Non-extensive Statistics to the Cosmological Lithium Problem

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ABSTRACT

Big Bang nucleosynthesis (BBN) theory predicts the abundances of the light elements D, ^{3}He, ^{4}He and ^{7}Li produced in the early universe. The primordial abundances of D and ^{4}He inferred from observational data are in good agreement with predictions, however, the BBN theory overestimates the primordial ^{7}Li abundance by about a factor of three. This is the so-called "cosmological lithium problem". Solutions to this problem using conventional astrophysics and nuclear physics have not been successful over the past few decades, probably indicating the presence of new physics during the era of BBN. We have investigated the impact on BBN predictions of adopting a generalized distribution to describe the velocities of nucleons in the framework of Tsallis non-extensive statistics. This generalized velocity distribution is characterized by a parameter q, and reduces to the usually assumed Maxwell-Boltzmann distribution for $q = 1$. We find excellent agreement between predicted and observed primordial abundances of D, ^{4}He and ^{7}Li for $1.069 \leq q \leq 1.082$, suggesting a possible new solution to the cosmological lithium problem.

Subject headings: cosmology: early universe — cosmology: primordial nucleosynthesis — plasmas

1. Introduction

First proposed in 1946 by George Gamow (Gamow 1946), the hot Big-Bang theory is now the most widely accepted cosmological model of the universe, where the universe expanded from a very high density state dominated by radiation. The theory has been vindicated by the observation of the cosmic microwave background (Penzias & Wilson 1965; Hinshaw et al. 2013), our emerging knowledge on the large-scale structure of the universe, and the rough consistency between calculations and observations of primordial abundances of the lightest elements in nature: hydrogen, helium, and lithium. The primordial Big-Bang Nucleosynthesis (BBN) began when the universe was 3-minutes old and ended less than half an hour later when nuclear reactions were quenched by the low temperature and density conditions in the expanding universe. Only the lightest nuclides (^{2}H, ^{3}He, ^{4}He, and ^{7}Li) were synthesized in appreciable quantities through BBN, and these relics provide us a unique window on the early universe. The primordial abundances of ^{2}H (referred to as D hereafter) and ^{4}He inferred from observational data are in good general agreement with predictions; however, the BBN theory overestimates the primordial ^{7}Li abundance by about a factor of three (Cyburt et al. 2003; Coc et al. 2004; Asplund et al. 2006; Sbordone et al. 2010). This is the so-called "cosmological lithium problem". Attempts to resolve this discrepancy using conventional nuclear physics have been unsuccessful over the past few decades (Angulo et al. 2005; Cyburt et al. 2008; Boyd et al. 2010; Wang et al. 2011; Scholl et al. 2011; Kirsebom & Davids 2011; Voronchev et al. 2012; Coc et al. 2012; Hammache et al. 2013; Pizzone et al. 2014; Famiano et al. 2016), although the nuclear physics solutions altering the reaction flow into and out of mass-7 are still being proposed (Cyburt & Pospelov 2009; Chakraborty et al. 2011). The dire potential impact of this longstanding issue on our understanding of the early universe has prompted the introduction of various exotic scenarios involving, for example, the introduction of new particles and interactions beyond the Standard Model (Pospelov & Pradler 2010; Kang et al. 2012; Coc et al. 2013; Yamazaki et al. 2014; Kusakabe et al. 2014; Goudelis et al. 2016). On the observational side, there are attempts to improve our understanding of lithium depletion mechanisms operative in stellar models (Vauclair & Charbonnel 1998; Pinsonneault et al. 1999, 2002; Richard et al. 2005; Korn et al. 2006). This remains an important goal but is not our focus here. For the recent reviews on BBN and primordial lithium problem, please read articles written by Fields (2011) and Cyburt et al. (2016).

