Condensed Matter Physics, 1998, Vol. 1, No 1(13), p. 113–124

CONDENSED
MATTER
PHYSICS

# On temperature *versus* doping phase diagram of high critical temperature superconductors

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Received March 6, 1998

An attempt to describe the bell-shape dependence of the critical temperature of high- $T_c$ superconductors on the charge carrier density is made. It is proposed to explain its linear increase in the region of small densities (underdoped regime) by the role of the order parameter phase 2D fluctuations which become less at this density growth. The critical temperature suppression in the region of large carrier densities (overdoped regime) is connected with the appearance (because of doping) of an essential damping of long-wave bosons which, within the framework of the model proposed, define the mechanism of indirect inter-fermion attraction.

**Key words:** *high-temperature superconductors, doping, critical temperature, phase diagram, pseudogap*

**PACS:** *67.20.+k, 74.20.-z, 74.25.-q, 74.72.-h*

## 1. Introduction

In spite of the intensive study of high-$T_c$ superconductors (HTSCs), the comprehension of many regularities which define their physical behaviour has not been achieved. First of all, it is put down to the HTSC normal state properties, the description of which within the framework of the standard Fermi-liquid theory proves to be impossible if the carrier concentration $n_f$ in the corresponding samples is such that the resulting critical temperature $T_c$ in them is less than some optimal value $T_c^{\max}$ characteristic of every compound. The latter appears in all (practically, without exception) HTSC copper oxides because, as it is well-known (see, for example, review [1]), the conductivity (consequently - superconductivity) of these initially quasi-2D antiferromagnetic insulators results from their doping by donor (Nd, Pr) or acceptor (Sr, Ba, O) ions.

 

Then, temperature $T_c$ being formed within the increase of the carrier density (or, in other words, Fermi energy) in a system also rises. This growth, however, rather quickly stops and, after some (not large upon $n_f$ values) part of "saturation", a drop to zero of the function $T_c(n_f)$ follows, which thus acquires a bell-shape form. As a result, HTSC compounds with a relatively large concentration of itinerant carriers (the so-called overdoped regime) become non-superconducting metals the behaviour of which to a certain degree can be explained on the basis of the conventional theory of Fermi liquid [2]. The corresponding carrier concentration values for the underdoped regime are such that $T_c(n_f)$ increase is observed for $n_f$ from $\simeq 0.04 - 0.08$ carrier per $CuO_2$ layer cell (i.e. from the threshold of the insulator-metal transition) to $T_c^{\max} \simeq T_c(0.15-0.18)$; temperature $T_c(n_f)$ becomes zero when $n_f \simeq 0.25 - 0.28$ [3–5]. If the initial part of function $T_c(n_f)$, where $\mathrm{d}T_c(n_f)/\mathrm{d}n_f \simeq \mathrm{const} > 0$ [6,7], can be (at least qualitatively) interpreted (see [8–10] and review [11]) proceeding from the theory of crossover from the Bose-Einstein condensation of separate composite bosons (local pairs) and their superfluidity to the superconductivity of the BCS type, then the reason of $T_c(n_f)$ suppression in the BCS carrier concentration region remains, in fact, unknown yet.

From the very beginning the above-mentioned behaviour of $T_c(n_f)$ was ascribed to strong electron-electron correlations (see, for example, book [12]) and to the filling of usually narrow Hubbard subbands by itinerant carriers. This question was intensively investigated by Prof. I.V.Stasyuk and his collaborators [13–16].

However, a little bit later it becomes evident that carrier concentrations corresponding to the disappearance of superconductivity are so small that such a filling of the whole valent (in fact, conduction) band in HTSCs is impossible. The alternative version of the HTSC effect disappearance most consistently considered in [17,18] (see also [1,2]) consists in the assumption that a bare Fermi level in HTSCs proves to be in the vicinity of the extended saddle point of the electronic spectrum. According to this point of view, at $n_f$ growth the Fermi energy $\epsilon_\mathrm{F} \simeq \boldsymbol{k}_\mathrm{F}^2/2m$ ($\boldsymbol{k}_\mathrm{F}$ – Fermi momentum, $m$ – effective mass) passes the van Hove singularity in the conduction band density of states and, hence, $T_c(n_f)$ reveals its maximum. If it were so, function $T_c(n_f)$ being similar to a BCS one, would have an exponential dependence on $n_f$ far from $T_c^{\max}$ which, as it is seen from the experiments and has been already mentioned, is not confirmed by observations. On the other hand, the very appearance of the van Hove spectrum singularity requires fine-tuning (for example, because of a rather definite ratio between the hole next-near and the nearest hopping constants) which seems to be hard to achieve even in one compound, to say nothing of many.

At the same time less attention was paid to the idea that the weakness of superconductivity can be caused by a "feedback" effect, or the changes in the spectrum $\omega(\boldsymbol{k})$ of intermediate bosons, the exchange of which results in fermion pairing. These bosons in HTSCs can be attributed to phonons (similarly to the BCS-Eliashberg model) the role of which is actively advocated by Ginzburg (see, for example, his review [19]), magnons, or spin fluctuations, [20,21], quadripole *dd*-excitons (the Gaididei-Loktev-Weber mechanism (see [1])), plasmons [22], etc.

