value for the force scale (s) ranging between s = 0.2 and s = 0.3 delivered good results during test runs. We then optimize the imperfect mesh using the PR algorithm until the desired relative planarity value is reached. For double glazed insulating unit a value of 1% may be appropriate whilst single glazing units can tolerate 3% non- planarity. The procedure of attracting and planarizing is then repeated until the mesh is close enough to the target surface.

4 CASE STUDY

4.1 Westfield Shopping Center

The undulating roof surface of the Glazed Roof at the shopping centre Westfield/London is inspired by the wave movements of water in nature. Designed by the Buchan Group International pty Ltd and Benoy and fabricated by seele the roof shell covers the area of approx. 5,500m². The triangulated steel structure is based on a plan grid of equilateral triangles which is then projected on the doubly curved roof surface. The result of this meshing process is 2250 unique glazing units, 12,000 individual beams and 7,000 individual nodes. The beams are formed by welded box sections with common outer cross section dimension and varying plate thicknesses. The nodes are fabricated using CNC milling and composed of 20 individual parts in order to from a bolted connection.

In plan the roof is C - shaped with 72m long and 24m wide horizontal legs. The upright long side measures 120m and is 24m wide as well. The surface comprised of a series of waves joined with dome shaped forms at the corners.

In the following chapter we will investigate an alternative PQ meshes solution for the roof glazing.

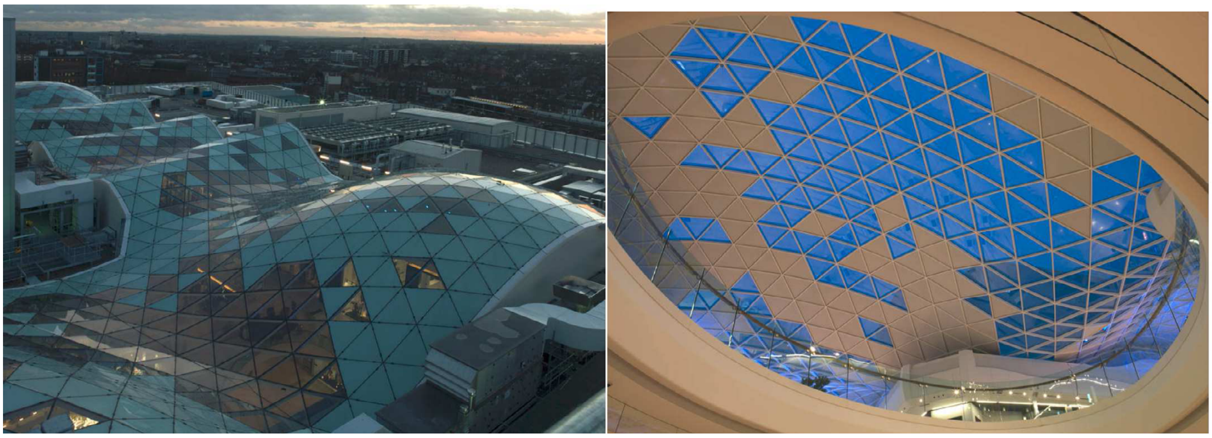

Figure 1+2: Westfield Shopping Centre Gridshell Roof ©seele

4.2 LPC Graph

In order to inform the layout of the starting mesh we compute the Lines of Principle Curvature (LPC) graph of the roof surface (Figure 3). The Lines of Principle Curvature forming a network of curves which is most likely to produce planar quad faces. We can observe a similar behavior of the LPC's in the wave parts of the surface. The maximum LPC's are generally following the valleys and ridges of the waves. The maximum LPC's then necessarily are crossing the valleys and ridges orthogonally. This suggests that this part of the surface is fairly suitable for a meshing which is very close to the LPC graph.

In the dome areas we can observe a different behavior of the LPC's. At both domes the LPC's show a very noisy behavior. Several umbilical points distort the graph so that is not suitable as a literal guide for the start meshing.

