A FUNDAMENTAL UNDERSTANDING OF ROTORDYNAMICS FROM A LIGHT RUB (BOUNCE)

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ABSTRACT

Machinery users, designers, and analysts are both alarmed and fascinated by the occasional occurrences of contact between a rotating shaft and a stationary part. "Rubs," a common description for such contact, are typically categorized into two classes according to the level of interface pressure generated by contact. Very high contact pressure accompanied by high frictional force is classified as a "hard rub" while a "light rub" indicates low contact pressure with a frictional force that may be insignificant. Light rubs that result in half frequency whirl are discussed. The author explains the phenomena using simple spring mass systems and generally understood terms such as critical speed, unbalance response, and contact stiffness. Some design parameters are included that may be used to affect rotor response to light rubs.

INTRODUCTION

Half frequency whirl is a phenomena that the author has encountered all to often during 15 years of rotordynamic work. When it occurs, it is very persistent and doesn't go away. Conventional wisdom has always suspected that rubs were involved, but there was no clear explanation for what was observed. It seemed reasonable to expect that if vibration occurred, it would occur at one of the system natural frequencies, but no matter how the natural frequencies changed due to size and design of machine, the problem always occurred at exactly one-half of running speed.

Since system natural frequencies were not the same for all machines troubled by half frequency whirl, was there something else the machines had in common? One thing was that the whirl frequency was always half of running speed, and this relationship did not change even when the operating speed of a troubled machine was changed. The exact half frequency component may be tied to a constantly triggered exciting mechanism, maybe rotor unbalance. If system dynamics after impact could prevent the rotor from making contact on the following revolution of the shaft, a repeatable pattern could exist as long as the shaft finally made contact after a whole number of shaft revolutions, the same spot on the shaft would hit each time. The mathematical analysis developed to support this theory is described.

Machines used as examples for this study include a large between bearing hot gas expander with tilt pad bearings, a large between bearing process gas compressor, and a small high speed single stage overhung compressor both with pressure dam bearings. Cures ranged from increasing rotor-to-casing clearances to modifying bearings for increased system damping.

THEORY

A distinction is made between the terms "light rub" and "bounce" in describing what takes place in half frequency whirl. A light rub implies that frictional forces play a role in exciting the rotor during half frequency whirl. This is most certainly the case with a hard rub, where the response is quite different. Frictional forces are a major driving force in hard rubs. But, this is not the case in half frequency whirl. If a frictional force of any significance is present in half frequency whirl, it diminishes response by adding to system damping. The term "bounce" ignores friction and is a more appropriate description of what happens during half frequency whirl.

Half frequency whirl begins when a rotor whirling from unbalance bumps into a stationary interference point. The rotor bounces off the contact point and the resulting shaft vibration causes the shaft to miss the contact point every other revolution of the shaft. The bouncing results in a whirl orbit which is much larger than would occur without the impact, indicating energy has been added to the system. The stationary part contacted can not be the source of this additional energy, because it is an energy conservative spring. It gives back only what was put in. Where does this additional energy come from?

Certain conditions must be met to provide the additional energy:

- The first rotor-bearing critical speed must be less than half running speed.

- The stiffness added during impact with the stationary part must be sufficient to move the first rotor-bearing critical speed above half running speed.

Assuming these parameters are met, which is the case for most high speed machines, the additional energy needed comes from the reaction of rotor unbalance during impact. When the rotor hits the stationary part, an additional stiffness is added to the system. This causes a sudden increase in the rotor critical speed. An adjustment in magnitude and orientation of the forces on the rotor (damping, unbalance and restoring spring force) are required to again reach equilibrium. For example, in the extreme case of low damping and sufficient impact stiffness to move the critical speed from below half running speed to above running speed, the phase relation between the rotor displacement and rotor unbalance needs to change by 180 degrees during impact to reestablish equilibrium of forces. The rotational inertia of the rotor prevents the phase shift from happening by a simple rotation of the rotor, so the rotor tries to jump across to the other side of the orbit to affect the shift (Figure 1). This causes the rotor to come out of the impact zone with a higher velocity, more energy, than when it entered.

