POLARIZATION FEATURES OF ACOUSTIC SPECTRA IN UNIAXIAL AND BIAXIAL NEMATIC LIQUID CRYSTALS

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The results of investigation of uniaxial and biaxial nematic liquid crystals dynamics with molecules of the various forms are presented. These condensed matters possess internal spatial anisotropy and for their adequate description introduction of additional dynamic quantities is necessary. They are vectors of spatial anisotropy and conformational degrees of freedom. Investigation of dynamics of the given condensed matters is based on Hamiltonian formalism in which framework the nonlinear dynamic equations for uniaxial and biaxial nematic liquid crystals are derived. Spectra of collective excitations are obtained and their polarization features are investigated.

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1. INTRODUCTION

Nematic liquid crystals are orientationally ordered anisotropic liquids with spontaneously broken symmetry to rotations in configuration space. Besides, they possess the internal molecular structure capable to be deformed in the process of evolution, which also must be considered at the macroscopic level. Hence, for the description of dynamics of such condensed matters introduction of the additional variables connected both with broken symmetry and with the form and the size of molecules is necessary.

In the given work features of dynamics of uniaxial and biaxial nematic liquid crystals taking into account internal structure are considered, and possibility of distribution of collective excitations and their polarization structure is studied.

2. FEATURES OF DYNAMICS AND POLARIZATION STRUCTURE OF ACOUSTIC SPECTRA OF UNIAXIAL NEMATIC LIQUID CRYSTALS

2.1. Nematics with rod-like molecules

For uniaxial nematics along with usual dynamic variables - densities of mass ρ, momentum π_i and entropy σ, the additional parameter - unit vector of spatial anisotropy (director) $\mathbf{n}(\mathbf{x})$ is introduced [1]. Using Hamiltonian approach of [2, 3] we obtain dynamic equations of uniaxial nematic with rod-like molecules [4]:

$$\begin{gathered}\dot{\rho} = -\nabla_i \pi_i, \quad \dot{\pi}_i = -\nabla_k t_{ik}, \quad \dot{\sigma} = -\nabla_k (\sigma v_k), \\ \dot{l} = -v_s(\mathbf{x}) \nabla_s l(\mathbf{x}) + l(\mathbf{x}) n_i(x) n_j(x) \nabla_j v_i(\mathbf{x}), \\ \dot{n}_j = -v_s \nabla_s n_j + \delta^{\perp}_{ij}(\mathbf{n}) n_k \nabla_k v_i. \end{gathered} \quad (1)$$

Momentum flux density looks like

$$\begin{gathered} t_{ik} = P\delta_{ik} + \frac{\partial \varepsilon}{\partial \pi_k}\pi_i + \frac{\partial \varepsilon}{\partial \nabla_k n_l}\nabla_i n_l + \frac{\partial \varepsilon}{\partial l} n_i n_k l \\ + n_k \delta^{\perp}_{il}(\mathbf{n}) \left(\frac{\partial \varepsilon}{\partial n_l} - \nabla_j \frac{\partial \varepsilon}{\partial \nabla_j n_l} \right). \end{gathered} \quad (2)$$

Here $P \equiv -\varepsilon + \frac{\delta H}{\delta \underline{\zeta}_a}\underline{\zeta}_a + \frac{\partial \varepsilon}{\partial \nabla_l n_i}\nabla_l n_i$ is pressure, $v_i \equiv \pi_k/\rho$ is macroscopic velocity, $\delta^{\perp}_{ik}(\mathbf{n}) = \delta_{ik} - n_i n_k$, ε is energy density, l is length of a molecule.

