

# FATIGUE DAMAGE OF STEAM TURBINE BLADE CAUSED BY FREQUENCY SHIFT DUE TO SOLID BUILDUP—A CASE STUDY

by

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## ABSTRACT

The case history is presented of a large steam turbine that experienced blade damage after about eight years of service. The probable cause and subsequent solution to the problem are addressed. A discussion of the metallurgical analysis of the damage, Campbell diagram, SAFE diagram and modal test results is provided. The damage is believed to have been caused by a solid build up at the blade root that shifted a natural frequency into an interference region, and changed the response characteristic of the bladed disk assembly.

Proper blade design with conservative stress levels and separation of natural frequency from exciting frequency is important for reliability. If solid build up is expected, or is experienced during operation, the blade design should be reviewed for possible interference, and its effect on the resulting life of the blading evaluated.

## INTRODUCTION

The case history is presented of an investigation into blade damage of a steam turbine rotor. The turbine was commissioned

in 1973 and drives a centrifugal compressor. The rotor has eleven stages producing approximately 45,000 hp at a rated continuous speed of 3830 rpm. The design operating speed range is between 2870 rpm and 4028 rpm. The damage occurred on the spare rotor after approximately eight years of service. Two of the 166 blades used on the eighth stage of the rotor were found cracked, at approximately 0.75 in radially up from the blade/root interface.

The purpose of damage analysis in such a situation is two fold. The first, and most important step is to determine the actual cause of the damage. This is often accomplished by analytical and metallurgical analysis. However, on occasions it is not always possible to conclusively determine the actual cause. On these occasions, a hypothesis into the possible cause of damage is often established. The second step requires that after the cause leading to the damage has been established, a recommendation for change in design should be made that considers all present and future requirements.

The tools to analyze blades have improved through the years and phenomenon that could not had been analyzed only a few years ago can now be examined. Frequency measurement tools and techniques have also improved and are commonly used today. In addition, a lot of experience has been gained from past design failures and successes that can be applied to understand and solve present day problems. Therefore, when troubleshooting design problems, one should take into consideration the years and condition of operating service when applying these newly developed analysis techniques. These same analytical tools should then be applied to demonstrate the reliability of the modified design.

The new concept of the SAFE diagram [1, 2] and an assessment method of relative reliability based on expected life of blading (of the original and modified blade designs) were applied to the subject case. This clearly demonstrates the need for such techniques to fully understand the cause of damage, and for a reliable solution thereof. Thus, the strategy followed in this investigation was to

- Conduct a metallurgical evaluation of the damaged pieces to determine the mechanism of damage;
- Review past operating history;
- Review the original design data and reevaluate the stresses;
- Recalculate the natural frequencies of the blades; and
- Conduct a modal test of the blades in the failed stage.

After careful assessment of these data, the cause of the damage was understood, and a design solution recommended.

## ANALYSIS

### *Metallurgical Evaluation*

Two broken blades were identified after visual inspection of the rotor. The damage occurred about 3/4 in above the root platform. There was some evidence of heavy deposits on the blades. Samples of the deposit from two locations, one at the base of the blade and the other at the midspan of the blade to shroud area, were obtained. The visual appearance of the first sample was of a mixture of black and white and the second sample was all black.

The blades of the stage were sequentially numbered from one to 166. The broken blades occupied locations 94 and 123. Both blades also occupied the end location of a six blade packet.

Magnetic particle inspection further revealed a small crack in another blade located in position 45. This blade also occupied the end location in a packet. The fracture surface of blade 94 revealed the macroscopic characteristics of high cycle fatigue

crack propagation. One crack origin was identified on the fracture surface of blade 94, while two crack origins were identified on blade 123. The fracture origin on blade 94 had a flat, smooth, brittle appearance. River markings flowing out of a flattened area at the edge of the airfoil surface were also visible. The fatigue origin on blade 123 was located close to the trailing edge of the airfoil. High cycle fatigue marks radiating from a corrosion pit were observed, as was a secondary crack that started in the vicinity of the leading edge of the airfoil. Evidence of fatigue crack progression was absent. A characteristic thumbnail in the overload area of fracture surface was exhibited. The crack in blade number 45 showed a macroscopic high cycle fatigue characteristic. It also showed the characteristic thumbnail shape. Details of high cycle fatigue were absent. The fracture surface looked as old as those observed in the other blades. Several corrosion pits on the fracture surface were observed.

X-ray electron dispersive spectroscopy (EDS) analysis was performed on the deposit samples collected. The sample from the midspan of blade to shroud area indicated the presence of iron and noncorrosive metallic elements. The EDS analysis of the second sample from the base of the blade and wheel-rim area revealed the presence of sulphur, calcium and sodium. These elements produced medium to large peaks on the EDS spectrum. The deposit was also viewed as a back scatter image and the material showed patches of low average atomic number. A spectrum of low atomic numbers showed a significant quantity of sodium, as shown in Figure 1. Sodium is considered to be an aggressive corrodant causing stress corrosion cracking. Chlorine was not discovered, even though pitting was observed on the fracture surface. However, a similar analysis done at an earlier time revealed the presence of chlorine but not sodium. A chemical analysis of the material taken from blade 123 is shown in Table 1. Hardness testing and microstructure analyses confirmed that the blade material met the required specification.