It must be noted, however, that, if a phonon spectrum does not at all (or very weakly) depend on doping, then long-wave excitations of the magnetic subsystem ($dd$-excitons, as excitations over a magnetic background, can also be considered here) are strongly suppressed because of the long-range magnetic order destruction in the metallic phase of HTSCs [1,12].

Namely, spin waves are subjected to a most appreciable doping effect; their low-frequency region (for the wave vectors less than some characteristic value $k_{\rm min}$) acquires a diffusion form, or becomes overdamped. In the insulating phase of HTSCs, as it is shown in [23], $k_{\rm min} \sim n_f$; in their metallic phase the magnon damping becomes even stronger and $k_{\rm min} \simeq 2k_{\rm F} \sim n_f^{1/2}$ [24,1], which is completely in line with the degradation of the correlation magnetic length $\xi_{\rm mag} \sim n_f^{-1/2}$ measured in HTSCs [25]. In that way the long-wave damping $\gamma(\boldsymbol{k})$ increases up to such high values ($\gamma(\boldsymbol{k}) \gg \omega(\boldsymbol{k})$) that corresponding intermediate bosons (most probably, spin fluctuations) cannot participate in interaction transfer, "being out".

The solution of the self-consistent magneto-electronic problem as a whole (i.e. the explicit solution of the equation of superconductivity similar to the Eliashberg equation) is scarcely possible now. Therefore, an attempt is made below to consider the simplest model with an indirect inter-fermion attraction provided by intermediate massive bosons with a long-wave cut which is proportional to $k_{\rm F}$. The carriers are supposed to appear in the system due to doping which corresponds to a generally accepted scenario of metallization of copper oxides. For the sake of simplicity we shall omit any other dampings (in particular, the carrier damping because of the disorder the effect of which was analysed, for example, in [26]).

Following [9,11], we shall also suppose that in a model 2D system (in fact, all HTSCs can be with good accuracy referred to this kind), the superconducting condensate is formed in a way principally different from the ordinary one. In such a case one has to distinguish the order parameter formation temperature from the real (observable) critical temperature in a sense that the absolute value of the former does not become zero at and above $T_c$.

## 2. Model and main equations

The model Hamiltonian density of the electron-phonon system can be written in the well-known form:

$$H(x) = -\psi_\sigma^\dagger(x)\left(\frac{\nabla^2}{2m} + \mu\right)\psi_\sigma(x) + H_{\rm ph}(\varphi(x)) + g_{\rm ph}\psi_\sigma^\dagger(x)\psi_\sigma(x)\varphi(x), \quad x = \boldsymbol{r}, t, \tag{2.1}$$

where $H_{\rm ph}$ is the Hamiltonian of free phonons which will be described more precisely below; $\psi_\sigma(x)$ and $\varphi(x)$ are the fermion and boson field operators, respectively; $m$ is the effective mass of the Fermi-particles, $\sigma =\uparrow,\downarrow$ - their spin variable, and $g_{ph}$ is the electron-phonon coupling constant; $\mu$ in (2.1) is the chemical potential of fermions which fixes their average density in the system; we put $\hbar = k_{\rm B} = 1$.

As it was mentioned in the Introduction, we shall model a boson-exchange interaction taking into account the dependent on doping saturation of long-wave bosons. Under the above mentioned assumptions about a gradual non-participation of a part of the bosons in the attracting interaction formation, the simplest way to describe it is by using a free phonon propagator in the form:

$$D(\omega,\boldsymbol{k}) = -\frac{2\omega(\boldsymbol{k})}{\omega^2-\omega^2(\boldsymbol{k})+i\delta}\theta(k-k_{\rm min}), \quad \delta\to 0, \tag{2.2}$$

where $\omega(\boldsymbol{k})$ is the boson dispersion law, and $\theta(k)$ is the step function. As it was pointed out above, $\boldsymbol{k}_{\rm min}$ in (2.2) is some characteristic wave vector which separates the region of overdamped ($k<k_{\rm min}$) and long-lived ($k>k_{\rm min}$) intermediate bosons (here: phonons by definition). Although the magnetic correlation length measurements result in $k_{\rm min}=2k_{\rm F}$, we shall adopt a more general (or soft) relation supposing that $k_{\rm min}=\alpha k_{\rm F}\equiv\alpha\sqrt{2m\epsilon_{\rm F}}$ where $\alpha$ is some free parameter.

It is very important that the Hamiltonian (2.1) is invariant with respect to the global symmetry transformations

$$\psi_\sigma(x)\to\psi_\sigma(x)e^{i\theta}, \quad \psi^\dagger_\sigma(x)\to\psi^\dagger_\sigma(x)e^{-i\theta}, \tag{2.3}$$

which in a 2D case (unlike a 3D one) remain unbroken and the phase transition is here accompanied by a change in the correlation function behaviour only.

