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Enhancement of the Josephson current by magnetic field in superconducting tunnel structures with a paramagnetic spacer

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The dc Josephson critical current of a (S/M)IS tunnel structure in a parallel magnetic field is investigated (here S is a superconductor, S/M is a proximity-coupled S and paramagnetic metal M bilayer, and I is an insulating barrier). We consider the case when, due to Hund's rule, in the metal M the effective molecular interaction aligns the spins of the conduction electrons antiparallel to the localized spins of magnetic ions. It is predicted that for the tunnel structures under consideration there are conditions such that the destructive action of the internal and the applied magnetic fields on Cooper pairs is weakened, and increase of the applied magnetic field causes field-induced enhancement of the critical tunnel current. The experimental realization of this interesting effect of the interplay between superconductivity and magnetism is also discussed.

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1. Introduction

In ferromagnetic (F) metals the exchange field H_E acting on the spin of the conduction electrons via the exchange interaction with the magnetic moments of ions is, in general, so large as to inhibit superconductivity. When an external magnetic field is applied, superconductivity is suppressed due to orbital and spin pair breaking effects, as well. However, there are magnetic metals, such as (EuSn)Mo_6S_8 [1,2] or $HoMo_6S_8$ [3], where an applied magnetic field can induce superconductivity. Several mechanisms that may enable superconductivity to develop in a ferromagnet or a paramagnet have been investigated in more or less detail (see [4,5] and references therein). One of them is the so-called Jaccarino–Peter effect [6]. It takes place in those para- and ferromagnetic metals, in which, due to the Hund coupling energy, the exchange interaction, $Js\mathbf{S}$, orients the spins s of the conduction electrons antiparallel to the spins $\mathbf{S}$ of rare-earth magnetic ions. The effective field acting on the spin of a conduction electron is $\mu_B H + g\mu_B J \langle S \rangle$, with $J < 0$ (μ_B is the Bohr magneton, g is the g factor). In such magnetic metals the exchange field $g\mu_B J \langle S \rangle$ can be reduced by an external magnetic field $\mu_B H$, so that the destructive action of both fields on the conduction electrons can be weakened or even canceled. If, in addition, these metals posses an attractive electron–electron interaction, as, for example, in pseudoternary compounds [5], it is possible to induce bulk superconductivity by a magnetic field.

In this report, we consider the dc Josephson effect for a tunnel structure where one electrode is a proximity-coupled bilayer of a superconducting film (S) and a paramagnetic metal (M), while the second electrode is an S layer. The system is under the effect of a weak external magnetic field, which by itself is insufficient to destroy superconductivity. The dc critical current of such a junction has been calculated using an approximate microscopic treatment based on the Gor'kov equations. We discuss the case when in the M metal the localized paramagnetic moments of the ions, oriented by magnetic field, exert an effective interaction $Js\mathbf{S}$ on the spins of the conduction electrons. The latter, whether it arises from the usual exchange interaction or due to configuration mixing, according to Hund's rules, is of the antiferromagnetic type, i.e., $J < 0$. In particular, such an M metal could be a layer of pseudoternary compounds like (EuSn)Mo_6S_8 or $HoMo_6S_8$. (While experimentally the Jaccarino–Peter phenomenon was observed [1–5] for paramagnets, this mechanism is applicable both to ferromagnetic and paramagnetic metals, and both type of the magnetic orders will be assumed here.) We demonstrate that in the region where the destructive action of the fields on both tunnel electrodes is decreased, an increase of the magnetic field causes enhancement of the Josephson critical current.

2. The model

The system we are interested in is the (S/M)IS layered structure of a superconducting S/M bilayer and S films separated by a very thin insulating (I) barrier (see Fig. 1). The S/M bilayer consists of proximity—coupled superconducting and paramagnetic metals in good electrical contact. It is assumed that the thicknesses of the S layers are smaller than the superconducting coherence length and that the thickness of the magnetic layer is smaller than the condensate penetration length, i.e., $d_S << \xi_S$ and $d_M << \xi_M$. Here $\xi_{S(M)}$ is the superconducting coherence length of the S(M) layer; $d_{S(M)}$ is the thickness of the S(M) layer. In this case, the superconducting order parameter may be regarded as being independent of the coordinates, and the influence of the magnetic layer on the superconductivity is not local. Other physical quantities characterizing the S/M bilayer are modified, as well. Such an approach was recently discussed in [7,8] for SFIFS structures, and, as was demonstrated, under these assumptions, a thin S/F bilayer is equivalent to a superconducting ferromagnetic film with a homogeneous superconducting order parameter and an effective exchange field. Similarly, we can consider the S/M bilayer as a thin SM film which is characterized by the effective values of the superconducting order parameter Δ_{eff}, the coupling constant γ_{eff}, and the exchange field $H_{E\text{eff}}$ that are determined by the following relations:

$$\Delta_{\text{eff}}/\Delta = \gamma_{\text{eff}}/\gamma = \nu_S d_S(\nu_S d_S + \nu_M d_M)^{-1}, \quad (1)$$

$$H_{E\text{eff}}/H_E = \nu_M d_M(\nu_S d_S + \nu_M d_M)^{-1}, \quad (2)$$

where ν_S and ν_M are the densities of quasiparticles states in the superconductor and magnetic metals, respectively; γ is the coupling constant in the S metal. We emphasize that *the superconductivity of the M metal is due to the proximity effect*. The applied magnetic field is too weak to induce superconducting properties through the Jaccarino–Peter scenario, if the M metal is a pseudoternary compound. While in the latter case the M metal can posses a nonzero electron–electron interaction, we will neglect this interaction, assuming for the M layer a vanishing value of the bare superconducting order parameter $\Delta_M^0 = 0$, so that relation (1) still remains valid.

The system is under the effect of a parallel magnetic field H. We will also assume that the thicknesses of the SM and S films are smaller than the London penetration depth λ_{SM} and λ_S, respectively. Then the magnetic field is homogeneous in both electrodes. The conditions $d_S << \xi_S$, $d_M << \xi_M$ ensure that orbital effects can be neglected, as well. The longitudinal dimension of the junction, W, is assumed to be much less than the Josephson penetration depth, $W << \lambda_J$, so that a flux quantum cannot be trapped by the junction: $HW(d_M + 2d_S + t) << \phi_0$, where ϕ_0 is the flux quantum and t is the thickness of the insulator.

If the transparency of the insulating layer is small enough, we can neglect the effect of a tunnel current on the superconducting state of the electrodes and use the relation of the standard tunnel theory [9], according to which the distribution of the Josephson current density $j_T(x)$ flowing in the z direction through the barrier (see Fig. 1) takes the form $j_T(x) = I_C \sin\varphi(x)$. Here $\varphi(x)$ is the phase difference of the order parameter across the barrier, while the Josephson current density maximum I_C is determined by the properties of the electrodes. In this report we present the results of a calculation of the critical current I_C for the tunnel junction under consideration.

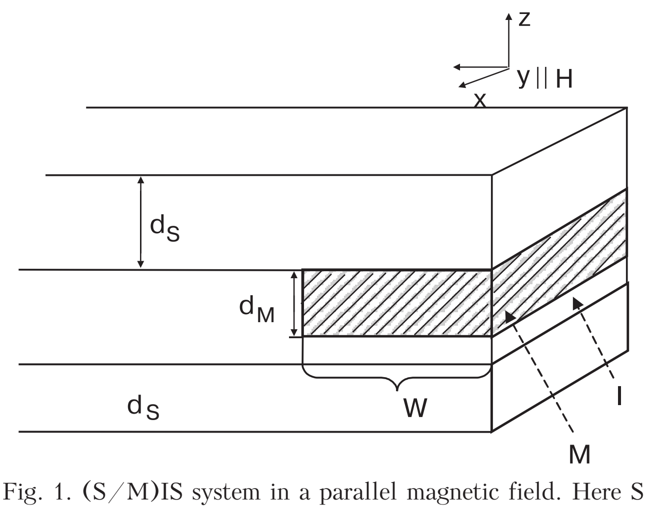

Fig. 1. (S/M)IS system in a parallel magnetic field. Here S is a superconductor; M is a magnetic metal; I is an insulating barrier; W is the longitudinal dimension of the junction.

