

# THE DESIGN OF SUPPORT STRUCTURES FOR ELEVATED CENTRIFUGAL MACHINERY

by

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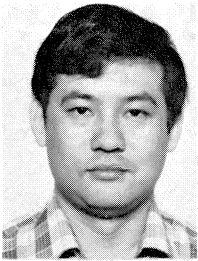
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## ABSTRACT

This paper briefly discusses the computer aided analysis and design of elevated turbomachinery support structures. The paper is focused upon structural behavior and is presented from the civil engineer's viewpoint. The major topics are the steps associated with a typical structural design. The history of elevated turbomachinery support structure designs based on past experience is also presented. Justifications to perform a rigorous computer aided analysis are given along with a succinct summary of the mathematical theory that enables the engineer to solve the large complex problem that represents the elevated turbomachinery support structure. Attributes of the computer aided technique are explained by presenting excerpts from typical example problems. Included in the presentation are superstructure and foundation drawings, dynamic load derivation or acquisition, mathematical modeling techniques, kinematic condensation, a sampling of computer output, and means of interpreting the computer results. This sampling includes computer output and graphical representation of mode shapes accomplished by computer techniques. Finally, this paper offers the civil engineer's view of possible improvements of cooperation between structural designers and manufacturers to better achieve the sound structural behavior of elevated turbomachinery support structures.

## INTRODUCTION

The support structure for a centrifugal train usually consists of massive concrete members. This structure is subjected to dynamic forces and high frequency vibrations. Excessive vibration of the structure could be troublesome for personnel or cause damage to the machine. To ensure trouble-free operation of the machine, it is essential that the design of such a support structure considers both the static and dynamic effects. Because of the nature of the structure and the degree of refinement required, the dynamic analysis of these structures is very complex and the tuning of the structure is difficult. However, these large turbomachines perform essential functions in the overall operation of petrochemical facilities, and are associated with tremendous initial investment. Failure of one of these machines would mean the shutdown of operation for a major

portion of a plant. Furthermore, tuning a structure after erection is usually difficult because the alteration of some of its dynamic characteristics requires gross changes in geometry and mass. In view of the important role these machines play in the overall operation of an installation, it is obvious that the rigorous dynamic analysis of the support structure is fully justified and the most advanced methods should be used.

This paper presents the dynamic analysis methods employed for elevated turbomachine support structures. Included in the presentation are: a history of design methods used, a brief review of the theory of structural dynamics, schemes of modeling these structures for computer programs, samples of computer outputs, and means of interpreting the computed results. Finally, this paper gives the civil engineer's view of possible improvements in coordination of structural designers and manufacturers to ensure sound structures for centrifugal trains.

## HISTORY OF DESIGN METHODS USED

In the early 1960's, turbomachines were typically high speed and low horsepower. During this era, it was general practice to use a "Rule of Thumb Method" to account for the dynamic effects in designing the support structure. A few of these rules were: to maintain certain ratios between the width of foundation and the height of structure, and between the weight of structure and the weight of machine; to reduce allowable stresses of the material normally used for static loads; to impose a certain percentage of the machine weight as a horizontal or vertical static load to simulate the dynamic effects. Some of these rules are still utilized today for initial sizing of the support structure. Toward the end of the 1960's, attention was given to more rigorous dynamic analysis methods, even though none of the compressor support structures had experienced any problems. From the late 1960's until the early 1970's, the design approach which was utilized was the one suggested by the ASME paper entitled "Foundations for High Speed Machines" by J. S. Sohre [4]. This paper provides an approximate design method for elevated support structures for centrifugal machines. This method simplifies a complex problem involving many degrees of freedom to a model with a few degrees of freedom, so that some of the dynamic design information can be calculated by hand. Although this method is less crude than the earlier approaches and has yielded satisfactory foundations, questions were raised about the accuracy of results obtained since some of the initial assumptions of the design method had been violated.

In recent years, analytical methods utilizing the digital computer have become increasingly sophisticated. Since the early 1970's, computer programs to predict the dynamic behavior of support structures for the slow speed centrifugal trains have come to the forefront. This is the topic of this paper.

## BRIEF REVIEW OF THEORY OF STRUCTURAL DYNAMICS

In order to better understand the computer analysis method, let us briefly review the theory of structural dynamics. The dynamic analysis of a support structure should include the determination of mode shapes and natural frequencies, as well as the predictions of the amplitude of vibration in the structure due to the dynamic loads.

