PIPING VIBRATION ANALYSIS

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ABSTRACT

Excessive piping vibrations are a major cause of machinery downtime, leaks, fatigue failures, high noise, fires, and explosions in refineries and petrochemical plants. Excessive vibration levels usually occur when a mechanical natural frequency of the piping system is excited by some pulsation or mechanical source. The vibration mode shapes usually involve lateral vibrations and/or shell wall radial vibrations.

Simplified methods are presented for analyzing lateral and shell wall piping vibrations and judging their severity. The methods are thought to be conservative and are intended to be used as screening criteria to determine if more sophisticated analyses, such as computer stress modelling or strain gage testing are necessary. Frequency factors for calculating the mechanical natural frequencies for the classical piping configurations (uniform straight beams) and various piping bend configurations are presented. Factors are presented to compensate the natural frequency calculations for concentrated and distributed weight effects.

The relationships between piping vibration displacement, velocity and stress are presented and criteria for judging the severity of piping vibration in terms of the endurance stress limit are shown. The mechanisms that can excite piping vibrations will be discussed, as well as methods for controlling their severity.

INTRODUCTION

Piping vibration failures have been one of the major causes of downtime, fires and explosions in industrial plants over the past 30 years. For example, one piping failure at a petrochemical plant in 1974 caused over \$114,000,000 in property damage [1], due to an explosion. In nuclear pressurized water reactor power plants, over 80 cases of cracks or leaks occurred in the piping systems of charging pumps over a two-year period [2]. Therefore, it is vitally important that piping vibration amplitudes in a system be evaluated to determine if the levels are acceptable. If the vibrations levels are judged to be excessive, the piping configuration, support structure, span length, or material may have to be modified to make the system acceptable. Alternately, if these factors cannot be changed, the excitation mechanisms must be altered or eliminated. In order to make practical modifications to the piping to solve vibration problems, it is necessary to understand all the principles involved in the determining the natural frequencies and the excitation sources that cause the problems.

Methods are presented for calculating the natural frequencies. The relationship between vibration amplitude and dynamic stress is shown. In addition, the excitation sources that cause excessive vibrations and methods for minimizing their harmful effects are discussed.

Vibration problem areas of typical piping systems include the excitation of the following:

- Piping span natural frequencies
- Piping shell wall circumferential and axial natural frequencies

• Piping appurtenances (vent and drain lines, gage, and test connections)

• Valves and valve components

 Reciprocating compressor cylinder and manifold bottle natural frequencies

The principles involved in understanding the behavior of piping vibrations of the components listed above are covered.

CALCULATION OF PIPING NATURAL FREQUENCIES [3-8]

To ensure that piping systems are free from excessive vibrations, it is necessary that the individual piping spans not be mechanically resonant to system excitation frequencies generated by compressors, pumps, flow excitation mechanisms, etc. To accomplish this, the frequencies of the excitation forces and the mechanical natural frequencies of the piping must be calculated. With experience, simplified design procedures can be used to evaluate the piping system with a minimum of detailed computer analyses. For complex systems, stress analysis computer programs should be used to evaluate piping system reliability.

Straight Piping Spans

Actual piping span natural frequencies deviate from the theoretical beam natural frequencies, since the configurations that exist in typical plant piping have boundary conditions that differ from ideal values. Nevertheless, ideal beam theory gives a valuable starting point for understanding piping vibration behavior.

The natural frequency of any piping span can be calculated if the frequency factor, the span length, the diameter, wall thickness and the weight per length are known. For a straight uniform piping span, the natural frequency can be calculated using the following relationship:

$$f_{o} = \frac{\lambda}{2\pi} \sqrt{\frac{gEI}{\mu l^{4}}}$$
(1)

where:

 f_0 = Span natural frequency, Hz

 $g = Gravitation constant, 386 in/sec^2$

E = Modulus of elasticity, psi

- I = Moment of inertia, in⁴
- 1 =Span length, in.
- λ = Frequency factor, dimensionless
- μ = Weight per unit length of beam (including fluid and insulation), lbs/in.
- ρ = Density, lbs/in³
- A = Pipe cross-sectional area, in^2

By substituting in material properties for steel, $E = 30 \times 10^6$ lb/in², $\rho = 0.283$ lb/in³, and g = 386 in/sec², Equation 1 can be simplified to:

$$f_{o} = 223\lambda \frac{k}{L^{2}}$$
(2)

where:

k = radius of gyration, inches

L= length of span, ft

Note that this equation does not include the weight of the fluid and the insulation. The frequency factors (λ) for calculating the first two natural frequencies for ideal straight piping spans are given in terms of the overall span length in Figure 1.

	Piping Freque		uency Deflection Stress*		Velocity Stress*		
	Configuration	tion Factor		Factor		Factor	
		lst	2nd	lst	2nd	lst	2nd
Ĵ	Fixed-Free	3.52	22.4	366	2295	219	219
	Simply Supported	9.87	39.5	1028	4112	219	219
<u>}</u>	Fixed - Supported	15.4	50.0	2128	6884	290	290
€	Fixed-Fixed	22.4	61.7	2935	8534	275	290
ĴB	L-Bend Out	16.5	97.6	1889	13996	241	301
A+B = L A=8	L-Bend In	59.4	75.5	7798	9575	276	266
A C	U-Bend Out	18.7	111.6	2794	14511	314	273
/// /// A+B+C = L A=B=C	U-Bend In	23.7	95.8	3751	8722	332	191
<u>}</u> ∎	Z-Bend Out	23.4	34.2	3522	4133	317	254
C A+B+C = L A=B=C	Z-Bend In	22.4	96.8	3524	8933	331	194
ABBC = L ABBC	3-D Bend	20.6	27.8	3987	4752	407	359
	Formula	$f = \frac{\lambda}{2\pi} \sqrt{\frac{gEI}{\mu l^4}}$		$S = K_d y_{\tilde{L}^2}^D SCF$		$S = K_v V SCF$	
	Reference	Eq. 1		Eq. 10		Eq. 13	

*Steel Piping (E = 30×10^6 psi, $\rho = 0.283$ lb/in³)

Figure 1. Frequency Factors and Stress Factors for Uniform Steel Pipe Configurations.