In this work we suggest one solution to the lithium problem that arises in a straightforward, simple manner from a modification of the velocity distributions of nuclei during the era of BBN. In the BBN model, the predominant nuclear-physics inputs are thermonuclear reaction rates (derived from cross sections). In the past decades, great efforts have been undertaken to determine these data with high accuracy (e.g., see compilations of Wagoner (1969); Caughlan & Fowler (1988); Smith et al. (1993); Angulo et al. (1999); Descouvemont et al. (2004); Serpico et al. (2004); Xu et al. (2013)). A key assumption in all thermonuclear rate determinations is that the velocities of nuclei may be described by the classical Maxwell-Boltzmann (MB) distribution (Rolfs & Rodney 1988; Iliadis 2007). The MB distribution was derived for describing the thermodynamic equilibrium properties of the ideal gas, and was verified by a high-resolution experiment at a temperature of $\sim$ 900 K about 60 years ago (Miller & Kusch 1955). However, it is worth asking: Do nuclei still obey the classical MB distribution in the extremely complex, fast-expanding, Big-Bang hot plasma? Indeed, Clayton et al. (1975) adopted a similar approach when addressing the solar neutrino problem prior to the unambiguous measurement of neutrino flavor change by Ahmad et al. (2001).

Whatever the source of the distortions from MB, one expects that the distribution should still maximize entropy. Hence, to account for modifications to the classical MB velocity distribution, one may use Tsallis statistics (also referred to as non-extensive statistics) (Tsallis 1988), which is based on the concept of generalized non-extensive entropy. The associated generalized velocity distribution is characterized by a parameter q and reduces to the MB distribution for $q = 1$. Tsallis statistics has been applied in a host of different fields, including physics, astronomy, biology and economics (Gell-Mann & Tsallis 2004).

2. Thermonuclear reaction rate

It is well-known that thermonuclear rate for a typical $1+2 \rightarrow 3+4$ reaction is usually calculated by folding the cross section $\sigma(E)_{12}$ with a MB distribution (Rolfs & Rodney 1988; Iliadis 2007)

$$\langle\sigma v\rangle_{12} = \sqrt{\frac{8}{\pi\mu_{12}(kT)^3}} \int_0^\infty \sigma(E)_{12} E \exp\left(-\frac{E}{kT}\right) dE, \tag{1}$$

with k the Boltzmann constant, μ_{12} the reduced mass of particles 1 and 2. In Tsallis statistics, the velocity distribution of particles can be expressed by Tsallis (1988)

$$f_q(\mathbf{v}) = B_q \left(\frac{m}{2\pi kT}\right)^{3/2} \left[1-(q-1)\frac{m\mathbf{v}^2}{2kT}\right]^{\frac{1}{q-1}}, \tag{2}$$

where B_q denotes the q-dependent normalization constant. With this velocity distribution, the non-extensive thermonuclear rate (Iliadis 2007) for a typical $1+2 \rightarrow 3+4$ reaction, where both reactants and products are nuclei, can be calculated by:

$$\langle\sigma v\rangle_{12} = B_q\sqrt{\frac{8}{\pi\mu_{12}}} \times \frac{1}{(kT)^{3/2}} \times \int_0^{E_{\max}} \sigma_{12}(E) E \left[1-(q-1)\frac{E}{kT}\right]^{\frac{1}{q-1}} dE, \tag{3}$$

with $E_{\max}=\frac{kT}{q-1}$ for $q > 1$ and $+\infty$ for $0 < q < 1$. Here, the $q < 0$ case is excluded according to the maximum-entropy principle (Tsallis 1988; Gell-Mann & Tsallis 2004). Usually, one defines the $1+2 \rightarrow 3+4$ reaction with positive Q value as the forward reaction and the corresponding $3+4 \rightarrow 1+2$ reaction with negative Q value as the reverse one. Under the assumption of classical statistics, the ratio between reverse and forward rates is simply proportional to $\exp(-\frac{Q}{kT})$ (Iliadis 2007). With Tsallis statistics, however, the reverse rate is expressed as:

$$\langle\sigma v\rangle_{34} = c \times B_q\sqrt{\frac{8}{\pi\mu_{12}}} \times \frac{1}{(kT)^{3/2}} \times \int_0^{E_{\max}-Q} \sigma_{12}(E) E \left[1-(q-1)\frac{E+Q}{kT}\right]^{\frac{1}{q-1}} dE, \tag{4}$$

where $c=\frac{(2J_1+1)(2J_2+1)(1+\delta_{34})}{(2J_3+1)(2J_4+1)(1+\delta_{12})}(\frac{\mu_{12}}{\mu_{34}})^{3/2}$. All parameters in Eqs. (1–3) are well-defined in Iliadis (2007). For a reaction $1+2 \rightarrow 3+\gamma$, we assume the photons obey the Planck radiation law (Iliadis 2007; Torres et al. 1997, 1998) and use the approximation of $e^{E\gamma/kT}-1 \approx e^{E\gamma/kT}$ (Mathews et al. 2011) when calculating the corresponding reverse rate.