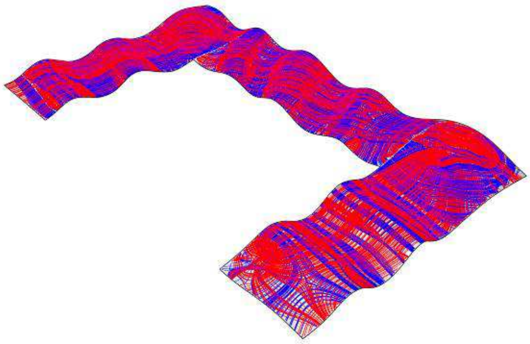

Figure 3: LPC Graph

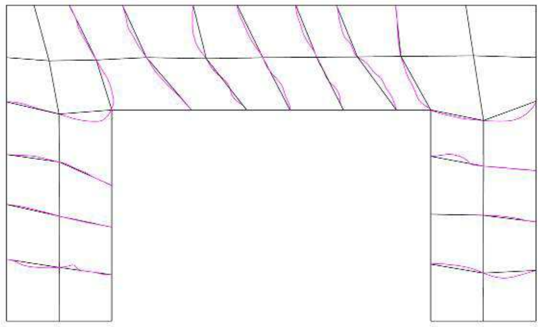

Figure 4: Guide LPC's

4.3 Starting Mesh

For the generation of the 2D starting mesh we will pick a series of maximum LPC's as guides. As shown in Figure 4 the guides are running in equal distance parallel to the wave ridges or valleys. The vertices of the crude topology mesh are the located at the ends and mid points of the guides. At the two domed areas we locate the vertices more freely in a centred position in order to receive an almost orthogonal layout of the mesh faces.

In order to populate the mesh faces we perform a Catmull Clarke subdivision to the crude mesh faces. We apply 3 subdivision iterations to receive 2304 faces with an average edge length of 2.00 m (Figure 5).

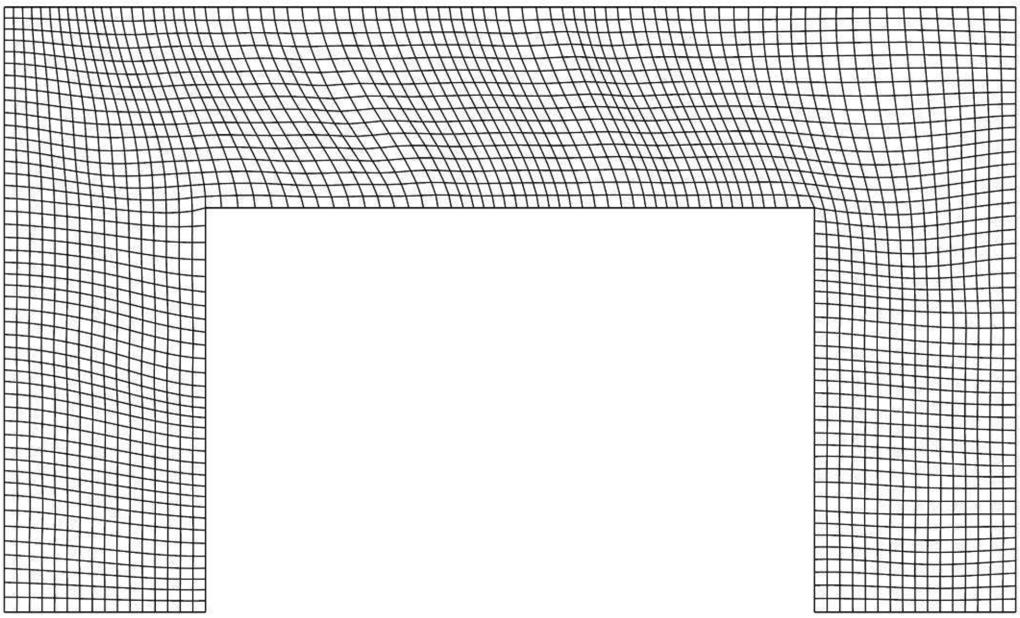

Figure 5: Subdivided Starting Mesh

4.4 Optimization

The target surface covers the entire plan perimeter of the staring mesh and the largest distance mesh vertex to surface closest point count to 6.8m prior to optimization. Figure 6 shows the attracting force vectors of all vertices in their full length.

For the approximation process we choose a force scale (s) of s = 0.3 and set the planarity target to 1%.

