Fizika Nizkikh Temperatur, 2004, v. 30, No. 3, p. 288–294

Josephson and spontaneous currents at the interface between two *d*-wave superconductors with transport current in the banks

Yu.A. Kolesnichenko, A.N. Omelyanchouk, and S.N. Shevchenko

B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Lenin Ave., 61103, Kharkov, Ukraine
E-mail: omelyanchouk@ilt.kharkov.ua

Received November 6, 2003

A stationary Josephson effect in the ballistic contact of two d-wave superconductors with different orientation of the axes and with transport current in the banks is considered theoretically. We study the influence of the transport current on the current–phase relation of the Josephson and tangential currents at the interface. It is demonstrated that the spontaneous surface current at the interface depends on the transport current in the banks due to the interference of the angle-dependent condensate wave functions of the two superconductors.

PACS: **74.50.+r**, 74.76.Bz

1. Introduction

It was shown that in the ground state of the contact of two d-wave superconductors with different orientation of the axes there is a current tangential to the boundary [1–8]. For the particularly interesting case of $\pi/4$ misorientation the ground state is twofold degenerate: there are the tangential currents in opposite directions at $\varphi = \pm\pi/2$ in the absence of Josephson current. The probabilities of finding the contact in one of the two states are equal and the corresponding tangential current is referred to as the spontaneous one. It was proposed to use such two-state quantum systems for quantum computation [9–11]. It is of interest to study the possibility of controlling this system by the external transport current, which is the motivation of the present work.

In the described problem of the Josephson contact of two d-wave superconductors with transport current in the banks the resulting tangential current is not a sum of the spontaneous and transport current. In the paper [12] we have studied the simpler case of the contact of two s-wave superconductors with a transport current flowing in the banks. It was shown that the presence of magnetic field [13–16], of transport superconducting current [12], or of the current in normal layer [17,18] in a mesoscopic Josephson junction can significantly influence the current–phase characteristics, current distribution etc.

In the present problem the Josephson current is defined by the interference of the angle-dependent condensate wave functions of the two superconductors. There are two factors of anisotropy which define the angle dependence of the order parameter: the pairing anisotropy and the transport current. Thus, it is natural to expect that the resulting interference current (which has both normal and tangential components) is parametrized by the external phase difference φ and by the value of the transport current (or by the superfluid velocity v_s). The presence of these two controlling parameters can be useful in the applications of Josephson junctions of high-T_c superconductors.

In Sec. 2 we derive basic equations to describe the ballistic planar Josephson junction of two differently orientated d-wave superconductors with homogeneous current in the banks. These equations are solved analytically in Sec. 3. Then we study in Sec. 4 the influence of the transport current on the Josephson current and vice versa at the interface. In the Appendix the order parameter and the current density in the homogeneous situation are considered.

2. Model and basic equations

We consider a model of the Josephson junction as an ideal plane between two singlet (particularly, d-wave) superconductors with different orientation of the axes (see Fig. 1). The pair breaking and the scattering at the junction as well as the electron scattering in the bulk of metals are ignored. We did not take into account the possibility of the generation of a subdominant order parameter, which results in decreasing of the amplitude of the current [7]. The c axes of both superconductors are parallel to the interface. The c axis direction is chosen as the z axis. The a and b axes are situated in the xy plane. In the banks of the contact a homogeneous current flows with a superconducting velocity $\mathbf{v}_s$. We consider the superfluid velocity $\mathbf{v}_s$ in the left (L) and right (R) superconducting half-spaces to be parallel to each other $\mathbf{v}_{sL} \| \mathbf{v}_{sR}$ and to the boundary; we choose the y axis along $\mathbf{v}_s$ and the x axis perpendicular to the boundary; $x = 0$ is the boundary plane.