Table 1: Nuclear reactions involved in the present BBN network. The non-extensive Tsallis distribution is implemented for the 17 principal reactions shown in the bold face. The listed flux Ratio is the time-integrated reaction flux calculated with the non-extensive Tsallis distribution (with $q = 1.0755$) relative to that with the classical MB distribution ($q = 1$). The references are listed for each reaction in the square brackets.

Reaction	Ratio	Reaction	Ratio
$^{1}\mathbf{H}(n,\gamma)^{2}\mathbf{H}$ (Hara et al. 2003)	1.02	$^{2}\mathrm{H}(n,\gamma)^{3}\mathrm{H}$ (Wagoner 1969)	1.09
$^{2}\mathbf{H}(p,\gamma)^{3}\mathbf{He}$ (Descouvemont et al. 2004)	0.81	$^{3}\mathrm{He}(n,\gamma)^{4}\mathrm{He}$ (Wagoner 1969)	1.10
$^{2}\mathbf{H}(d,n)^{3}\mathbf{He}$ (Descouvemont et al. 2004)	1.12	$^{3}\mathrm{He}(^{3}\mathrm{He},2p)^{4}\mathrm{He}$ (Caughlan & Fowler 1988)	1.54
$^{2}\mathbf{H}(d,p)^{3}\mathbf{H}$ (Descouvemont et al. 2004)	0.91	$2^{4}\mathrm{He}(n,\gamma)^{9}\mathrm{Be}$ (Caughlan & Fowler 1988)	0.62
$^{3}\mathbf{H}(d,n)^{4}\mathbf{He}$ (Descouvemont et al. 2004)	1.02	$^{6}\mathrm{Li}(p,\gamma)^{7}\mathrm{Be}$ (Xu et al. 2013; He et al. 2013)	0.59
$^{3}\mathbf{H}(\alpha,\gamma)^{7}\mathbf{Li}$ (Descouvemont et al. 2004)	0.60	$^{6}\mathrm{Li}(n,\gamma)^{7}\mathrm{Li}$ (Malaney & Fowler 1989)	0.47
$^{3}\mathbf{He}(n,p)^{3}\mathbf{H}$ (Descouvemont et al. 2004)	1.11	$^{6}\mathrm{Li}(n,\alpha)^{3}\mathrm{H}$ (Caughlan & Fowler 1988)	0.47
$^{3}\mathbf{He}(d,p)^{4}\mathbf{He}$ (Descouvemont et al. 2004)	0.84	$^{7}\mathrm{Li}(n,\gamma)^{8}\mathrm{Li}$ (Wagoner 1969)	1.06
$^{3}\mathbf{He}(\alpha,\gamma)^{7}\mathbf{Be}$ (Descouvemont et al. 2004)	0.37	$^{8}\mathrm{Li}(n,\gamma)^{9}\mathrm{Li}$ (Li et al. 2005)	1.06
$^{7}\mathbf{Li}(p,\alpha)^{4}\mathbf{He}$ (Descouvemont et al. 2004)	0.61	$^{8}\mathrm{Li}(p,n)2^{4}\mathrm{He}$ (Wagoner 1969)	1.07
$^{7}\mathbf{Be}(n,p)^{7}\mathbf{Li}$ (Descouvemont et al. 2004)	0.39	$^{9}\mathrm{Li}(p,\alpha)^{6}\mathrm{He}$ (Thomas et al. 1993)	1.07
$^{3}\mathbf{H}(p,\gamma)^{4}\mathbf{He}$ (Dubovichenko 2009)	0.69	$^{9}\mathrm{Be}(p,\alpha)^{6}\mathrm{Li}$ (Caughlan & Fowler 1988)	1.01
$^{2}\mathbf{H}(\alpha,\gamma)^{6}\mathbf{Li}$ (Angulo et al. 1999; Xu et al. 2013; Anders et al. 2014)	0.43	$^{9}\mathrm{Be}(p,d)2^{4}\mathrm{He}$ (Caughlan & Fowler 1988)	0.97
$^{6}\mathbf{Li}(p,\alpha)^{3}\mathbf{He}$ (Angulo et al. 1999; Xu et al. 2013)	0.36		
$^{7}\mathbf{Be}(n,\alpha)^{4}\mathbf{He}$ (King et al. 1977)	0.35		
$^{7}\mathbf{Li}(d,n)\mathbf{2}^{4}\mathbf{He}$ (Caughlan & Fowler 1988)	0.53		
$^{7}\mathbf{Be}(d,p)\mathbf{2}^{4}\mathbf{He}$ (Caughlan & Fowler 1988; Parker 1972)	0.11		