The mode shape, sometimes called the characteristic shape, is the deflected shape which the structure will tend to take when it is excited at the corresponding natural frequency.

Each mode shape is presented in terms of relative values of displacement.

For an elastic structure having multiple degrees of freedom, the differential equation of free vibration motion can be represented in matrix form as:

$$M \ddot{y} + C \dot{y} + K y = 0 \quad (1)$$

Where:

vectors  $y$ ,  $\dot{y}$ ,  $\ddot{y}$  are the displacement, velocity and acceleration of the system, respectively.

matrices  $M$ ,  $C$ ,  $K$  are the mass, damping coefficient and stiffness of the system, respectively.

This equation can be reduced to the "eigenvalue problem" form by neglecting the damping term and assuming the vibration in harmonic mode. Ignoring damping should yield less than 1% error in the results. The solutions of Equation 1 result in frequencies and mode shapes of the system.

The forced vibration comes about when a harmonic force is imposed on the structure. The general differential equation for the forced vibration motion is:

$$M \ddot{y} + C \dot{y} + K y = P \sin(\omega t + \phi) \quad (2)$$

Where:

$y$ ,  $\dot{y}$ ,  $\ddot{y}$ ,  $M$ ,  $C$ ,  $K$  are the same as in Equation (1).

$P$  and  $\omega$  are the amplitude and frequency of the harmonic load.

$t$  is time, and  $\phi$  is phase angle.

There are many techniques [3] for solving this equation, and its solutions are the deformation, velocity and acceleration of the system. This paper uses the computed deformation to measure the soundness of the structure, since compressor/turbine requirements for support structures are generally given in terms of maximum deflection. Alternatively, the velocity and acceleration could be used for this purpose. Damping effects should be considered in solving Equation 2.

The support structure for a centrifugal machine generally has a high number of degrees of freedom. It becomes unmanageable to solve Equation 2 for such a system by hand, but can be carried out by utilizing computer aided techniques.

## DYNAMIC ANALYSIS PROCEDURES

Having reviewed the basic theory of dynamic analysis, we now present the computer-aided technique currently utilized to analyze support structures for slow speed centrifugal machines. In general, the analysis proceeds in three steps:

1. Model the structure as a system of points and lines.
2. Perform the modal analysis to determine natural frequencies and mode shapes of the structure.
3. Perform the harmonic analysis to predict the amplitude of vibration in the structure due to dynamic loads.

A flow diagram of this analysis technique is shown in Figure 1, and details of each procedure are given in the following sections.

### *Modeling of the Structure*

The modeling of the structure is a very important step since the skill with which this is accomplished determines the accuracy of the total analysis. It should be noted that one never analyzes a structure, but only the mathematical model which represents it.