We describe the coherent current state in the superconducting ballistic structure in the quasiclassical approximation by the Eilenberger equation [19,20]

$$\mathbf{v}_F \frac{\partial}{\partial \mathbf{r}} \hat{G} + [\tilde{\omega}\hat{\tau}_3 + \hat{\Delta}, \hat{G}] = 0, \tag{1}$$

where $\tilde{\omega} = \omega_n + i\mathbf{p}_F \mathbf{v}_s$, $\omega_n = \pi T(2n+1)$ are the Matsubara frequencies, $\hat{G} = \hat{G}_\omega(\mathbf{v}_F, \mathbf{r}) = \begin{pmatrix} g & f \\ f^+ & -g \end{pmatrix}$ is the energy-integrated Green function, and $\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix}$. Equation (1) should be supplemented by

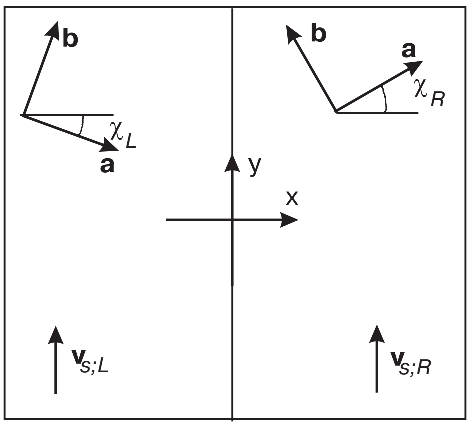

Fig. 1. Geometry of the contact of two superconductors with different orientation of the axes and different transport currents (superfluid velocities $\mathbf{v}_{s;L,R}$) in the banks.

the equation for the order parameter (the self-consistency equation):

$$\Delta(\mathbf{v}_F, \mathbf{r}) = \pi N_0 T \sum_\omega \langle V(\mathbf{v}_F, \mathbf{v}'_F) f(\mathbf{v}'_F, \mathbf{r}) \rangle_{\mathbf{v}'_F}, \tag{2}$$

N_0 is the density of states at the Fermi level and $\langle \ldots \rangle_{\mathbf{v}_F}$ denotes averaging over directions of $\mathbf{v}_F$; $V(\mathbf{v}_F, \mathbf{v}'_F)$ is a pairing attractive potential. For the bulk d-wave superconductor it is usually assumed that $\Delta(\theta) = \Delta_0(T, \mathbf{v}_s)\cos 2\theta$, $V(\mathbf{v}_F, \mathbf{v}'_F) = V_d \cos 2\theta \cos 2\theta'$, where the angle θ defines a direction of the velocity $\mathbf{v}_F$. Solutions of Eqs. (1), (2) must satisfy the conditions for the Green functions and gap function in the banks far from the interface:

$$g(\mp\infty) = \frac{\omega_{L,R}}{\Omega_{L,R}}, \tag{3}$$

$$f(\mp\infty) = \frac{\Delta(\mp\infty)}{\Omega_{L,R}}, \tag{4}$$

$$\Delta(\mp\infty) = \Delta_{L,R} \exp(\pm i\varphi/2). \tag{5}$$

Here $\omega_{L,R} = \omega_n + i\mathbf{p}_F \mathbf{v}_{s;L,R}$, $\Omega_{L,R} = \sqrt{\omega_{L,R}^2 + \Delta_{L,R}^2}$; φ is the phase difference between the left and right superconductors, which parametrizes the Josephson current state. The angles $\chi_{L,R}$ define the orientation of the crystal axes **a** and **b** in the left and right half-spaces (see Fig. 1). The angle between the axes of the right and left superconductors (the misorientation angle) is $\delta\chi = \chi_R - \chi_L$.

Provided we know the Green function $\hat{G}$, we can calculate the current density:

$$\mathbf{j}(\mathbf{r}) = -2\pi i e N_0 T \sum_\omega \langle \mathbf{v}_F g(\mathbf{v}_F, \mathbf{r}) \rangle_{\mathbf{v}_F}. \tag{6}$$

For singlet superconductors it is usually assumed that $\Delta(-\mathbf{v}_F) = \Delta(\mathbf{v}_F)$, and we therefore have:

$$f^+(\omega, -\mathbf{v}_F) = f^+(-\omega, \mathbf{v}_F) = f^*(\omega, \mathbf{v}_F), \tag{7}$$

$$g(\omega, -\mathbf{v}_F) = -g(-\omega, \mathbf{v}_F) = g^*(\omega, \mathbf{v}_F). \tag{8}$$

