ASSESSING ROTOR STABILITY USING PRACTICAL TEST PROCEDURES

by

Kenneth E. Atkins

Senior Project Engineer

Engineering Dynamics, Incorporated

San Antonio, Texas

and

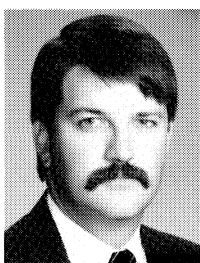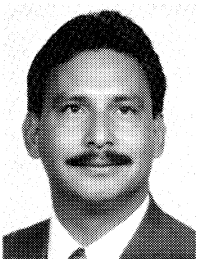**Robert X. Perez**

Senior Reliability Engineer

Citgo Petroleum Corporation

Corpus Christi, Texas

Kenneth E. (Ken) Atkins is a Senior Project Engineer with Engineering Dynamics, Incorporated, and has experience in performing lateral and torsional critical speed analyses, rotor stability analyses, and the evaluation of structural vibration problems using finite element methods. He has been actively involved in field troubleshooting of a wide variety of rotordynamics, structural, and piping problems.

Mr. Atkins received a B.S. degree in Engineering Science from Trinity University in 1978. He is a member of ASME and a registered Professional Engineer in the State of Texas.

Robert X. Perez is a Senior Reliability Engineer for the Citgo Corpus Christi refinery. He has previously written and has been a coauthor on several papers in the field of machinery vibration. His primary duties include troubleshooting rotating equipment problems, leading root-cause failure analysis teams, and serving as the plant consultant in the areas of vibration analysis and rotordynamics. He was instrumental in the implementation of a computer-based vibration monitoring program, and was the first in his plant to promote the use of Weilbull Analysis techniques to analyze mechanical failure data.

Mr. Perez is a registered Professional Engineer in the State of Texas and a member of ASME and the Vibration Institute. He received his BSME degree from Texas A&M University, and an MSME from the University of Texas.

ABSTRACT

Perturbation testing of an industrial centrifugal compressor in an at-speed balancing chamber is discussed in PERTURBATION TESTING FOR ASSESSING ROTOR STABILITY. The instrumentation and data acquisition techniques are described, as well as test results from five different rotor-bearing configurations. The test results were used to choose a bearing design which optimized the rotor stability characteristics. A method determining when full scale perturbation testing is warranted, based on risk management assessment techniques, is described in WHEN IS PERTURBATION TESTING WARRANTED? Methods of quantifying risks are discussed, as well as determining which machines are good candidates for testing.

PERTURBATION TESTING FOR ASSESSING ROTOR STABILITY

The evaluation of rotor stability is an important aspect of turbomachinery rotor design. Self- excited rotor instabilities have been responsible for millions of dollars of downtime, maintenance costs, and lost production. For this reason, much effort has been directed to understanding and predicting rotor stability performance. As a result, rotor stability calculations have improved significantly in recent years, with much research effort devoted to understanding the destabilizing effects of oil ring seals, impeller-diffuser interaction, labyrinth seals, fluid film bearings, etc.

In conjunction with the improvements in mathematical models and computer techniques for calculating rotor behavior, improvements in test equipment (transducers, signal analyzers, portable computers, etc.) have made it practical to obtain meaningful measurements concerning rotor stability on full scale industrial machines. Here, a description is given of a series of perturbation tests performed on a high speed centrifugal compressor in an at-speed vacuum balance chamber. The equipment used was readily available and involved relatively little additional cost over the normal high speed balancing procedure. Additional time in the vacuum test stand would be eight hours or less.

Test Procedures

The rotor tested was a five stage centrifugal compressor designed for 2,800 hp at 12,000 rpm (Figure 1). The design includes five shoe load-on-pad (5SLOP) bearings, which are intended to improve rotor stability (Figure 2). This rotor is typical of a large number of process compressors which operate above their second critical speed and can experience excessive vibration due to self-excited nonsynchronous instabilities. The unstable mode generally corresponds to the first forward whirl mode or damped eigenvalue that, depending on the relative stiffness of the shaft and the 5SLOP bearings, has an undamped two-dimensional mode shape, as shown in Figure 3. Although many factors influence stability, significant insight into the system stability can be gained by evaluating the rotor alone in its bearings. Rotor stability is usually expressed in terms of the logarithmic decrement or "log dec" [1]. The term "mechanical log dec," or δ_M, will be used to refer to the stability characteristics of the rotor alone in its bearings with no other destabilizing effects from aerodynamic loading, seals, etc., considered. The δ_M of the rotor-bearing system can be calculated from

the computer model and, as will be shown here, measured using practical techniques.

The basic concept of the test was to operate the rotor at speed in a vacuum-balancing chamber and then attempt to excite the first forward whirl mode by sweeping a controlled excitation source in the expected frequency range. Similar tests on laboratory test rotors have yielded meaningful results [2]. Full scale tests on operating machines have also been conducted [3]. It was felt that perturbation testing on a rotor on the test stand before installation would provide meaningful data that would help optimize the design.
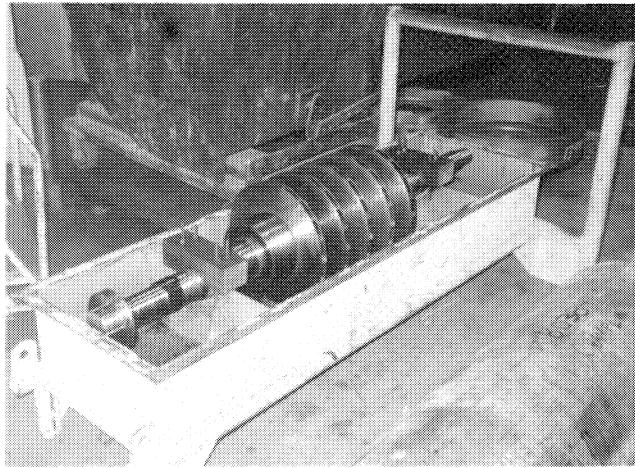