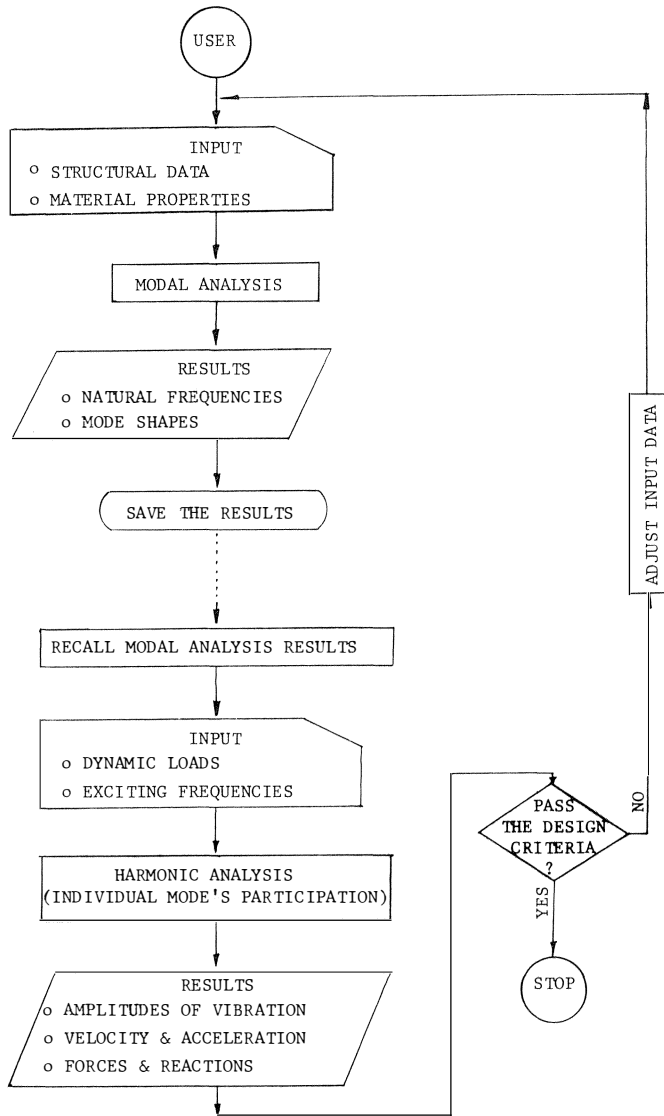


Figure 1. Flow Diagram of the Dynamic Analysis Technique.

Prior to modeling the structure, engineers must determine the general structural configuration which can be estimated by using layouts such as the height of support points, and cutouts and blockouts for surface condensers, nozzles, lube lines, etc. To model a structure, the beams and columns are usually idealized as lines, while the slabs and walls are simulated by finite elements. The member lines and/or finite elements are connected at the nodal points which represent the centers of joint blocks. In order to incorporate the size effects of joint blocks in the analysis, it is advisable to represent the member properties for the in-the-joint portion as being rigid. Since the structure is supported on a foundation mat, it is also necessary to include the flexibility of the soil or piles beneath the mat. This will require many nodal points on the mat to describe the interaction between the soil and/or piles and the foundation mat.

Figure 2 shows a typical configuration of an elevated turbomachine structure supported on a concrete mat, and Figure 3 illustrates its computer model. Other structural data required for computer analysis include the material properties (E, G,

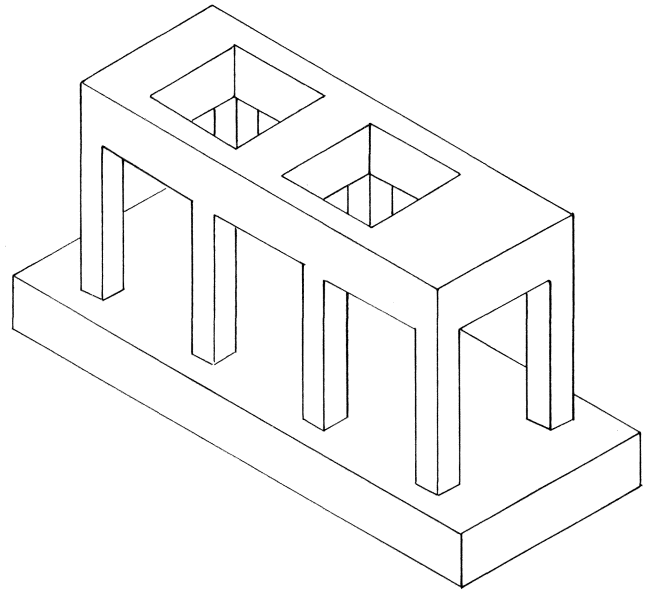


Figure 2. Isometrics of a Typical Support Structure for Turbomachinery.

density, . . .), cross-sectional properties of the members (moment of inertia, shear area, cross section area), support conditions, and the external mass imposed on the structure.

At each nodal point, the structure may be given as many as six degrees of freedom — three translation components and three rotation components. If the model contains a large number of nodal points, the computational and interpretative efforts will be very large. Thus an extremely accurate model could be constructed but would contain so many nodal points that it would be impractical because of the vast computer resources required, and therefore, high computer cost. A special technique, called condensation, can be used to reduce the total number of degrees of freedom. This involves a particular coor-

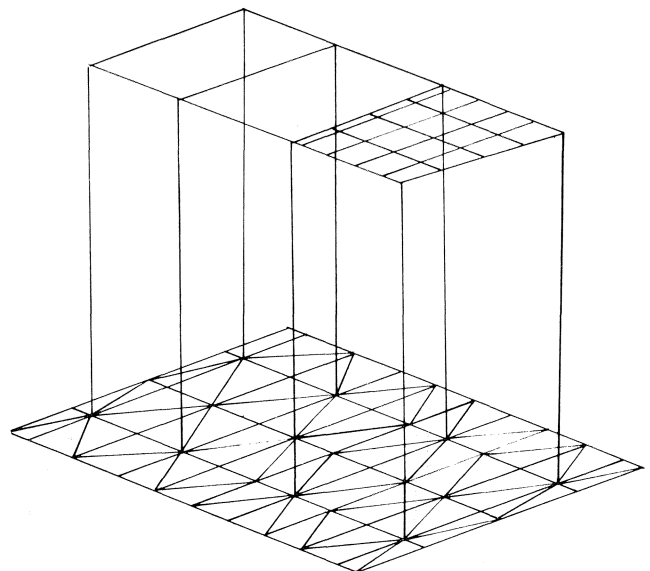


Figure 3. Computer Model.