Making use of Eq. (8), we can rewrite Eq. (6) as

$$\mathbf{j}(\mathbf{r}) = -j_0 \frac{T}{T_c} \sum_{\omega>0} \langle \hat{\mathbf{v}}_F \operatorname{Im} g(\mathbf{r}) \rangle_{\mathbf{v}_F}, \quad j_0 = 4\pi|e|N(0)v_F T_c. \tag{9}$$

3. Analytical solution of the Eilenberger equation

In this paper we consider the problem non-self-consistently: we assume the superconducting velocity $\mathbf{v}_s$ is homogeneous and that the order parameter Δ is constant in the two half-spaces:

$$\mathbf{v}_s(\mathbf{r}) = \begin{cases} \mathbf{v}_{s;L}, & x<0 \\ \mathbf{v}_{s;R}, & x>0 \end{cases}, \quad \Delta(\mathbf{r}) = \begin{cases} \Delta_L \exp(i\varphi/2), & x<0 \\ \Delta_R \exp(-i\varphi/2), & x>0 \end{cases}. \tag{10}$$

As was shown in the papers [7], the self-consistent consideration of a Josephson junction of d-wave superconductors does not differ qualitatively from the non-self-consistent one. In the paper [7] the authors compare numerically the self-consistent solution with the non-self-consistent one. The self-consistency of the solution allows one to take into account the suppression of the order parameter at the interface; the major effect of this is the reduction of the current [7].

Equation (1) together with Eqs. (3)–(5) and (10) yields for the left and right superconductors:

$$g_{L,R}(x) = \frac{\omega_{L,R}}{\Omega_{L,R}} + C_{L,R} \exp\left(-\frac{2|x|}{|v_x|}\Omega_{L,R}\right), \tag{11}$$

$$f_{L,R}(x) = \frac{\Delta_{L,R}}{\Omega_{L,R}} e^{-\mathrm{sgn}(x) i\varphi/2} - C_{L,R} \frac{\mathrm{sgn}(x)\eta\Omega_{L,R} + \omega_{L,R}}{\Delta_{L,R}} \times$$

$$\times \exp\left(-\frac{2|x|}{|v_x|}\Omega_{L,R}\right) e^{-\mathrm{sgn}(x) i\varphi/2}, \tag{12}$$

where $\eta = \mathrm{sgn}(v_x)$. Making use of the continuity condition, we obtain the expression for the g function at the interface:

$$g(0) = \frac{\Omega_L \omega_R + \Omega_R \omega_L - i\eta\Delta_L\Delta_R \sin\varphi}{\Omega_L\Omega_R + \omega_L\omega_R + \Delta_L\Delta_R\cos\varphi}. \tag{13}$$

Equations (9) and (13) allow us to calculate the Josephson current $j_J = j_x(x=0)$ and the tangential current $j_y(x=0)$ at the interface. We emphasize that these equations are valid for describing the current at the interface of two singlet superconductors with different orientation of the axes and with different transport currents in the banks. The contact of conventional superconductors was considered in [12] and in the present paper we study the contact of d-wave superconductors, for which the order parameter is $\Delta_{L,R}(\theta) = \Delta_0(T, \mathbf{v}_{s;L,R}) \cos 2(\theta - \chi_{L,R})$. The consideration presented here can be also used to consider the contact of g-wave superconductors or an s-wave/d-wave contact, etc.

As we restrict ourselves to the non-self-consistent model we should calculate the order parameter $\Delta_0 = \Delta_0(T, \mathbf{v}_s)$ in the bulk d-wave superconductor. That is the subject of the Appendix.