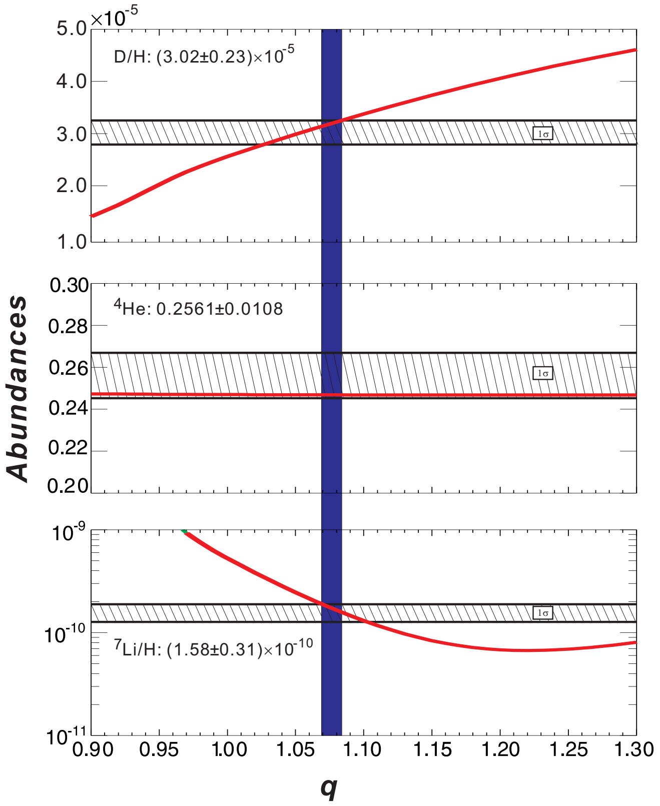

Fig. 1.— Predicted primordial abundances as a function of parameter q (in red solid lines). The observed primordial abundances (Olive et al. 2012; Aver et al. 2010; Sbordone et al. 2010) with 1σ uncertainty for D, ^{4}He, and ^{7}Li are indicated as hatched horizontal bands. The vertical (blue) band constrains the range of the parameter q to $1.069 \leq q \leq 1.082$. Note that the 'abundance' of ^{4}He exactly refers to its mass fraction.

3. Impact of non-extensive statistics on BBN

A previous attempt to examine the impact of deviations from the MB distribution on BBN (Bertulani et al. 2013) only used non-extensive statistics for forward rates and did not consider the impact on reverse rates. Here, we have for the first time used a non-extensive velocity distribution to determine thermonuclear reaction rates of primary importance to BBN in a consistent manner. With these non-extensive rates, the primordial abundances are predicted by a standard BBN code by adopting the up-to-date cosmological parameter $\eta = (6.203 \pm 0.137) \times 10^{-10}$ (Hinshaw et al. 2013) for the baryon-to-photon ratio, and the neutron lifetime of $\tau_n = (880.3 \pm 1.1)$ s (Olive et al. 2014).

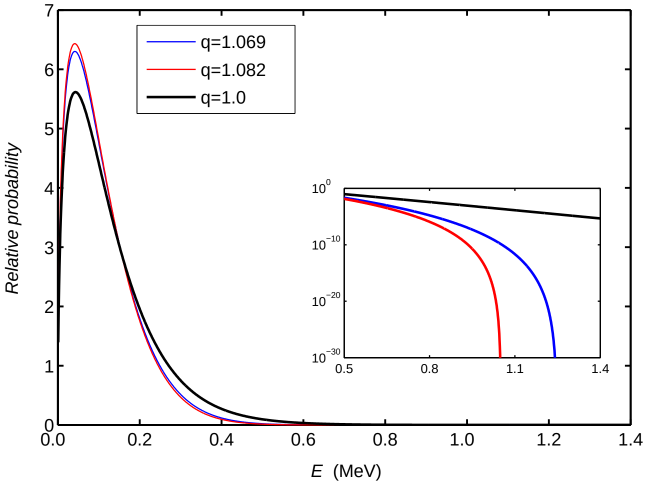

Fig. 2.— Normalized relative probabilities for non-extensive energy distributions and for the standard MB distribution ($q = 1$) at temperature of 1 GK. The enlarged insert plot shows the tails, which are cut off at $E_{\rm max} = kT/(q-1)$ for the non-extensive distributions.

