

Unknown Weak Signal Detection Based on Duffing Oscillator

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Abstract: Chaotic systems are widely used to detect the weak signal of the noise background. The effect of the noise in the detection is often ignored because of the characteristics of sensitive to certain signals and inert to noise. However, it's found that chaotic system has low noise immunity for some different variance of noise after a large number of experiments, which makes mistake for the signal to be measured. In this paper, the affection of noise to weak signal detection is analyzed. The method of using cross-correlation detection system to process the signal is proposed, which can suppress noise. Also, the cyclic algorithm is introduced to the chaotic array which is universal poor and difficult to achieve with the application. This method is simple to operate and the simulation results have high accuracy. Copyright © 2014 IFSA Publishing, S. L.

Keywords: Duffing oscillator, Noise, Chaotic array, Signal-to-noise ratio, Cross-correlation detection.

1. Introduction

With the continuous improvement of human science and technology and the deep research of many domains such as biomedicine, physics, optics, quantum mechanics and chemistry, weak signal detection requirements to further improvement, especially in the vibration measurement, biomedical, fault diagnosis system and many other research areas. We often need to determine whether there is a weak signal. The traditional linear filtering method often fails to detect weak signal in the strong noise background [1]. So looking for a new detection method becomes a pressing task.

Chaotic oscillator detection system not only sensitive to initial conditions and parameters, but also to inert to periodic signal whose frequency is largely different with the chaotic oscillator inherent frequency, which possesses huge advantage in weak signal acquisition [2-5]. Kurt Wiesenfeld has studied that period-doubling system as small-signal amplifiers to amplify weak signal and then detect it [6]. Ray Brown has proposed that it's sensitive dependence on initial conditions nature's sensory device [7]. Li Yue has risen applying a special kind of two coupled Duffing oscillator system to detect periodic signals under the background of strong colored noise [8]. Xie Tao has introduced a method

of weak signal detecting based on periodic region of chaotic oscillator [9]. Wei Hengdong has introduced that weak signal is detected by Duffing oscillator based on Hamiltonian [10]. Lai Zhihui proposed a new method of detection weak characteristic signals based on the scale transformation of Duffing oscillator and can detect harmonic signal with any frequency and phase [11]. Li Yifang introduced the combination of autocorrelation and chaotic oscillator phase transformation theory to detect the weak vital periodic signals [12]. These methods are on the premise of ignoring the noise impact. In fact some noise will affect the test results.

The influence of noise on the chaotic oscillator system of weak signal detection is analyzed and the method that the signal detected process by cross-correlation detecting is introduced. And the loop algorithm is introduced into the chaotic oscillator detection system. This method is easy to detect and have strong practicability.

2. Principle of Duffing Detection Signal

The basic model of Duffing system is:

$$\ddot{x}(t) + k\dot{x} + ax(t) + bx^3(t) = \gamma \cos(t), \quad (1)$$

where k is the damping coefficient, the external force $\gamma \cos(t)$; the k , b and γ are all greater than zero; a is less than zero (generally $a=-1$); the term $ax(t)+bx^3(t)$ is nonlinear restore force.

In order to detect any frequency signal by the Duffing equation, let $t = \omega\tau$, we can get:

$$\dot{x}(t) = \frac{dx(t)}{dx} = \frac{1}{\omega} \frac{dx(\tau)}{d\tau} = \frac{1}{\omega} \dot{x}_\tau(\tau)$$

$$\ddot{x}(t) = \frac{d\dot{x}(t)}{dx} = \frac{1}{\omega^2} \frac{d^2x_\tau(\tau)}{d\tau^2} = \frac{1}{\omega^2} \ddot{x}_\tau(\tau)$$

(2) is obtained.

$$\begin{aligned} \frac{1}{\omega^2} \ddot{x}_\tau(\tau) + \frac{k}{\omega} \dot{x}_\tau(\tau) - x_\tau(\tau) \\ + x_\tau^3(\tau) = \gamma \cos(\omega\tau) \end{aligned}, \quad (2)$$

We can get from eq. (2):

$$\begin{cases} \dot{x}_1 = \omega x_2 \\ \dot{x}_2 = \omega(-kx_2 - ax_1 - bx_1^3 + \gamma \cos(\omega t)) \end{cases}, \quad (3)$$

Calculating the Melnikov function based (3), the threshold existing chaos is [13]:

$$\frac{\gamma}{k} = R(\omega) = -\frac{4\cosh(\pi\omega/2)}{3\sqrt{2}\pi\omega}, \quad (4)$$