In the particular case considered below in detail we have $\mathbf{v}_{s;L} = \mathbf{v}_{s;R} = \mathbf{v}_s$ and denote $\tilde{\omega} = \omega_n + i\mathbf{p}_F\mathbf{v}_s$, $\Omega_{L,R} = \sqrt{\tilde{\omega}^2 + \Delta_{L,R}^2}$; in this case we obtain

$$g(0) = \frac{\tilde{\omega}(\Omega_L + \Omega_R) - i\eta\Delta_L\Delta_R\sin\varphi}{\Omega_L\Omega_R + \tilde{\omega}^2 + \Delta_L\Delta_R\cos\varphi}. \tag{14}$$

In the absence of the transport current ($\mathbf{v}_s = 0$) in this expression: $\tilde{\omega} = \omega_n$ [7].

We should also clarify the sign of the square root in $\Omega_{L,R}$. To make the solution (11) convergent, we must require $\mathrm{Re}\,\Omega_{L,R} > 0$, which fixes the sign of the square root in $\Omega_{L,R}$ to be sgn $(\omega\mathbf{p}_F\mathbf{v}_{s;L,R})$. Moreover, this requirement, as can be shown, provides the supplementary condition on Re g: $\mathrm{sgn}(\mathrm{Re}\, g) = \mathrm{sgn}(\omega)$.

4. Influence of the transport current on the Josephson and spontaneous currents at the interface

Further we study the Josephson contact for the definite case: $\mathbf{v}_{sL} = \mathbf{v}_{sR} = \mathbf{v}_s$ and $\chi_L = 0$ and $\chi_R = \pi/4$.

For small values of v_s (in the approximation linear in $p_F v_s/T_c$) we can state the following approximate relations (which are valid for values of φ in the vicinity of $\pm\pi/2$):

$$j_J(-\mathbf{v}_s, \varphi) \simeq j_J(\mathbf{v}_s, \varphi),$$

$$j_y(-\mathbf{v}_s, \varphi) \simeq -j_y(\mathbf{v}_s, -\varphi),$$

and for the difference $\delta j \equiv j(\mathbf{v}_s) - j(\mathbf{v}_s = 0)$:

$$\delta j_J(-\varphi) \simeq -\delta j_J(\varphi), \quad \delta j_y(-\varphi) \simeq \delta j_y(\varphi),$$

while at $\mathbf{v}_s = 0$

$$j_J(-\varphi) = -j_J(\varphi), \quad j_y(-\varphi) = -j_y(\varphi).$$

In the linear approximation the shift current δj_y is an even function of φ, in contrast to $j_y(\mathbf{v}_s = 0)$. For the spontaneous current (at $\varphi = \pm\pi/2$) the shift currents δj_y are equal:

$$j_y(\varphi = \pm\pi/2) = j_S + \delta j_y. \tag{15}$$

$$j_S(-\pi/2) = -j_S(\pi/2), \quad \delta j_y(-\pi/2) = \delta j_y(\pi/2).$$

In a nonlinear treatment these shift currents are different for the two cases and are discussed below.

In Figs. 2, 3 we plot the normal (Josephson) and tangential components of the current densities at the interface plane as functions of the phase difference

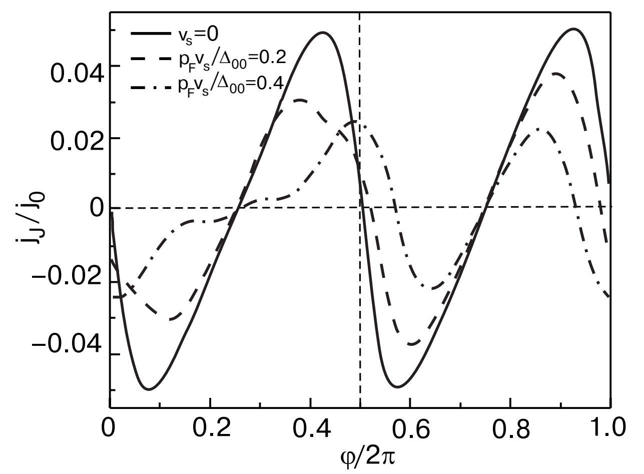

Fig. 2. Josephson current density through the interface j_J versus phase φ $(\chi_L = 0, \chi_R = \pi/4, T = 0.1T_c)$; $\Delta_{00} = \Delta_0(T = 0, v_s = 0) = 2.14T_c$.