Figure 1. Five Stage Centrifugal Compressor Rotor—2,800 HP @ 12,000 RPM.
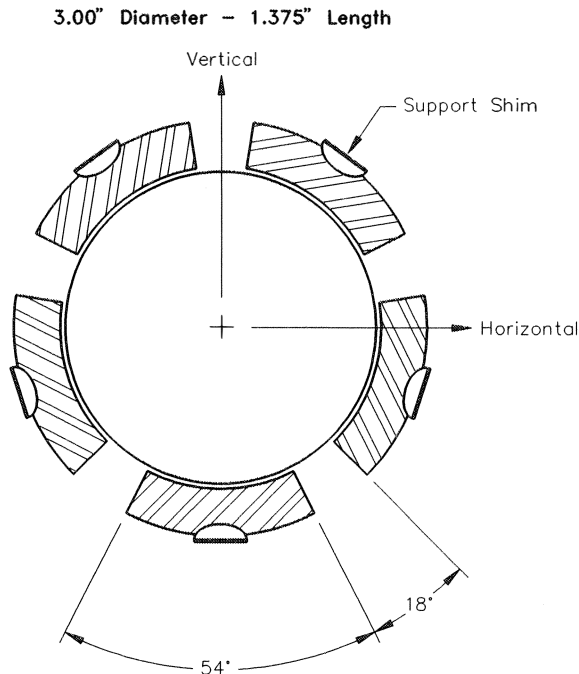

Figure 2. 5 SLOP Tilt Pad Journal Bearing.

Hardware

The rotor was prepared for normal at-speed balancing in the vacuum bunker (Figure 4). Appropriate bearing retainers were
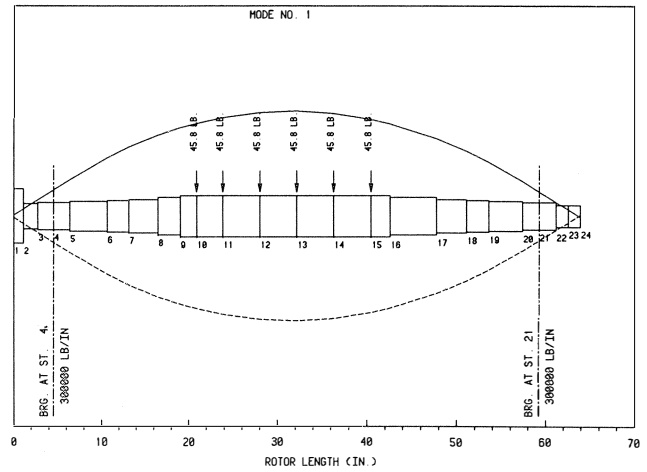

Figure 3. Mode Shape at First Natural Frequency.

machined to fit the balance machine pedestals. Noncontacting proximity probes were installed to measure the vibrational response at the standard probe surfaces near each radial bearing. Two additional probes were mounted near the rotor midspan, since the expected mode shape would have the highest amplitude at this location.
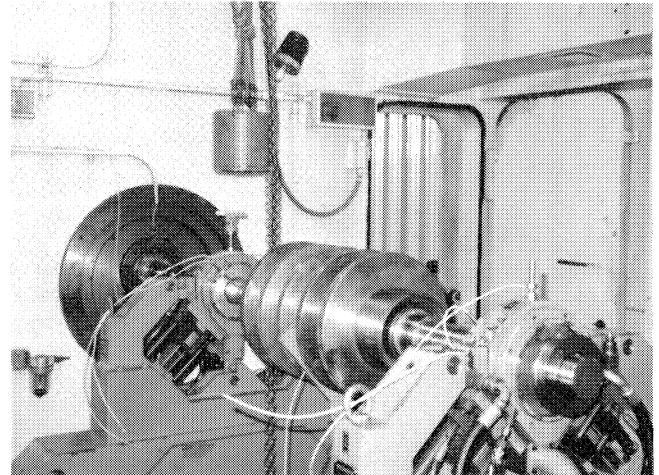

Figure 4. Rotor Mounted in Vacuum Balance Bunker.

A small electrodynamic shaker was used to provide the excitation force. A mounting bracket was fabricated to attach the shaker to one of the bearing housings. A load cell was used to measure the input force. A photograph of the shaker installed on the bearing housing is shown in Figure 5.

The shaker was capable of generating 50 lb (p-p) of force from 5.0 to 20,000 Hz when driven by an appropriate audio amplifier. The actual frequency range of interest was approximately 60 to 85 Hz, based on the rotor's calculated damped eigenvalues. The frequency sweep was provided by a signal generator with ramp input capability.

The remainder of the test hardware included an FFT analyzer, a digital frequency counter, an FM tape recorder, appropriate signal conditioners and power supplies for the force and displacement transducers, and a portable 386 class microcomputer for instrument control and analysis of the acquired data. A schematic of the instrumentation setup is shown in Figure 6.
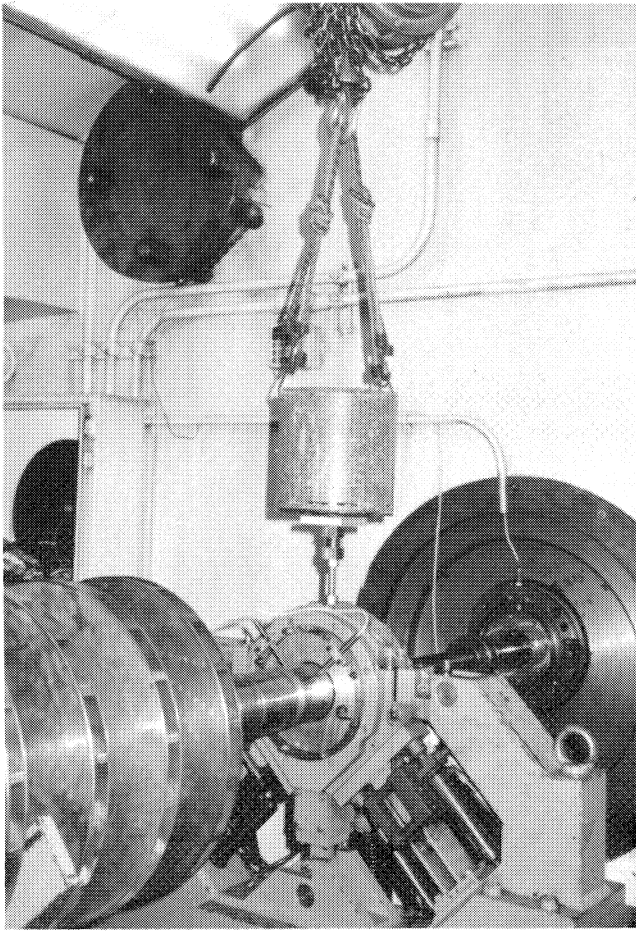
Figure 5. Shaker Mounted on Inboard Bearing Pedestal.