dinate transformation which results in a condensed set of coordinates or equations. Its effect on the modal analysis will be discussed in the next section.

#### Modal Analysis

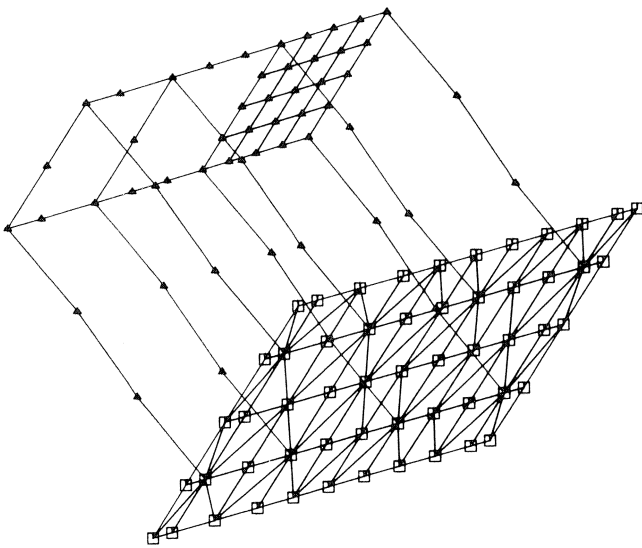
The next step is to determine the natural frequencies and the mode shapes by performing modal analysis.

Results from the modal analysis of a typical problem are presented graphically in Figures 4 and 5. Figure 4 shows a simple mode shape, while Figure 5 illustrates a complex mode shape. The simple modes are simple oscillations such as translation, rocking or torsional movement either of the entire structure or individual elements, and the higher order mode shapes are more complex configurations.

It has been observed (and is in agreement with the theory) that the simple mode shapes are much more excitable than the more complex mode shapes. In other words, the simple modes would contribute more to the total vibration of a support structure than the more complex modes. Therefore, it is necessary to avoid the frequencies of the simple modes in the operational speed ranges of the machine by adjusting the member properties through geometric changes. Some more complex modes will usually lie within the operational speed range and should be adjusted only if they prove to cause excessive vibration. The condensation technique discussed in the previous section will eliminate calculations of the independent mode shapes at the condensed nodal points and of the specified directions. Thus, this technique must be handled with care even though it is useful to reduce the total degrees of freedom of the structural model.

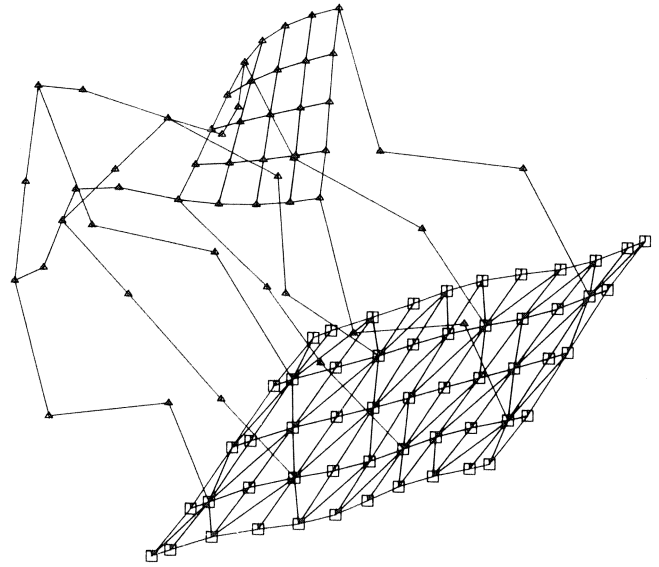
#### Harmonic Analysis

Centrifugal machines can impose continuous forces at the rotor bearings. This out of balance force is assumed to be harmonic and its magnitude is usually furnished by the manufacturer. The amplitude of vibration of the structure in response



MODE 2

Figure 4. Simple Mode Shape.