The conclusions of this evaluation were

- the blade material was within specification and no metallurgical defects were detected that could explain the damage.
- the blade failures were due to high cycle fatigue, possibly compounded by corrosion, since a primary crack was observed to grow out of a corrosion pit.

### *Review of Operational History*

It is difficult to discuss every minute detail of the operation of the turbine. However, the concern relating to the solid build up

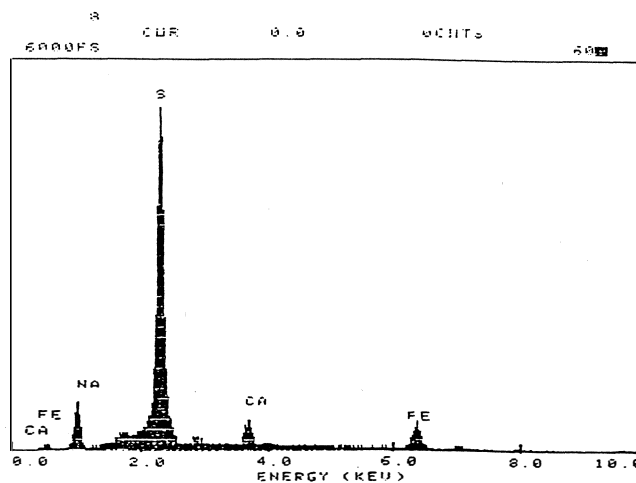


Figure 1. Low Atomic Number Material Decorating Airfoil/Wheel Rim Sample.

Table 1. Chemical Analysis Results.

| ELEMENT    | SPECIFICATION |       | BUCKET # 123 |
|------------|---------------|-------|--------------|
|            | MIN           | MAX   |              |
| CARBON     | 0.15          | 0.13  |              |
| MANGANESE  | 1.00          | 0.51  |              |
| PHOSPHORUS | 0.40          | 0.022 |              |
| SULFUR     | 0.03          | 0.002 |              |
| SILICON    | 0.50          | 0.020 |              |
| CHROMIUM   | 11.50 - 13.00 | 11.90 |              |
| MOLYBDENUM | N/A           | 0.13  |              |
| NICKEL     | 0.50          | 0.39  |              |
| ALUMINUM   | 0.30          | 0.20  |              |

was investigated. The solid buildup at the blade roots and wheel juncture could have resulted from an approximate one week period when the turbine was being operated on steam that was being desuperheated by a secondary makeup water source which contained some chemicals. This occurred one year before the blade damage.

Review of the Original Design Data

During an uprate of this machine in 1983, Campbell, SAFE and Goodman diagrams were generated. These are provided in Figures 2, 3, and 4. Review of these diagrams shows that there are no interferences between low harmonics of the running speed (up to 15 times running speed) and any of the blade natural frequencies. These diagrams also show no interference between the one times nozzle passing ( $1 \times$  NPF) excitation and the blade frequencies. The Campbell diagram shows many interferences between  $2 \times$  NPF excitation and the higher order modes of the

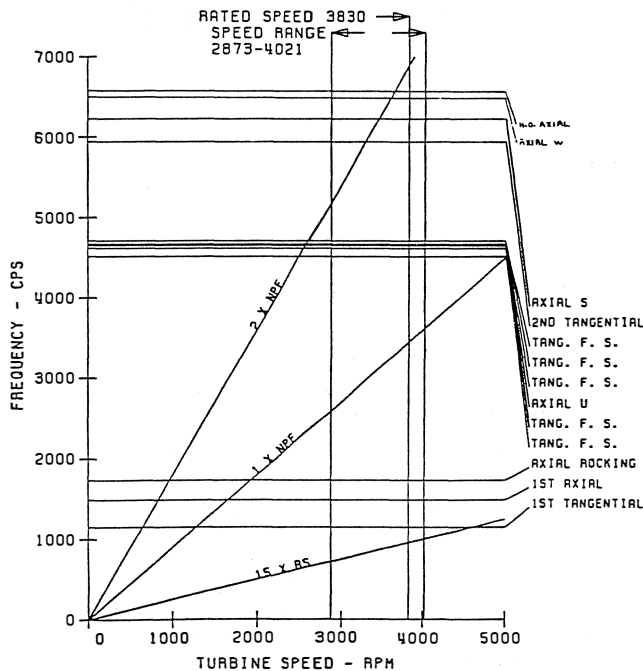


Figure 2. Campbell Diagram of Original Design Using NASTRAN.

blade. However, the SAFE diagram shows only one interference. This interference is between a higher order axial mode. Since higher order NPF excitations are weak, the calculated stress levels are low. The calculated factor of safety was higher than that required by the applicable design guideline.