Figure 6. Instrumentation Schematic.

Software

The test software consisted of data acquisition and data analysis modules. With the rotor spinning at selected (constant) speeds, the shaker frequency was ramped through the range of interest. The data acquisition functions involved programming the system controller as a tracking filter to measure the transfer function (displacement/force) amplitude and phase as a function of shaker frequency.

The acquisition software was used to store the frequency response functions at each speed on disk files. The analysis software used was a modal analysis package [4] capable of curve fitting the measured data and estimating the system damping ratio, along with other modal parameters. Rotor stability was evaluated by computing the mechanical log dec (δ_M) from the estimated modal damping as shown in Equation 1.

$$Q_m = \frac{1}{2DR} \qquad (1)$$

where: Q_m = system quality factor or amplification factor

$DR = \frac{c}{c_c}$ = critical damping ratio – from modal curve fits

The mechanical log dec (δ_m) was then estimated from Equation 2.

$$\delta_m = \frac{\pi}{Q_m} \text{ (based on single degree of freedom system)} \qquad (2)$$

Test Results

The perturbation tests were conducted on both the main rotor and the spare rotor with various bearing configurations. The 5SLOP bearings were of the ball and socket design, with provisions for adjusting preload by changing the shim thickness behind the spherical seat of each pad (Figure 2). The pads were machined with a specified bore, and various bearing configurations were obtained by utilizing different sets of shims. Since the spare rotor had slightly undersized journals, this added another variable. The bearing configurations tested are summarized in Table 1.

Table 1. Summary of Rotor-Bearing Configurations Tested.

Test Number	Assembled Clearance (mils)	Preload	Rotor	Shaker Location
1	7.0	0.30	Main	Inboard
2	7.0	0.30	Main	Outboard
3	5.0	0.50	Main	Inboard
4	8.0	0.20	Main	Inboard
5	6.0	0.45	Spare	Inboard
6	8.5	0.23	Spare	Inboard

As noted in Table 1, the majority of the tests were performed with the shaker mounted on the inboard bearing housing. One test was performed with the shaker on the outboard bearing to verify that the test results were basically insensitive to the choice of which bearing was excited. This is thought to be due to the mode shape of interest which has approximately equal modal amplitudes near the bearing locations.

The frequency response functions were measured for several bearing configurations and are shown graphically in Figures 7, 8, 9, 10, and 11. A Bodé plot (amplitude and phase *vs* speed) was generated at each rotor speed for each bearing configuration. These plots generally showed the classical amplitude and phase *vs* frequency characteristics. It was observed that the frequency peaks became sharper at higher rotor speeds, indicating reduced effective damping. This correlates with both calculated and measured
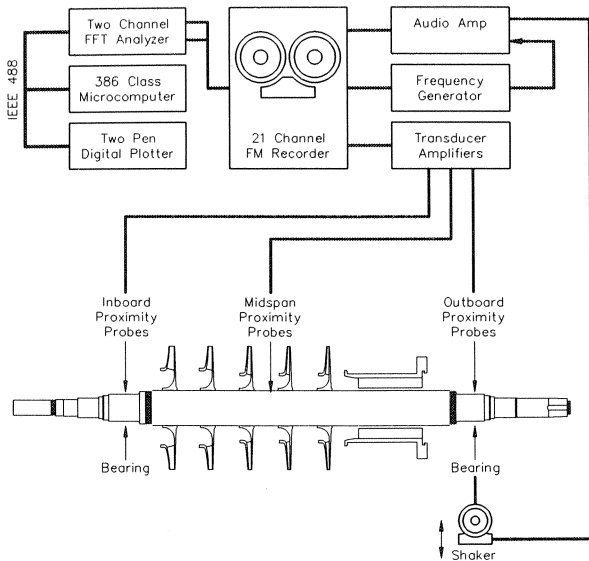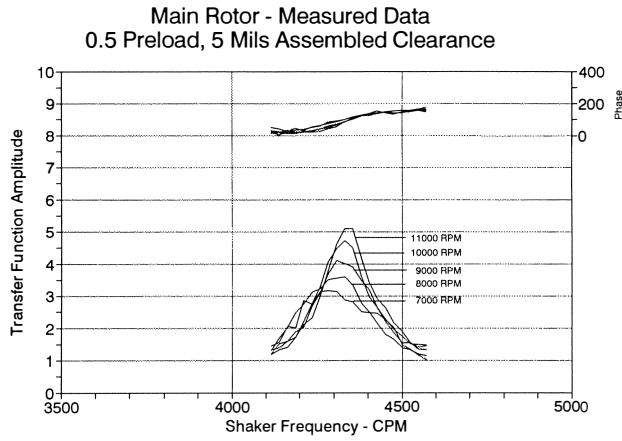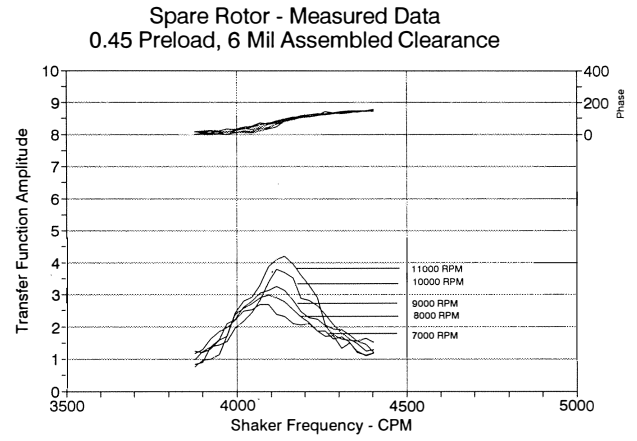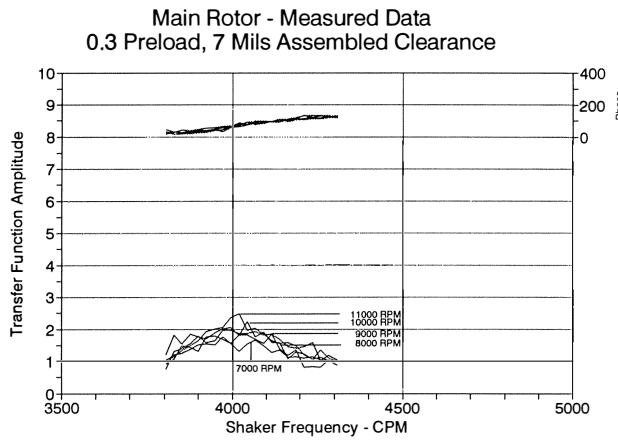
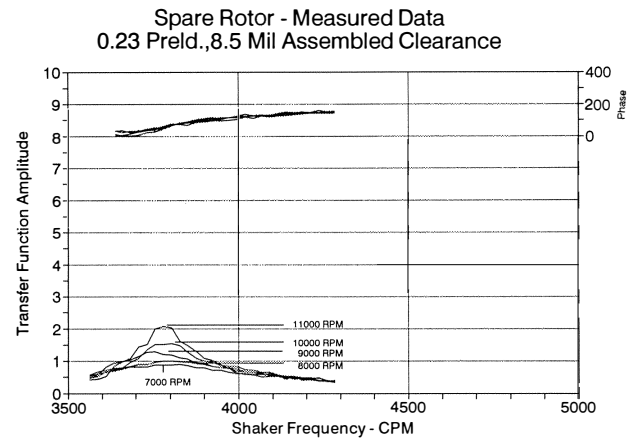*Figure 7. Main Rotor, 5.0 Mil Bearings, 0.5 Preload.*