The $T-n_f$ phase diagram of the system can be calculated by using the Hubbard-Stratonovich method generalized by the case of a non-local (indirect) interaction (the so-called auxiliary bilocal field method). For finding the grand partition function $Z$ it is useful to pass to Nambu spinors: $\Psi^\dagger=(\psi^\dagger_\uparrow,\psi_\downarrow)$ and its conjugated one. After performing an integration over bosonic fields it is easy to obtain the Lagrangian

$$L=\Psi^\dagger(x)[-\partial_\tau+(\frac{\nabla^2}{2m}+\mu)\tau_z]\Psi(x)$$
$$-\frac{1}{2}\Psi(x_1)\Psi^\dagger(y_1)\tau_z K(x_1,y_1;x_2,y_2)\Psi(x_2)\Psi^\dagger(y_2)\tau_z \tag{2.4}$$

of the system where an integration over repeated indices is supposed. The kernel $K$ in (2.4) will be defined below.

Let us introduce the pairing order parameter

$$\phi(x_1,y_1)=K(x_1,y_1;x_2,y_2)\Psi(x_2)\Psi^\dagger(y_2)\tau_z$$
$$\equiv\tau_+\phi(x_1,y_1)+\tau_-\phi^*(x_1,y_1), \tag{2.5}$$

where $\tau_+=\frac{1}{2}(\tau_x+i\tau_y)$, $\tau_-=\frac{1}{2}(\tau_x-i\tau_y)$ (and $\tau_z$ in (2.4)) are the Pauli matrices.

Then, adding a zero term to the Lagrangian $L$

$$\frac{1}{2}[\phi(x_1,y_1)-K(x_1,y_1;x'_1,y'_1)\Psi(x'_1)\Psi^\dagger(y'_1)\tau_z]K^{-1}(x_1,y_1;x_2,y_2)[\phi(x_2,y_2)$$
$$-K(x_2,y_2;x'_2,y'_2)\Psi(x'_2)\Psi^\dagger(y'_2)\tau_z]$$

for the purpose of cancelling the four-fermion interaction, one comes to the expression

$$L(x_1,y_1;x_2,y_2) = \Psi^\dagger(x_1)\left[-\partial_\tau + \left(\frac{\nabla^2}{2m}+\mu\right)\tau_z - \tau_+\phi(x_1,y_1)\right.$$
$$\left.-\tau_-\phi^*(x_1,y_1)\right]\Psi(y_1) + \frac{1}{2}\phi(x_1,y_1)K^{-1}(x_1,y_1;x_2,y_2)\phi(x_2,y_2) \quad (2.6)$$

for the Lagrangian needed. The Fourier transformation of $K$ can be written as

$$K(x_1,y_1;x_2,y_2) = \int \frac{\mathrm{d}^3P\mathrm{d}^3p_1\mathrm{d}^3p_2}{(2\pi)^9}K_P(p_1;p_2)\exp\left[-iP\left(\frac{x_1+y_1}{2}-\frac{x_2+y_2}{2}\right)\right.$$
$$\left.-ip_1(x_1-y_1)-ip_2(x_2-y_2)\right],$$

($p_i = (\boldsymbol{p}_i,\omega_i)$ where $i=1,2$ and $P=(\boldsymbol{P},\omega)$ are the relative and the centre of mass momenta, respectively). Supposing now that $K_P(p_1;p_2)$ is $P$- independent we pass to the standard kernel form

$$K(p_1;p_2) = g_{\mathrm{ph}}^2 D(p_1-p_2), \quad (2.7)$$

which corresponds to the indirect inter-fermion interaction.

The partition function can be written as

$$Z = \int \mathcal{D}\Psi^\dagger\mathcal{D}\Psi\mathcal{D}\phi\mathcal{D}\phi^*\exp\left[-\beta\int L(\Psi^\dagger,\Psi,\phi^*,\phi)\mathrm{d}x\mathrm{d}y\right]$$
$$\equiv \int \mathcal{D}\phi\mathcal{D}\phi^*\exp(-\beta\Omega[\mathcal{G}]), \quad \beta\equiv 1/T,$$

In the last expression $\Omega[\mathcal{G}]$ is a thermodynamic potential which in the "leading order" on $g_{\mathrm{ph}}$ has the form:

$$\beta\Omega[\mathcal{G}] = -\mathrm{Tr}\left[\mathrm{Ln}\mathcal{G}^{-1} + \frac{1}{2}(\phi K^{-1}\phi)\right], \quad (2.8)$$

in which

$$\mathcal{G}^{-1} = -\left[\partial_\tau - \left(\frac{\nabla^2}{2m}+\mu\right)\tau_z - \tau_+\phi - \tau_-\phi^*\right] \quad (2.9)$$

is a full fermion Green function. After the direct minimization of the potential (2.8) it is easy to obtain an equation for the auxiliary $\phi$-field (or order parameter):

$$\delta\Omega/\delta\phi = 2\phi - tr\int\frac{\mathrm{d}^2k\mathrm{d}\omega}{(2\pi)^3}K(p;\boldsymbol{k},\omega)\mathcal{G}(\boldsymbol{k},\omega)\tau_x = 0. \quad (2.10)$$