MODE 57

Figure 5. Complex Mode Shape.

to this force can be obtained by performing harmonic analysis. Input data to this analysis are: machine speed, magnitude and direction of the unbalanced forces, location of forces, critical damping ratio, and the results of modal analysis. The transverse forces are applied in two orthogonal directions to represent the rotating vector of the unbalanced force. A damping ratio of two to four percent of critical damping for concrete structures is used. It is very important to include the damping ratio in predicting the amplitude of vibration. The effect of damping on the structural dynamic behavior is depicted in Figure 6. The x-axis of this figure is the ratio of exciting frequency to a particular natural frequency, while the y-axis represents the ratio of the dynamic vibration due to unbalanced force at a given frequency to the deflection that would be obtained if the dynamic force were applied as a static load. Theoretically, the structure with no damping would oscillate at an infinite amplitude if an harmonic load were imposed in the proper direction and which matched the natural frequency of a particular mode. Inspection of Figure 6 indicates that damping can drastically reduce the amplitude of vibration.

Ordinarily the machine will not operate at a constant speed, and some natural frequencies of the support structure will fall within the operational speed range of the machine. Hence, the practice has been to perform a harmonic analysis by imposing the dynamic loads on the structure at the nominal operating speed, as well as at each of the natural frequencies which are within normal operational speed range.

Results from the harmonic analysis consist of the amplitude of vibration, velocity and acceleration, forces and reactions.

#### Interpretation of Results

The computer analysis will result in a large amount of output. Table 1 illustrates a representative computer output of dynamic deflections and the corresponding phase angles.

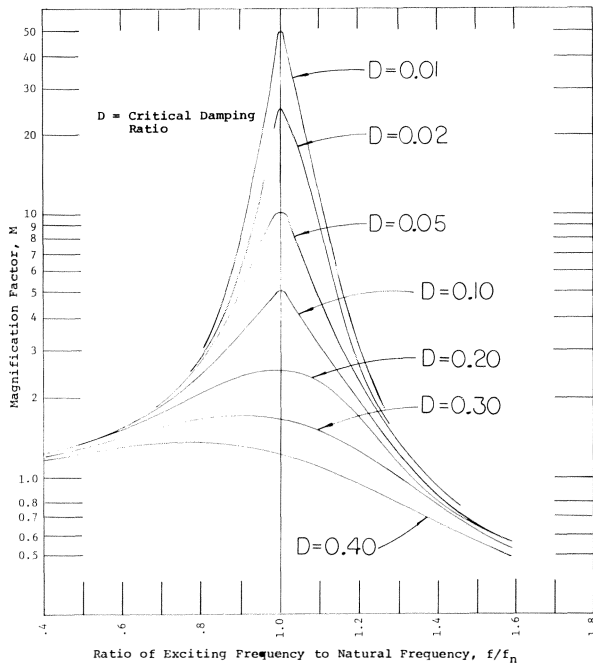


Figure 6. Effect of Damping on the Structural Dynamic Behavior.

These dynamic deflections are obtained by a technique known as modal superposition. Table 2 shows the participation of each mode that yields the deflection of Table 1.

The response spectrum of the maximum dynamic deflections for a given point in the structure is then produced for frequencies within the operating speed range. A typical example is shown in Figure 7. The spectrum is analyzed and compared to the design criterion. If the response is greater than the design criterion, the mode shapes causing the high response are reviewed from the above output. The member sizes are then adjusted to reduce the response and a new modal and harmonic analysis is performed to verify the effect of the adjustments.

Design Criteria

The main design criteria for turbomachine support structure are the limits of allowable movement the turbomachine can tolerate without damage or other duress. The effects of movement at various frequencies upon human beings, machines, and the structure can be found in numerous literature articles [1,3]. One criterion is shown in Figure 8 [1]. If one picks the speed of 3000 RPM, and adjusts for centrifugal machine by a "service factor" of 1.6, it can be seen that 0.37 mil (= 0.23x1.6) single amplitude vibration is approximately the upper limit for a "no fault" machine and is a safe allowable deflection criterion for continuous operation of most turbomachine structures. It is important to point out that this deflection criterion is a limiting factor for good machine operation, but not for the safety of the support structure. As a point