Piping Bends

The natural frequencies of selected pipe configurations with piping elbows (L-bends, U-bends, Z-bends, and three dimensional bends) were analyzed using a finite element program (ANSYS) to generate frequency factors for the first two modes. In this analysis, a curved beam (elbow) element was used so that more accurate frequency factors for the piping configurations could be established. In doing so, the frequency factors may be slightly different from other published data for square corner beams or beams without cylindrical cross sections. The frequency factors were generated for a range of aspect ratios to develop sufficient information so that the natural frequency of piping spans could be approximated regardless of the configuration. The accuracy of the analysis was verified by comparison of the frequency factors with the theoretical values at the limits of the aspect ratios. The frequency factors as a function of the aspect ratios of the leg lengths are given in Figures 2, 3, 4, 5, 6, 7, 8.



Figure 2. Frequency Factors for Uniform L-Bend Piping Configurations.



Figure 3. Frequency Factors for Uniform U-Bend Piping Configurations for First Out-of-Plane Mode.



Figure 4. Frequency Factors for Uniform U-Bend Piping Configurations for First In-Plane Mode.



Figure 5. Frequency Factors for Uniform Z-Bend Piping Configurations for First Out-of-Plane Mode.

Effect of Concentrated Masses

Applying energy methods [3], it can be shown that the first natural frequency of a beam with a concentrated load can be calculated by:

$$f_{p} = \frac{f_{o}}{\sqrt{1 + B \frac{P}{W}}}$$
(3)

where:

 $f_p = Pipe span natural frequency with concentrated weight, Hz$

 $\rm f_{o}$ = Pipe span natural frequency without concentrated weight, Hz

P = Concentrated weight, lbs

- W= Weight of beam span, lbs
- B = Weight correction factor, dimensionless

Weight correction factors to be used in calculating the natural frequencies of ideal piping spans for weights at the maximum deflection locations are given in Figure 9. If two weights are located in one span, the following equations can be used to calcu-



Figure 6. Frequency Factors for Uniform Z-Bend Piping Configurations for First In-Plane Mode.



Figure 7. Frequency Factors for Uniform 3D-Bend Piping Configurations for First Mode.



Figure 8. Frequency Factors for Uniform 3D-Bend Piping Configurations for Second Mode.

late the effect of the second weight. The frequency for one weight P_1 is:

$$f_1 = \frac{f_o}{\sqrt{1 + B \frac{P_1}{W}}} \tag{4}$$

If the second weight in the span is considered by itself, the equation is:

$$f_2 = \frac{f_o}{\sqrt{1 + B \frac{P_2}{W}}}$$
(5)

The frequency for the span with both weights can be obtained from the following equation:

$$f_1^2 + 2 = \frac{1}{\frac{1}{f_1^2} + \frac{1}{f_2^2} - \frac{1}{f_o^2}}$$
(6)

Correlation of Calculated and Measured Natural Frequencies

Theoretical beam natural frequency calculations can be corrected to make them agree more closely with measured field data [7]. The correction factors, given in Table 1, are based on the non-ideal end conditions typically encountered in actual piping installations.

The procedures for calculating the natural frequency of piping spans can be used to select clamp spacings which ensure that the piping spans will be resonant above some selected frequency. The use of these correction factors will normally give answers that are within 15 percent of measured values. For the majority of piping vibration problems, this accuracy should be sufficient.



Figure 9. Weight Correction Factors for Uniform Piping Configurations.

Table 1. Natural Frequency Correction Factors For Piping EndConditions.

Рір Туре	ing Configuration e of End Conditions	Natural Frequency Calculated on Basis Of Fixed-Ends	End Correction Factor
Straight	Welded-Welded	Fixed-Fixed	0.9 - 1.0
Straight	Welded-Supported	Fixed-Fixed	0.8
Straight	Supported-Supported	Fixed-Fixed	0.7
Straight	Welded-Free	Fixed-Free	0.5 - 1.0
Straight	Supported-Free	Fixed-Free	0.3 - 0.7
Bends	Welded-Welded	Fixed-End	1.0
Bends	Welded-Supported	Fixed-End	0.8
Bends	Supported-Supported	Fixed-End	0.7
Bends	Welded-Change of Plane	Fixed-End	0.8
Bends	Supported-Change of Plane	Fixed-End	0.7
Straight Pipe with Valve		Fixed-Fixed With L = Span Length - Valve Length	1.0

Shell Wall Vibrations

High frequency piping shell wall vibrations can be caused by excitation of circumferential radial mode natural frequencies [6]. The nodal patterns are illustrated in Figure 10 for a simply supported cylinder showing the combination of the lateral beam vibration modes and circumferential modes. A number of theories are used to calculate the natural frequencies and the stresses due to shellwall vibration. According to Blevins [6], the Flugge and Sanders shell theories are generally felt to be the most accurate.



Figure 10. Axial and Circumferential Nodal Patterns for Simply Supported Cylinder Without Axial Constraint.

The curvature of the shell couples the flexural and extensional vibrations and considerably complicates the analysis of shell vibrations. The shell theories describe the motion of the shell in terms of an eighth-order differential equation. Because of the complexity of the shell equations and their solutions, few closedform solutions are available for the natural frequencies and mode shapes of shells.

Blevins [6] gives the following relationship for calculating the natural frequencies for cylindrical shells of infinite length:

$$f_{i} = \frac{\lambda_{i}}{2\pi R} \left[\frac{E}{\gamma (1 - \nu^{2})} \right]^{1/2}$$
(7)

$$\lambda_i = \frac{1}{12^{1/2}} \frac{h}{R} \frac{i(i^2 - 1)}{(1 + i^2)^{1/2}}; i = 2, 3, 4...$$
(8)

where:

 f_i = Shell wall natural frequency, Hz

 λ_i = Frequency factor, dimensionless

R = Mean radius of pipe wall, inches

 ν = Poisson's ratio

- γ = Mass density of pipe material, lb-sec²/in⁴
- h = Pipe wall thickness, inches

 $i = Mode number, 1, 2, \ldots$

Arnold and Warburton [9] investigated the effects of the end conditions on the natural frequencies of shells and compared measured test results with their calculations. Fung, Sechler and Kaplan [10] included the effects of internal pressure on the natural frequencies of shells.