Using (2.10) one can easily arrive at the well-known Cornwell-Jackiw-Tomboulis formula for the effective action in the one-loop approximation [27]:

$$\beta\Omega(\mathcal{G}) = -\mathrm{Tr}\mathrm{Ln}\mathcal{G}^{-1} + \frac{1}{2}\mathrm{Tr}\mathcal{G}K\mathcal{G},$$

or, taking into account (2.9) (or (2.10)),

$$\beta\Omega(\mathcal{G}) = -\mathrm{Tr}\left[\mathrm{Ln}\mathcal{G} + \frac{1}{2}(\mathcal{G}\mathcal{G}_0^{-1} - 1)\right]. \tag{2.11}$$

To investigate the possibility of the condensate formation in a 2D system it is convenient, in accordance with [9], to pass to a modulus-phase parametrization of the order parameter (cf. (2.3), where $\theta = \mathrm{const}$):

$$\phi(x,y) = \rho(x,y)\exp[-i(\theta(x)+\theta(y))/2] \tag{2.12}$$

with a simultaneous Nambu spinor transformation

$$\Psi^\dagger(x) = \chi^\dagger(x)\exp[i\theta(x)\tau_z/2], \tag{2.13}$$

corersponding to "separation" of bare fermions on their neutral $\chi(x)$ and charge $\theta(x)$ parts (fermi- and bose-ones, respectively).

In the approximation that $\rho(x,y) = \rho = \mathrm{const}$ (see [28]) and spatial $\theta$-fluctuations are small, one can (using (2.12) and (2.13)) obtain the following expressions for $\mathcal{G}$ and $\Omega$ which are defined in (2.9) and (2.11):

$$\begin{aligned}\mathcal{G}^{-1} \;=\; & -\Bigg[\partial_\tau - \tau_z\left(\frac{\nabla^2}{2m}+\mu\right) + i\tau_x\rho \\ & -\tau_z\left(\partial_\tau\theta + \frac{\nabla\theta^2}{2m}\right) - i\left(\frac{\nabla^2\theta}{2m} + \frac{\nabla\theta\nabla}{m}\right)\Bigg] \equiv G^{-1}(\rho) - \Sigma(\partial\theta)\end{aligned} \tag{2.14}$$

and $\Omega = \Omega_{\mathrm{kin}}(\rho,\nabla\theta) + \Omega_{\mathrm{pot}}(\rho)$ with $\Omega_{\mathrm{pot}}(\rho)$ which is defined by (2.11) at $\nabla\theta = 0$ and

$$\begin{aligned}\beta\Omega_{\mathrm{kin}}(\rho,\nabla\theta) = \mathrm{Tr}[G\Sigma - G_0\Sigma + \frac{1}{2}G\Sigma G\Sigma - \frac{1}{2}G_0\Sigma G_0\Sigma \\ +\tau_x\frac{1}{2}i\rho G(G\Sigma + G\Sigma G\Sigma)] = \frac{T}{2}\int_0^\beta \mathrm{d}\tau\int \mathrm{d}^2 r J(\mu,T,\rho(\mu,T))(\nabla\theta)^2,\end{aligned} \tag{2.15}$$

where the effective neutral fermion stiffness

$$\begin{aligned}J(\mu,T,\rho(\mu,T)) = \frac{1}{8\pi}\left(\sqrt{\mu^2+\rho^2} + \mu + 2T\ln\left[1+\exp\left(-\frac{\sqrt{\mu^2+\rho^2}}{T})\right)\right]\right) \\ -\frac{T}{4\pi}\left[1 - \frac{\rho^2}{4T^2}\frac{\partial}{\partial(\rho^2/4T^2)}\right]\int_{-\mu/2T}^{\infty}\mathrm{d}x\frac{x+\mu/2T}{\cosh^2\sqrt{x^2+\rho^2/4T^2}}\end{aligned} \tag{2.16}$$

was introduced.

The evident analogy with the XY-model (two-component order parameter in 2D space) gives an equation for temperature $T_{\mathrm{BKT}}$ of the Berezinskii-Kosterlitz-Thouless phase transition in the system, namely, (see Chapter 15 in book [29]):

$$\frac{\pi}{2}J(\mu,T_{\mathrm{BKT}},\rho(\mu,T_{\mathrm{BKT}})) = T_{\mathrm{BKT}}. \tag{2.17}$$

(Remember that temperature $T_{\mathrm{BKT}}$ plays the role of a critical one in 2D metals.)