Linearization of (1) near equilibrium state leads to the system of linear and homogeneous equations

$$D_{ij}(\mathbf{k},\omega)\, \delta v_j(\mathbf{k},\omega) = 0, \quad (3)$$

which have a nontrivial solution for the vanishing of the determinant

$$\det D_{ij} = \omega^6 + \omega^4 I_4 + \omega^2 I_2 = 0, \quad (4)$$

where ω is frequency, $\mathbf{k}$ is wave vector and

$$\begin{gathered} I_4(\mathbf{k}) = -\mathbf{k}^2 c^2 - c^2 \lambda (\mathbf{kn})^2 \le 0, \\ I_2(\mathbf{k}) = c^4 \lambda \left(\mathbf{k}^2 - (\mathbf{kn})^2 \right) (\mathbf{kn})^2 \ge 0. \end{gathered} \quad (5)$$

Here $\lambda \equiv \frac{l^2}{\rho c^2}\frac{\partial^2 \varepsilon}{\partial l^2} > 0$ and c is sound velocity in usual liquid. From (4) it is clear, that in unixial nematic with rod-like molecules the propagation of two acoustic oscillation modes $\omega^2_{\pm}(\mathbf{k}) = c^2_{\pm}(\mathbf{k}) k^2$ is possible corresponding to the first and second sound. The solution with a sigh (+) corresponds to the first sound analogous to that in usual liquid. The solution with a sigh (-) is new branch of excitations caused by conformational degree of freedom - rod-like molecule length. In spherical system of coordinates

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$\mathbf{kn} = k\cos\theta$, where θ is polar angle, hence, velocities $c_\pm$ look like [2]:

$$c_\pm(\theta) = \frac{c}{\sqrt{2}}\left[1 + \lambda\cos^2\theta \pm\left[\left(1+\lambda\cos^2\theta\right)^2 - \lambda\sin^2 2\theta\right]^{1/2}\right]^{1/2}. \tag{6}$$

Calculated angular values of extremum points ($\theta = \pi/4$) for sound velocity c_- coincide with experimental data [5, 6].

Let's consider solutions of (3) corresponding to modes $\omega = kc_\pm$. Expression for $\delta v_j^{(\pm)}(\mathbf{k})$ we are looking for in the form of decomposition on three orthogonal vectors:

$$\delta v_j^{(\pm)}(\mathbf{k}) = k_j \delta v_{||}^{(\pm)}(\mathbf{k}) + [\mathbf{k}\times\mathbf{n}]_j \delta v_{1\perp}^{(\pm)}(\mathbf{k}) + \frac{[[\mathbf{k}\times\mathbf{n}]\times\mathbf{k}]_j}{k}\delta v_{2\perp}^{(\pm)}(\mathbf{k}).$$

From (3) we find, that $\delta v_{1\perp}^{(\pm)}(\mathbf{k}) = 0$. Then

$$\delta v_j^{(\pm)}(\mathbf{k}) = k_j \delta v_{||}^{(\pm)}(\mathbf{k}) + \frac{[[\mathbf{k}\times\mathbf{n}]\times\mathbf{k}]_j}{k}\delta v_{2\perp}^{(\pm)}(\mathbf{k}).$$

Hence, solutions corresponding to the first and second sounds are superposition of longitudinal and transversal components and the relation of these amplitudes has the form:

$$\frac{\delta v_{2\perp}^{(\pm)}}{\delta v_{||}^{(\pm)}} = \frac{-k^5\left(c_\pm^2 - c^2\right) + \lambda c^2 k k_{||}^4}{\lambda c^2 k_{||}^3 k_\perp^2} \equiv f_\pm(\theta). \tag{7}$$

Using (6) we rewrite (7) in terms of polar angle:

$$f_\pm(\theta) = \frac{1+\lambda\cos^2\theta\cos 2\theta}{\lambda\sin 2\theta\sin\theta\cos^2\theta} \mp \frac{\sqrt{\left(1+\lambda\cos^2\theta\right)^2 - \lambda\sin^2 2\theta}}{\lambda\sin 2\theta\sin\theta\cos^2\theta}.$$

At $\lambda << 1$ the relation of amplitudes for the first sound $f_+(\theta)$ becomes simpler:

$$f_+(\theta) = -\frac{\lambda\cos^5\theta}{4\sin^2\theta}.$$

We can conclude, that at $\theta \to 0$ function $f_+(\theta) \to \infty$, hence, at such values of polar angle sound is transversal. At $\theta \to \pi/2$ function $f_+(\theta) \to 0$, hence, at such values of polar angle sound is longitudinal. For the second sound the relation of amplitudes $f_-(\theta)$ looks like

$$f_-(\theta) = \frac{1-\lambda\sin^2\theta\cos 2\theta}{\lambda\cos\theta\sin^4\theta}. \tag{8}$$

We can conclude, that at $\theta \to 0$ and $\theta \to \pi/2$ function $f_-(\theta) \to \infty$, hence, at such values of polar angle sound is transversal.