Let $a=-1$, $b=1$, fixed k (when $k=0.5$, the phase transition is obvious [14]), we can obtain $\gamma_c=0.3766$ by calculating (3). When $\gamma=0$, we can have the saddle point (0, 0) and focal point ($\pm 1, 0$) in the plane. Point (x, x') will stay at any of the two focuses. When $\gamma \neq 0$, the system will appear complex dynamics state. When the value of γ is small, the phase track presents the attractor in the meaning of Poincare mapping, the phase point will oscillate periodically near the two focuses. When $\gamma > \gamma_c$ (γ_c can be got by (4)), with the increasing of γ , we can see homoclinic track (Fig. 1), period-doubling bifurcation (Fig. 2) and chaotic state (Fig. 3). γ can variety in a large range to keep the system in chaotic state. If γ continues increasing, the system enters critical state (Fig. 4). The system will present big scale periodical state when $\gamma > \gamma_d$ (Fig. 5).

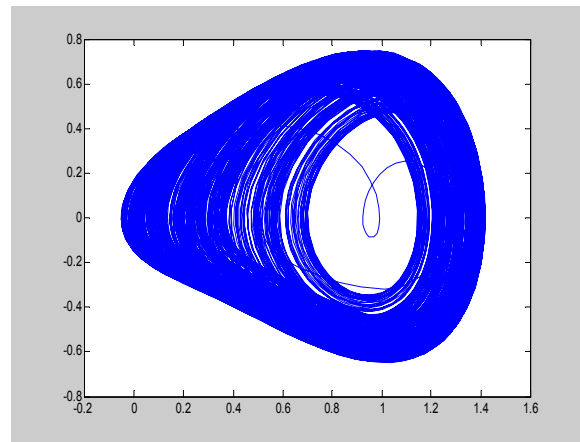


Fig. 1. Phase plane track diagram – Homoclinic track ($\gamma=0.38$, $\omega=1\text{rad/s}$).

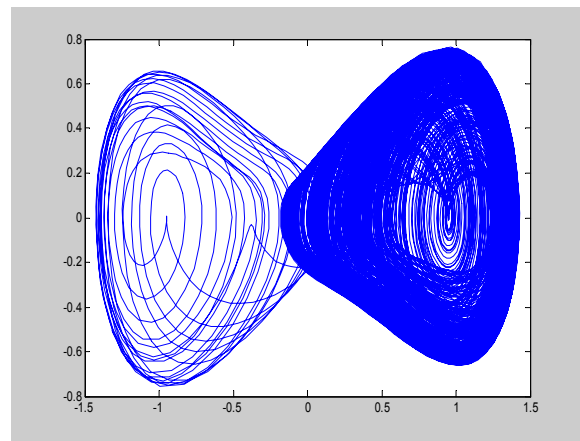


Fig. 2. Phase plane track diagram – Period-doubling bifurcation ($\gamma=0.39$, $\omega=1\text{rad/s}$).

For chaotic oscillator sensitive to periodic signal and inserting to noise, weak signal detected is based on the changing of the system phase plane. There is an intermittent chaos state between chaotic state and big scale periodical state. Adjust parameters and

make the system into intermittent chaos state. Then add the signals to be measured to the chaotic system. If the state changes, we can see that the weak signal is detected.

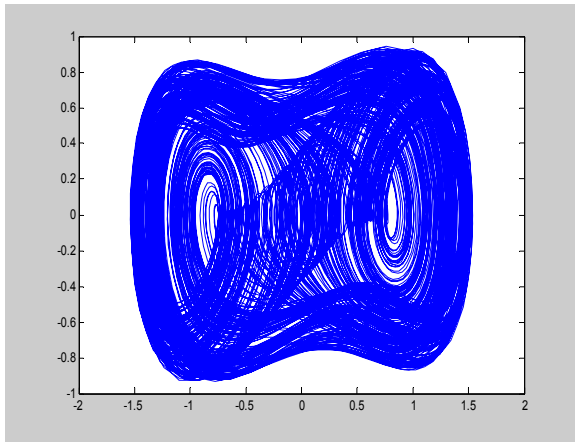


Fig. 3. Phase plane track diagram – Chaotic state ($\gamma=0.62$, $\omega=1\text{rad/s}$).