The parameters $\mu$ and $\rho$ in (2.17), being dependent on $T$, are still unknown; therefore, it is necessary to obtain equations which connect them with carrier density $n_f$. The first one follows from (2.10) for $\rho \neq 0$:

$$1 = T \sum_{m=-\infty}^{\infty} \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{K(\omega_m)}{\omega_m^2 + \xi^2(\boldsymbol{k}) + \rho^2}, \tag{2.18}$$

where $\omega_n = (2n+1)\pi T$ is the Matsubara fermion frequencies, $\xi(\boldsymbol{k}) = \boldsymbol{k}^2/2m - \mu$ and the Einstein model was used for phonon dispersion: $\omega(\boldsymbol{k}) = \omega_0$. The dependence on parameter $\alpha$ in equation (2.18) is preserved through kernel $K$ (see (7) and (2)).

The second one is defined by the condition $V^{-1}\partial\Omega_{\mathrm{pot}}(\rho)/\partial\mu = -n_f$ ($V$ is the volume of the system) which results in the well-known number equation:

$$\sqrt{\mu^2 + \rho^2} + \mu + 2T \ln\left[1 + \exp\left(-\frac{\sqrt{\mu^2 + \rho^2}}{T}\right)\right] = 2\epsilon_{\mathrm{F}}, \tag{2.19}$$

where the equality $\epsilon_{\mathrm{F}} = \pi n_f/m$ was used; it is correct for free 2D fermions with the quadratic dispersion law.

Thus, we have obtained a self-consistent set of equations (17)-(19) needed to investigate the phase diagram of a 2D metal with an arbitrary carrier density. The last parameter $\rho$ defines such metal superconducting properties.

## 3. Phase diagram of a system

As it can be seen from the previous Section (see also [9,11]), there exist two characteristic temperatures in a system: $T_\rho$, where formally the complete order parameter given by equation (2.5) arises, but its phase is a random quantity, i.e. $\langle\phi(x,y)\rangle = 0$, and another one, $T_{\mathrm{BKT}} < T_\rho$, where the phase of the order parameter becomes ordered, so that $\langle\phi(x,y)\rangle \neq 0$. It must be, however, stressed that temperature $T_\rho$ is not a real critical temperature; it only denotes the characteristic region where the modulus of the order parameter achieves its maximal growth at $T$ decreasing. Unlike $T_\rho$, temperature $T_{\mathrm{BKT}}$ does correspond to the phase transition when correlators $\langle\exp[i\theta(\boldsymbol{r},\tau)]\exp[i\theta(\boldsymbol{r}')]\rangle$ as functions of $|\boldsymbol{r} - \boldsymbol{r}'|$ change their behaviour [29].

Let us find the $n_f$-dependence of temperatures $T_\rho$ and $T_{\mathrm{BKT}}$. “Effective” temperature $T_\rho$ can be estimated from the set (2.16)-(2.19) in the mean-field approximation by putting $\rho = 0$. Another temperature $T_{\mathrm{BKT}}$ follows from the equations (17) and (19).

It is impossible to solve the equations obtained analytically, so we shall do that by numerical calculation. Nevertheless, some asymptotical expressions for these temperatures as functions of $n_f$ can be found in the analytical form:

i) at $\epsilon_{\mathrm{F}}/\omega_0 \to 0$ one obtains $T_{\mathrm{BKT}} = \epsilon_{\mathrm{F}}/8$, and $T_\rho$ satisfies the simple mean-field equation $T_\rho \ln(T_\rho/\epsilon_{\mathrm{F}}) = \omega_0 \exp(-2/\lambda)$, where $\lambda = g_{\mathrm{ph}}^2 m/2\pi$ is the dimensionless coupling constant.

ii) $\epsilon_{\rm F} \to \epsilon_{\rm F}^{\rm cr}$; the critical point $\epsilon_{\rm F} = \epsilon_{F}^{\rm cr}$ (or $n_f = n_f^{\rm cr}$ at which $T_\rho = T_{\rm BKT} = 0$ can be found from equations (17)-(19). This unknown energy is the solution of the equation

$$1 = \frac{\lambda}{2} \ln \frac{(W - \epsilon_{\rm F})[(\alpha-1)\epsilon_{\rm F} + \omega_0]}{(\alpha-1)\epsilon_{\rm F}(W - \epsilon_{\rm F} + \omega_0)} \to \frac{\lambda}{2} \ln \frac{(\alpha-1)\epsilon_{\rm F} + \omega_0}{(\alpha-1)\epsilon_{\rm F}}|_{W\to\infty}. \qquad (3.1)$$