2.2. Nematics with disc-like molecules

The direction of orientation of such liquid crystals is defined by unit vector of a normal to molecule plane. Studying of dynamic behavior of uniaxial nematic with disc-like molecules we will carry out similar to earlier considered case of uniaxial nematic with rod-like molecules. Dynamic equations of uniaxial nematic with disc-like molecules are as follows [4]:

$$\begin{aligned} &\dot\rho = -\nabla_i\pi_i, \quad \dot\pi_i = -\nabla_k t_{ik}, \quad \dot\sigma = -\nabla_k(\sigma v_k), \\ &\dot d = -v_s\nabla_s d - d\delta_{lk}^\perp(\mathbf{n})\nabla_k v_l, \\ &\dot n_j = -v_s\nabla_s n_j - n_i\delta_{j\lambda}^\perp(\mathbf{n})\nabla_\lambda v_i, \end{aligned} \tag{9}$$

$$t_{ik} = P\delta_{ik} + \tfrac{\partial\varepsilon}{\partial\pi_k}\pi_i + \tfrac{\partial\varepsilon}{\partial\nabla_k n_l}\nabla_i n_l + \tfrac{\partial\varepsilon}{\partial d}d\delta_{lk}^\perp(\mathbf{n}) + n_k\delta_{il}^\perp(\mathbf{n})\left(\tfrac{\partial\varepsilon}{\partial n_l} - \nabla_j\tfrac{\partial\varepsilon}{\partial\nabla_j n_l}\right).$$

Here d is molecule diameter. Dispersion equation has the form (4), where coefficients $I_a,\ a = 2.4$ are as follows:

$$\begin{aligned} &I_4(\mathbf{k}) = -\mathbf{k}^2c^2 - c^2\lambda\left(\mathbf{k}^2 - (\mathbf{kn})^2\right) < 0, \\ &I_2(\mathbf{k}) = c^4\lambda\left(\mathbf{k}^2 - (\mathbf{kn})^2\right)(\mathbf{kn})^2 > 0, \end{aligned} \tag{10}$$

where $\lambda = \frac{d^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial d^2} > 0$. From here we come to acoustic spectra $\omega_\pm^2(\mathbf{k}) = c_\pm^2(\mathbf{k})k^2$. In this case also two anisotropic velocities of acoustic waves exist [2]:

$$c_\pm(\theta) = \frac{c}{\sqrt{2}}\left[1 + \lambda\sin^2\theta \pm \left[\left(1+\lambda\sin^2\theta\right)^2 - \lambda\sin^2 2\theta\right]^{1/2}\right]^{1/2}. \tag{11}$$

Polarization structure of the received spectra of collective excitations looks like

$$\delta v_j^{(\pm)}(\mathbf{k}) = k_j\delta v_{||}^{(\pm)} + \frac{[[\mathbf{k}\times\mathbf{n}]\times\mathbf{k}]_j}{k}\delta v_{2\perp}^{(\pm)},$$

and relation of amplitudes has the form

$$\frac{\delta v_{2\perp}^{(\pm)}(\mathbf{k})}{\delta v_{||}^{(\pm)}(\mathbf{k})} = \frac{-k^5\left(c_\pm^2 - c^2\right) + \lambda c^2 k k_\perp^4}{\lambda c^2 k_{||} k_\perp^4} \equiv g_\pm(\theta). \tag{12}$$

Using (11) we rewrite (12) in terms of polar angle:

$$g_\pm(\theta) = \frac{1}{\lambda\sin 2\theta\sin^3\theta}\left(1 - \lambda\sin^2\theta\cos 2\theta\right) \mp \frac{\sqrt{\left(1+\lambda\sin^2\theta\right)^2 - \lambda\sin^2 2\theta}}{\lambda\sin 2\theta\sin^3\theta}.$$