3. Critical current

Insofar as the exchange field and the external magnetic field act only on the spin of electrons, we can write the Gor'kov equations for the S and SM layers in the magnetic field in the form

$$(i\varepsilon_n + \xi - \sigma H_{S(SM)})\hat{G}_{\varepsilon S(SM)} + \hat{\Delta}_{S(SM)}\hat{F}_{\varepsilon S(SM)} = 1, \quad (3)$$

$$(-i\varepsilon_n + \xi - \sigma H_{S(SM)})\hat{F}_{\varepsilon S(SM)} + \hat{\Delta}_{S(SM)}\hat{G}_{\varepsilon S(SM)} = 0, \quad (4)$$

where $\xi = \varepsilon(p) - \varepsilon_F$, ε_F is the Fermi energy; $\varepsilon(p)$ is the quasiparticle spectrum; $\sigma = \pm 1$; $\varepsilon_n = \pi T(2n+1)$, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ are Matsubara frequencies; T is the temperature of the junction (here and below we have taken the system of units with $\hbar = \mu_B = k_B = 1$); $H_{SM} = H_{E\text{eff}} - H$ is the resultant magnetic field in the SM bilayer (the subscript SM) and $H_S = H$ is the magnetic field in the S layer (the subscript S); G_ε and F_ε are normal and anomalous Green functions. The equations are also supplemented with the well-known self-consistency equations for the order parameters. In the case of conventional singlet superconducting pairing, when $\hat{\Delta} = i\sigma_y\Delta$ (σ_y is Pauli matrix), one can easily find (see, e.g., [8]):

$$\ln\left(\frac{\Delta_0}{\Delta_{S(SM)}}\right) = \int_0^{\omega_D} \frac{dx}{\sqrt{x^2 + \Delta^2_{S(SM)}}}\left\{\frac{1}{\exp[\beta\sqrt{x^2+\Delta^2_{S(SM)}} - H_{S(SM)}] + 1} + \frac{1}{\exp[\beta\sqrt{x^2+\Delta^2_{S(SM)}} + H_{S(SM)}] + 1}\right\} \quad (5)$$

where $\Delta_0 = \Delta(0,0)$ is the BCS gap at zero temperature and in the absence of both the applied and the exchange fields; ω_D is the Debye frequency; $\beta = 1/T$; $\Delta_{SM}(T, H_{SM})$ and $\Delta_S(T, H_S)$ are the superconducting order parameters of the SM and S electrodes, respectively. If $H_{S(SM)} = 0$, formula (5) is reduced to Eq. (16.27) of Ref. 10.

In accordance with the Green's function formalism, the critical current of the SMIS junction can bewritten as follows:

$$I_C = (2\pi T/eR_N)\,\mathrm{Sp}\sum_{n,\sigma} f_{SM}(H_{SM}) f_S(H_S), \quad (6)$$

where R_N is the contact resistance in the normal state and $f_{\varepsilon SM(S)}$ are anomalous Green functions averaged over energy ξ. From Eqs. (3) and (4) one can easily find that:

$$f_{\varepsilon SM(S)} = \Delta[(\varepsilon_n + i\sigma H_{SM(S)})^2 + \Delta^2]^{-1/2}. \quad (7)$$

Using Eqs. (6) and (7), after summation over spin index, we find for the reduced (i.e., $eR_N\{4\pi T\Delta_0^2\}^{-1} I_C$) quantity

$$j_C(T,H) = \Delta_{SM}(T, H_{SM})\Delta_S(T,H)\Delta_0^{-2} \times$$
$$\times \mathrm{Re}\sum_n \{[(\varepsilon_n - i(H_{E\text{eff}} - H))^2 + \Delta^2_{SM}(T, |H_{E\text{eff}} - H|)][(\varepsilon_n + iH)^2 + \Delta^2_S(T,H)]\}^{-1/2}. \quad (8)$$

The Josephson critical current of the junction, as function of the fields and temperature, can be calculated using formula (8) and self-consistency equation (5). In the general case, the dependence of the superconducting order parameter on effective field can be rather complicated due to the possibility of transition to the inhomogeneous (Larkin–Ovchinnikov–Fulde–Ferrell) phase [11,12]. We will not touch upon this scenario here, restricting the consideration below to the region with the homogeneous superconducting state. Even in this case at arbitrary temperatures the values of $\Delta_{SM}(T, |H_{E\text{eff}} - H|)$ and $\Delta_S(T,H)$ can be determined only numerically. The phase diagram of a homogeneous superconducting state in the H–T plane has been obtained earlier (see, e.g., [8]). At finite temperatures, it is found that $\Delta(T,H)$ has a sudden drop from a finite value to zero at a threshold of H, exhibiting a first-order phase transition from a superconducting state to a normal state. Using these results, from Eq. (5) we take only one branch of solutions, corresponding to a stable homogeneous superconducting state. It should be also noted that, insofar as $H_E \propto \langle S\rangle$, a self-consistency equation should be used for $H_{E\text{eff}}$, as well. However, we will suppose that $H_{E\text{eff}}$, while being much smaller than in an isolated M film, is still larger than $\Delta_{SM}(T, |(H_{E\text{eff}} - H)|)$ for the full temperature region of the homogeneous superconducting state. So, proceeding in such a way as to tackle the new physics, we will ignore the temperature dependence of $H_{E\text{eff}}$ in Eq. (8).