For a steel pipe with Poisson's ratio of 0.3, the equation can be simplified to:

$$\mathbf{f_i} = \frac{\lambda_i}{R} [33749] \tag{9}$$

The frequency factor is a function of the thickness divided by the mean radius; therefore, the natural frequency varies linearly with the shell wall thickness and is inversely proportional to the square of the radius.

VIBRATION-INDUCED STRESS LEVELS

In order to determine if piping vibration amplitudes are acceptable, the resultant dynamic stresses caused by the vibrations must be compared to the allowable endurance stress limit. To accomplish this, the maximum stress in a piping span vibrating at resonance must be expressed as a function of the dynamic deflection or velocity measured at the maximum vibration point within the span.

There have been attempts to develop criteria for acceptable piping vibration levels as a function of frequency. Probably, the most widely used are the vibration amplitude *vs* frequency charts [4, 7] that were developed, based on experience in the petrochemical industry. These amplitude versus frequency charts are used as a screening criteria in the evaluation of piping systems experiencing high vibration levels.

The material presented in this section gives the relationship between vibration and stress in typical piping configurations and presents definitive methods for evaluating piping system reliability using the actual vibration-induced stresses.

Vibration Displacement Amplitude Vs Frequency Criteria

The vibration versus frequency criteria chart given in Figure 11 can be used as a first evaluation of the severity of a piping vibration problem. These curves are based on experience and have been used in the petrochemical industry for over 25 years with good success [4, 7]. These criteria are very conservative for long flexible piping spans, such as those used in centrifugal



equipment plant piping. They are not applicable to shell wall vibrations.

The authors' experiences have shown that, whenever piping vibration amplitudes at the measured frequencies are greater than the danger line, piping failures are a typical occurrence. When vibration levels were below the design line, very few failures have occurred. Therefore, these vibration versus frequency criteria can serve as a good starting point in evaluating piping vibrations to screen those systems that need further analyses.

Stress as a Function of the

Vibration Displacement (Deflection) Amplitude

A better method to evaluate the severity of piping vibration deflection amplitudes is to compare the maximum resonant vibration-induced dynamic stresses to an allowable endurance limit stress. There is general agreement that the low cycle fatigue curves for carbon steel given in the ASME USAS B31.7-1969 can be used to obtain an acceptable endurance limit stress [11]. ANSI/ASME Code OM3-1987 [12] uses this stress versus cycles-to-failure curve as a basis for specifying criteria for evaluating the vibration-induced stresses in nuclear power plant piping for preoperational and startup testing. API Standard 618 [13] uses the same data to specify the allowable dynamic stress level for steel pipes as a design requirement.

OM3 is the first code that has attempted to establish a method for evaluating piping vibration-induced stresses based on measured resonant vibration amplitudes or velocities. The methodology used in the code involves a three-step process in determining the acceptability of piping vibrations. The first step is categorized as Vibration Monitoring Group 3 (VMG3) and involves a visual or perception walkdown of the piping to determine if the vibrations are acceptable, based on the experience of the analyst with the type of piping system being examined. If, in the judgement of the analyst, the vibrations are not obviously safe, the piping is judged to be in the Vibration Monitoring Group 2 (VMG2). In VMG2, the acceptability of the piping vibrations are judged by conservatively estimating the vibrationinduced stresses by measuring the vibrations and calculating the vibration-induced stresses by simplified methods. These methods are based on modelling the vibration portion of the piping via a simple beam analogy and determining the vibration limits in terms of the displacement or velocity. (Note that this paper gives the information required to make these calculations.) The third category is Vibration Monitoring Group 1 (VMG1) and involves a rigorous qualification method requiring that the vibrational stresses be determined with a high degree of accuracy. VMG1 qualification may involve a detailed correlation between analysis and experimental results or instrumentation of the piping with a sufficient number of strain gages to determine the magnitude of the highest stresses. In VMG1, computer models of the system are developed, in conjunction with the measured vibration amplitudes, to predict the maximum vibrationinduced stresses.

The same methodology is used in evaluating piping vibrations throughout the petrochemical industry; however, the methodology is not specifically detailed in an applicable code. Olson [2] has compared the acceptable vibrations determined by using the ANSI/ASME Code OM 3 [12], which is based on stress, to the amplitude versus frequency curves presented in Figure 11.

The vibration-induced stress in a piping span vibrating at resonance has been shown to be related to the maximum vibration amplitude (deflection) in the span [4, 5, 6, 7]. The relationship is given in the equation below:

Figure 11. Allowable Piping Vibration Levels Vs Frequency.

$$S = K_{d} y \frac{D}{L^2} (SCF)$$
(10)

where,

- S = Dynamic stress, psi
- K_d = Deflection stress factor
- y = Maximum vibration amplitude (deflection) measured between nodes (normally at supports), mils
- D = Outside pipe diameter, inches

$$L = Span length, ft$$

SCF = Stress concentration factor

The deflection stress factor is a function of the boundary conditions and the vibration mode shape at resonance. Blevins [6] gives the normalized vibratory mode shapes at resonance for the classical beams. This data can be used to generate the deflection stress factors for these modes using the methods presented by Wachel [4]. The deflection stress factors for the first two modes of the ideal classical beams and the piping configurations with elbows are also given in Figure 1. These factors are used to calculate the stress at the piping span natural frequency and the stress has to be corrected if the pipe is vibrating at a different frequency. Mode correction factors are given in a later section.