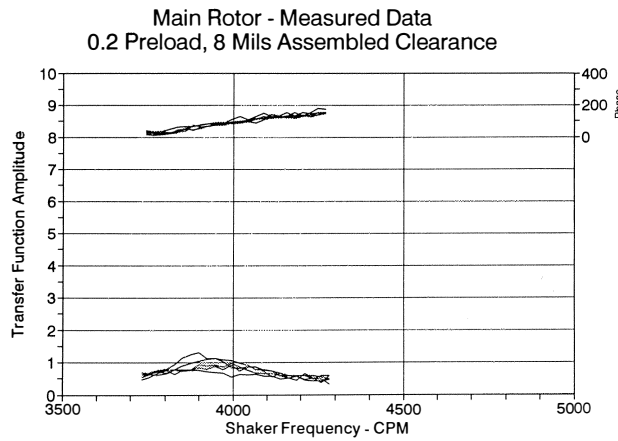*Figure 8. Main Rotor, 7.0 Mil Bearings, 0.3 Preload.*

Figure 9. Main Rotor, 8.0 Mil Bearings, 0.2 Preload.

behavior of rotors as the log dec decreases with increasing speed until the threshold speed is reached, at which the rotor becomes unstable.

Another key observation from the measured results is that the effective damping improved with higher clearance and lower preload bearings. This also agrees with the authors' experience in both calculating and measuring stability characteristics of rotors of this type.

Figure 10. Spare Rotor, 6.0 Mil Bearings, 0.45 Preload.

Figure 11. Spare Rotor, 8.5 Mil Bearings, 0.23 Preload.

The modal analysis software was used to curve fit the measured frequency response functions and compute the damping ratio for each case. Plots of the curve fit data are included in Figures 12, 13, 14, 15, and 16. The modal parameters are presented in Table 2, along with δ_M values computed using a computer model of the rotor and its bearings. The general trends in the stability characteristics in the absence of aerodynamic effects were in close agreement.
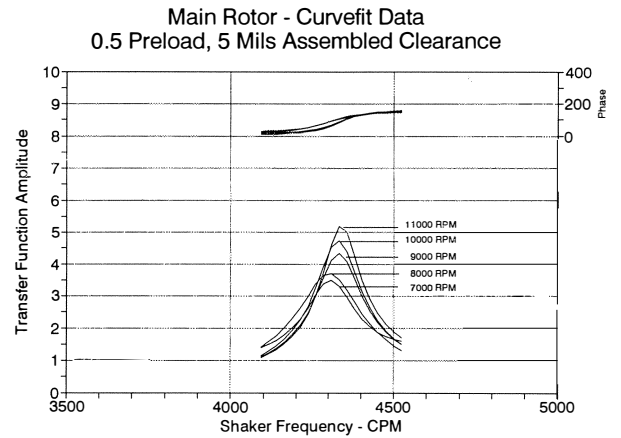

Figure 12. Curvefit Data, Main Rotor with 5.0 Mil Bearings.

Table 2. Summary of Test Results.