TABLE 1. DYNAMIC DEFLECTIONS AND CORRESPONDING PHASE ANGLES (Sample Computer Output)

LOAD	JOINT	FREQ. (CY/11")	TYPE	X TRANS.	Y TRANS.	Z TRANS.	X ROT.	Y ROT.	Z ROT.		
42	GL0	2829.00	REL DISP	MOD	0.052	1.547	0.020	0.000	0.000	0.000	
			PHA	225.653	236.047	-32.436	118.626	237.082	197.614		
		3536.00	REL DISP	MOD	0.041	1.296	0.018	0.000	0.000	0.001	
				PHA	-84.711	264.789	-30.653	118.612	-67.388	256.545	
		3713.00	REL DISP	MOD	0.048	1.437	0.005	0.000	0.000	0.001	
				PHA	172.653	174.901	151.953	-7.389	174.863	169.166	
		3211.92	REL DISP	MOD	0.072	3.285	0.039	0.000	0.000	0.002	
				PHA	178.254	-29.646	123.515	-42.122	140.862	-56.082	
	43	GL0	2829.00	REL DISP	MOD	0.054	1.414	0.020	0.000	0.000	0.000
				PHA	67.648	213.899	-26.529	118.626	237.082	197.614	
			3536.00	REL DISP	MOD	0.014	1.296	0.018	0.000	0.000	0.001
					PHA	176.825	252.158	-30.653	118.612	-67.388	256.545
		3713.00	REL DISP	MOD	0.006	1.437	0.002	0.000	0.000	0.001	
				PHA	158.310	174.278	123.501	-7.389	174.863	169.166	
		3211.92	REL DISP	MOD	0.108	4.220	0.029	0.000	0.000	0.002	
				PHA	-62.971	-32.022	116.959	-42.122	140.862	-56.082	
44		GL0	2829.00	REL DISP	MOD	0.006	1.469	0.020	0.000	0.000	0.000
				PHA	145.382	225.165	-26.514	118.626	237.082	197.614	
			3536.00	REL DISP	MOD	0.021	1.369	0.018	0.000	0.000	0.001
					PHA	256.315	261.647	-30.648	118.612	-67.388	256.545
		3713.00	REL DISP	MOD	0.027	1.515	0.002	0.000	0.000	0.001	
				PHA	170.955	174.194	122.963	-7.389	174.863	169.166	
		3211.92	REL DISP	MOD	0.048	3.896	0.029	0.000	0.000	0.002	
				PHA	256.395	-31.174	116.955	-42.122	140.862	-56.082	
	45	GL0	2829.00	REL DISP	MOD	0.032	1.576	0.020	0.000	0.000	0.000
				PHA	225.653	235.265	-26.519	118.626	237.082	197.614	
			3536.00	REL DISP	MOD	0.041	1.445	0.018	0.000	0.000	0.001
					PHA	-84.711	263.878	-30.653	118.612	-67.388	256.545
		3713.00	REL DISP	MOD	0.048	1.592	0.002	0.000	0.000	0.001	
				PHA	172.653	174.118	123.501	-7.389	174.863	169.166	
		3211.92	REL DISP	MOD	0.072	3.574	0.029	0.000	0.000	0.002	
				PHA	178.254	-30.172	116.959	-42.122	140.862	-56.082	
46		GL0	2829.00	REL DISP	MOD	0.050	1.195	0.020	0.000	0.000	0.000
				PHA	-12.794	243.589	126.755	49.993	178.414	66.345	
			3536.00	REL DISP	MOD	0.071	0.877	0.224	0.000	0.000	0.000
					PHA	-48.990	72.502	9.654	-30.602	21.675	263.139
		3713.00	REL DISP	MOD	0.146	1.234	0.159	0.000	0.000	0.000	
				PHA	-38.599	-9.173	5.710	130.024	4.301	219.526	
		3211.92	REL DISP	MOD	1.741	1.002	0.541	0.000	0.000	0.000	
				PHA	10.329	70.702	18.692	5.580	48.941	223.834	
	47	GL0	2829.00	REL DISP	MOD	1.044	1.350	0.976	0.000	0.000	0.000
				PHA	187.517	242.005	126.755	49.993	178.414	66.345	
			3536.00	REL DISP	MOD	0.087	0.985	0.224	0.000	0.000	0.000
					PHA	-28.565	74.461	9.654	-30.602	21.675	263.139
		3713.00	REL DISP	MOD	0.163	1.246	0.159	0.000	0.000	0.000	
				PHA	-32.117	-9.671	5.710	130.024	4.301	219.526	