At $\lambda << 1$ the relation of amplitudes for the first sound $g_+(\theta)$ becomes simpler:

$$g_+(\theta) = -\frac{\lambda\sin^4\theta}{4\cos\theta}.$$

We can conclude, that at $\theta \to 0$ function $g_+(\theta) \to 0$, hence, in this case sound is longitudinal. At $\theta \to \pi/2$ function $g_+(\theta) \to \infty$, hence, in this case sound is transversal. At $\lambda << 1$ expression for c_- does not depend on molecule form, so the relation (8) is identical for uniaxial nematics with rod-like and disc-like molecules.

3. FEATURES OF DYNAMICS AND POLARIZATION STRUCTURE OF ACOUSTIC SPECTRA OF BIAXIAL NEMATIC LIQUID CRYSTAL

3.1. Nematics with ellipsoidal molecules

In the case of biaxial nematic with ellipsoidal molecules the set of thermodynamic variables contains additionally two unit and orthogonal vectors of spatial anisotropy $\mathbf{n}(\mathbf{x}),\mathbf{m}(\mathbf{x})$ and three conformational parameters $u(\mathbf{x}), v(\mathbf{x}), p(\mathbf{x})$ describing sizes of long and short molecule axes and an angle between them. Acting further similarly to previously considered case of uniaxial nematics, we obtain dynamic equations of biaxial nematic with ellipsoidal molecules [4]:

$$
\begin{aligned}
&\dot{\rho} = -\nabla_i \pi_i, \quad \dot{\pi}_i = -\nabla_k t_{ik}, \quad \dot{\sigma} = -\nabla_k(\sigma v_k),\\
&\dot{n}_j(\mathbf{x}) = -v_s(\mathbf{x})\nabla_s n_j(\mathbf{x}) - F_{i\lambda j}(\mathbf{x})\nabla_\lambda v_i(\mathbf{x}),\\
&\dot{m}_j(\mathbf{x}) = -v_s(\mathbf{x})\nabla_s m_j(\mathbf{x}) - G_{i\lambda j}(\mathbf{x})\nabla_\lambda v_i(\mathbf{x}),\\
&\dot{u}(\mathbf{x}) = -v_i(\mathbf{x})\nabla_i u(\mathbf{x}) - F_{ij}(\mathbf{x})\nabla_j v_i(\mathbf{x}),\\
&\dot{v}(\mathbf{x}) = -v_i(\mathbf{x})\nabla_i v(\mathbf{x}) - G_{ij}(\mathbf{x})\nabla_j v_i(\mathbf{x}),\\
&\dot{p}(\mathbf{x}) = -v_s(\mathbf{x})\nabla_s p(\mathbf{x}) - H_{ij}(\mathbf{x})\nabla_i v_j(\mathbf{x}), \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
t_{ik} = {} & P\delta_{ik} + \tfrac{\partial\varepsilon}{\partial\pi_k}\pi_i + \tfrac{\partial\varepsilon}{\partial\nabla_k n_l}\nabla_i n_l + \tfrac{\partial\varepsilon}{\partial\nabla_k m_l}\nabla_i m_l +\\
& + \tfrac{\partial\varepsilon}{\partial u}F_{ik} + \tfrac{\partial\varepsilon}{\partial v}G_{ik} + \tfrac{\partial\varepsilon}{\partial p}H_{ik} +\\
& + F_{ikl}\left(\tfrac{\partial\varepsilon}{\partial n_l} - \nabla_j \tfrac{\partial\varepsilon}{\partial\nabla_j n_l}\right) + G_{ikl}\left(\tfrac{\partial\varepsilon}{\partial m_l} - \nabla_j \tfrac{\partial\varepsilon}{\partial\nabla_j m_l}\right),
\end{aligned}
$$