($W$ is a conduction bandwidth determined by the evident condition: $W = \boldsymbol{k}_{\rm B}^2/2m$, where $\boldsymbol{k}_{\rm B}$ is the Brillouin wave vector). In other words, it follows from (2.18) that because the long waves phonons begin to quit the interaction transfer, there exists (at $\alpha > 1$ only), a point where both $T_\rho$ and $T_{BKT}$ temperatures become zero, which means that superconductivity (but not conductivity) is suppressed. Near this point the temperatures have the following behaviour ($W \to \infty$):

$$T_\rho = \omega_0 / \ln \frac{4\epsilon_{\rm F}^{\rm cr}[(\alpha-1)\epsilon_{\rm F}^{\rm cr} + \omega_0]}{\lambda\omega_0(\epsilon_{\rm F}^{cr} - \epsilon_{\rm F})};$$

$$T_{\rm BKT} = \epsilon_{\rm F}^{\rm cr} \left[ \frac{9}{144}(\alpha-1)^4 \left( \frac{\lambda\omega_0(\epsilon_{\rm F}^{\rm cr} - \epsilon_{\rm F})}{\epsilon_{\rm F}^{\rm cr}[(\alpha-1)\epsilon_{\rm F}^{\rm cr} + \omega_0]} \right)^2 \right]^{1/5}.$$

The results of the numerical calculations of equations (17)-(19) are shown in figure 1.

It is seen that due to the long-wave phonon suppression (in fact, supposed to be strong), functions $T_\rho(n_f)$ and $T_{\rm BKT}(n_f)$ quickly acquire a bell-like shape at $n_f$ variation. This "bell" proves to be non-symmetrical, and its height, width and to some extent form depend on $\alpha(> 1)$. In a certain sense, such a result is surprising because, as it is generally accepted, not long-, but short-wave intermediate bosons play the main role in the attraction which appears in the BCS-Eliashberg model due to the electron-phonon (or any other boson) interaction. The sensitivity of superconducting properties of a 2D metal to a long-wave part of the intermediate boson spectrum is rather unusual and allows one to hope that a more accurate consideration also results in the similar effect.

## 4. Conclusions

The existence of two different temperatures in underdoped HTSCs is now a well established fact. They, as it is shown in many papers (see review [11]), are one of the consequences of two-dimensionality of their electronic and magnetic properties when the "ordering" of the order parameter modulus and phase takes place at different temperatures. In a pure 2D system temperature $T_{\rm BKT}$, as it was mentioned, has to be considered as a critical one, and in the region $T_{\rm BKT} < T < T_\rho$ the so-called pseudogap and also a normal phase are formed in underdoped HTSCs. It is destroyed when $T > T_\rho$ or $n_f$ becomes so large (overdoped regime) that the chemical potential of fermions and the Fermi energy are indistinctive ($\mu = \epsilon_{\rm F}$). The aim of this paper is to demonstrate that in a superconducting system with