TABLE 2. DYNAMIC PARTICIPATION OF EACH MODE (Sample Computer Output)

LOAD	JOINT	FREQ (CY/TIM)	MODE	TYPE	X TRANS.	Y TRANS.	Z TRANS.	X ROT.	Y ROT.	Z ROT.
			33	RDSP MOD	0.000	0.000	0.000	0.000	0.000	0.000
				PHA	109.992	-70.008	109.992	109.992	109.992	109.992
			34	RDSP MOD	0.000	0.000	0.000	0.000	0.000	0.000
				PHA	88.896	88.896	268.896	268.896	268.896	268.896
			35	RDSP MOD	0.000	0.000	0.000	0.000	0.000	0.000
				PHA	268.990	268.990	268.990	88.990	268.990	88.990
			36	RDSP MOD	0.000	0.000	0.000	0.000	0.000	0.000
				PHA	-32.658	147.342	147.342	-32.658	147.342	-32.658
44	GLU	2829.00	1	RDSP MOD	0.000	0.000	0.135	0.000	0.000	0.000
				PHA	0.008	180.008	180.008	180.008	0.008	180.008
			2	RDSP MOD	0.002	0.000	0.000	0.000	0.000	0.000
				PHA	264.291	84.291	264.291	264.291	264.291	84.291
			3	RDSP MOD	0.000	0.000	0.008	0.000	0.000	0.000
				PHA	0.678	0.678	0.678	180.678	180.678	0.678
			4	RDSP MOD	0.001	0.000	0.127	0.000	0.000	0.000
				PHA	180.819	180.819	0.819	180.819	180.819	180.819
			5	RDSP MOD	0.011	0.001	0.000	0.000	0.000	0.000
				PHA	-79.118	100.882	-79.118	-79.118	-79.118	100.882
			6	RDSP MOD	0.015	0.050	0.000	0.000	0.000	0.000
				PHA	91.122	91.122	91.122	-88.878	91.122	91.122
			7	RDSP MOD	0.001	0.000	0.405	0.000	0.000	0.000
				PHA	181.682	1.682	1.682	1.682	1.682	1.682
			8	RDSP MOD	0.001	0.000	0.108	0.000	0.000	0.000
				PHA	183.712	3.712	3.712	3.712	3.712	3.712
			9	RDSP MOD	0.000	0.001	0.106	0.000	0.000	0.000
				PHA	5.339	185.339	185.339	185.339	185.339	5.339
			10	RDSP MOD	0.003	0.005	0.003	0.000	0.000	0.000
				PHA	15.732	15.732	15.732	15.732	195.732	195.732
			11	RDSP MOD	0.004	0.206	0.025	0.000	0.000	0.000
				PHA	-69.604	-69.604	-69.604	110.396	-69.604	-69.604
			12	RDSP MOD	0.049	3.973	0.010	0.000	0.000	0.001
				PHA	243.663	243.663	63.663	243.663	63.663	243.663
			13	RDSP MOD	0.001	2.405	0.002	0.000	0.000	0.001
				PHA	75.943	75.943	75.943	255.943	75.943	75.943
			14	RDSP MOD	0.007	0.502	0.001	0.000	0.000	0.000
				PHA	260.926	80.926	260.926	80.926	260.926	80.926
			15	RDSP MOD	0.000	0.001	0.008	0.000	0.000	0.000
				PHA	-48.466	-48.466	131.534	-48.466	131.534	-48.466
			16	RDSP MOD	0.003	0.172	0.001	0.000	0.000	0.000
				PHA	265.702	265.702	265.702	85.702	265.702	265.702
			17	RDSP MOD	0.000	0.000	0.000	0.000	0.000	0.000
				PHA	-75.530	-75.530	104.470	-75.530	104.470	104.470
			18	RDSP MOD	0.000	0.000	0.000	0.000	0.000	0.000
				PHA	-85.357	-85.357	-85.357	94.643	-85.357	-85.357
			19	RDSP MOD	0.000	0.000	0.000	0.000	0.000	0.000
				PHA	202.682	202.682	202.682	202.682	202.682	202.682

of interest, 0.37 mil is approximately 1/10th the thickness of an ordinary bond paper. Thus, with this stringent deflection criterion, the authors contend the support structure should be analyzed for those conditions at the expected long term continuous force and not at an emergency or upset loading conditions. This continuous force should be based upon the maximum level of vibration at which the machine should be operated continually. This level of vibration and its corresponding magnitude of dynamic force should be determined by the plant owner with the recommendation of the manufacturer.