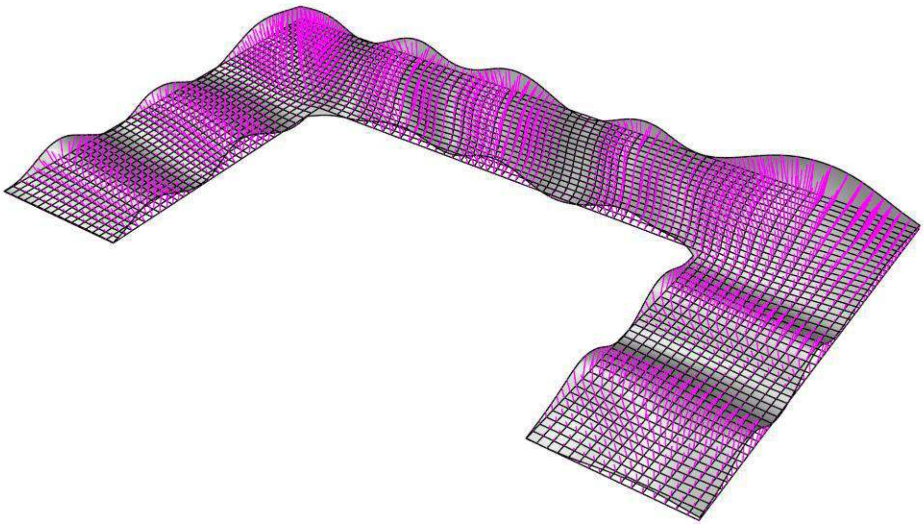

Figure 6: Attracting Vectors Starting Mesh

After the attracting imperfections are applied to the target surface the most non-planar facet are located in the dome areas and count to 7.78% (Figure 7). The afterwards applied PR algorithm requires 1045 cycles and moves the mesh up to 0.27m back towards its initial state prior attraction.

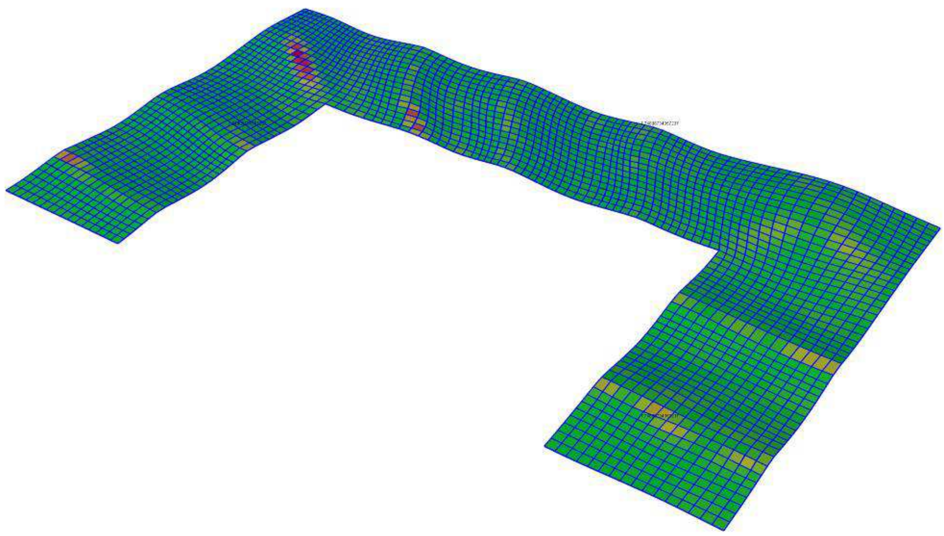

Figure 7: Planarity of Imperfect Mesh

In order to keep the mesh faces smooth and continuously aligned we apply smoothing iterations after the attractions and before planarization.

The procedure: attraction - smoothing - planarization is then repeated until the mesh traveled does not get closer to the target surface. The maximum distance to target surface ($d_{ts,max}$) is measured after each loop. In the following figures the deformation of the mesh during 30 attraction loops in steps of 5 loops are displayed. The displayed maximum relative non-planarity counts to the preset 1% target.