For the piping configurations with elbows, the stress deflection factors were calculated with the finite element program ANSYS. The accuracy of the calculations was verified by comparison with the factors obtained for the classical beams. The plots of the deflection stress factors are given in Figures 12, 13, 14, 15,, 16, 17, and 18 for the various piping configurations with bends for the out-of-plane and the in-plane modes. The stress used in the calculations was the maximum resultant stress since vibrations in piping configurations with bends cause multidirectional stresses.

The allowable vibration amplitude can be calculated based on the endurance limit. OM3 specifies 10000 psi zero to peak as the allowable endurance limit for carbon steel and specifies that the minimum safety factor is 1.3 which equates to 7690 psi zero to peak. API 618 states that the stresses shall be less than 26000 psi peak to peak, or 13000 psi zero to peak, and is normally used with a safety factor of 2. The allowable vibration y_a in mils is given by:



Figure 12. Deflection Stress Factors for Uniform Steel L-Bend Configurations.



Figure 13. Deflection Stress Factors for Uniform Steel U-Bend Configurations for First Out-of-Plane Mode.

$$y_{a} = \frac{S_{a}}{(SCF)(SF)} \left(\frac{L^{2}}{K_{d}D}\right)$$
(11)

where:

 S_a = Allowable stress, psi

SCF = Stress concentration factor

SF = Safety factor

 K_d = Deflection stress factor



Figure 14. Deflection Stress Factors for Uniform Steel U-Bend Configurations for First In-Plane Mode.

If the API 618 allowable of 26000 psi peak to peak is used as the endurance limit combined with a stress concentration factor of 4.33, a safety factor of 2, and a stress deflection factor of 3000



Figure 15. Deflection Stress Factors for Uniform Steel Z-Bend Configurations for First Out-of-Plane Mode.



Figure 16. Deflection Stress Factors for Uniform Steel Z-Bend Configurations for First In-Plane Mode.

(applicable for a fixed-fixed pipe), the allowable vibration in peak to peak mils can be calculated. Equation (11) becomes:

$$y_a = \frac{L^2}{D}$$
 "Rule of Thumb" (12)

This can be used (conservatively) as a screening criteria for straight runs of piping or for piping with bends, based on the deflection stressfactors given in Figures 1 and 12–18. The span length is the length between measured vibration nodes which



Figure 17. Deflection Stress Factors for Uniform Steel 3D-Bend Configurations for First Mode.



Figure 18. Deflection Stress Factors for Uniform Steel 3D-Bend Configurations for Second Mode.

are normally at the supports. This criteria is overly conservative for cantilever beams.

Stress as a Function of the Vibration Velocity

In a piping span vibrating at resonance, it is also possible to relate the maximum stress to the measured velocity [4]. In order to develop a closed-form solution of the dynamic stress as a function of the velocity, the radius of gyration has to be expressed as a function of the outside diameter of the pipe. A comparison of the radius of gyration for different sizes of pipe versus the simplified equation of 0.34 D_o where D_o is the outside pipe diameter shows that, for a significant range of pipe sizes, this

simplified equation is within a few percent for pipe schedules from 10 to 160 [4]. By making the substitution of $0.34 \ D_o$ for the radius of gyration, the relationship of the maximum pipe velocity in the span to stress can be developed. The results show that the stress in an ideal beam is equal to a constant, K_v multipled by the maximum velocity measured in the piping span.

The velocity stress factors for the first two modes are given in Figure 1 for the classical types of straight spans as defined by the end conditions.

For the piping configurations with piping elbows, the velocity stress factors were also calculated in the analysis which developed the frequency factors and the deflection stress factors. The velocity stress factors are given in Figures 19–25 for the first two modes for the various aspect ratios of the leg lengths.



Figure 19. Velocity Stress Factors for Uniform Steel L-Bend Configurations.

The actual maximum span stress is equal to the velocity stress factor times the maximum measured velocity times the stress concentration factor. This equation for the stress is:

$$S = K_{v} \times V \times SCF$$
(13)

where:

- S = Dynamic stress, psi
- K_v = Velocity stress factor
- SCF = Stress concentration factor
- V = Maximum velocity in pipe span, in/sec

The allowable velocity is also a function of the endurance limit and is given in the Equation (14). To account for system unknowns, it is necessary to include a safety factor, usually 2 for fatigue analysis.

$$V_{a} = \frac{S_{a}}{K_{v} \times SF \times SCF}$$
(14)



Figure 20. Velocity Stress Factors for Uniform Steel U-Bend Configurations for First Out-of-Plane Mode.



Figure 21. Velocity Stress Factors for Uniform Steel U-Bend Configurations for First In-Plane Mode.

where:

 V_a = Allowable vibration velocity in pipe span, in/sec

- S_a = Allowable endurance limit stress, psi
- K_{v} = Velocity stress factor
- SCF = Stress concentration factor
- SF = Safety factor

In calculating allowable vibration, it is customary to use the zero to peak stress allowable, since velocity is always expressed as zero to peak. Based on an allowable endurance limit of 13000 psi zero to peak, a maximum velocity stress constant of 318, a



Figure 22. Velocity Stress Factors for Uniform Steel Z-Bend Configurations for First Out-of-Plane Mode.



Figure 23. Velocity Stress Factors for Uniform Steel Z-Bend Configurations for First In-Plane Mode.

stress concentration factor of 5, and a safety factor of 2, the allowable velocity is equal to:

$$V_{a} = \frac{13000}{(318 \times 2 \times 5)}$$
(15)

$$V_a = 4$$
 in/sec zero to peak (16)

Other Considerations for Vibration-Induced Stresses

It is necessary to consider other factors, such as concentrated or distributed weights, and responses at frequencies other than the first natural frequency when calculating the actual dynamic stresses of a piping span.

For evaluating the maximum vibration-induced stresses, the equation becomes:

$$\mathbf{S}_{\mathrm{m}} = (\mathbf{S}) (\mathbf{SCF}) \mathbf{K}_1 \mathbf{K}_2 \mathbf{K}_3 \tag{17}$$

where:

- $S_m = Maximum dynamic stress, psi$
- S = Dynamic stress calculated at the natural frequency, psi



Figure 24. Velocity Stress Factors for Uniform Steel 3D-Bend Configurations for First Mode.