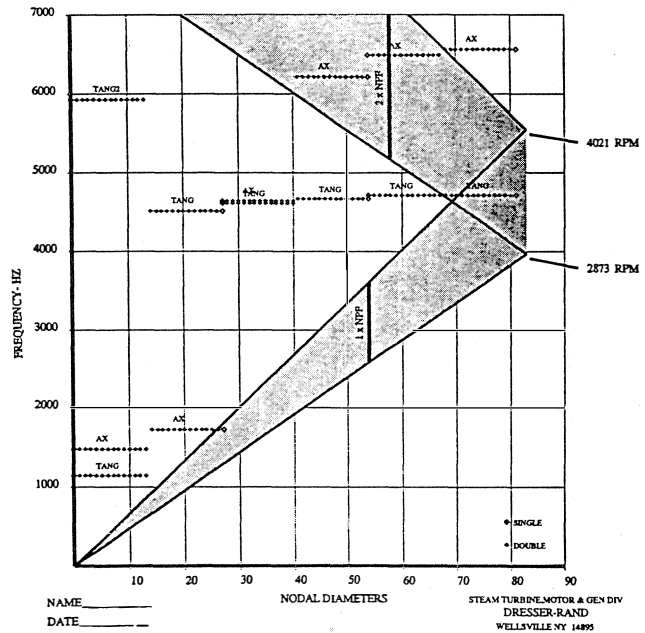


Figure 3. SAFE Diagram of Original Design.

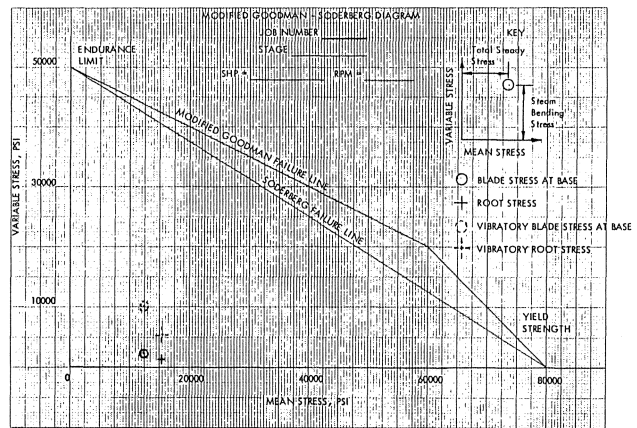


Figure 4. Goodman Diagram for Original Design.

New Analysis

The finite element program "NASTRAN" was used to calculate the natural frequencies at the time of the uprate in 1983. The analyses conducted during this investigation were performed by a general purpose finite element program "ANSYS" [3] and a finite element program "TBDYNE" [4] purposely developed for analyzing blades and bladed disk systems. A packet containing six blades was modelled in "TBDYNE" and the Campbell and SAFE diagrams generated based on the results are shown in Figure 5 and 6, respectively.

The entire bladed disk was also modelled and analyzed using "ANSYS." This model includes the flexibility of the disk, which was neglected in all previous analyses. The SAFE diagram

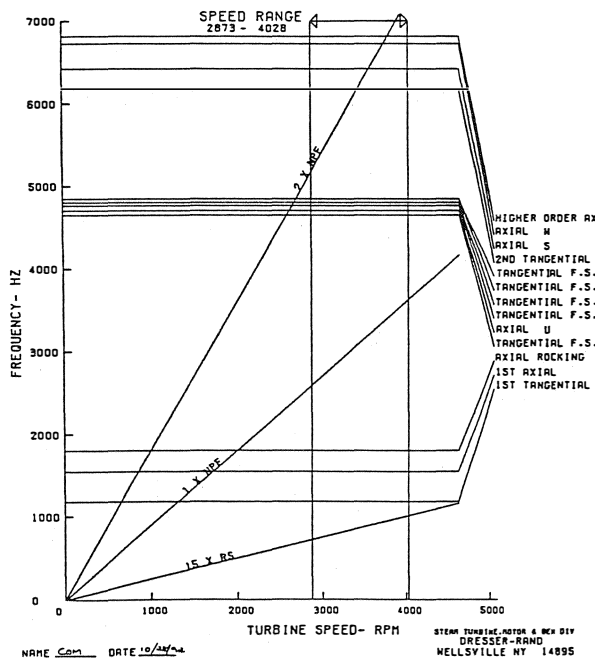


Figure 5. Campbell Diagram of Original Design Using ANSYS.

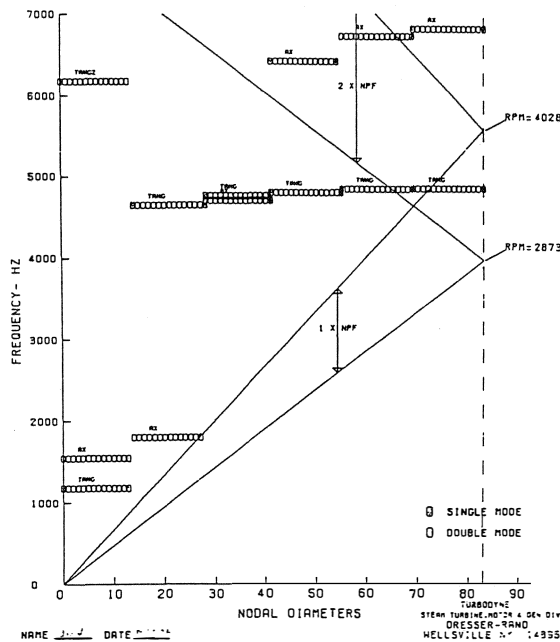


Figure 6. SAFE Diagram of Original Design Using ANSYS.

generated, based on the results of this analysis, is shown in Figure 7. A typical axial vibration mode of the bladed disk system is shown in Figure 8. This mode is identified as a three nodal diameter mode (3ND). This nomenclature of identifying mode shapes of a bladed disk system is described by Dello [5].