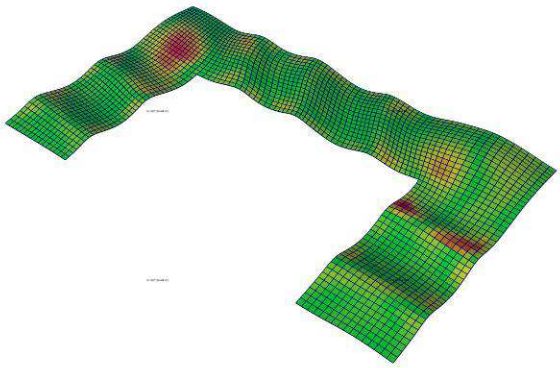

Figure 8: Loop5, $d_{ts,max}$ = 2,35m

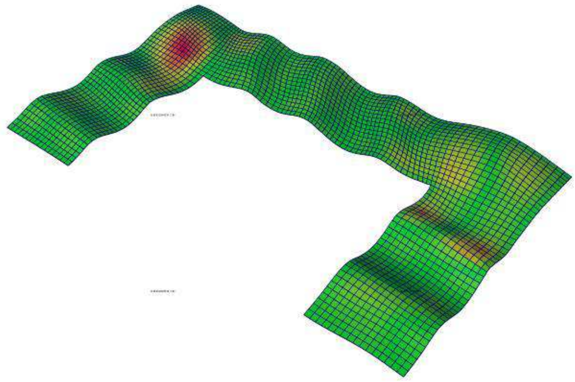

Figure 9: Loop10, $d_{ts,max}$ = 1,15m

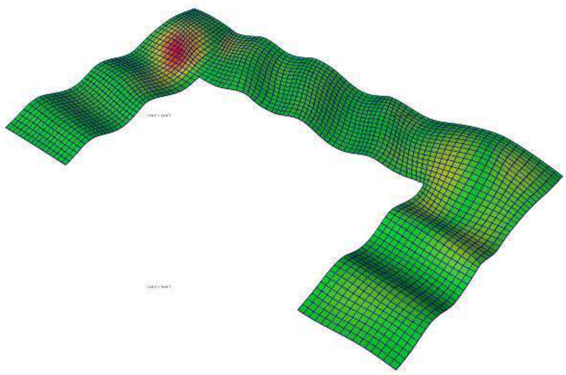

Figure 10: Loop15, $d_{ts,max}$ = 0,95m

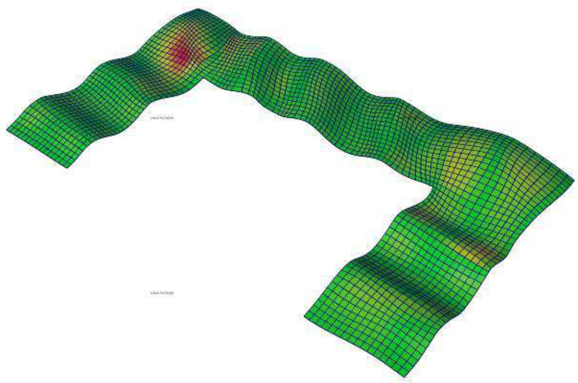

Figure 11: Loop20, $d_{ts,max}$ = 0,91m

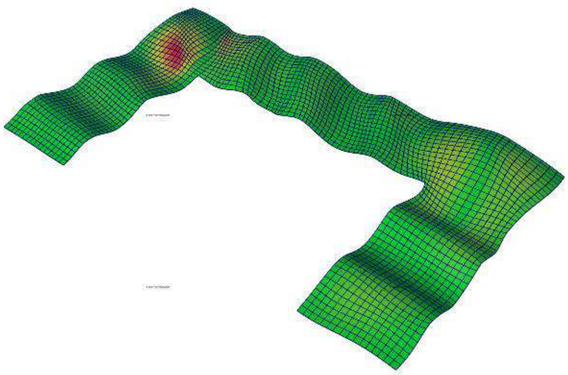

Figure 12: Loop25, $d_{ts,max}$ = 0,92m

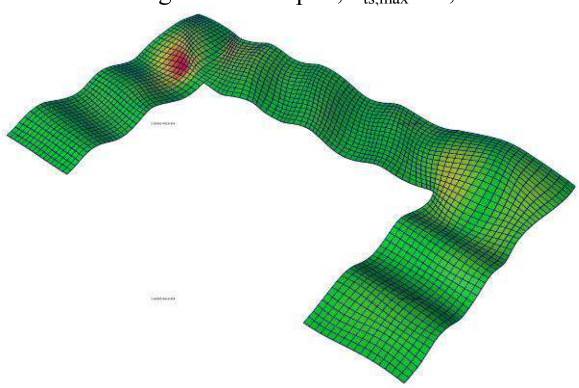

Figure 13: Loop30, $d_{ts,max}$ = 1,02m

We can observe that the optimized mesh gets closest to the target surface after 20 loops. Then the process stagnates between 20 to 25 loops keeping a similar distance to the target. After 25 loops the mesh starts departing again and hence we stopped the optimization after 30 loops. This behavior suggests that at least a local optimum was reached.

In the dome area the mesh faces get compresses (figure 14) or stretched (figure 15) in order to get closer to the target surface. This can be interpreted as the digital counterpart to physical sheet metal forming using molds.

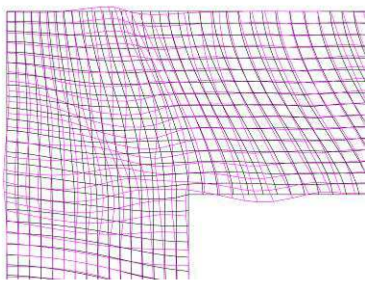

Figure 14: Mesh compression

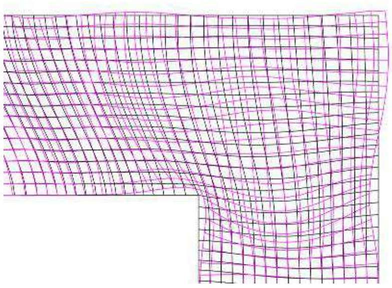

Figure 15: Mesh stretch

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