Figure 25. Velocity Stress Factors for Uniform Steel 3D-Bend Configurations for Second Mode.

- K_1 = Stress correction factor for concentrated weights
- K_2 = Stress correction factor for mode/frequency considerations
- K_3 = Stress correction factor for pipe contents and insulation

Stress Weight Correction Factors (K₁)

When a concentrated weight is located in a pipe span vibrating at its first natural frequency, the stresses calculated using the uniform beam equations can be in error; therefore, correction factors must be applied to the stress calculations. The stress weight correction factors given in Figure 26 can be used to compensate the dynamic stress calculations for the effect of concentrated weights on piping spans with the classical boundary conditions. Two curves are presented, one which gives the factors for correcting the calculated stresses based on measured vibration deflection and the other for correcting the calculated stresses based on measured vibration velocity. To obtain these factors, concentrated weights were placed at the vibration antinode locations for the cantilever, simply-supported, fixed-supported, fixed-fixed, L-Bend, U-Bend, Z-Bends and 3D-Bends piping configurations. The curves are composite curves which should be conservative for piping spans vibrating at their first natural frequency. This factor increases the stress; therefore, the allowable vibration is reduced whenever a concentrated weight is present.



Figure 26. Stress Weight Correction Factors (k_1) .

For these curves, if the concentrated weight is not exactly at the antinode, the weight correction factor can be approximated by linear interpolation.

Mode Correction Factor (K_2)

If the piping span is not vibrating at its first lateral bending mode, mode correction factors which depend upon whether the span is vibrating above or below the first natural frequency must be applied.

If the piping span is vibrating below its lowest lateral beam vibration mode, the mode shape will be similar in shape to the static deflection mode shape and will gradually change to the vibration first mode shape as the frequency approaches the first natural frequency. Data analyses performed to determine the mode correction factors are summarized in Figure 27 [4] for the deflection stress method. These data show that the stresses calculated using the vibration deflection for the piping span vibrating at its first mode will be within a few percent of the correct stress for most piping configurations.



Figure 27. Off-Resonant Mode Stress Correction Factors for Deflection-Stress Calculations (K_2) .

If the piping span is vibrating at frequencies higher than the first natural frequency, the calculated stresses based on the deflection mode shape at the first natural frequency can be in error. The mode correction factor needs to be developed for a particular configuration if this method is to be used for frequencies that are greater than approximately 50 percent above the first natural frequency.

If the stresses are calculated using the velocity stress method, the mode correction factor from Figure 28, given as a function of the frequency ratio for the classical beams, can be used [4]. It can be seen that the mode correction factor is inversely proportional to the frequency; the lower the frequency, the higher the mode correction factor. This means that the velocity stress calculations should not be used for frequencies below the first natural frequency unless the exact mode correction factor is developed. This is recognized in OM3 which specifies that the stresses should be multiplied by the ratio of the natural frequency to the exciting frequency.

When the excitation frequency is higher than the first natural frequency, Figure 28 shows that the stresses calculated at the first natural frequency would be conservative.

If the piping span is vibrating at its second natural frequency, the deflection and/or velocity stress factor given for the second mode should be used to calculate the stresses. The natural frequency for the span length used should match the measured frequency or the resulting calculations could be in error. In complex piping systems, this can be a problem, since adjacent spans can cause severe off-resonance vibrations.

Stress Correction Factor for Weight of Pipe Contents and Insulation (K_3)

The primary effect of the increased weight of piping contents and insulation is to lower the mechanical natural frequency, since the insulation and contents do not add appreciable stiff-



Figure 28. Off-Resonant Mode Stress Correction Factors for Velocity-Stress Calculations (K_3 .

ness. In the original derivation of the natural frequency, the linear density was in the denominator of the square root function. Therefore, the natural frequency is reduced by the ratio of the square root of the original linear weight per unit length to the new overall weight per unit length.

For the stress calculations using the deflection stress factors, the correction factor is 1. For the stress calculations using the velocity stress factors, the correction factor will be equal to:

$$K_3 = \sqrt{1 + \frac{w_c}{w_p} + \frac{w_i}{w_p}}$$
(18)

where:

 w_c = Weight per unit length of contents

 w_i = Weight per unit length of insulation

 w_p = Weight per unit length of pipe

A screening velocity value can be obtained by assuming values of the correction factors that are maximum. OM3 arrived at a screening criteria of 0.5 in/sec; however, this number has proven to be very conservative in many piping systems [14]. OM3 uses a factor of K_3 of 1.5 and a K_1 of 8. The concentrated weight correction factor K_1 of 8 that is used was based on a concentrated weight to span weight of 20 to 1. In practice, the ratio is more likely to be less than 3 to 1. For a maximum concentrated weight to span weight of 3, K_1 would be approximately 2.7, and the screening criteria would be 1.5 in/sec. If vibration measurements indicate that the screening criteria is exceeded, the actual stress factors and the correction factors should be applied for the span and the acceptability of the vibrations based on these numbers.

Shell Wall Vibration-Induced Stresses

When the high frequency piping shell wall vibrations (axial and circumferential mode shapes) are excited, the equations relating vibration displacement, velocity and acceleration to the piping stress based on the lateral beam vibration between supports do not apply. Therefore, the equations relating vibration amplitudes to stress for these modes must be determined in order to assess the reliability of the vibrations. The stresses in the shell wall are given by the following isotropic stress strain relationships:

$$S_{\varkappa} = \frac{E}{1 - \nu^2} \left(\epsilon_{\varkappa} + \nu \epsilon_{\theta} \right)$$
(19)

$$S_{\theta} = \frac{E}{1 - \nu^2} \left(\epsilon_{\theta} + \nu \epsilon_{\varkappa} \right) \tag{20}$$

where:

 S_{\varkappa} = Lateral bending stress, psi

 S_{θ} = Circumferential stress, psi

 ν = Poisson's ratio=0.3 for steel

 ϵ_{\varkappa} = Strain in axial direction

 ϵ_{θ} = Strain in circumferential direction

Mikasinovic [15] presented an expression relating vibration velocity measured on the cylindrical shell wall to dynamic strain:

$$V = \frac{C_{\epsilon}}{2\pi} \tag{21}$$

where:

V = Vibration velocity, in/sec, zero to peak

$$C = Velocity of sound in metal \sqrt{[Eg/p]=202,284 in/sec for steel]}$$

 ϵ = Dynamic strain, in/in

The assumptions made in the derivation above are that the vibration measurements are peak measurements and several resonant modes are involved, such that the peak vibration velocity is approximately the same around the circumference and along the axial length of the piping between the constraints. It is not known how practical this assumption is, since the vibration conditions in a given pipe length are a function of the piping configuration, the wall thickness, and the internal driving forces. Mikasinovic tested different pipe sizes, wall thicknesses and end conditions with satisfactory results.

Using this formula, it would be possible to relate the vibration velocity to the fatigue endurance limit. In ANSI/ASME OM3, the allowable endurance limit stress is 10000 psi, and the minimum safety factor is 1.3, which makes the allowable stress equal to 7690 psi zero to peak. For the shell wall vibrations, the maximum stress concentration factor in the heat-affected zone of a weld would be 5. This means that the allowable stress could be as low as 1538 psi zero to peak. If we divide the stress by the elastic modulus of 30,000,000 psi, the allowable strain is obtained at 51.3 microstrain (in/in $\times 10^{-6}$). This value is consistent with the experience of the authors [16].

Using this value for the acceptable strain, the allowable velocity is equal to:

$$V_{a} = \frac{(202284) (51.3 \times 10^{-6})}{2\pi}$$
(22)

$$V_a = 1.7$$
 inches per second zero to peak (23)

If the stress concentration factor is less than the maximum, the allowable vibration velocity would be higher by the ratio of the actual stress concentration factor to 5. For a butt weld, the stress concentration factor is approximately 2; therefore, the allowable velocity would be 4.1 ips.

The vibrational velocity of the shell wall is also related to the sound pressure level (C weighting); however, no closed-form solution exists. Field experience with strain gages installed on piping with high frequency, broad band vibrations has shown that the sound pressure level (SPL) measured approximately 1 inch away from the pipe wall is proportional to the dynamic strain. Although the relationship between dynamic strain and SPL amplitude is not exact, the overall levels as presented below have been used to estimate the severity of shell wall vibrations and as a screening method to help determine where strain gages should be installed on a piping system to determine the safety factor.

Piping SPL and Strain Criteria

When the SPL is measured with the sound pressure meter using C weighting approximately 1.0 in from the vibrating pipe wall, the following criteria have been found to be applicable:

130 dB is equivalent to approximately 100 microstrain

136 dB is equivalent to approximately 200 microstrain

In addition to the criteria outlined above, it has been shown by field experience that allowable strain levels can be specified [16]. These allowable strain levels (ϵ) are given below:

$$\epsilon < 100$$
 microstrain Safe (24)

100 microstrain $< \epsilon < 200$ microstrain Marginal (25)

$$\epsilon > 200$$
 microstrain Excessive (26)

These strain limits are based on measurements that are located away from the high intensified stress locations, such as the heat-affected zone. Typically, the strain gages are installed about one-half inch away from the weld. This strain limit criteria is equivalent to an allowable stress of 3000 psi peak to peak.

VIBRATION EXCITATION SOURCES

Piping vibrations are most often excited by pulsation forces inside the piping or, secondarily, by mechanical excitation from machinery unbalanced forces and moments at one and two times the running speed. Potential excitation sources are included in the following list and are also summarized in Table 2.

• Mechanical energy from machinery unbalanced forces and moments

• Pulsations generated by reciprocating compressors and pumps

• Pulsations generated by centrifugal compressors and pumps

- · Pulsations generated by flow through or across objects
- Pulsations generated by pressure drop at restrictions
- Pulsations generated by cavitation and flashing

• Pulsations generated by waterhammer and surge

Pulsation Generating Mechanisms

Reciprocating Compressors and Pumps

The intermittent flow of a fluid through compressor or pump cylinder valves generates fluid pulsations which are related to a number of parameters, including operating pressures and temperatures, horsepower, capacity, pressure ratio, clearance volumes, phasing between cylinders, fluid thermodynamic properties, and cylinder and valve design. Pulsations are generated at discrete frequency components corresponding to the multiples of operating speed.

The pulsation amplitudes depend on the magnitude of the pulsation generated and the reflected amplitudes of the frequency components as they interact with the acoustical resonances in the system.

Pulsation amplitudes can be predicted by modelling the acoustic characteristics of the piping, the pulsations generated by the compressor or pump and the interaction of the two. Digital [5] and analog simulation techniques [7] have been developed to model the piping and the pulsation generating characteristics of compressor and pump systems. The analog technique, which was developed in the 1950s, solves the differential equations by building electrical models of the piping and the compressors and pumps. In the digital technique, the differential equations of the acoustic phenomena are solved directly with complex matrix algorithms using modern high speed computers.

Centrifugal Compressors and Pumps

Pulsation amplitudes generated by centrifugal machines generally occur at one times running speed and blade passing frequency and their multiples. They are a function of the radial vibrations, the radial impeller clearance, seal and wear rings clearances, the symmetry of the impeller, diffuser and case, and the volute characteristics. As operating conditions deviate from the design or best efficiency point, a variety of secondary flow patterns [17] may produce additional pressure fluctuations.

Significant low frequency pulsations can also be produced as a result of dynamic interaction of the acoustical response of the piping, the head-flow curve of the unit, the dynamic flow damping, and the location of the unit in the piping geometry [7, 18].

Flow Through or Across Objects

Flow through a restriction or past an obstruction or restriction in the piping may produce turbulence or flow-induced pulsations [19, 20]. These flow generated pulsations (commonly called Strouhal excitation) produce noise and vibration at frequencies which are related to the flow velocity and geometry of the obstruction.