**Modal Testing**

A modal test was conducted on the undamaged blade packets before the blades were removed from the wheel. This was done by exciting a blade packet with an impact hammer and measuring the response with accelerometers. The modal test system

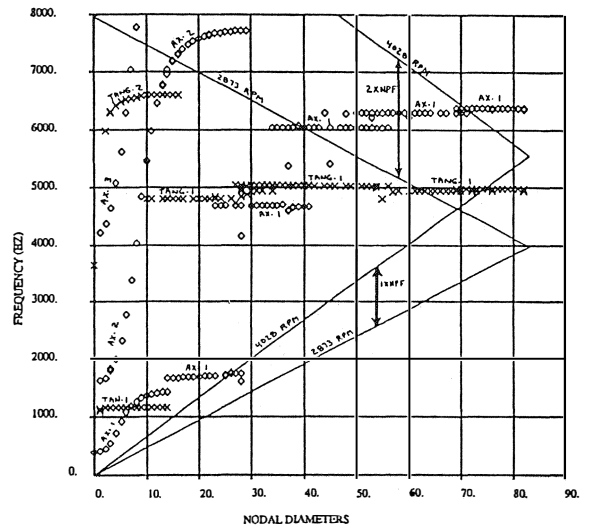


Figure 7. SAFE Diagram of Original Design Using ANSYS for Complete Bladed Disk.

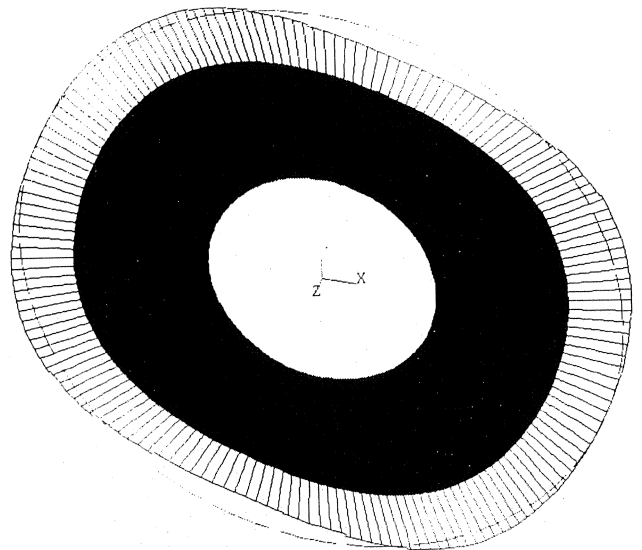


Figure 8. Three Nodal Diameter Mode of the Bladed Disk.

consisted of a spectrum analyzer, a modal data manager and a modal analysis software package running on a 330 computer. A block diagram of the modal test system is represented in Figure 9. The interrelationship between the modeshape of a packet with the overall mode shape of a disk is displayed in Figure 10, as it relates to testing. The blade packet was excited in both the axial and tangential directions. The force exerted by the hammer was measured by a force gauge. Two miniature accelerometers, one in each direction (axial and tangential) were mounted on the blade to record the response in these directions. The response was measured at the base, mid height and tip of all the blades in the packet. There were a total of 18 data points for a single six blade packet. Natural frequencies of the system are calculated at each peak of the response. Utilizing the software, the frequency response data was reduced to yield mode shapes at each discrete frequency where peaks occurred. A modal survey was also conducted at the disk circumference to identify lower frequency modes that are disk dominated. This test was performed in two directions, axial and radial, which is necessary to define the

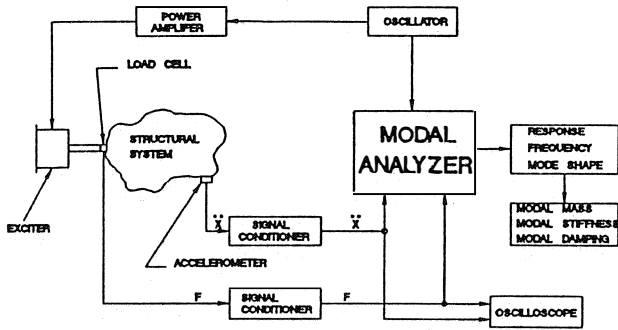


Figure 9. A System Block Diagram for Modal Testing.