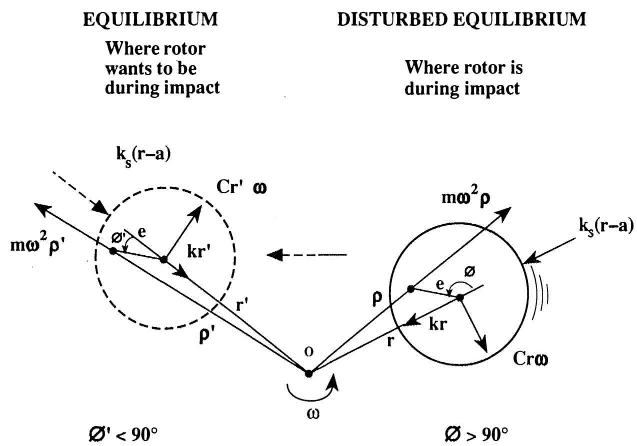

Figure 1. Impact Upsets Balance of Forces.

So energy input comes from a shift in critical speeds, but why does this result in half frequency whirl? To cause half frequency whirl, the resulting additional rotor vibration from impact must be such that when added to the normal unbalance orbit, it prevents impact every other revolution. This is accomplished when the free vibration of the impacted rotor occurs at a frequency less than half running speed, the first critical speed of the rotor. With this slow response, the impacted rotor is still moving away from the impact zone when the normal unbalance orbit tries but fails to return the rotor to impact. The rotor returns to impact every other revolution (Figure 2 and Figure 3). If the rotor first critical speed is equal to

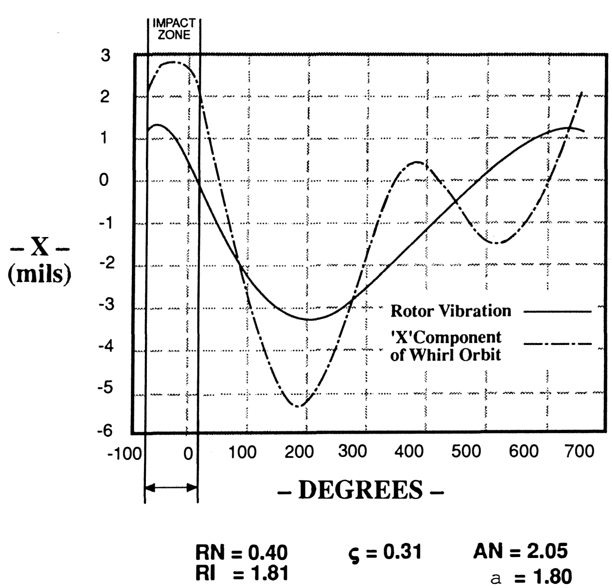

Figure 2. Rotor Vibration / "X" Component.

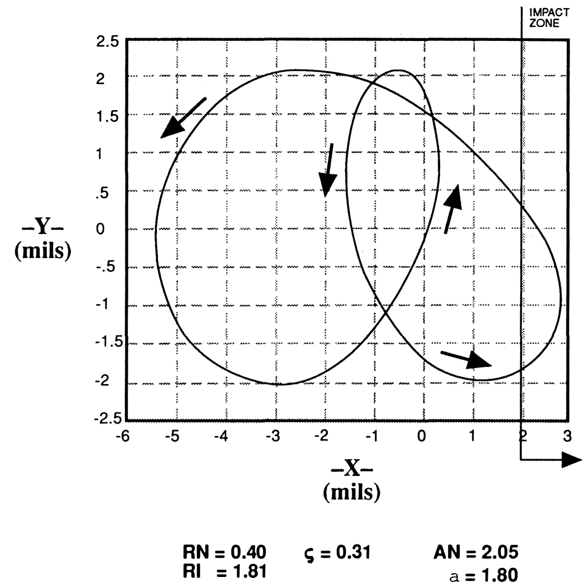

Figure 3. Half Frequency Whirl Orbit.

or greater than half running speed, the response is too fast and half frequency whirl does not occur. The impacted rotor vibration comes into phase with the normal unbalance orbit and impact occurs every revolution (Figure 4 and Figure 5).