where $P \equiv -\varepsilon + \frac{\delta H}{\delta\zeta_a}\zeta_a + \frac{\partial\varepsilon}{\partial\nabla_l n_i}\nabla_l n_i + \frac{\partial\varepsilon}{\partial\nabla_l m_i}\nabla_l m_i$ is pressure, F_{ij}, G_{ij}, H_{ij} and $F_{ijk}, G_{ijk}, H_{ijk}$ are some functions of $\mathbf{n}(\mathbf{x}),\mathbf{m}(\mathbf{x})$ and $u(\mathbf{x}), v(\mathbf{x}), p(\mathbf{x})$. Linearization of (13) near equilibrium state leads to the system of linear and homogeneous equations

$$\delta v_j(\mathbf{k},\omega) D_{ij}(\mathbf{k},\omega) = 0. \qquad (14)$$

Condition for the existence of a nontrivial solution of (14) is the vanishing of the determinant $\det\hat{\mathbf{D}} = \omega^6 + \omega^4 I_4 + \omega^2 I_2 + I_0 = 0$, where coefficients $I_a, a = 0,2,4$ are some functions of $\mathbf{k},\mathbf{F},\mathbf{G},\mathbf{H}$ and $\mathbf{F},\mathbf{G},\mathbf{H}$ are some functions of $\mathbf{n}(\mathbf{x}),\mathbf{m}(\mathbf{x}),\mathbf{k}$ and parameters λ_α, $\alpha = 1,2,3$:

$$\lambda_1 \equiv \frac{u^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial u^2} > 0, \quad \lambda_2 \equiv \frac{v^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial v^2} > 0,$$

$$\lambda_3 \equiv \frac{p^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial p^2} > 0.$$

As a result we come to bicubic dispersion equation

$$\omega^6 + I_4(\mathbf{k},\theta,\varphi)\omega^4 + I_2(\mathbf{k},\theta,\varphi)\omega^2 + I_0(\mathbf{k},\theta,\varphi) = 0. \qquad (15)$$

From (15) it is clear, that in biaxial nematic with ellipsoidal molecules in general case propagation of three acoustic oscillation modes $\omega^2_{1,2,3}(\mathbf{k}) = c^2_{1,2,3}(\mathbf{k})k^2$ is possible corresponding to the first, second and third sounds. Detailed analysis of the obtained spectra is given in [4].

Let's consider solutions of (14) corresponding to modes $\omega^2_{1,2,3} \equiv c^2_{1,2,3}(\theta,\varphi)k^2$. Expression for $\delta v_j^{(1,2,3)}(\mathbf{k})$ we are looking for in the form of decomposition on three orthogonal vectors:

$$
\begin{aligned}
\delta v_j^{(1,2,3)}(\mathbf{k}) = {} & k_j \delta v_{||}^{(1,2,3)}(\mathbf{k}) + [\mathbf{k}\times\mathbf{l}]_j \delta v_{1\perp}^{(1,2,3)}(\mathbf{k}) +\\
& + \tfrac{[[\mathbf{k}\times\mathbf{l}]\times\mathbf{k}]_j}{k}\delta v_{2\perp}^{(1,2,3,)}(\mathbf{k}).
\end{aligned}
$$

From (14) we find that these solutions are superposition of one longitudinal and two transversal components. It can be shown, that at $\theta \to 0$ sounds are cross-polarized with components $\delta v_{1\perp}^{(1,2,3)}(\mathbf{k}), \delta v_{2\perp}^{(1,2,3,)}(\mathbf{k})$; at $\theta \to \pi/2$ sounds are cross-polarized with component $\delta v_{2\perp}^{(1,2,3,)}(\mathbf{k})$.