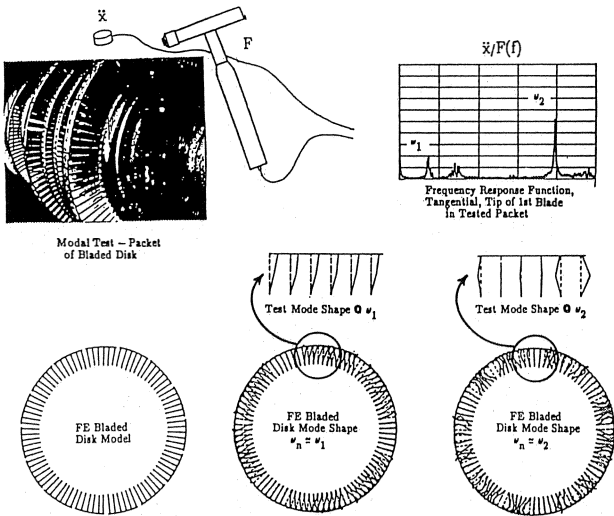


Figure 10. Relationship Between Mode Shape of a Packet with the Mode Shape of the Complete Bladed Disk.

nodal diameters and nodal circles content of a mode shape. Some representative frequency response functions (FRF) from the modal test are provided in Figures 11, 12, and 13. Two mode shapes of a single blade packet are shown in Figures 14 and 15. In Figure 14, the displacement is in the tangential direction with the displacements of all blades in phase. This mode is called the first tangential mode. The mode shown in Figure 15 is an axial "W" mode. In this mode, the displacement is in the axial direction and the shape of the displaced shroud band is in the form of a "W." The mode shape shown in Figure 16 is that of the rim of the disk. The displacement shows the characteristic of a  $\sin 2\theta$ . This mode is known as a two nodal diameter (2ND) mode. Experience with modal test results and their correlation with analytical predictions have been good, particularly for a single blade and a blade packet. In the case of packeted bladed disks, the correlation for well spaced modes has, in general, been good. However, for high density modes, although the natural frequency correlation has been acceptable, the precise definition of mode shapes poses some difficulty. The shape has been identified qualitatively which matches with analytical prediction.

**Correlation**

It is prudent to correlate the results obtained using current analysis techniques with those from the original analysis. This provides understanding of the original design and establishes a basis for using the latest technology to arrive at a modification, if needed. A direct comparison of the results of Figures 2 and 3

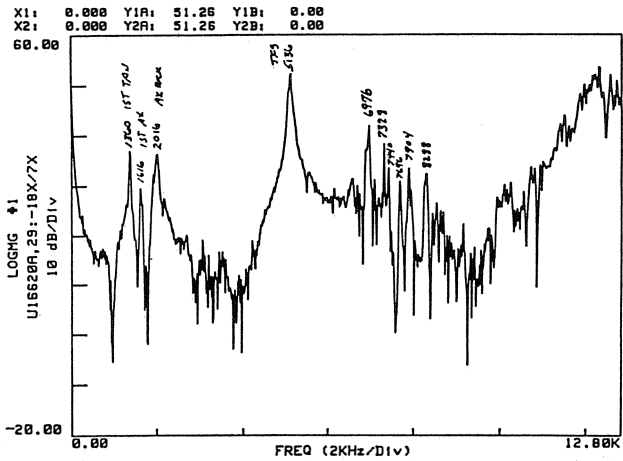


Figure 11. Tangential Response of a Packet.

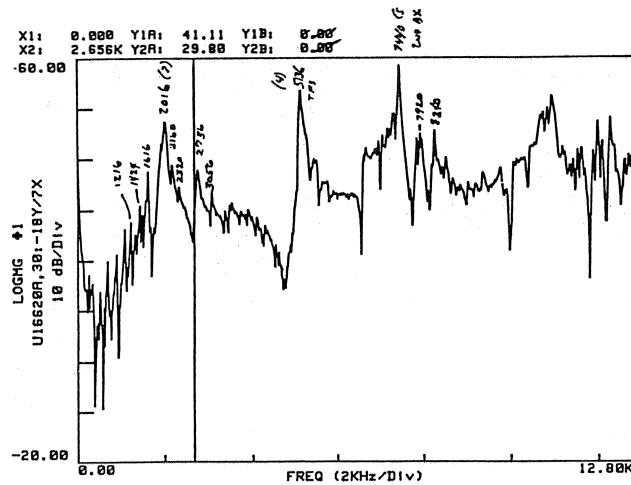


Figure 12. Axial Response of a Packet.

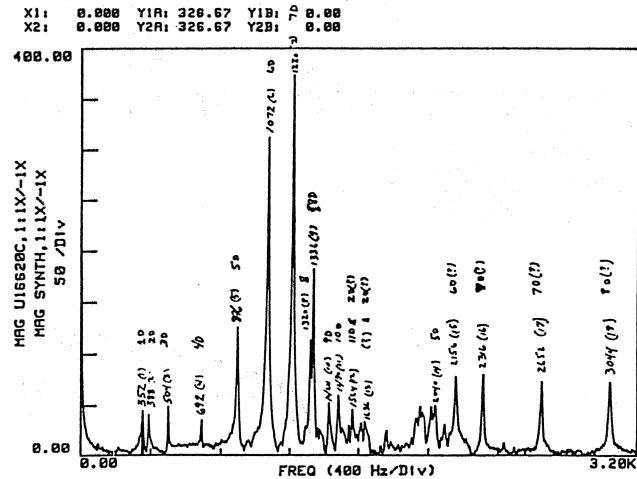


Figure 13. Axial Response of the Disk.