The acoustical modes of a piping system and the location of the turbulent excitation have a strong influence on the frequency and amplitude of the vortex shedding. The frequencies generated by the turbulent energy are centered around a frequency which can be determined by the following equation:

$$f_s = \frac{S_n V}{D}$$
(27)

where:

- f_s = Strouhal vortex frequency, Hz
- S_n = Strouhal number, dimensionless (0.2 to 0.5)
- V = Flow velocity in the pipe, ft/sec
- D = Characteristic dimension of the obstruction, ft

Table 2. Piping Vibration Excitation Sources.

	Description of				
Generation Mechanism	Excitation Forces	Excitati	on Frequencies	Piping Response	Typical Problems
 MECHANICAL INDUCED A. Machinery Unbalanced Forces & Moments 	High Level, Low Frequency	$f_1 = \frac{1N}{60}$ $f_2 = \frac{2N}{50}$		Mechanical and/or Piping Resonance of Piping System	Foundation Resonances
B. Structure Bourne	Low Level	$f = \frac{N}{60}$			Vents & Drains Instrumentation Lines
2. PULSATION INDUCED A. Reciprocating Compressors	High Pressure Pulsations, Low Frequency	$f = \frac{nN}{60}$	$n = 1,2,3, \pmod{m}$ N = Speed, rpm	Mechanical and/or Acoustic Resonance of Piping System	Piping System Fatigue Failures, Excessive Loads to Rotating Equipment, Damaged Supports/ Restraints
B. Reciprocating Pumps	High Pressure Pulsations, Low Frequency	$f = \frac{nNP}{60}$	P = Number of Pump Plungers	Mechanical and/or Acoustic Resonance of Piping Systems	Cavitation on Suction Piping Fatigue Failures
C. Centrifugal Compressors & Pumps	Low Pressure Pulsations, High Frequency	$f = \frac{nN}{60}$		Complex Vibration Modes	lligh Acoustic Energies (Noise) Piping System Failures, Excessive Loads to Rotating Equipment,
		$f = \frac{nBN}{60}$ $f = \frac{nvN}{60}$	B = Number of Blades v = Number of Volutes or Diffuser Vanes		Small Branch Connection Failures
3. GASEOUS FLOW EXCITED A. Flow Through Pressure Letdown Valves or Re- strictions/Obstructions	High Acoustic Energy, Mid to High Broad Band Frequencies	$f = S \frac{V}{D}$	S = Strouhal Number 0.2 - 0.5 V = Flow Velocity ft/sec D = Restriction Diameter, ft.	Complex Vibration Modes in Both Longitudinal and Circumferential Directions	Fatigue Failures of Large Diameter Piping Downstream of High Capacity Pressure Letdown Valves, Small Branch Connection Failures, Flange Leakage
B. Flow Past Stubs	Moderate Acoustic Energy Mid to High Frequencies	$f = S \frac{V}{D}$	S = 0.2 - 0.5 D = Stub Diameter, ft.	Acoustic Resonance of Short Stubs	Fatigue Failure of Stub Connection to Main Run, Valve Chatter
 LIQUID (OR MIXED PHASE) A. Flow Turbulence Due to Quasi Steady Flow (e.g. Fluid Solids Lines) 	FLOW EXCITED Random Vibrations, Low Frequency	f = 0 - 30 Hz	(Typically)	Low Frequency Line Movements at Mechanical Natural Frequencies	Excessive Loads on Piping Supports and Restraints
B. Cavitation and Flashing	High Acoustic Energy, Mid to High Frequencies	Broad Band		Complex Vibration Modes in Both Longitudinal and Circumferential Directions	Fatigue Failures, Small Branch Connection Failures
5. PRESSURE SURGE/ IIYDRAULIC HAMMER	Transient Shock Loading	Discrete Events		lligh Impact Loads to Piping and Restraints	Excessive Piping/Structure Loads Due to Quick Valve Closures or Rapid Pump Starts/Stops

For flow over tubes, D is the tube diameter, and for excitation by flow past a branch pipe, D is the diameter of the branch pipe.

Pressure Drop Through Restrictions

Pressure regulators, flow control valves, relief valves, and pressure letdown valves produce pulsations (noise) associated with turbulence and flow separation, and the relatively broad band frequency spectrum is characteristically centered around a frequency corresponding to a Strouhal number of approximately 0.2.

Cavitation and Flashing

Flashing and cavitation can occur in the low pressure region of liquid system pressure control valves when the pressure drops below the vapor pressure. When cavitation occurs, a gas bubble is formed and moves with the flow. As the pressure increases, the pressure rises above the vapor pressure, the gas bubble collapses, and a high amplitude shock pulse results in the fluid.

To avoid flashing after a restriction, sufficient back pressure should be provided by taking pressure drop at several locations. Alternately, the restriction could be located near an open end so that the flashing energy can dissipate into a larger volume.

Hydraulic Waterhammer and Surge

Starting and stopping pumps with the attendant fast opening and closing of valves is a major cause of severe transient pressure

surges in piping systems. Increasing the closure time of valves can reduce the severity of the surge pressure. Methods are available to evaluate the severity of waterhammer in a particular piping configuration for various closure rates [21].

Centrifugal compressors and pumps can sometimes surge when they are operating at a low flow, high-head condition. The flow-versus-head curve can actually cause backflow to occur and significant pulsations can be generated which are a function of the piping acoustical natural frequencies and the overall impedance characteristics [18].

Coupling Mechanisms

For vibrations to occur, there must be an energy generating source plus a coupling mechanism to convert the pressure forces into shaking forces. Therefore, in evaluating the piping vibration characteristics of an installation, it is essential to understand the coupling mechanisms which cause shaking forces to occur in the piping system.

Pressure pulsations couple to produce shaking forces at piping bends, closed ends of vessels and headers, discontinuities or changes in the piping diameters and at restrictions, such as orifices, valves, and reducers. In a continuous straight pipe of constant diameter, pulsations will not produce a significant vibration excitation force.