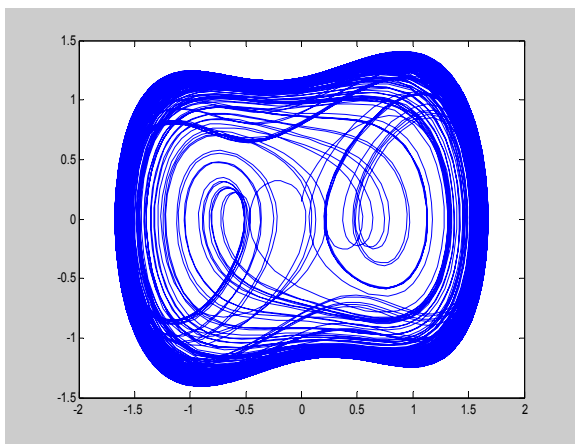


Fig. 4. Phase plane track diagram – critical state ($\gamma=0.8254$, $\omega=1\text{rad/s}$).

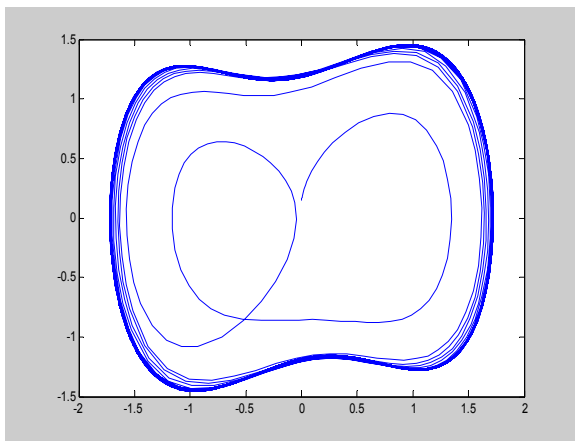


Fig. 5. Phase plane track diagram – Big scale periodical state ($\gamma=0.84$, $\omega=1\text{rad/s}$).

After introduction of the strong noise and a small amplitude periodic signal having a little angular frequency difference with the inner driving force to disturb eq.(3) and some transformations in time scale, the Holmes Duffing equation is changed to:

$$\ddot{x}(t) = -\omega k \dot{x}(t) + \omega^2 [-ax(t) - bx^3(t) + \gamma \cos(\omega t) + f \cos(\Omega t + \varphi) + n(t)] \quad (5)$$

Seeing:

$$\begin{aligned} & \gamma \cos(\omega t) + f \cos(\Omega t + \varphi) + n(t) \\ &= \gamma \cos(\omega t) + f \cos(\Omega t) \cos(\varphi) - f \sin(\Omega t) \sin(\varphi), \quad (6) \\ &= F(t) \cos(\omega t + \Phi(t)) + n(t) \end{aligned}$$

We can get:

$$F(t) = \sqrt{\gamma^2 + 2\gamma f \cos(\Delta\omega t + \varphi) + f^2}, \quad (7)$$

$$\Phi(t) = \arctan \left[\frac{f \sin(\Delta\omega t + \varphi)}{\gamma + f \cos(\Delta\omega t + \varphi)} \right], \quad (8)$$

where $F(t)$ is the total external force initial phase angle $\Phi(t)$, $\Delta\omega = |\Omega - \omega|$. For $f \ll \gamma$, so the effect of $\Phi(t)$ can be ignored. When $\Delta\omega = 0$ and $\pi - \cos^{-1} \frac{f}{2\gamma} \leq \varphi \leq \pi + \cos^{-1} \frac{f}{2\gamma}$, the system will keep on the chaotic state all along [15]. And the phase transition will take place only φ is not in this regime. Fix k, a, b , when the external signal frequency is same as reference signal, namely $\Omega = \omega$, adjust γ to γ_0 , make the system in the intermittent chaotic state. We can get that the amplitude of the signal to be measured: $f = \gamma_d - \gamma_0$.

When the external signal is not the same as reference signal, namely $\Delta\omega \neq 0$, we can get by the expression of the $F(t)$: $\gamma - f \leq F(t) \leq \gamma + f$. When their directions tend to consistency, the resultant vector makes the total drive force amplitude larger than in some time range, the oscillator transits into the exterior trajectory period 1 motion and $F(t)$ will be in a certain period of area greater than γ_d . When their directions tend to deviates from each other, the resultant vector makes the total drive force amplitude smaller than in some time range, the oscillator degenerates into the original chaotic motion. In this way, the intermittent chaos (periodic motion one moment, chaotic motion the next) appears in the oscillator. When there is a small angular frequency difference between inside driving signal and disturbing signal, $F(t)$ will move larger and smaller than in cycles.