with those of Figures 5 and 6 show insignificant differences in the natural frequencies. The SAFE diagram of Figure 7 shows basically the same results except at the low order modes where the disk's flexibility plays a major role. Therefore, the analytical

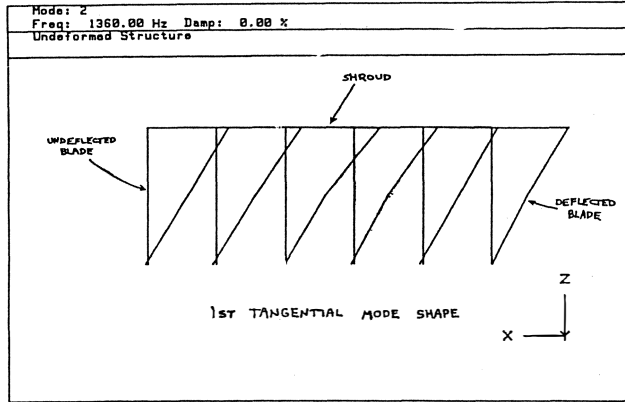


Figure 14. First Tangential Mode Shape of a Packet.

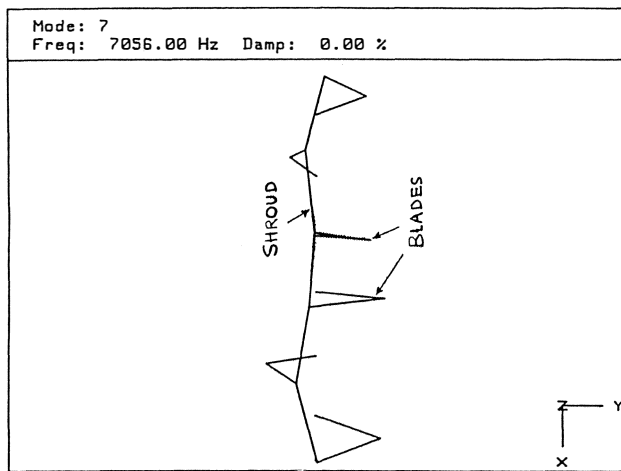


Figure 15. Axial "W" Mode Shape of a Packet.

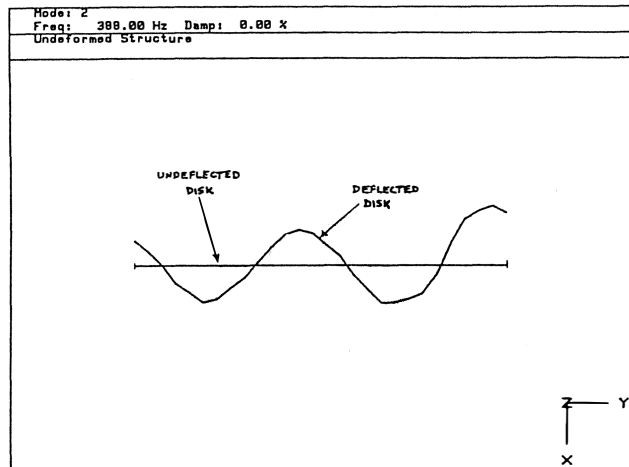


Figure 16. Two Nodal Diameter Mode Shape of the Disk.

blade frequency predictions using TBDYNE and ANSYS programs show a good consistency in comparison and basically predict the results originally obtained by using NASTRAN.

After establishing the above correlation of analytical results, the next phase was to do the same with the experimental results. A SAFE diagram is presented in Figure 17 with the modal test

results overlaid on the analytically predicted results for a six blade packet (as shown in Figure 6). The experimental results are consistently higher than the analytical results. All the analytically predicted and experimental results are listed in Table 2. Past experience of the correlation between analytical predictions and experimental results has been very good. However, such difference has occasionally been observed. This difference is due to a change in the root attachment boundary conditions with time. The clearance between the blade root and the disk groove is small, and steam deposit build up or a material corrosion/oxidation build up can fill these clearances. This can stiffen the support of the root in the groove, and hence raise the blade frequency. To investigate this possibility, a dynamic analysis was performed to see how sensitive the blade frequency is to changing root conditions. The results are listed in Table 3 for different root support conditions together with the results of the modal test. This comparison shows that the modal test results fall somewhere between the blade root being considered at the standard location (upper root lock) and the wheel outside diameter. It seems plausible that the solid deposit build up in the root could have contributed to this condition and raised the blade frequency.

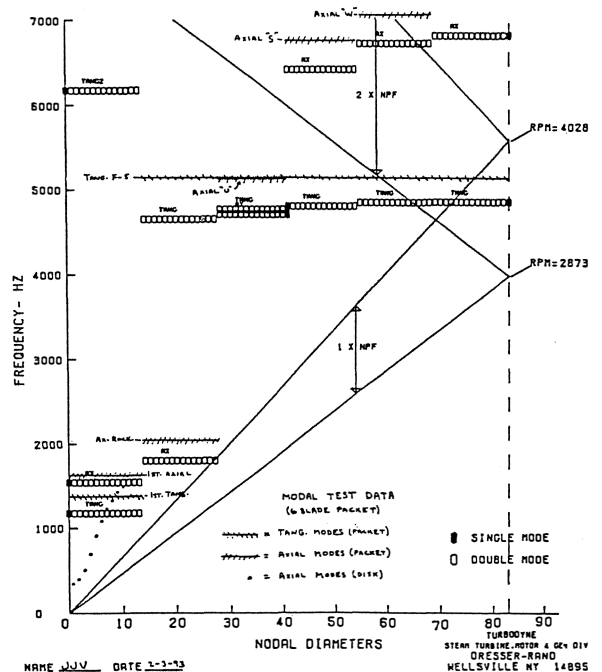


Figure 17. SAFE Diagram of the Original Design with Test Data.