The reaction network involves totally 30 reactions with nuclei of $A \leq 9$ (see Table 1). Here, the thermonuclear (forward and reverse) rates for those 17 principal reactions (with bold face in Table 1) have been determined in the present work using non-extensive statistics, with 11 reactions of primary importance (Smith et al. 1993) and 6 of secondary importance (Serpico et al. 2004) in the primordial light-element nucleosynthesis. The standard MB rates have been adopted for the remaining reactions, as they play only a minor role during BBN. Our code gives results in good agreement with the previous BBN predictions (Bertulani et al. 2013; Coc et al. 2012; Cyburt et al. 2016) if $q = 1$, as seen in Table 2.

It shows that the predicted and observed abundances (Olive et al. 2012; Aver et al. 2010; Sbordone et al. 2010) of D, ^{4}He and ^{7}Li fall into agreement (within 1σ uncertainty of observed data) when a non-extensive velocity distribution with $1.069 \leq q \leq 1.082$ is adopted, as shown in Fig. 1 and Table 2. As the reliability of primordial ^{3}He observations is still under debate (Coc et al. 2012), we do not include this species in the figure. In this calculation, the predicted ^{3}He abundance for the above range of q agrees at the 1.8σ level with an abundance of ^{3}He/H = 1.1(2) (Bania et al. 2002) observed in our Galaxy's interstellar medium. Thus, we have found a possible new solution to the cosmological lithium problem without introducing any exotic theory. Figure 2 illustrates the level of deviation from the MB energy distribution implied by $q = 1.069$ and 1.082 at 1 GK.

The agreement of our predicted ^{7}Li abundance with observations can be attributed to the reduced production of ^{7}Li and radioactive ^{7}Be (which decays to ^{7}Li) when $q > 1$. Production of these species is dominated by the radiative capture reactions ^{3}H(α,γ)^{7}Li and ^{3}He(α,γ)^{7}Be, respectively. The forward alpha-capture rates of these reactions decrease for $q > 1$ due to the decreased availability of high energy baryons relative to the MB ($q = 1$) distribution (see Fig. 2). On the other hand, the reverse photodisintegration rates are independent of q due to our adoption of Planck's radiation law for the energy density of photons. As a result, the net production of ^{7}Li and ^{7}Be decreases, giving rise to concordance between predicted and observed primordial abundances. Figure 3 shows the time and temperature evolution of the primordial abundances during BBN calculated with the MB and the non-extensive distributions (with average value of q allowed, $q = 1.0755$). It can be seen that the predicted ^{7}Be (ultimately decaying to ^{7}Li) abundance with $q = 1.0755$ is reduced significantly relative to that with $q = 1$, and ultimately the ^{7}Li problem can be solved in this model.

The time-integrated reaction fluxes are calculated within the frameworks of classical MB and non-extensive distributions, respectively. Figure 4 displays the reaction network for the most important reactions that occur during BBN with a non-extensive parameter of $q = 1.0755$, where the reaction fluxes are scaled by the thickness of the solid lines. It demonstrates, in particular, that for q within our allowed range, the fluxes of the main reactions responsible for the net production of ^{7}Be (such as ^{3}He(α,γ)^{7}Be and ^{7}Be(n,p)^{7}Li) are reduced by about 60% relative to fluxes determined using $q = 1$. Thus, it results in an ultimate smaller predicted ^{7}Li abundance, which is consistent with observations.

Table 2: The predicted abundances for the BBN primordial light elements. The observational data are listed for comparison.