Though a large number of experiments, the intermittent chaotic state is able to be clearly identified, when $\Delta\omega \leq 0.03$ [16]. Common ratio 1.03 geometric columns can be chosen for the array of natural frequency oscillator. Assuming that the frequency of the signal to be measured is between

1 rad/s and 10 rad/s, 180 oscillators can be chosen, $\omega_1=1, \omega_2=1.03, \dots, \omega_{180}=10.0946$. When the frequency of the signal to be measured is equal to a certain oscillator when signal is added, the oscillator becomes periodic motion or intermittent chaotic movements. When the frequency of the signal to be measured is not equal to any oscillator frequency, intermittent motion occurs only between two adjacent oscillators. So the frequency of the signal to be measured can be calculated.

The above methods, there are two issues need to be considered in practical: 1) To determine the impact of noise on the test results, the impact can not be ignored for the weak signal with large noise background. 2) On the project, the problem is how the frequency of the signal to be measured is achieved using chaotic array detection.

3. Improved Methods

3.1. The Impact of Noise on Chaotic Oscillator and Cross-Correlation Technique

Chaotic system is immune to noise of zero mean, so the influence of the noise on the system is generally ignored. However, after a large number of experiments, we found that some intensity noise will impact on the phase state of the detection system. When setting the amplitude of the cycle force $\gamma=0.82544187$, system is in the critical state. Adding two different noise, from Fig. 6, the noise variance $\sigma^2=0.0001$, the system is on the critical state. However, when the noise variance $\sigma^2=0.001$, the system is on the large scale periodic state.

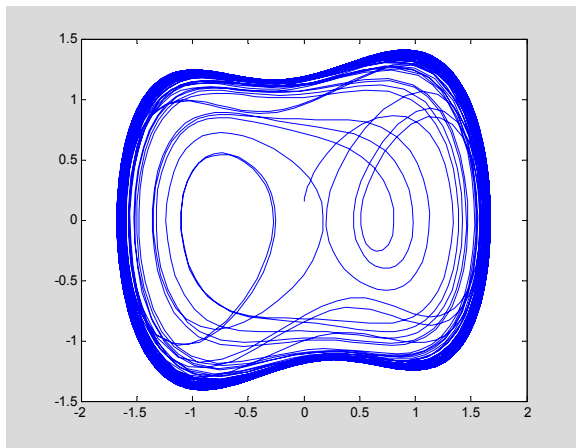


Fig. 6 (a). The phase plane plot $\sigma^2=0.0001$.

Thus, the state of the system will change under the influence of a certain intensity noise. We may mistake that some useful weak signal exists in the signals in experiments and reduces the detection performance of weak signal. Add signal detected

with different power noise background to the system. Then calculate the detection SNR of the system. The formula of SNR is:

$$SNR = 10 \log \frac{\text{power of signals}}{\text{power of noise}}, \quad (9)$$

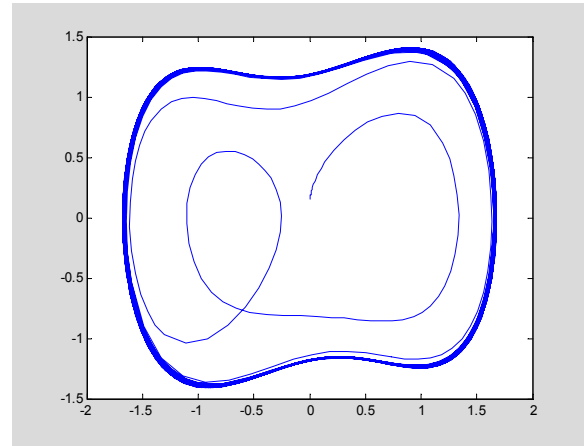


Fig. 6 (b). The phase plane plot $\sigma^2=0.001$.

Thus, the state of the system will change under the influence of a certain intensity noise. We may mistake that some useful weak signal exists in the signals in experiments and reduces the detection performance of weak signal. Add signal detected with different power noise background to the system. Then calculate the detection SNR of the system.

A large number of experiments show that, detection SNR is -69 dB when the noise variance is 0.0001, while the noise variance is 0.001, the detection SNR is only -27 dB. Therefore, this method is not effective for weak signal detection under some noise intensity.