φ at low temperature. In the absence of the transport current: (i) $\mathbf{j}$ is an odd function of φ; (ii) the normal component of the current (Josephson current) is π-periodic; (iii) in the equilibrium state at $\varphi = \pm\pi/2$: $j_J = 0$, $j_y(\pm\pi/2) = j_S = \mp|j_S|$. In the latter case the tangential current exists in the absence of the Josephson current; for that reason it is referred to as the spontaneous current. The presence of the transport current breaks the symmetry relations (i)–(iii). There is a nonzero Josephson current at $\varphi = 0, \pi$. How the transport current influences the spontaneous current (i.e., the tangential current at $\varphi = \pi/2$ and $\varphi = -\pi/2$) is shown in Fig. 4. The shift of the two values of the current for small values of v_s (in the linear in $p_F v_s/T_c$ approximation) is equal (see Eq. (15)); however, at values $v_s \sim 0.2\Delta_{00}/p_F$ the shift current (i.e., the difference $j_y(\mathbf{v}_s) - j_S(\mathbf{v}_s = 0)$) is of different sign for the two currents and in the directions opposite to j_S.

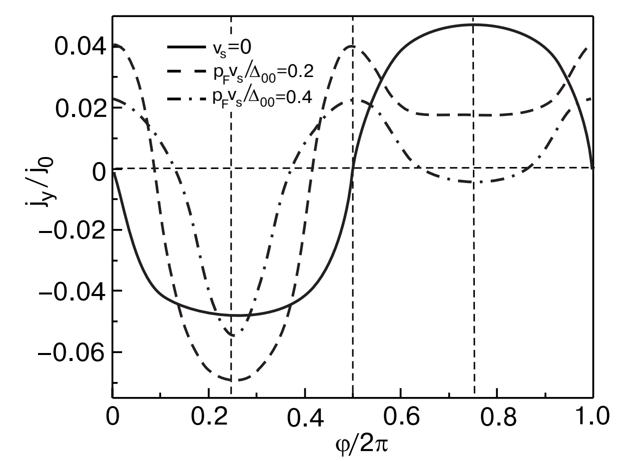

Fig. 3. Tangential current density at the interface j_y versus phase $\varphi(\chi_L = 0, \chi_R = \pi/4, T = 0.1T_c)$.

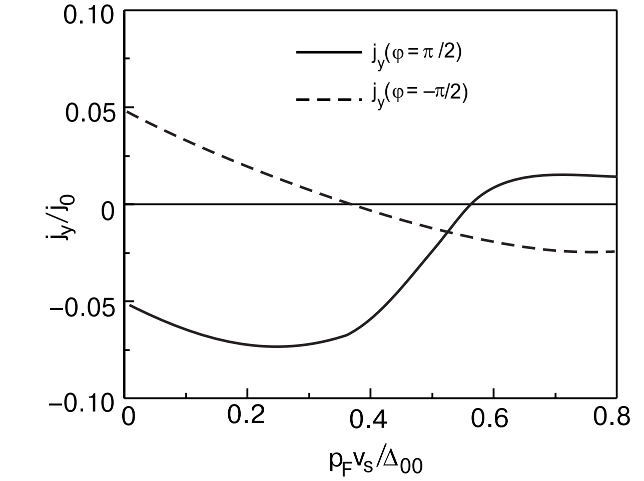

Fig. 4. Tangential current density at the interface j_y for two values of the phase difference (spontaneous current) versus superfluid velocity v_s $(\chi_L = 0, \chi_R = \pi/4, T = 0.1T_c)$.