Rotor	Clearance/ Preload (mils)	Speed (rpm)	Peak Freq. (Hz)	Damping %	Q	δ_m
Main	5.0/0.50	7000	71.7	1.73	28.9	0.109
		8000	71.7	2.14	23.4	0.135
		9000	72.2	1.58	31.6	0.099
		10000	72.1	1.53	32.7	0.096
		11000	72.3	1.30	38.5	0.082
Main	7.0/0.30	7000	67.6	3.71	13.5	0.233
		8000	67.5	3.09	16.2	0.194
		9000	67.3	3.16	15.8	0.199
		10000	67.7	2.80	17.9	0.176
		11000	67.3	2.51	19.9	0.158
Main	8.0/0.20	7000	63.7	6.13	8.16	0.385
		8000	64.9	4.27	11.71	0.268
		9000	66.0	4.46	11.21	0.280
		10000	65.0	2.37	21.10	0.149
		11000	65.3	2.78	17.99	0.175
Spare	6.0/0.45	7000	67.9	2.33	21.5	0.146
		8000	68.1	2.54	19.7	0.160
		9000	68.5	2.13	23.5	0.134
		10000	68.7	1.84	27.2	0.116
		11000	68.9	1.64	30.5	0.103
Spare	8.5/0.23	7000	63.4	5.43	9.21	0.341
		8000	63.5	4.23	11.82	0.266
		9000	62.6	3.51	14.25	0.221
		10000	63.0	2.50	20.00	0.157
		11000	63.0	1.95	25.64	0.123

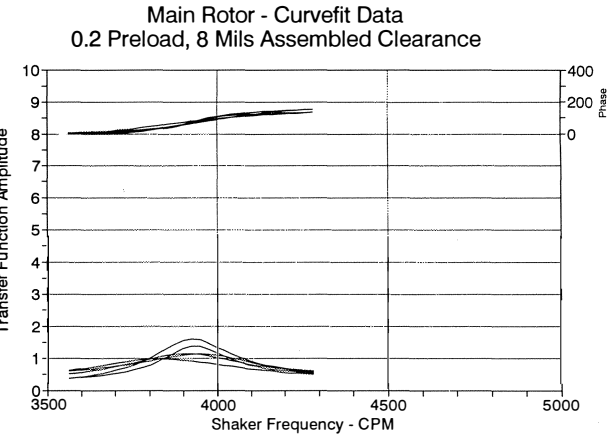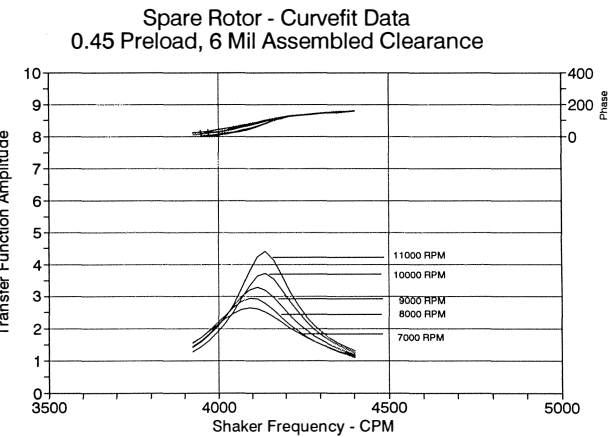

Figure 14. Curvefit Data, Main Rotor with 8.0 Mil Bearings.

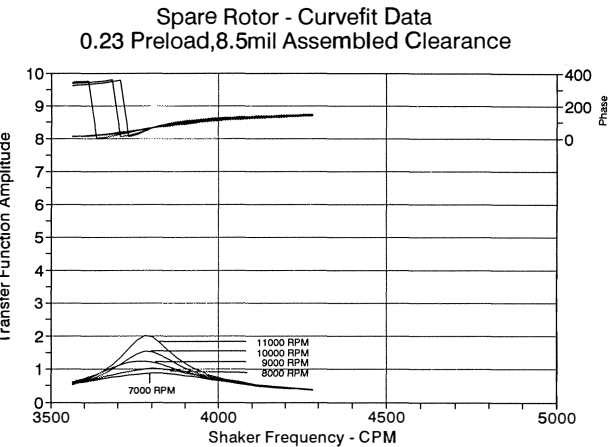Figure 15. Curvefit Data, Spare Rotor with 6.0 Mil Bearings.

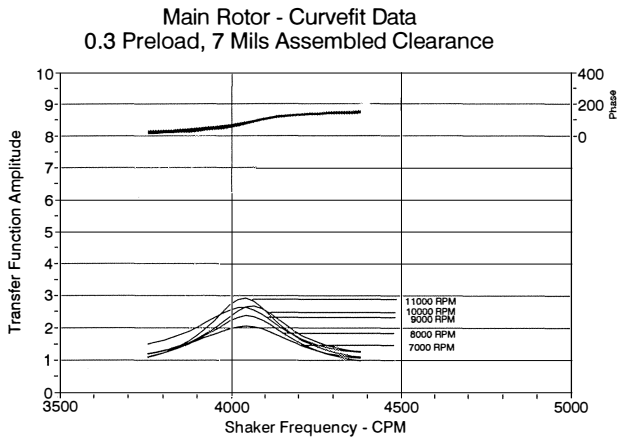Figure 13. Curvefit Data, Main Rotor with 7.0 Mil Bearings.

Figure 16. Curvefit Data, Spare Rotor with 8.5 Mil Bearings.

One other interesting comparison was made between the measured results obtained using hard pedestals or soft pedestals. The majority of the tests were conducted using hard pedestals, which were felt to represent the installed rotor configuration. When the balance machine pedestals were in the "soft" setting, the peak frequency and apparent damping were reduced and the vibration amplitude increased, as can be seen by comparing Figures 17 and 18.