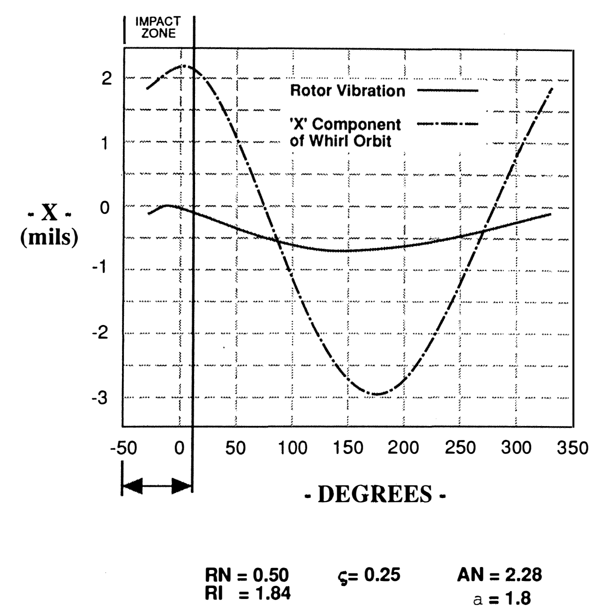

Figure 4. Rotor Vibration / "X" Component.

A similar argument might be made for the possible existence of a 1/N whirl, which would be reinforced every N revolutions, N being a whole number. This requires the first rotor-bearing critical speed to be below 1/N of running speed. The requirement that N be a whole number is necessary for periodic motion to occur. The vibration is reinforced by the same intensity of impact every N revolutions. The same spot on the rotor makes impact because a whole number of revolutions takes place. This synchronized excitation reinforces the steady state motion.

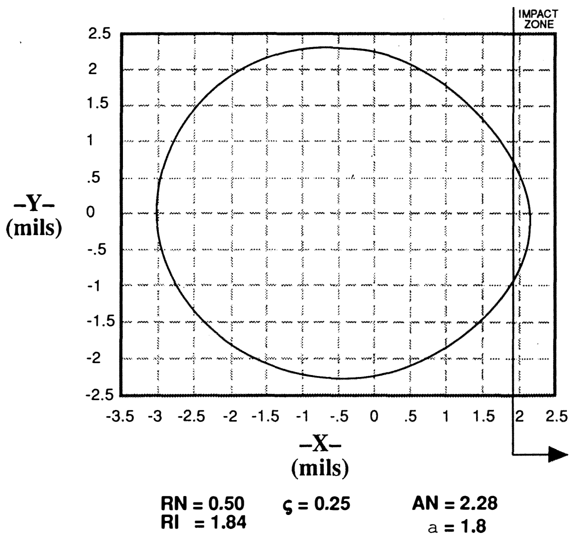

Figure 5. Whirl Orbit without Half Frequency.

The whirl direction in half frequency whirl is almost always forward, in the direction of whirl from unbalance, and as mentioned before, the impact is more of a bounce than a rub. Friction forces are probably quite small, which would explain why half frequency whirl does not typically "clear" itself. The rotor does not wear away the stationary contact point but merely bounces off it.