We also note the following relations for $\mathbf{v}_s \neq 0$: 1) $j_J(\varphi = \pi) = -j_J(\varphi = 0) \neq 0$ (the presence of the transport current induces a nonzero Josephson current in the absence of an external phase difference); 2) $j_J\left(\varphi = \pm\dfrac{\pi}{2}\right) = 0$, $\dfrac{dj_J}{d\varphi}\left(\varphi = \pm\dfrac{\pi}{2}\right) > 0$ (the transport current does not change the values of equilibrium phase difference, at $\varphi = \pm\pi/2$); 3) $j_y(\varphi = \pi) = j_y(\varphi = 0) \neq 0$. This last relation concerns the interesting phenomena studied in [12]: for some values of phase difference (here in the vicinity of $\varphi = 0, \pi$) the interference of the angle-dependent condensate wave functions results in the appearance of an additional tangential current with the direction opposite to the transport current in the banks. We emphasize that the resulting tangential current is not the sum of the spontaneous current and the transport current [12]. Thus, the transport current drastically influences both the tangential (spontaneous) and Josephson currents.

We can write down explicitly an expression for the current for temperatures close to the critical (so close that $\Delta_0, p_F v_s \ll T_c$). From Eq. (14) we have:

$$\operatorname{Im} g(0) \simeq \Delta_L \Delta_R \left[-\eta \frac{1}{2\omega_n^2} \sin\varphi + \frac{\mathbf{p}_F \mathbf{v}_s}{\omega_n^3} \cos\varphi + \eta \frac{3}{2} \frac{(\mathbf{p}_F \mathbf{v}_s)^2}{\omega_n^4} \sin\varphi + \eta \frac{\Delta_L \Delta_R}{8\omega_n^4} \sin 2\varphi \right]. \quad (16)$$

At $\chi_L = 0$ and $\chi_R = \pi/4$ this results in the following:

$$\mathbf{j} = \mathbf{j}_J + \mathbf{j}_S + \tilde{\mathbf{j}}, \quad (17)$$

φ

$$\mathbf{j}_J = -\frac{1}{3024\pi} j_0 \frac{\Delta_0^4}{T_c^4} \sin 2\varphi \cdot \mathbf{e}_x, \tag{18}$$

$$\mathbf{j}_S = -\frac{1}{60\pi} j_0 \frac{\Delta_0^2}{T_c^2} \sin\varphi \cdot \mathbf{e}_y, \tag{19}$$

$$\tilde{\mathbf{j}} = \frac{3}{560\pi} j_0 \frac{\Delta_0^2}{T_c^2} \frac{(p_F v_s)^2}{T_c^2} \sin\varphi \cdot \mathbf{e}_y. \tag{20}$$

Here $\Delta_0 = \Delta_0(T, v_s)$ and is defined by Eq. (25). In particular, at $v_s = 0$ this gives:

$$j_J = -1.7 \cdot 10^{-2} j_0 \left(1 - \frac{T}{T_c}\right)^2 \sin 2\varphi, \tag{21}$$

$$j_S = -6.6 \cdot 10^{-2} j_0 \left(1 - \frac{T}{T_c}\right) \sin\varphi. \tag{22}$$

We note that $\tilde{\mathbf{j}} = -\frac{9}{28}\frac{(p_F v_s)^2}{T_c^2}\mathbf{j}_S$. It follows that the effect of transport current on the spontaneous tangential current at $T \sim T_c$ is to reduce its value by a small shift. It is remarkable that the current tangential to the boundary contains only corrections of the second order in the parameter $p_F v_s / T_c$.* If $\chi_L = 0$ and $\chi_R = \delta\chi \neq \pi/4$, the integration of the second term in Eq. (16) would give us the factor $\pi \cos^2 \delta\chi - \pi/2$, which is zero for $\delta\chi = \pi/4$; this term at $\delta\chi = 0$ and $\varphi = 0$ gives the homogeneous current density (Eq. (26)).

The integration of the first term in Eq. (16) gives us the factor $\cos 2\delta\chi$ for the x component of the current and $\sin 2\delta\chi$ for the y component. In the case of $\delta\chi = \pi/4$ this term gives only the tangential component. As a consequence $j_S >> j_J$ (see Eqs. (21), (22)).

It was discussed above that the terms linear in $p_F v_s / T_c$ result in a uniform shift of j_S. We can see that nonlinear terms result in a shift of different sign, and in both cases in the direction opposite to j_S (see Eq. (20)). This in part explains the nonmonotonic behavior of j_y (see Fig. 4). The fact that the presence of the transport current significantly changes the tangential (spontaneous) currents might be used for its control, which is important in view of their possible application for quantum computation [9–11].