Conclusions

Based on the test and analysis results presented herein, the following conclusions are made concerning the perturbation test procedures:

• The perturbation excitation technique for determining the rotor stability margin was practical in that the test equipment used was readily available and would be applicable to most at-speed balancing facilities. Relatively little additional cost over the normal balancing procedure would be incurred. For this rotor, the additional time in the vacuum test stand was less than eight hours per rotor.
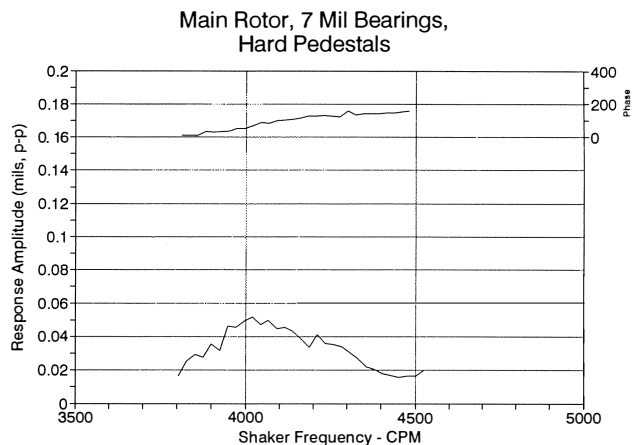
Main Rotor, 7 Mil Bearings,
Hard Pedestals

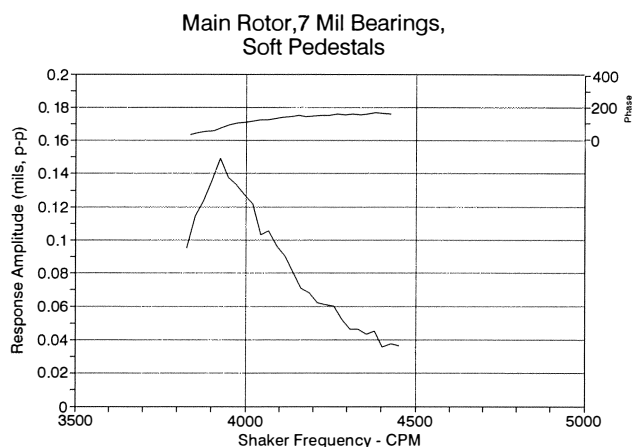Figure 17. Main Rotor, 7.0 Mil Bearings, Hard Pedestals.

Main Rotor,7 Mil Bearings,
Soft Pedestals

Figure 18. Main Rotor, 7.0 Mil Bearings, Soft Pedestals.

• Analysis of the tests results verified that the stability (δ_M) calculations agreed favorably with the measured results. The results showed trends for improved stability with higher clearance and lower preload bearings.

• The procedure provided a means for optimizing bearing parameters based on measured data. This increased the level of confidence that the best bearing (for stability) was selected.

WHEN IS PERTURBATION TESTING WARRANTED?

Managing Risk

The two major roles of managers in maintenance organizations involve the judicious use of resources to reduce maintenance expenses and maximize the availability of process equipment. While equipment maintenance expenses are readily analyzed and understood, the impact of equipment reliability on maintenance expenses is not always clear. One means of addressing equipment reliability and its impact on the maintenance and process bottom line is to review the magnitude of risk present in the equipment population. Risk, which is defined as exposure to the chance of injury or loss, is an intangible consequence of operating process equipment. However, there are some categories of equipment that represent higher levels of risk than others. For example, a high pressure vessel in a corrosive environment represents a higher risk than a low pressure vessel in a mild environment.

Although risk assessment techniques are relatively new to the process industry, this methodology has been employed by the aerospace, defense, and nuclear industries for decades [5]. Due to the potentially enormous costs associated with catastrophic failures, such as personal injury lawsuits, loss of company goodwill, and government fines, the process industry is now applying risk analysis techniques in the design of new plants. These techniques are now finding their way into the fields of reliability and maintenance engineering. To keep up with this evolution, plant engineers must be aware of these methods and determine if they are worthwhile in their sphere of responsibility.

One type of risk that is present in every plant with high speed equipment is rotordynamic instability. Those who have experienced instability problems know that the downtime involved can be costly in terms of production losses and modification costs. With the trend of today's equipment towards higher operating speeds, higher operating pressures, and more lightly constructed shafts, along with the move toward gas seals, the risk of rotor instability is greater now than ever.

To assess the risk of instability, plant engineers should review equipment that falls in the following categories:

• Unspared Process Compressors—Equipment outages in this category have the highest impact on process unit availability, and can result in considerable economic losses.

• Compressors with a History of Stability Problems—These are compressors which have known problems, but are not completely understood.

Compressors that have been or are about to be modified—These compressors are potential problems, especially if the modifications are destabilizing in nature (i.e., new bearings, gas seals, etc.).

• New Equipment—Again, equipment in this category represents potential problems.

Compressors that fall into one or multiple categories should be regarded as candidates representing sizable economic risk. While compressors in these categories are not automatically at risk, the plant engineers should address them when conducting a comprehensive risk assessment audit.

Quantifying Risk

To quantify risk, one must know the frequency that an event occurs per unit time, if the unit is spared or unspared, and the cost of the event. Frequency is usually expressed in the number of events per unit time, such as 0.5 failures/yr, and the cost of the event should include the maintenance cost of repair and the production losses resulting from the event. If, for example, an unspared compressor goes unstable once every 100 years, the frequency of this event is 0.01 events per year. If the cost of the repair and process outage is $1,000,000, then the annual economic risk of having a rotordynamic instability problem for such a machine is:

$$0.01/\text{yr} * \$1,000,000 = \$10,000/\text{yr}.$$

If the frequency of instability related problems increases, then annual economic risk also increases. This frequency (λ) can be obtained from historical data. Consider, for example, a machine that experiences problems after runs of one year, five years, and three years. By plotting this data on a hazard plot (see Figure 19), several conclusions can be obtained. First, the MTBF (mean time between failures) can be extracted, which in this case is 2.8 years. The second conclusion resulting from the hazard plot is that the failures are random and not related to wearout. The inverse of the best fit line slope on the hazard plot is known as the shape parameter, denoted as β. A $\beta = 1$, implies that failures are occurring randomly. A $\beta > 1$ implies a wearout failure mechanism is responsible for failures, and a $\beta < 1$ suggests that the failure rate decreases

with time. A β of 1.0, in this example, suggests that these incidents of instability are perhaps related to random changes in process conditions or random surge events. If, on the other hand, the failure distribution suggests that the failure mode is related to some form of wearout (i.e., $\beta > 1.0$), the equipment engineer should be spurred to investigate the failure mechanism in detail before pursuing a purely rotordynamic solution. Perhaps, if a wearout mechanism is identified, the part which is wearing out can be modified, and the equipment run length can be extended. Once the MTBF is obtained, the economic risk can be calculated. Note that λ is simply 1/MTBF. (This method of statistically analyzing failure data has been suggested by Corley [6].)