The turbine at the time of damage was reported to be running at 3930 rpm. Examination of the SAFE diagram (Figure 17) indicates an interference of the blade frequency of the axial "W" mode with  $2 \times$  NPF at 3930 rpm. In summary, the following conclusions can be drawn

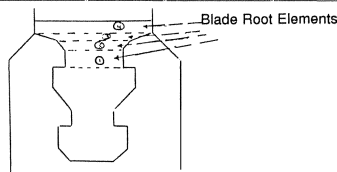
- There is a possible shift in the blade resonant frequency due to solid buildup.
- No resonance is predicted at operating speed without this shift.
- The location and direction of cracks match with the axial "W" mode shape.
- Failure was due to high cycle fatigue.

Table 2. Comparison of Blade Frequencies Estimated by Different Methods.

| FREQUENCY, HZ     | NASTRAN PACKET (1983) | TBDYNE PACKET | ANSYS BLADED DISK | MODAL TEST |
|-------------------|-----------------------|---------------|-------------------|------------|
| FIRST TANGENTIAL  | 1150                  | 1179          | 1175              | 1360       |
| FIRST AXIAL       | 1489                  | 1546          | 1430              | 1616       |
| AXIAL ROCKING     | 1736                  | 1800          | 1750              | 2016       |
| TANG. FIXED-SUPP. | 4517                  | 4655          | 4795              | 5136       |
| TANG. FIXED-SUPP. | 4616                  | 4770          | 4935              | 5136       |
| TANG. FIXED-SUPP. | 4669                  | 4808          | 4982              | 5136       |
| TANG. FIXED-SUPP. | 4709                  | 4848          | 5017              | 5136       |
| TANG. FIXED-SUPP. | 4710                  | 4855          | 5033              | 5136       |
| AXIAL " U "       | 4650                  | 4709          | 4661              | 5120       |
| SECOND TANGENTIAL | 5938                  | 6171          | 6584              | NOT FOUND  |
| AXIAL " S "       | 6227                  | 6418          | 6025              | 6752       |
| AXIAL " W "       | 6497                  | 6723          | 6280              | 7056       |
| H.O. AXIAL        | 6577                  | 6815          | 6365              | NOT FOUND  |

Table 3. Blade Frequency Variation with Root Modelling.

| FREQUENCY, HZ     | MODAL TEST | TBDYNE STD. FIXITY | TBDYNE FIXED @ ELEMENT #4 | TBDYNE FIXED @ ELEMENT #3 |
|-------------------|------------|--------------------|---------------------------|---------------------------|
| FIRST TANGENTIAL  | 1360       | 1179               | 1340                      | 1321                      |
| FIRST AXIAL       | 1616       | 1546               | 2110                      | 2075                      |
| AXIAL ROCKING     | 2016       | 1800               | 2314                      | 2280                      |
| TANG. FIXED-SUPP. | 5136       | 4655               | 5334                      | 5269                      |
| TANG. FIXED-SUPP. | 5136       | 4770               | 5531                      | 5450                      |
| TANG. FIXED-SUPP. | 5136       | 4808               | 5577                      | 5495                      |
| TANG. FIXED-SUPP. | 5136       | 4848               | 5616                      | 5534                      |
| TANG. FIXED-SUPP. | 5136       | 4855               | 5627                      | 5545                      |
| AXIAL " U "       | 5120       | 4709               | 5396                      | 5332                      |
| SECOND TANGENTIAL | -----      | 6171               | 7137                      | 7034                      |
| AXIAL " S "       | 6752       | 6418               | 7915                      | 7811                      |
| AXIAL " W "       | 7056       | 6723               | 8497                      | 8388                      |
| H.O. AXIAL        | -----      | 6815               | 8653                      | 8542                      |



- Existence of corrosion pits indicate that the steam environment decreased the fatigue strength of the blade material.
- This stage runs in wet steam which will allow the concentration of any contaminants.
- Based on the reported flow through the turbine at the time of failure, the expected steady stress was higher than design.

To make the stage more reliable for future operation, it was decided to design a blade without this sensitivity to change in resonance condition. This was achievable either by changing the number of nozzles or by redesigning the blade to change the natural frequency. A decision was made to change the airfoil to offer more reliability and provide better stage efficiency. The SAFE diagram of this design is given in Figure 18 with a minimum operating speed of 3300 rpm rather than 2870 rpm as originally specified. This is due to the fact that a change in the minimum operating speed was anticipated for future operation.