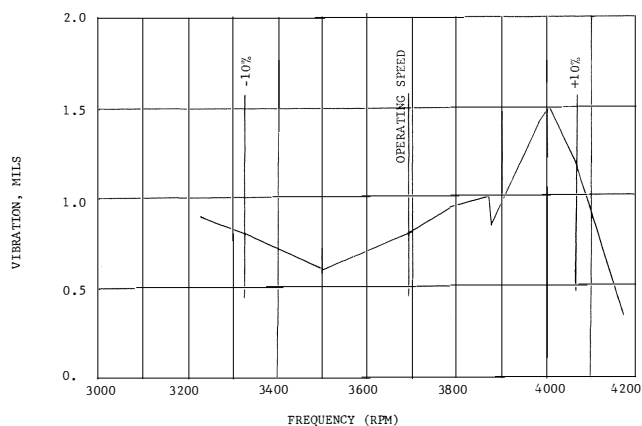


Figure 7. Typical Response Spectrum.

### DATA COORDINATION-MANUFACTURERS AND STRUCTURAL ENGINEERS

Since the accuracy of the computerized technique is dependent upon proper input, the structural engineer must have a good understanding of the information furnished by the manufacturer. This requires, in many cases, close coordination of the structural engineers and the manufacturer's engineers.

The structural engineer is expecting the manufacturer to provide the loads that represent the dead weight of the machine distributed to the point of support (i.e., the bottom of the base plate of the machine). He is also anticipating that these will be provided as separate loading by the manufacturer in order to properly input these as static masses in the modal analysis.

The structural engineer also needs the steady state normal operating dynamic loads reduced to the point of support of the machine. These loads should include any effects the machine might have in reducing or increasing the dynamic load that occurs at the rotor bearing point. Since the machine speed can vary, the manufacturer should indicate how the magnitudes of these loads change with speed and the limits of the normal operating speed range.

It is also necessary that the manufacturer and structural engineer agree that the cutouts for pipe nozzles should leave enough space to provide concrete members in the structure which will not have frequencies in the range of operation speeds. Due to the constraints of machinery, we have had some difficulties with the support beam between the turbine

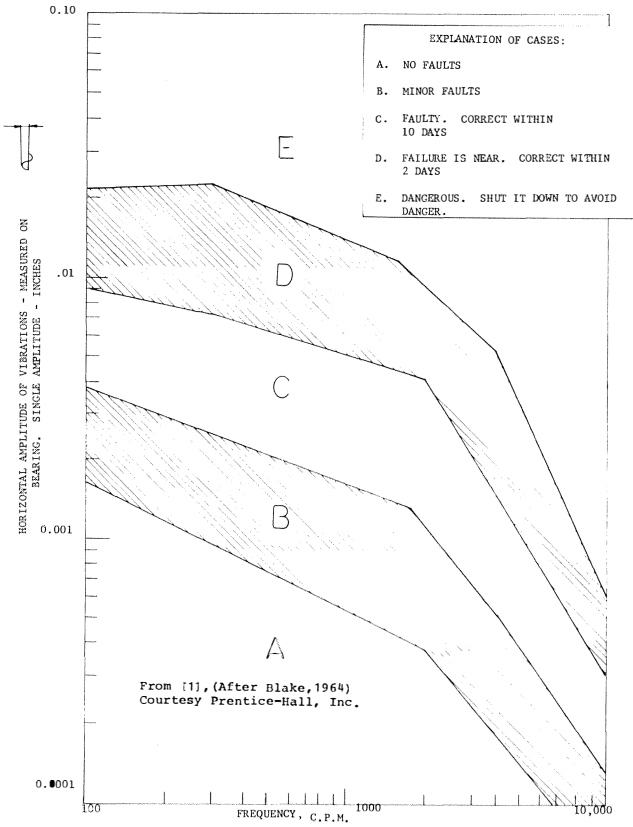


Figure 8. Design Criteria.

and compressor. Sometimes enough space to provide a structurally adequate beam with a natural frequency outside the operating speed range was difficult to obtain. However, by working with the manufacturer's engineers, we were able to work out an adequate solution.

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