The advantage of chaotic detection of weak signal is extraction signal ability. And the conventional algorithm is suppression noise technology. Cross-correlation detection technology is using the characteristics of periodic signal and random noise, and removing the noise by the relevant operation to achieve and the cycle of signal is not changed.

The input signal has following: $y(t)=x(t)+n(t)=asint+n(t) (0 \leq t \leq T)$, the reference signal is $s(t)=bsint$. The cross-correlation function is defined:

$$\begin{aligned} R_{xy}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T y(t)s(\tau-t)dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)s(\tau-t)dt + \\ &\quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T n(t)s(\tau-t)dt, \quad (10) \\ &= \frac{ab}{2} \cos \tau + n'(\tau) \\ &= R_{xx}(\tau) + R_{sn}(\tau) \end{aligned}$$

where $R_{sx}(\tau)$ is the cross-correlation function of $x(t)$ and $n(t)$; $R_{sn}(\tau)$ is the cross-correlation function of $s(t)$ and $n(t)$ [17].

Due to the noise is unrelated with the reference signal, so $R_{sx}(\tau)=0$. But T can not be infinite length, so the $R_{sn}(\tau)$ is not zero in actual calculation. It shows residual noise. On the power, $n'(t) < n(t)$, it shows that the noise has been restrained in a certain degree.

The advantage of cross-correlation algorithm only accepts the output associating with the local signal and suppresses all forms of noise that not relating to the reference signal. But this method needs a local signal. That is the restrictions on detecting the unknown signals.

3.2. Improving Chaotic Detection System

Chaotic array method can effectively detect the unknown signal frequency. However, the chaotic array element is so numerous that it's different to detect and universal poor. For this situation, Loop algorithm method is introduced to the chaos detection system. The key step is determined whether into the intermittent chaos. The accuracy of the system is greatly improved by using Lyapunov exponent as deterministic basis of chaotic state [18-21].

The study show that when $\Delta\omega \leq 0.03$, the intermittent chaotic state could be clearly identified, so we get 0.06 for the cycle of step length. After getting the period T of intermittent chaos movement, we can use the formula $T=2\pi/\Delta\omega$ to get the difference of the test signal and the frequency of the driving force. Then calculate the frequency of the test signal. This method can detect any unknown signal frequency. Test duration is determined by the size of the unknown signal frequency.

The specific steps are as follows:

- 1) Adjust the value of γ based on Lyapunov exponent, modulate of the system in the critical state[22-24].
- 2) Input to-be-detected signal which is buried in the noise.
- 3) Observe the state of system. If the state changes to the large scale periodic state, we can record the times of the system cycling. If not, add ω to $\omega+0.06$ and repeat steps (2). Meanwhile, the times of repeating is taken notes.
- 4) Detect critical state motion cycle T .

4. Simulation Experiment

This simulation model like Fig. 7.

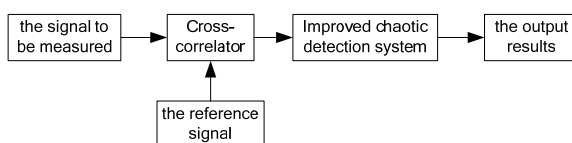
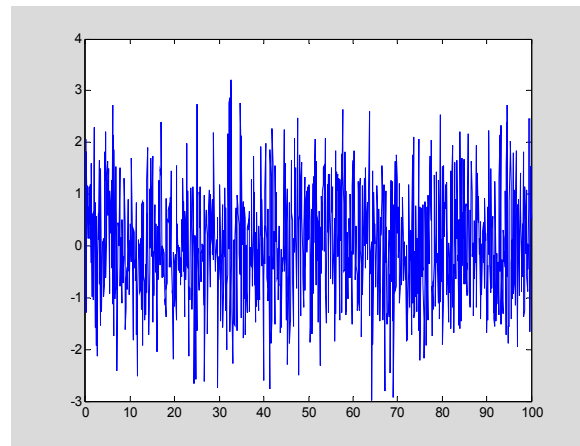


Fig. 7. Schematic diagram of the chaotic detection system.