ANALYSIS

Equations of Motion

The model used in this analysis is a single mass rotor with no cross coupling effects. Frictional forces and bearing cross coupling terms were not included to simplify the analysis by decoupling the equations of motion. For further simplification without loss of generality, the impact point is located on the positive X-axis and is simulated by an additional spring.

$$M \cdot DDX + C \cdot DX + K \cdot X = M \cdot e \cdot \omega^2 \cdot COS(\omega t) \quad \text{Between impacts}$$
$$M \cdot DDY + C \cdot DY + K \cdot Y = M \cdot e \cdot \omega^2 \cdot SIN(\omega t)$$

$$M \cdot DDX + C \cdot DX + K \cdot X + Ks \cdot (X - a) = M \cdot e \cdot \omega^2 \cdot COS(\omega t) \quad \text{During impact}$$
$$M \cdot DDY + C \cdot DY + K \cdot Y = M \cdot e \cdot \omega^2 \cdot SIN(\omega t)$$

Complete general solutions were obtained for between impacts and during impact. The resulting constant coefficients were evaluated by matching displacements and velocities at the points entering and leaving the impact zone.

Solutions

Between impacts:

$$X = (NC \cdot COS(RDN \cdot \omega t) + NS \cdot SIN(RDN \cdot \omega t)) \cdot EXP(-\zeta \cdot RN \cdot \omega t) + AN \cdot COS(\omega t - \phi N)$$

$$Y = AN \cdot SIN(\omega t - \phi N)$$

During impact:

$$X = (IC \cdot COS(RDI \cdot \omega t) + IS \cdot SIN(RDI \cdot \omega t)) \cdot EXP(-\zeta \cdot RN \cdot \omega t) + AI \cdot COS(\omega t - \phi I) + AK$$

$$Y = AN \cdot SIN(\omega t - \phi N)$$

All solutions are steady state and periodic. There are three zones of rotor behavior depending on system damping.

- For a critical damping ratio exceeding 0.50, there is no half frequency whirl present.
- For a critical damping ratio of 0.50 to approximately 0.20, the rotor may whirl at running speed or with half frequency. However, the running speed orbit is not a stable equilibrium, if perturbed the rotor goes into half frequency whirl. The amplitude of the half frequency whirl orbit increases with decreasing damping as expected. For the typical system parameters considered herein, the X amplitude of the orbit for a critical damping ratio of 0.20 is roughly three times that occurring for a critical damping ratio of 0.50.

The shape of the orbit changes as damping is decreased. At a critical damping ratio of 0.50 there is a loop inside a larger outer loop. Sort of a distorted circle within a distorted circle. As damping is decreased, the inner loop gets very narrow in the X direction and at a critical damping ratio of 0.20, the inner loop moves outside what was the outer loop. Now the orbit looks something like a figure eight with the impacted end flattened.

- For critical damping ratio of 0.20 or less, orbit shape remains an unsymmetrical figure eight and, somewhat surprisingly, there is very little additional increase in whirl orbit amplitude. The orbit is not much larger with zero damping than occurred for a critical damping ratio of 0.20. If perturbed, the rotor returns to the half frequency steady state orbit indicating a stable equilibrium.

It is interesting to note that a critical damping ratio above 0.20 for the first critical speed of a rotor is not typical. Values are usually 0.10 or less. Typical values of stiffness and mass were used in the model, but to get the whirl orbit shape and size similar to that observed in the field required higher than expected damping. This may be from a deficiency in the model caused by ignoring cross coupling terms or the nonlinearities of fluid film bearings in the presence of large whirl orbits.

As previously mentioned, energy is added from the unbalance force seeking a new equilibrium position during impact. The additional energy is consumed by system damping through a larger orbit amplitude, so an energy balance is retained. The following expression computes the work done (energy) in the X direction. Ignore work done in the Y direction, because it does not change from impact, since the equations of motion are uncoupled.

$$\text{Work} = \int_{0}^{4\pi} Me\omega^2 \cos(\theta) \frac{dx}{d\theta} d\theta$$

where

Between impacts:

$$\frac{dx}{d\theta} = [(RDN \cdot NS - \zeta RN \cdot NC)\cos(RDN \cdot \theta) - (RDN \cdot NC + \zeta RN \cdot NS)\sin(RDN \cdot \theta)]\exp[-\zeta RN\theta] - AN \cdot \sin(\theta - \phi N)$$