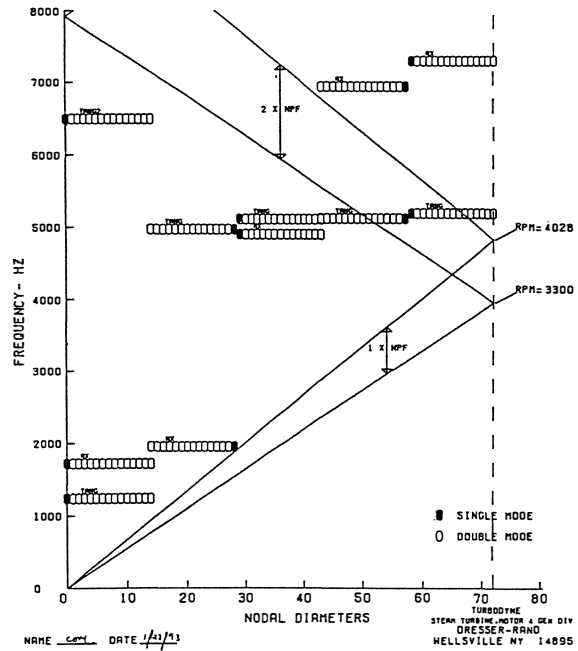


Figure 18. SAFE Diagram of the Replacement Blade.

## LIFE EVALUATION

The analysis reported in this section was not performed at the time of the blade redesign. The reason for this was that the method required to predict blade life was under development, and in the process of being correlated with past experience. At this time, the design evaluation was based on a predicted factor of safety, which is a function of dynamic stress, steady stress and material property. However, it is desirable to have an estimate of fatigue life of the design in terms of vibration cycle or, better yet, in number of years. The fatigue life of a blade depends on the operational history, the effect of the environment on material property and the steady and vibrational stresses. Thus, the accurate prediction of life in a real operational mode is difficult to determine. Nevertheless, the method can be used to compare different designs based on life assessment. Another level of uncertainty is due to the fact that nominal values of variables are used in the analysis, and a variation in limits from the nominal magnitude is possible. Since the values of these parameters are random, within the prescribed limits, the response also becomes random. Under such circumstances, the deterministic life is not very meaningful. The prediction of a probability of achieving the expected life is desired that allows a reliability evaluation of a design to be made. This type of method has been used often in design [6, 7].

The method used is a combination of Soderberg's factor of safety, the strain-life relationship of the material and a reliability type calculation. This method was used to determine the expected life of the original design after the shift in the natural frequency. The expected life of the replacement design was then compared with that of the original design. The predicted life of the damaged blade was a few days with 99 percent probability, while the life of the replacement blade was calculated to be more than thousands of years (with 99 percent probability). The result should not be taken as an absolute value but it provides a relative comparison of the damaged blade life with that of the replacement blade.

## SUMMARY

The damage investigation of the blade utilized several new design tools developed over the years. These tools helped to establish the cause of the damage and also allowed a thorough evaluation of the replacement design. The SAFE diagram was successfully used in identifying the natural frequency and its mode shape, which was in resonance with the exciting force. This was accomplished by comparing the frequency and mode shape of the bladed disk with the exciting frequency and spatial shape of the force ( $2 \times$  NPF). This analysis showed that the axial "W" mode's coincidence with the exciting force was the likely cause of damage.

Dynamic analysis of blades was performed using three different finite element programs, results of which agreed with each other. These results showed that the axial "W" mode should not be in resonance with the  $2 \times$  NPF excitation. However, the result of a modal test conducted on the damaged disk showed a higher frequency than the analytical prediction. Based on the measured frequency, a resonant condition was predicted at operating speed. However, this machine ran successfully for approximately eight years.

If a resonant condition had been present at the time the machine was first run, the damage would have occurred earlier. Therefore, a possibility of change in dynamic behavior over the years was investigated. The metallurgical investigation reported solid buildup, which could have resulted from temporary operation on steam that was being desuperheated by a secondary makeup water source that contained some chemicals. A chemical analysis of the solid revealed the possibility that sodium and/or chlorine would have been present at some time during operation of the machine. This reasoning led to an analysis to predict the influence of solid build up in the root area on blade frequency. The result of this analysis showed the possibility that solid build up can raise the blade frequency to the level that was measured in the modal test. Moreover, it is known that the presence of chemical agents like sodium and chlorine can lower the fatigue property of blade materials.

Finally, for the replacement blade, the same analytical tool was used to evaluate its reliability. The SAFE diagram shows no resonance with  $2 \times$  NPF excitation.

A newly developed method to assess blade life, not available at the time of the redesign, was used to predict fatigue life of the original and the replacement blades. The results indicate that the original blade design would likely have a few days fatigue life under a true resonant condition. However, the blade ran successfully for eight years, therefore, it is unlikely that the resonant condition existed from the beginning of operation. The change

in blade frequency might have been gradual over many years or a few days, but as soon as it coincided with the exciting frequency, the damage to the blade occurred in a couple of days.

The results of this investigation show that whenever a change from the nominal design condition is experienced, at a minimum, a quick review of the impact on blade life should be conducted. A quick modal survey at the time of a scheduled maintenance is recommended, and if the frequency has changed, the effect on blade life should be estimated.

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