Nuclide	Coc et al. (2012)	Cyburt et al. (2016)	Bertulani et al. (2013)	This work		Observation
	(q=1)	(q=1)	(q=1)	q=1	q=1.069∼1.082	
^{4}He	0.2476	0.2470	0.249	0.247	0.2469	0.2561±0.0108 (Aver et al. 2010)
D/H($\times 10^{-5}$)	2.59	2.58	2.62	2.57	3.14∼3.25	3.02±0.23 (Olive et al. 2012)
^{3}He/H($\times 10^{-5}$)	1.04	1.00	0.98	1.04	1.46∼1.50	1.1±0.2 (Bania et al. 2002)
^{7}Li/H($\times 10^{-10}$)	5.24	4.65	4.39	5.23	1.62∼1.90	1.58±0.31 (Sbordone et al. 2010)

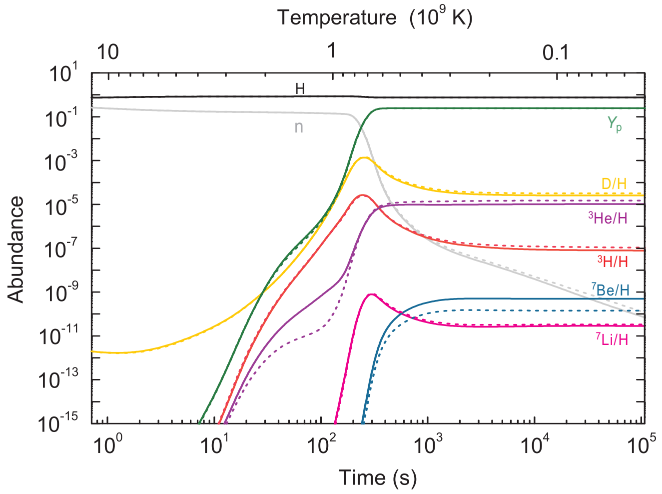

Fig. 3.— Time and temperature evolution of primordial light-element abundances during the BBN era. The solid and dotted lines indicate the results for the classical MB distribution ($q = 1$) and the non-extensive distribution ($q = 1.0755$), respectively.

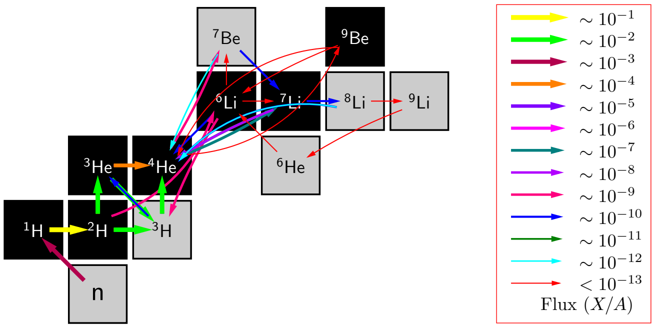

Fig. 4.— The time-integrated fluxes for primary reactions involved in BBN, as calculated using a non-extensive velocity distribution with $q = 1.0755$.

The corresponding flux ratios are listed in Table 1.

One can rationalize the above modified statistics based upon the following arguments. Since the nuclear reactions that lead to the production of ^{7}Li and ^{7}Be occur during the end of BBN, they are falling out of equilibrium and must be evolved via the Boltzmann equation. In general, the Boltzmann equations become a coupled set of partial-integral differential equations for the phase-space distributions and scattering of all species present. Here, we can reduce our consideration to the evolution of the distribution functions of the $A = 3, 4$ species contributing to the formation of $A = 7$ isotopes. For these species there are two competing processes. On the one hand the nuclear reaction cross sections favor the reactions among the most energetic ^{3}He, ^{3}H, and ^{4}He nuclei which would tend to diminish slightly the distributions in the highest energies. At the same time however, the much more frequent scattering of these nuclei off of ambient electrons and (to a lesser extent) photons will tend to restore the distributions to equilibrium. The competition between these two processes, plus the fact that the distributions of ^{3}He, ^{3}H are Fermi-Dirac will lead to a slight deviation from standard MB statistics.

4. Conclusion

We have studied the impact on BBN predictions of adopting a generalized distribution to describe the velocities of nucleons in the framework of Tsallis non-extensive statistics. By introducing a non-extensive parameter q, we find excellent agreement between predicted and observed primordial abundances of D, ^{4}He and ^{7}Li in the region of $1.069 \leq q \leq 1.082$ ($q = 1$ indicating the classical Maxwell-Boltzmann distribution), which might suggest a possible new solution to the cosmological lithium problem. We encourage studies to examine sources for departures from classical thermodynamics during the BBN era so as to assess the viability of this mechanism. Furthermore, the implications of non-extensive statistics in other astrophysical environments should be explored as this may offer new insight into stellar nucleosynthesis.

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