3.2. Nematics with discoidal molecules

Dynamic equations of biaxial nematic with discoidal molecules are as follows [4]:

$$
\begin{aligned}
&\dot{\rho} = -\nabla_i \pi_i, \quad \dot{\pi}_i = -\nabla_k t_{ik}, \quad \dot{\sigma} = -\nabla_k(\sigma v_k),\\
&\dot{n}_j(\mathbf{x}) = -v_s(\mathbf{x})\nabla_s n_j(\mathbf{x}) - f_{i\lambda j}(\mathbf{x})\nabla_\lambda v_i(\mathbf{x}),\\
&\dot{m}_j(\mathbf{x}) = -v_s(\mathbf{x})\nabla_s m_j(\mathbf{x}) - g_{i\lambda j}(\mathbf{x})\nabla_\lambda v_i(\mathbf{x}),\\
&\dot{q}(\mathbf{x}) = -v_i(\mathbf{x})\nabla_i q(\mathbf{x}) - f_{ij}(\mathbf{x})\nabla_j v_i(\mathbf{x}),\\
&\dot{t}(\mathbf{x}) = -v_i(\mathbf{x})\nabla_i t(\mathbf{x}) - g_{ij}(\mathbf{x})\nabla_j v_i(\mathbf{x}),\\
&\dot{p}(\mathbf{x}) = -v_s(\mathbf{x})\nabla_s p(\mathbf{x}) - h_{ij}(\mathbf{x})\nabla_i v_j(\mathbf{x}), \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
t_{ik} = {} & P\delta_{ik} + \tfrac{\partial\varepsilon}{\partial\pi_k}\pi_i + \tfrac{\partial\varepsilon}{\partial\nabla_k n_\lambda}\nabla_i n_\lambda + \tfrac{\partial\varepsilon}{\partial\nabla_k m_\lambda}\nabla_i m_\lambda\\
& + \tfrac{\partial\varepsilon}{\partial q}f_{ik} + \tfrac{\partial\varepsilon}{\partial t}g_{ik} + \tfrac{\partial\varepsilon}{\partial p}h_{ik} + f_{ik\lambda}\left(\tfrac{\partial\varepsilon}{\partial n_\lambda} - \nabla_j \tfrac{\partial\varepsilon}{\partial\nabla_j n_\lambda}\right)\\
& + g_{ik\lambda}\left(\tfrac{\partial\varepsilon}{\partial m_\lambda} - \nabla_j \tfrac{\partial\varepsilon}{\partial\nabla_j m_\lambda}\right).
\end{aligned}
$$

Here $\mathbf{n}(\mathbf{x}),\mathbf{m}(\mathbf{x})$ are unit and orthogonal vectors of spatial anisotropy, $q(\mathbf{x}), t(\mathbf{x}), p(\mathbf{x})$ are conformational parameters describing sizes of long and short molecule axes and an angle between them, f_{ij}, g_{ij}, h_{ij} and $f_{ijk}, g_{ijk}, h_{ijk}$ are some functions of $\mathbf{n}(\mathbf{x}),\mathbf{m}(\mathbf{x})$ and $q(\mathbf{x}), t(\mathbf{x}), p(\mathbf{x})$. Linearization of (16) near equilibrium state leads to the system of linear and homogeneous equations

$$\delta v_j(\mathbf{k},\omega) D_{ij}(\mathbf{k},\omega) = 0. \qquad (17)$$

Condition for the existence of a nontrivial solution of (17) is the vanishing of the determinant $\det\hat{\mathbf{D}} = \omega^6 + \omega^4 I_4 + \omega^2 I_2 + I_0 = 0$, where coefficients $I_a, a = 0,2,4$ are some functions of $\mathbf{k},\mathbf{f},\mathbf{g},\mathbf{h}$ and $\mathbf{f},\mathbf{g},\mathbf{h}$ are some functions of $\mathbf{n}(\mathbf{x}),\mathbf{m}(\mathbf{x}),\mathbf{k}$ and parameters λ_α, $\alpha = 1,2,3$:

$$\lambda_1 \equiv \frac{q^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial q^2} > 0, \quad \lambda_2 \equiv \frac{t^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial t^2} > 0,$$

$$\lambda_3 \equiv \frac{p^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial p^2} > 0.$$

As a result we come to bicubic dispersion equation

$$\omega^6 + I_4(\mathbf{k},\theta,\varphi)\omega^4 + I_2(\mathbf{k},\theta,\varphi)\omega^2 + I_0(\mathbf{k},\theta,\varphi) = 0. \qquad (18)$$

From (18) it is clear, that in biaxial nematic with discoidal molecules in general case propagation of three acoustic oscillation modes $\omega^2_{1,2,3}(\mathbf{k}) = c^2_{1,2,3}(\mathbf{k})k^2$ also is possible corresponding to the first, second and third sounds. Detailed analysis of the obtained spectra is given in [4].