During impact:

$$\frac{dx}{d\theta} = [(RDI \cdot IS - \zeta RN \cdot IC)\cos(RDI \cdot \theta) - (RDI \cdot IC + \zeta RN \cdot IS)\sin(RDI \cdot \theta)]\exp[-\zeta RN\theta] - AI \cdot \sin(\theta - \phi I)$$

The integrals were evaluated by making use of integration by parts.

$$uDv = D(uv) - vDu$$

The resulting expressions are not recorded here because they are quite lengthy and believed to be of questionable value to the reader. However, computational results comparing work done with and without impact and varying system damping are included in Table 1.

For the model used in this analysis, rotor vibration from impact occurs only in the X direction since cross coupling effects were not

Table 1. Work Done in "X" Direction after Two Shaft Revolutions (In·Lb).

	No Impact		With Impact	
Zeta	*Work* (in·lb)	*X Amp* (mils)	*Work* (in·lb)	*X Amp* (mils)
0.31	4.13	2.05	7.36	4.20
0.27	3.64	2.07	8.79	5.30
0.21	2.93	2.09	9.08	6.10
0.14	2.00	2.12	6.07	6.12
0.04	0.62	2.13	1.71	6.25
0.0	0.0	2.13	0.0	6.25

included. An actual half frequency whirl is shown in Figure 6 occurring on an overhung compressor with pressure dam bearings. It can be seen that the inner loop of the orbit is altered in both the X and Y directions by the impact indicating the presence of cross coupling effects. Mathematical investigations to date show the level of gyroscopics present with this overhung compressor has little effect on orbit shape but the bearing cross coupling stiffness terms are significant.

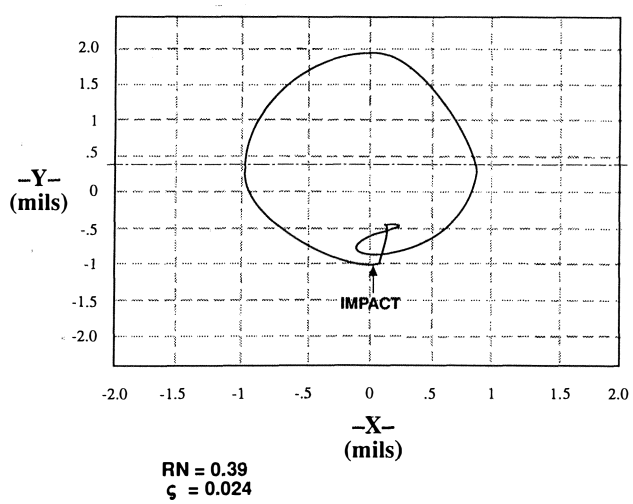

Figure 6. Actual Half Frequency Whirl Orbit.

The following are a few comments about the effect on the half frequency whirl orbit from the amount of rotor-to-stationary part interference and stiffness of the stationary contact. As should be expected, the greater the interference, the larger the response. The amount of interference used in the present analysis was 0.30 mils, to obtain vibration levels observed in the field. As to the stiffness of the stationary contact, the stiffer the contact, the more the impact critical speed is moved above half running speed. This results in more energy added during impact and a larger response. There must be an upper bound to this trend, probably when the stiffness is high enough to bring in the next higher critical speed. But it would seem that the most practical direction for improvement is to soften the stationary contact stiffness.

CONCLUSIONS

The present analysis indicates that if a rotor operates above the second critical speed and incurs a light rub with a stiff stationary part, it will very likely exhibit half frequency whirl with an enlarged whirl orbit. As long as the condition does not degrade into a hard rub, the resulting orbit can remain bounded and be a stable equilibrium. Results of this study show that the following criteria can be used to prevent or reduce sensitivity to half frequency whirl from light rubs.