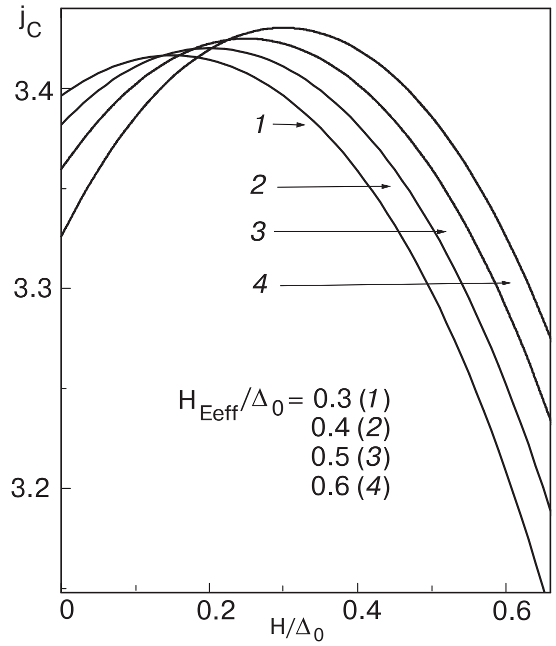

Fig. 2. Critical current of the SMIS tunnel junction versus external magnetic field for $T = 0.1T_C$, $\Delta_{SM}(0, 0) = \Delta_S(0, 0) = \Delta_0$, and different values of the effective exchange field in the SM bilayer.

Figures 2 and 3 show the results of numerical calculations of expression (8) for the Josephson critical current versus external magnetic field for the case of low $T = 0.1T_C$ and finite $T = 0.7T_C$ temperatures, and different values of the exchange field. To keep the discussion simple, for the SM and S layers we put $\Delta_{SM}(0, 0) = \Delta_S(0, 0) = \Delta_0$. As is seen in the figures, for some interval of the applied magnetic field enhancement of the dc Josephson current takes place in comparison with the case

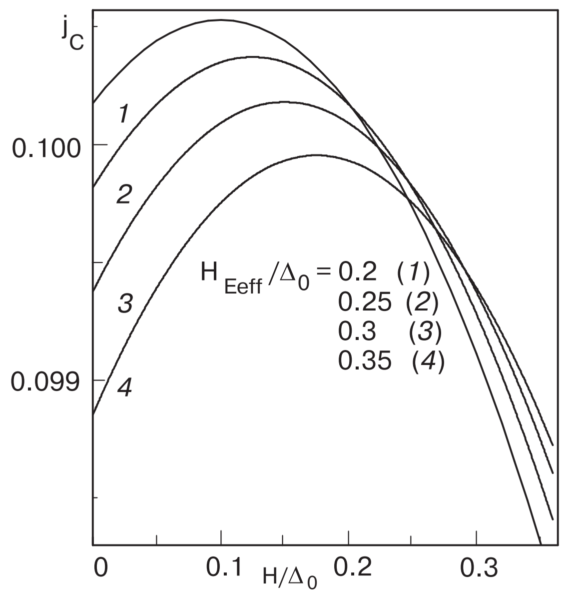

Fig. 3. Critical current of the SMIS tunnel junction versus external magnetic field for $T = 0.7T_C$, $\Delta_{SM}(0, 0) = \Delta_S(0, 0) = \Delta_0$ and different values of the effective exchange field in the SM bilayer.

$H = 0$. Note that the larger the effective field H_{Eeff} is, the larger the growth of the critical current that can be observed (compare, for example, the j_C curves for $H_{Eeff} = 0.4\Delta_0$ and $H_{Eeff} = = 0.6\Delta_0$ at $H = 0$ in Fig. 2). This behavior is also predicted by expression (8). The sudden breakoff in the $j_C(H)$ dependences in the presence of H is due to a first-order phase transition from a superconducting state with finite $\Delta(T, H)$ to a normal state with $\Delta(T, H) = 0$.