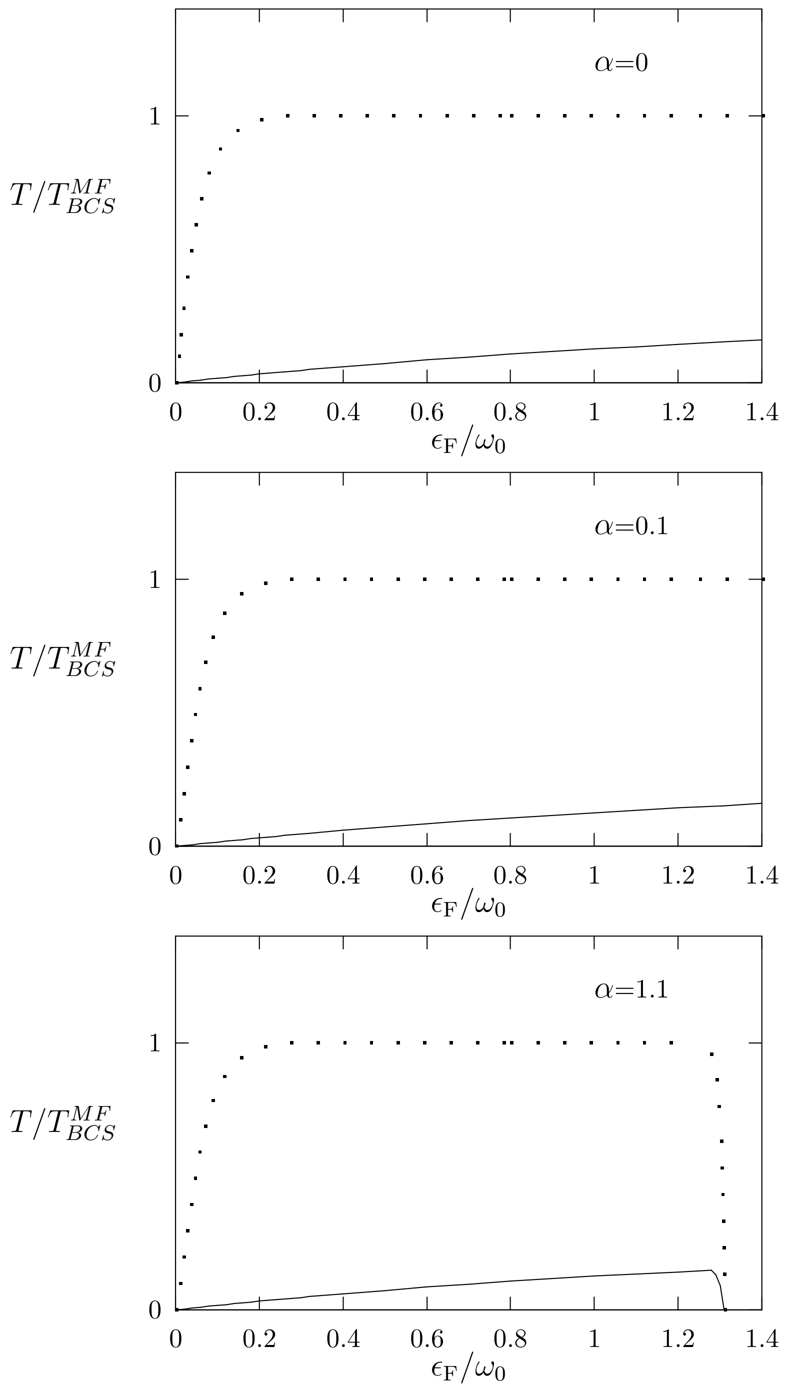


**Figure 1.** The characteristic patterns of the $T - n_f$ phase diagram of 2D metal with coupling constant $\lambda = 1$. Doted and solid lines define temperatures $T_\rho$ and $T_{\rm BKT}$, respectively.

an indirect attraction the role of long-wave bosons (for instance, phonons, spin fluctuations and so on) can be crucial in the region where the standard Fermi liquid theory becomes applicable.

In spite of some qualitative similarity between the experimental and the obtained pictures, the considered model is so simple (and even rough because propagator (2) is, in fact, postulated, while it must contain the boson damping only) that any quantitative use of it is almost senseless. Therefore, it must be generalized by taking into account such HTSC features as: quasi-two-dimensionality (which results in real $T_c$), intermediate boson dispersion $\omega(\boldsymbol{k})$ (as it takes place for spin fluctuations) and damping $\gamma(\boldsymbol{k})$; pairing anisotropy, etc. These problems will be considered separately.

## Acknowledgements

We are glad to dedicate this paper to a well-known Ukrainian theorist, corresponding member of the National Academy of Sciences of Ukraine, Prof. I.V.Stasyuk whose numerious critical remarks are always interesting and helpful for us.

## References

1. Loktev V.M. Mechanisms of high-$T_c$ superconductivity of copper oxides. // Low Temp. Phys., 1996, vol. 22 , No 1, p. 1-32.
2. Shen Z.-X., Dessau D.S. Electronic structure and photoemission studies of late transition - metal oxides - Mott insulators and high-temperature super-conductors. // Phys. Rep., 1995, vol. 253, No 1-3, p. 1-162.
3. Keimer B., Belk N., Birgeneau R.J., Cassano A., Chen C.Y., Greven M., Kastner M.A., Shirane G. Magnetic excitations in pure, lightly doped and weakly metallic $La_2CuO_4$. // Phys. Rev. B, 1992, vol. 46, No 21, p. 14034-14053.
4. Takagi H., Cava R.J., Marezio M., Batlogg B., Krajewski J.J., Peck W.F., Jr., Bordet P., Cox D.E. Disappearance of superconductivity in overdoped $La_{2-x}Sr_xCuO_4$ at structural phase boundary. // Phys. Rev. Lett., 1992, vol. 68, No 25, p. 3777-3780.
5. Nagano T., Tamioka Y., Nakayama Y., Kishio K., Kitazawa K. Bulk superconductivity in both tetragonal and orthorhombic solid solutions of $(La_{1-x}Sr_x)_2CuO_{4-\delta}$ at structural phase boundary. // Phys. Rev. B 1993, vol. 48, No 13, p. 9689-9696.
6. Uemura Y.J., Le L.P., Luke J.M., Sternlieb B,J., Wu W.D., Brewer J.H., Riesman T.M., Seaman C.L., Maple M.B., Ishikawa M., Hinks D.G., Jorgensen J.D., Saito G., Yamchi H. Basic similarities among cuprate, bismthate, organic, Chevrel-Phase and heavy-fermion superconductors shown by penetration- depth measurements. // Phys. Rev. Lett. 1991, vol. 66, No 20, p. 2665-2668.
7. Yamada K., Wakimoto S., Shirane G., Lee C.H., Kastner M.A., Hosoya S., Greven M., Endoh Y., Birgeneau R.J. Direct observation of a magnetic gap in superconducting $La_{1.85}Sr_{0.15}CuO_4$ ($T_c = 37.3$K). // Rev. Lett., 1995, vol. 75, No 8, p. 1626-1629.
8. Emery V., Kivelson S.A. Importance of phase fluctuations in superconductors with small superfluid density. // Nature, 1995, vol. 374, p. 434-437.