5. Conclusion

We have studied influence of the transport current, which flows in the banks, on the stationary Josephson effect in the contact of two d-wave superconductors. We have derived equations which allow general consideration of the contact of two singlet superconductors with different orientation of the axes and with different transport currents in the banks. In particular, we have studied the planar contact of two d-wave superconductors in the case of $\pi/4$ misorientation with equal transport currents in the banks. It was demonstrated that the current–phase relation drastically depends upon the value of the transport current. The ground state degeneracy in the absence of transport current (at $\varphi = \pm\pi/2$) is lifted at $v_s \neq 0$. The dependence of the shift current (which is the difference of the tangential current and the spontaneous one) on v_s is shown to be nonlinear. It is proposed to use the transport current for the control of qubits based on the contact of two d-wave superconductors.

We acknowledge support from D-Wave Systems Inc. (Vancouver).

Results of the present study were reported at the International Conferences: «Applied Electrodynamics of high-T_c Superconductors», IRE, Kharkov, Ukraine (May 2003) and «Basic Studies and Novel Applications», Jena, Germany (June 2003).

6. Appendix. Order parameter in the homogeneous current state

In this Section we study the homogeneous current state in the bulk d-wave superconductor (see also in [21]). We note that the order parameter Δ_0 is a function of temperature T, superfluid velocity v_s, and the angle χ between the crystallographic axis a and the direction of the superfluid velocity $\mathbf{v}_s$. For that we should solve Eqs. (2) and (9) with g and f given by Eqs. (3) and (4):

$$\frac{1}{\lambda} = 2T\sum_{\omega>0}\int_{-\pi/2}^{\pi/2} d\theta \,\mathrm{Re}\,\frac{\Delta^2(\theta)/\Delta_0^2}{\Omega},$$

$$\frac{\mathbf{j}}{j_0} = -\frac{T}{\pi T_c}\sum_{\omega>0}\int_{-\pi/2}^{\pi/2} d\theta \hat{\mathbf{v}}_F \,\mathrm{Im}\,\frac{\tilde{\omega}}{\Omega}.$$

Here $\lambda = N_0 V_d$, $\tilde{\omega} = \omega_n + i\mathbf{p}_F \mathbf{v}_s$, $\Omega = \sqrt{\tilde{\omega}^2 + \Delta^2}$, $\Delta(\theta) = \Delta_0(T, \mathbf{v}_s)\cos 2(\theta - \chi)$.

* There is also a term with the factor $\frac{p_F v_s}{T_c}\frac{\Delta_0^4}{T_c^4}$, which is neglected here. This term results in equal shifts of j_S for $\varphi = \pm\pi/2$.

For $T = 0$ (replacing $\pi T \sum_\omega$ by the integral $\int d\omega$) we obtain the equations for the order parameter Δ_0 and the current density j:

$$\ln\left(\frac{\Delta_{00}}{\Delta_0}\right) = \frac{2}{\pi}\int d\theta \left(\frac{\Delta(\theta)}{\Delta_0}\right)^2 \ln\left(\left|\frac{\mathbf{v}_s \mathbf{p}_F}{\Delta(\theta)}\right| + \sqrt{\left(\frac{\mathbf{v}_s \mathbf{p}_F}{\Delta(\theta)}\right)^2 - 1}\right), \tag{23}$$

here $\Delta_{00} = \Delta_0(T = 0, v_s = 0) = \xi\omega_c e^{-2/\lambda}$, $\xi = 4e^{-1/2}$,

$$\frac{j}{j_0} = -\frac{1}{4\pi}\frac{v_s p_F}{T_c} + $$

$$+\frac{1}{2\pi^2}\int d\theta |\cos\theta| \sqrt{\left(\frac{\mathbf{v}_s \mathbf{p}_F}{T_c}\right)^2 - \left(\frac{\Delta(\theta)}{T_c}\right)^2}. \tag{24}$$

In Eqs. (23) and (24) the integration is performed in the region where $\Delta(\theta)^2 < (\mathbf{v}_s \mathbf{p}_F)^2$ for $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