4. Discussion

As is well known [13,14], due to the difference in energy between spin-up and spin-down electrons and holes under the exchange field of a ferromagnet, a singlet Cooper pair, adiabatically injected from a superconductor into a ferromagnet, acquires a finite momentum. As a result, the proximity-induced superconductivity of the F layer is spatially inhomogeneous, and the order parameter contains nodes where the phase changes by π. In particular, the transport properties of tunnel SF structures have turned out to be quite unusual. The π state is characterized by a phase shift of π in the ground state of the junction and is formally described by a negative critical current I_C in the Josephson current–phase relation: $j(\varphi) = I_C \sin(\varphi)$. The π-phase state of an SFS weak link due to Cooper pair spatial oscillation was first predicted by Buzdin *et al.* [15,16]. Experiments that have by now been performed on SFS weak links [17,18] and SIFS tunnel junctions [19] directly prove the π-phase superconductivity.

There is another interesting case of a thin F layer, $d_F << \xi_F$, being in contact with an S layer. Insofar as the thickness of the F layer d_F is much less than the corresponding superconducting coherence length ξ_F there is spin splitting but no order parameter oscillation in the F layer. Surprisingly, but it was recently predicted [7,8,20–24] that for SFIFS tunnel structures with very thin F layers one can, on condition of parallel orientation of the F layers magnetization, turn the junction into the π-phase state with the critical current inversion; if the internal fields of the F layers have antiparallel orientation, one can even enhance the tunnel current. It is obvious that the physics behind the inversion and the enhancement of the supercurrent in this case differs from that proposed by Buzdin *et al*. Namely, in this case the π-phase state is due to a superconducting phase jump at the SF interface [21,24]. The exchange-field enhancement of the critical current for SFIFS tunnel structure can be qualitatively understood using the simple fact that the Cooper pairs consist of two electrons with opposite spin directions. Pair-breaking effects due to spin-polarized electrons are weaker in the antiparallel-aligned configuration, since the spin polarizations from the exchange fields of the F layers are of opposite signs and under certain conditions can cancel each other. More formally, one can show that the maximum of the supercurrent is achieved exactly at those values of the exchange field for which two singularities in the quasiparticle density of states do overlap [23].

We emphasize that the scenario of the magnetic-field enhancement of the critical current discussed here differs from those studied before for SFIFS tunnel structures. In our case the pair-breaking effect due to spin-polarized electrons is weakened in the SM electrode, since the spin polarizations from the exchange field of the magnetic ions and the applied field are of opposite signs and reduce each other. On the other hand, the paramagnetic effect induced by the external field is increased for the Cooper pairs of the S electrode if the applied field is increased. Competition between these two opposite effects determines the critical current behavior for the SMIS junction in magnetic field. In our case the mechanism described above is valid for the full temperature region of the homogeneous superconducting state (see, e.g., Fig. 3), while for the SFIFS system with antiparallel geometry—only at low temperature $T \ll T_C$ [7,8].

In conclusion, we have calculated the dc critical current of an (S/M)IS tunnel structure in which one electrode is a proximity-coupled bilayer of a superconducting film and a paramagnetic metal, while the second electrode is an S layer. The structure is under the effect of a weak parallel external magnetic field. In the magnetic metal the localized magnetic moments of the ions, oriented by the magnetic field, exert the effective interaction $Js\mathbf{S}$ on spins of the conduction electrons. The latter, whether it arises from the usual exchange interaction or due to configuration mixing, according to Hund's rules, is of the antiferromagnetic type, i.e., $J < 0$. In particular, such a film can be a layer of pseudoternary compounds like (EuSn)Mo_6S_8, $HoMo_6S_8$, etc. There are no specific requirements on the superconductor, so that it can be any superconducting film proximity coupled with the magnetic metal. Using approximate microscopic treatment of the S/M bilayer and the S layer, we have predicted the effect of magnetic-field-induced supercurrent enhancement in the tunnel structure. This striking behavior contrasts with the suppression of the critical current by magnetic field. The idea of using a magnetic material in which the effective magnetic interaction aligns the spins of the conduction electrons antiparallel to the localized spin of the magnetic ions in order to enhance superconductivity of superconductor—magnetic metal multilayered structures has not been considered before and, to the best of our knowledge, is new. The existing large variety of magnetic materials, the ternary compounds in particular, should allow experimental realization of this interesting new effect of the interplay between superconducting and magnetic orders.

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