9. Gusynin V.P., Loktev V.M., Sharapov S.G. Phase diagram of 2D metal with variable number of carriers. // JETP Lett., 1997, vol. 65, No 2, p. 182-188; On peculiarities of superconducting state formation in 2D metallic systems. // Low Temp. Phys., 1997, vol. 23 , No 8.
10. Abrikosov A.A. Size of BCS ratio in strongly underdoped high-$T_c$ cuprates. // Phys. Rev. B, 1997, vol. 55, No 10, p. R 6149-R 6151.
11. Loktev V.M., Sharapov S.G. Superconducting condensate formation in metallic systems with arbitrary carrier density. // Cond. Mat. Phys., 1997, No 11, p. 131-178.
12. Plakida N.M. High-temperature superconductors. Berlin, Springer Verlag, 1995.
13. Plakida N.M., Yushankhai V.Yu, Stasyuk I.V. On the role of kinematic and exchange interactions in superconducting pairing of electrons in the Hubbard model. // Physica C, 1989, vol. 160, No 1, p. 80-88.
14. Stasyuk I.V., Shvaika A.M., Schachinger E. On the electron spectrum of the Hubbard model including the interaction with loca anharmonic vibrations. // Physica C, 1993, vol. 213, p. 57-70.
15. Stasyuk I.V., Shvaika A.M. Dielectric properties and electron spectrum of the Müller model in HTSC theory. // Acta Phys. Polonica A, 1993, vol. 84, No 2, p. 293-313.
16. Stasyuk I.V., Shvaika A.M., Schachinger E. Electron spectrum of the Hubbard model with coupling to pseudospin degrees of freedom. // Physica B, 1994, vol. 194-196, p. 1965-1966.
17. Friedel J. The high-$T_c$ superconductors: a conservative view. // J. Phys.: Cond. Mat., 1989, vol. 1, No 42, p. 7757-7794.
18. Markiewicz R.S. Van Hove singularities and high-$T_c$ superconductivity: A review. // Int. J. Mod. Phys. B, 1991, vol. 5, No 12, p. 2037-2071.
19. Ginzburg V.L. Superconductivity and superfluidity (what is done and what is not done). // Usp. Fiz. Nauk, 1997, vol. 197, No 4, p. 429-454 (in Russian).
20. Kampf A.P. Magnetic correlations in high-temperature superconductivity. // Phys. Rep., 1994, vol. 249, No 4,5, p. 219-351.
21. Pines D. Understanding high temperature superconductivity: progress and prospects. // CNLS Newsletter, Los Alamos Nat. Lab., 1997, No 138.
22. Pashitskii E.A. Low frequency charge excitations in cuprate metal-oxide compounds. // Fiz. Nizkikh Temp., 1995, vol. 21, No 10, p. 995-1019; No 11, p. 1091-1137 (in Russian).
23. Ivanov M.A., Loktev V.M., Pogorelov Yu.G. Localization of spin excitations and disruption of long-range order in weakly doped $La_2CuO_4$. // JETP, 1992, vol. 74, No 2, p. 317-325.
24. Loktev V.M. About correlation length in magnetic subsystem of high-$T_c$ superconductors. // Supercond.: Physics, Chemistry, Technics, 1991, vol. 4, No 12, p. 2993-2296.
25. Thurston T.R., Birgeneau R.J., Gabbe D.R., Jenssen H.P., Kastner M.A., Picone P.J., Preyer N.W., Axe J.D., Böni P., Shirane G., Sato M., Fukuda K., Shamoto S. Neutron scattering study of soft optical phonons in $La_{2-x}Sr_xCuO_{4-y}$. // Phys. Rev. B, 1989, vol. 39, No 7, p. 4327-4333.
26. Loktev V.M., Pogorelov Yu.G. Metallization and superconductivity in doped metal-oxide layered compounds. // Physica C, 1996, vol. 272, p. 151-160.
27. Cornwell J.M., Jackiw R., Tomboulis E. Effective action for composite operators. // Phys. Rev. D, 1974, vol. 10, No 8, p. 2428-2445.
28. Loktev V.M., Sharapov S.G., Turkowski V.M. Crossover from supefluidity to super-

conductivity in 2D systems with indirect inter-carrier interaction. // J. Phys. Studies, 1997, vol. 1, No 3, p. 431–440.
29. Izyumov Yu.A., Skryabin Yu.N. Statistical mechanics of magnetically ordered systems. Moscow, Nauka, 1987 (in Russian).

## До фазової діаграми “температура -легування” надпровідників з високою критичною температурою

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Отримано 6 березня 1998 р.

Зроблена спроба опису дзвоноподібної залежності критичної температури високотемпературних надпровідників від густини носіїв заряду. Пропонується пояснити її лінійне зростання в області малих густин (недолегований режим) роллю $2D$ флуктуацій фази параметра порядку, які стають меншими при зростанні густини. Подавлення критичної температури в області великих густин носіїв (перелегований режим) пов’язаний з появою (завдяки легуванню) суттєвого затухання довгохвильових бозонів, які в рамках запропонованої моделі визначають механізм непрямого між-ферміонного притягання.

**Ключові слова:** *високотемпературні надпровідники, легування, критична температура, фазова діаграма, псевдограф*

**PACS:** *67.20.+k, 74.20.-z, 74.25.-q, 74.72.-h*