EVALUATION OF THE SEVERITY OF PIPING VIBRATION

When a vibration problem occurs, it is necessary to evaluate its severity and determine the most effective way to alleviate the problem. The first step is to make an initial survey or walkdown of the piping system to determine piping spans with high vibration levels. During the walkdown of the piping, it is necessary to look for common symptoms of piping vibrations problems. These include fatigue cracks in the piping, leaks at flanges, broken or loose pipe clamps or hangers, cracked concrete piers, rubbed weight supports (bright metal), damaged pressure gages, noise related to the pipe hitting its restraint, or high shell wall vibrations.

The second step is to make vibration measurements to evaluate specific piping spans that are thought to have excessive vibration amplitudes. The acceptability is judged by performing the calculations necessary to obtain the dynamic stresses using the simplified techniques presented in this paper. Since the relationships between vibration and stress were developed for resonant piping spans, the frequency factors presented can be used to verify that the span is at resonance.

If the system vibration characteristics are complex and it is desired to ensure the safety of the piping, it may be necessary to develop a computer model of the piping. This model could be forced to have the measured vibrations and the resultant stresses calculated. The acceptability of the vibrations could be judged by comparison of the calculated stresses to the material endurance limit. Strain gages can be installed at the suspected high stress locations to measure the dynamic strains which can be compared to the criteria presented herein.

SOLUTIONS TO PIPING VIBRATION PROBLEMS

Solutions to most piping vibration problems involve reducing the excitation forces, eliminating the coupling mechanisms, or eliminating the mechanical or pulsation resonances. The most effective solutions are those that eliminate the resonances since the amplification factors for mechanical resonances are typically 10 - 30. Amplification factors for pulsation resonances can be as high as 50, although the range of 10 - 30 is more typical.

Modifications to Solve Mechanical Resonances

Since the span natural frequency is an inverse function of the square of the span length, the most effective way to solve a mechanical resonance is to add pipe restraints, such as piers, supports or clamps to shorten the vibrating span. Many times, temporary bracing with hydraulic jacks, wooden beams and wedges can be used to confirm that a support at a particular location will reduce the vibrations.

Some of the general guidelines which can be used in selecting modifications to detune the mechanical resonances are outlined below:

• Pipe supports and clamps should be installed on one side of each bend, at all heavy weights, and at all piping discontinuities.

• The support and clamp stiffness should be adequate to restrain the shaking forces in the piping to the desired amplitudes and should be greater than twice the basic span stiffness in order to effectively enforce a node at the support location.

• Vents, drains, bypass, and instrument piping should be braced to the main pipe to eliminate relative vibrations between the small-bore piping and the main pipe.

• Restraints, supports, or gussets should not be directly welded to the pressure vessels or the piping unless they are sub-

jected to the appropriate heat treatment. It is more desirable to add a saddle-type clamp around the pipe and weld the braces to the clamp.

• Pipe guides with clearance are used as thermal expansion control devices and are generally ineffective in controlling piping vibrations.

• To resist vibration, the piping clamps should have contact with the pipe over 180 degrees of the circumference. Rubber or gasket-type material can be used between the clamp and the pipe to improve the contact.

• The piping span natural frequency should not be coincident with the excitation frequencies.

• In piping that has high shell wall vibrations, reduction of the vibrations and the noise can be accomplished by adding constrained-layer damping, if proper design procedures are used.

• In systems with pressure reducing valves, the wall thickness of the piping should be one-half inch or greater if there is a possibility of sonic flow downstream of the valve [20]. Full saddle reinforcement tees or welding tees should be used downstream of sonically choked valves or where there is a possibility of sonic flow occurring at the branch pipe intersection.

Solutions to Pulsation Resonances

When a pulsation resonance is found, acoustic changes to the piping system can be the most effective way to detune or reduce the amplitudes of the pulsations. Probably the most effective element that can be conveniently used in existing systems is an orifice plate, which is an acoustical resistance element, and is most effective when located at a pressure pulsation node. Generally, without additional information, an orifice plate with a diameter ratio of approximately 0.5 will give sufficient pressure drop (acoustical resistance) to evaluate whether such an acoustical modification will be an effective solution.

If orifice plates are ineffective or impractical due to the pressure drop, it may be necessary to install pulsation filters to reduce the amplitudes of the pulsations. These could be volume bottles, Helmholtz-type filters, gas/bladder type accumulators, etc. It may be necessary to acoustically model the piping system using digital or analog techniques to determine the level of changes that will be required to detune the system and solve the problem. A combination of mechanical and pulsation changes may be needed to reduce the severity of the problem to the point where the vibrations are acceptable.

CONCLUSIONS

Some of the basic principles necessary for understanding the behavior of piping vibrations and determining the acceptable vibration levels in piping spans have been presented. The information presented can be used by engineers to perform the following:

• Calculation of the first and second mechanical natural frequencies of uniform piping spans, including piping bends with various aspect ratios.

• Calculation of the maximum vibration-induced stresses based on the maximum measured vibration deflection within a uniform piping span.

Calculation of the maximum vibration-induced stresses based on the maximum measured vibration velocity within a uniform piping span. • Calculation of the maximum acceptable vibration deflection within a uniform piping span, based on the endurance stress limit.

• Calculation of the maximum acceptable vibration velocity within a uniform piping span, based on the endurance stress limit.

• Calculation of the effects of concentrated and distributed (contents and insulation) weights on the natural frequencies and the vibration-induced stresses.

• Calculation of the effects of off-resonant stresses.

• Determination of the acceptable screening criteria for a piping span to eliminate those spans with adequate safety factors so that detailed vibration and stress analyses will not have to be performed on all piping spans.

• Determination of the possible excitation sources that could be causing the piping vibrations.

• Determination of the possible mechanisms that could be coupling the pulsation or mechanical energy into shaking forces.

• Determination of the possible mechanical solutions to solve the vibration problem.

• Determination of the possible acoustical solutions to solve the pulsation problem which may be causing the vibrations.

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