In Figs. 5 we plot the order parameter $\Delta_0(T, \mathbf{v}_s)$ and the current density versus the superfluid velocity v_s for different angles χ at low temperature. For comparison we also plot the curves for the *s*-wave superconductor. A numerical analysis at low temperature shows that in spite of the strong anisotropy of the pairing potential, the order parameter Δ_0, the critical velocity v_s^{cr}, and the critical current j_c depend weakly on the angle χ between $\mathbf{v}_s$ and the crystallographic *a* axis (see Figs. 5 and in Ref. 21). Namely, the respective difference is maximal for $\chi = 0$ and $\chi = \pi/4$ and does not exceed 0.1. For small values of the superfluid velocity, i.e., in the approximation linear in the parameter $v_s p_F / T_c$, both Δ_0 and j are independent of χ.

For a temperature close to $T_c = \beta\omega_c e^{-2/\lambda} = 0.47\Delta_{00}$, where $\beta = (2/\pi)e^C$ (C is the Euler constant), both the gap function Δ_0 and current density j are independent of the angle χ:

$$\Delta_0^2 = \frac{32\pi^3}{21\zeta(3)} T_c^2 \left(1 - \frac{T}{T_c}\right) - \frac{4}{3}(p_F v_s)^2, \tag{25}$$

$$\frac{j}{j_0} = -\frac{7\zeta(3)}{32\pi^3}\frac{\Delta_0^2}{T_c^2}\frac{p_F v_s}{T_c}. \tag{26}$$

The temperature dependence of critical velocity v_s^{cr} follows from Eq. (25): $\frac{p_F v_s^{\mathrm{cr}}}{T_c} = \sqrt{\frac{8\pi^2}{7\zeta(3)}}\sqrt{1 - \frac{T}{T_c}}$.

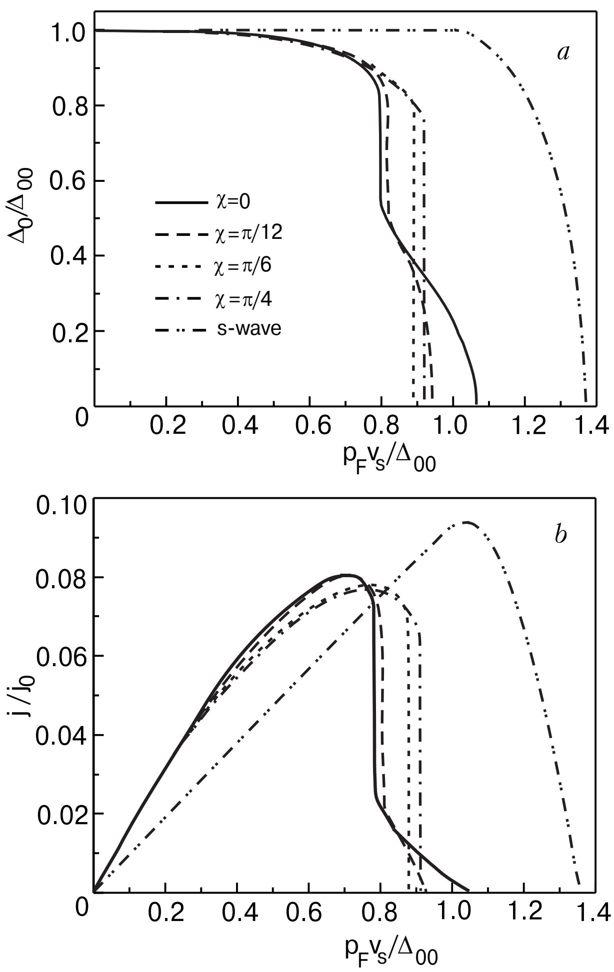

Fig. 5. Order parameter $\Delta_0(T, \mathbf{v}_s)$ (*a*) and current density (*b*) in the bulk *d*-wave superconductor versus superfluid velocity v_s for different angles χ between $\mathbf{v}_s$ and the **a** axis ($T = 0$).

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