Example of Cumulative Hazard Plot
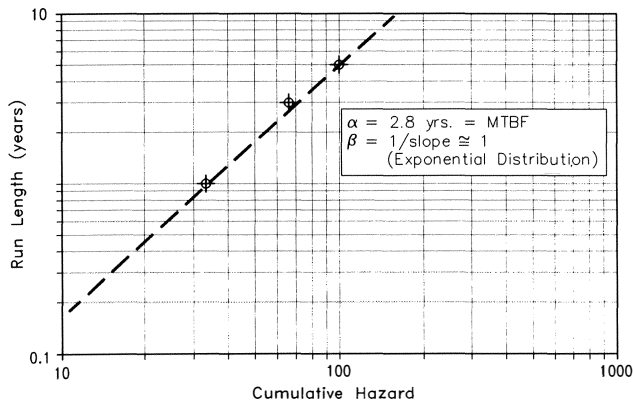### Outages are Due to Instability

Figure 19. Cumulative Hazard Plot.

If a plot of λ *vs* cost per event is constructed, a plot similar to Figure 20 is obtained. In this type of plot, an acceptable risk line is drawn to provide a means of assessing risk. For example, management may determine that an annual economic risk of $100,000/yr is acceptable per machine. A line matching these criteria is drawn in Figure 20. In this manner, it can be easily determined if a machine falls above or below the acceptance criterion of $100,000/yr. In the example above, with the MTBF of 2.8 yrs, assuming the same cost per event of $1,000,000, the annual economic risk is

$$1/2.8 * \$1,000,000 = \$360,000/yr.$$

which is well above the accepted maximum of $100,000. In this example, the plant engineer should actively pursue rotordynamic design modifications to eliminate the instability problem.

Instability Assessment Techniques

The above quantitative techniques are useful for evaluating risk if historical information is available, but if no historical data is available, or if the machine is new, then other analytical means of assessing the risk of instability are required. There are several methods, ranging from those that are rudimentary to rigorous, available to those evaluating new or modified equipment. For example, a rough estimate of rotor stability can obtained by reviewing several simplified rotordynamic parameters. These include:

$A = K_v/K_h$ bearing asymmetry ratio

$\eta = N/N_{cl}$ critical speed ratio

$K = 2K_b/K_s$ stiffness ratio

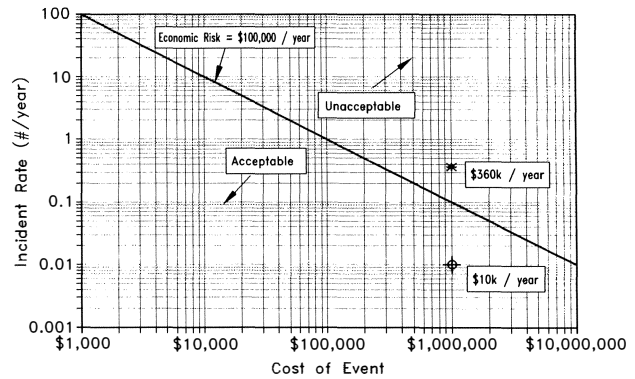## Risk Assesment Curve
Determining When Risk is Acceptable

Figure 20. Risk Assessment Curve.

$D = 7372\, C_b/(W_m N_{cl}\, (1 + K))$ damping factor

$AF = (1 + K)\, (1 + D^2)/D$ amplification factor

$S = (4/K)\, (\,|1-A\,|)$ bearing stability parameter

Where the terms are defined:

K_s = shaft stiffness, lb/in

AF = amplification factor

K_b = bearing stillness, lb/in

C_b = bearing damping, lb-sec/in

W_m = modal weight, lbs

N_{cl} = critical speed, cpm

N = operating speed, rpm

δ = logarithmic decrement

The following guidelines for these rotordynamic parameters should promote stability and increase the probability of successful operation:

- The critical speed ratio should be less than two
- The amplification factor should be less than five
- The stiffness ratio should be less than two
- The bearing stability parameter should be greater than 0.8.
- Values of S between 0.1 and 0.8 are considered marginal.

Another nonrigorous method of evaluating a rotor design was proposed by Kirk and Donald [7]. These researchers reviewed commercially available machines that operated successfully, along with those that were unsuccessful, in terms of their stable operation. This was done to determine if key parameters could be found to predict the likelihood of successful operation. They found that the parameters of N/N_{cl} (critical speed ratio) and $\Delta P \times P_2$ (pressure parameter) were key in determining the probability of success. The plot in Figure 21 was generated using actual data from back-to-back machines. Using this plot, a user can determine if a given machine with a given critical speed ratio (N/N_{cl}) and pressure parameter ($\Delta P \times P_2$) will have a tendency to be stable or unstable. This plot is based on new machines built to manufacturer's specifications, and it assumes that oil seal effects are insignificant.

As an example, a variable speed straight through machine with a maximum critical speed ratio of 3.33, a maximum ΔP of 500 psi, and a maximum discharge pressure of 1,700 psi was considered. Three operating points were investigated: point "A" corresponds to operation at 8,000 rpm; point "B" corresponds to operation at
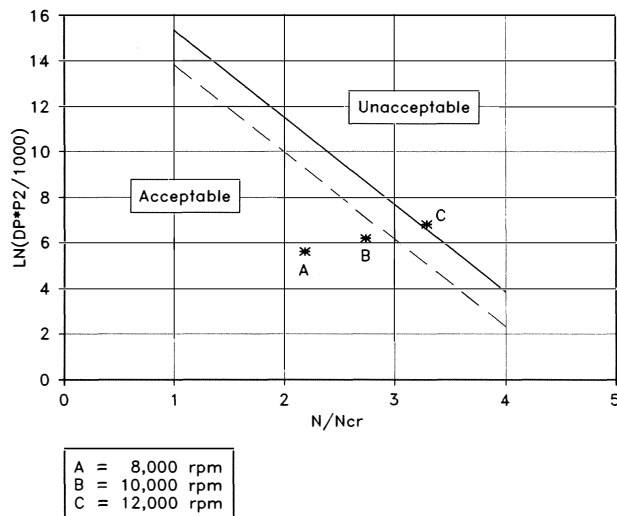
Figure 21. Stability Design Criteria.