TO PREVENT:

- Keep the rotor-bearing first critical speed above half of running speed.
- Eliminate possible contact.

If the above conditions cannot be met, then the only option left is to try and reduce half frequency whirl response.

TO REDUCE RESPONSE:

- Soften the possible contact points through the use of such things as spring back seals. This lessens the critical speed increase during impact which reduces the energy added from impact.
- Maximize system damping. It does not appear possible to add enough damping to prevent half frequency whirl, but any additional damping will help. However, it appears that system damping must exceed a critical damping ratio of 0.20 before significant improvement is obtained.

Most rotors prone to half frequency whirl from bounce are usually also susceptible to instability problems. It is fortunate that the above criteria on first critical speed to running speed ratio and system damping are the same criteria for improving rotordynamic stability in the presence of aerodynamic and fluid dynamic excitation.

For comparison, the troublesome machines mentioned earlier had the following frequency ratio's and system damping:

Between bearing hot gas expander	RN=0.43 & ζ=0.210
Between bearing compressor	RN=0.41 & ζ=0.006
Overhung compressor	RN=0.39 & ζ=0.024

Throughout this study, the first critical speed of the rotor was used in evaluating frequency ratios. This was done because of everyone's familiarity with the term critical speed, and in most cases this gives the proper result. However, the frequency ratios RN, RI, RDN, and RDI are really the undamped and damped first natural frequency of the rotor at running speed divided by running speed. This is frequently the same as the rotor's first critical speed over running speed, but not always. A subsynchronous natural frequency when at running speed can exist and not appear in a critical speed analysis. This occurs when a natural frequency tracks with speed but never coincides with running speed. Consequently, the output of a stability program evaluated at running speed should always be used to confirm the correctness of using the first critical speed of the rotor in meeting the above limits.

NOMENCLATURE

NC,NS	Constant coefficients in homogeneous solution before impact
IC,IS	Constant coefficients in homogeneous solution before impact
ω	Operating or running speed
t	Time
e	Unbalance eccentricity
M	First critical speed modal mass of rotor
KEQ	Equivalent rotor/bearing stiffness
CEQ	Equivalent rotor/bearing damping
Ks	Impact stiffness of stationary contact
a	Centered rotor clearance to impact point

AN	Unbalance amplitude before impact
ϕN	Unbalance phase before impact
AI	Unbalance amplitude during impact
ϕI	Unbalance phase during impact
KB	Stiffness during impact
AK	Static offset of whirl orbit during impact
RN	First undamped critical speed before impact over running speed
RI	First undamped critical speed during impact over running speed
ζ	Critical damping ratio
RDN	First damped critical speed before impact over running speed
RDI	First damped critical speed during impact over running speed

ADDITIONAL EQUATIONS USED

KB=KEQ+Ks

AK=Ks•a/KB

$RN=(KEQ/M)^{1/2}/\omega$

$RI=(KB/M)^{1/2}/\omega$

$ZETA=CEQ/(2\cdot M\cdot RN\cdot\omega)$

$RDN=(RN^2-(\zeta\cdot RN)^2)^{1/2}$

$RDI=(RI^2-(\zeta\cdot RN)^2)^{1/2}$

$DENN=(1-RN^2)^2+(2\cdot\zeta\cdot RN)^2$

$DENI=(1-RI^2)^2+(2\cdot\zeta\cdot RN)^2$

$NPC=e\cdot(RN^2-1)/DENN$

$NPS=e\cdot(2\cdot\zeta\cdot RN)/DENN$

$IPC=e\cdot(RI^2-1)/DENI$

$IPS=e\cdot(2\cdot\zeta\cdot RN)/DENI$

$\phi N=ATAN(NPS,NPC)$

$\phi I=ATAN(IPS,IPC)$

$AN=(NPC^2+NPS^2)^{1/2}$

$AI=(IPC^2+IPS^2)^{1/2}$

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