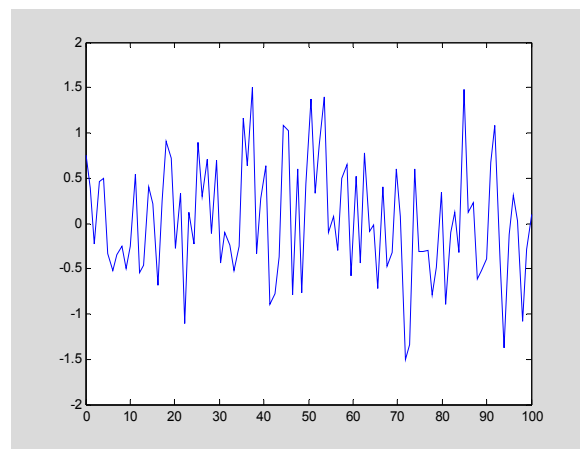
The signal to be measured is

$$g(t) = f \cos(\Omega t + \varphi) + n(t),$$

where $\Omega=1.1$ Hz, $f=0.025$ V, $n(t)$ is the noise whose mean value is 0 and variance is 0.01. The reference signal is $h(t)=\sin(\Omega t)$. Add $g(t)$ to the cross-correlation and the Fig. 8(b) is obtained. Comparing with the time domain waveform graph of signals without processing, we can get that the noise of the signal is restrained.



(a) The time domain waveform of the signal to be measured.



(b) Waveform cross-correlation after processing.

Fig. 8. Cross-correlation detection of the sine signal.

Thus, we can see that cross-correlation detection can well restrain noise and reduce the power and enhance SNR, especially suitable for the situation that the weak periodic signal is not related with noise.

Let the frequency of driving force is 1.08 Hz. Experiments are in accordance with the steps of the flow chart. After the system cycling 18 times, the system is in intermittent chaos state. And the system jump out of the loop. From Fig. 9, the total time is 1800 s and there are 6.5 intermittent periods. The period can be acquired: $T=1800/6.5=276.923$ s. So,

$$\begin{aligned}\Omega &= \omega + \Delta\omega = 1.08 + \frac{2\pi}{230.769}, \\ &= 1.10721\text{Hz}\end{aligned}\quad (11)$$

The error only is 0.00721 Hz compared with actual signal frequency. The detection effect is very good. So this method is not only small amount of calculation but also high accuracy.

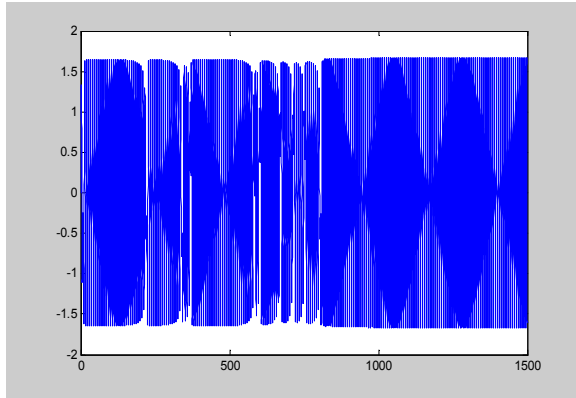


Fig. 9. Phase plane diagram of critical state in the circle detection.

5. Conclusions

From the perspective of noise, this article discussed the influence of different noise on chaotic system, and preprocessed signal by using cross-correlation method. This method can strongly reduce the noise power, and improve SNR and the accuracy of the detection. Besides, the traditional chaotic array detection method is improved in this article. The simple cycle algorithm introduced can detect the unknown weak signal frequency easily.

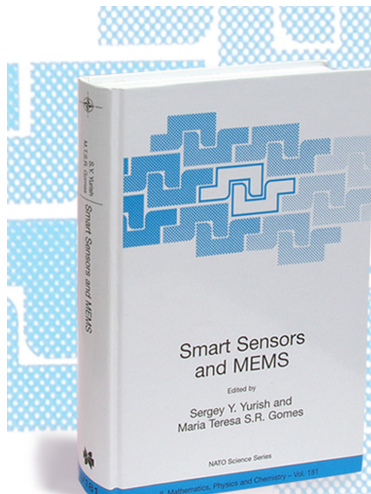
The cross-correlation detection need a reference signal which is similar with signal to be measured, so a frequency unknown signals to be measured should be added to the cycle chaotic array detection system. Calculate the frequency of the signal. Then, we can use this reference signal to detect the signals of other unknown parameters by cross-correlation detection calculation. This system is easy to be operated; the performance of the detection is well and also able to detect the test signals which contain a variety of frequency.

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