10,000 rpm; and point "C" corresponds to operation at 12,000 rpm. Operation below the solid line shown in Figure 21 will probably result in stable operation, and operation above the line will probably lead to unstable operation. The operating condition denoted by point "C" is clearly in the region where instability is considered likely. The dotted line was generated by using values of $0.75 \times N/N_{cl}$ for corresponding $DP \times P_2$ values. This denotes a region of questionable operation. Operation below this 75 percent line will provide an extra margin of safety. The user may wish to change this margin from 75 percent to 80 to 90 percent. In this example, additional analyses may be warranted to determine if the risk of instability is truly significant.

While the Kirk/Donald method is not rigorous, it does provide users a means of evaluating unproven equipment. The authors have found that some machines with histories of instability were questionable when reviewed using this method.

The most accurate analytical technique of determining the margin of stability is the computer stability analysis. This method, which utilizes a modified Myklestad-Prohl procedure, calculates the damped eigenvalues and the growth factor. The analysis results are typically plotted in terms of the logarithmic decrement *vs* either speed or aerodynamic cross-coupling (Figure 22). If the calculated log dec is found to be greater than 0.1, including all destabilizing
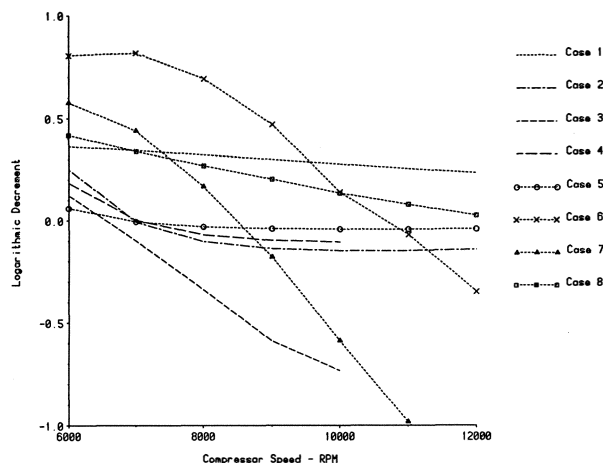

Figure 22. Stability Map.

effects, then the probability of stable rotor operation is considered to be high. However, one must remember that this is still only an estimate of the actual rotor log dec. It is known that the actual rotor log dec can be affected by factors unknown to the analyst, such as bearing misalignment, lack of support stiffness, seal misalignment or seal eccentricities, etc.

Candidates for Rotor Perturbation Testing

After conducting an economic risk audit of critical machines, how does the plant engineer decide which compressors are the most likely candidates for at-speed perturbation testing? The most obvious candidates for perturbation testing are new critical rotors that are already earmarked for high speed balancing. During the balancing procedure when the rotor is in the vacuum bunker, it can readily be instrumented for perturbation testing. As previously mentioned, it is estimated that the stability testing can be done in approximately eight hours.

Other candidates for stability testing are those machines that represent a high economic risk. As discussed above, machines with a history of instability, or those that are to be significantly modified, should be investigated in detail to determine the risk of instability. This investigation may be analytical or a measurement of actual log dec. In some cases, both analytical and field techniques may be used if the economic risk warrants the duplication of analysis.

Conclusions

Using the aforementioned methods of quantifying risk, a user can compare these risks with the cost of analytical and actual tests to determine if they are economical. This burden of assessing specific equipment risks, determining when risk is unacceptable, and determining when a detailed investigation is warranted inevitably falls on the plant engineer's shoulders. In most cases, however, the cost of these techniques are minor, when compared to the economic impact of instability in unspared process compressors.

REFERENCES

1. Lund, J. W., "Stability and Damped Critical Speeds of a Flexible Rotor in Fluid Film Bearings," ASME Paper No. 73—DET—103, Design Engineering Technical Conference, Cincinnati, Ohio (1973).

2. Muszynska, A., "Modal Testing of Rotor/Bearing Systems," Excerpted from Course Notes, Senior Mechanical Engineering Seminar, Bently Nevada Corporation, Minden, Nevada (June 1987).

3. Fujii, H., Hizume, A., Kanki, H., Ichimura, T., and Yamamoto, T., "Solving Nonsynchronous Vibration Problems of Large Rotating Machineries by Exciting Test in Actual Operating Condition," The International Conference on Rotordynamics, Tokyo, Japan (September 1986).

4. "STAR Reference Manual and Theory/Applications," Reference Material for the SMS Modal Analysis Software, Structural Measurement Systems, San Jose, California (1990).

5. Arendt, J. S., et. al., "A Risk Based Analysis of a Petroleum Refinery," Hazard Prevention Magazine (January/February 1985).

6. Corley, J. E., "Troubleshooting Turbomachinery Problems Using Statistical Analysis of Failure Data," *Proceedings of the Nineteenth Turbomachinery Symposium,* Turbomachinery Laboratory, Department of Mechanical Engineering, Texas A&M University, College Station, Texas (September 1990).

7. Kirk, R. G. and Donald, G. H., "Design Criteria for Improved Stability of Centrifugal Compressors," Rotor Dynamical Instability, presented at the ASME Applied Mechanics Bio-Engineering and Fluid Engineering Conference, Houston, Texas (1983).

BIBLIOGRAPHY

AIChE, Hazards Evaluation and Reliability Analysis of CPI Systems Course, Houston, Texas (March 1989).

Atkins, K. E., Perez, R. X., and Turner, D. M., "A Simple Procedure for Assessing Rotor Stability," Third International Symposium on Transport Phenomena and Dynamics of Rotating Machinery, Honolulu, Hawaii (1990).

Wachel, J. C., Atkins, K. E., et. al., Rotordynamics of Machinery, Engineering Dynamics, Incorporated, Technical Report 86-334 (1986).