Let's consider solutions of (17) corresponding to modes$\omega^2_{1,2,3} \equiv c^2_{1,2,3}(\theta,\varphi)\,k^2$. Expression for $\delta v_j^{(1,2,3)}(\mathbf{k})$ we are looking for in the form of decomposition on three orthogonal vectors:

$$\delta v_j^{(1,2,3)}(\mathbf{k}) = k_j \delta v_{||}^{(1,2,3)}(\mathbf{k}) + [\mathbf{k}\times\mathbf{l}]_j \delta v_{1\perp}^{(1,2,3)}(\mathbf{k}) + \frac{[[\mathbf{k}\times\mathbf{l}]\times\mathbf{k}]_j}{k}\delta v_{2\perp}^{(1,2,3,)}(\mathbf{k})\,.$$

From (17) we find that like in the previous case of biaxial nematic with ellipsoidal molecules these solutions are superposition of one longitudinal and two transversal components. It can be shown, that at $\theta \to 0$ sounds are cross-polarized with components $\delta v_{1\perp}^{(1,2,3)}(\mathbf{k})\,, \delta v_{2\perp}^{(1,2,3,)}(\mathbf{k})$; at $\theta \to \pi/2$ sounds are cross-polarized with component $\delta v_{2\perp}^{(1,2,3,)}(\mathbf{k})$.

4. CONCLUSIONS

On the basis of Hamiltonian approach the dynamic theory of uniaxial and biaxial nematic liquid crystals with molecules of different geometry is constructed. For all types of liquid crystals nonlinear dynamic equations are derived, acoustic spectra of collective excitations are received and their polarization structure is studied. It is shown that in the case of uniaxial nematics the first sound is mostly longitudinal and the second one is mostly transversal; for biaxial nematics the first, second and third sounds mostly possess transversal polarization.

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ПОЛЯРИЗАЦИОННЫЕ ОСОБЕННОСТИ АКУСТИЧЕСКИХ СПЕКТРОВ В ОДНООСНЫХ И ДВУХОСНЫХ НЕМАТИЧЕСКИХ ЖИДКИХ КРИСТАЛЛАХ

М.Ю. Ковалевский, Л.В. Логвинова, В.Т. Мацкевич

Представлены результаты исследования динамики одноосных и двухосных нематических жидких кристаллов с молекулами различной формы. Эти конденсированные среды обладают внутренней пространственной анизотропией, и для их адекватного описания необходимо введение дополнительных динамических переменных. Ими являются векторы пространственной анизотропии и конформационные степени свободы. Исследование динамики данных конденсированных сред базируется на гамильтоновом формализме, в рамках которого выведены нелинейные уравнения динамики для одноосных и двухосных нематических жидких кристаллов. Получены спектры коллективных возбуждений и исследованы их поляризационные особенности.

ПОЛЯРІЗАЦІЙНІ ОСОБЛИВОСТІ АКУСТИЧНИХ СПЕКТРІВ В ОДНОВІСНИХ ТА ДВОВІСНИХ НЕМАТИЧНИХ РІДКИХ КРИСТАЛАХ

М.Ю. Ковалевський, Л.В. Логвінова, В.Т. Мацкевич

Представлено результати досліджень динаміки одновісних та двовісних нематичних рідких кристалів з молекулами різної форми. Ці конденсовані середовища мають внутрішню просторову анізотропію, тож для їх адекватного опису необхідно введення додаткових динамічних змінних. Ними є вектори просторової анізотропії та конформаційні ступені свободи. Дослідження динаміки даних конденсованих середовищ базується на гамільтоновому формалізмі, у рамках якого виведені нелінійні рівняння динаміки для одновісних та двовісних нематичних рідких кристалів. Отримано спектри колективних збуджень та досліджено їх поляризаційні особливості.