METHODOLOGY OF OPEN BLADED IMPELLER RESONANCE IDENTIFICATION

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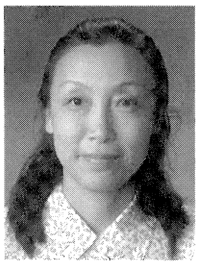

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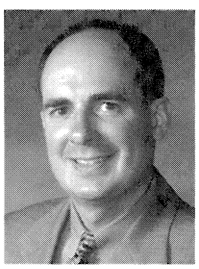

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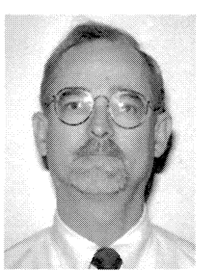

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ABSTRACT

Open bladed impellers (impellers without shrouds) with high rotational speeds are a vital component of centrifugal compressors. The impeller reliability is directly related to the compressors' successful operation and environmental safety. Assuming that the impeller is designed within the mechanical limits of its construction material, one of the most common causes of impeller failure is fatigue. There are primarily two types of fatigue failures: blade failures and disk failures. Metallurgical inspection of failed impellers can be used to identify fatigue as the root cause. It is imperative, however, that rigorous engineering analysis is applied in the initial design phase to prevent resonance that will result in impeller failures.

To meet the objective of designing reliable and trouble-free impellers, blade and disk modal analyses are of the utmost importance. Development of a Campbell diagram is a typical means of identifying potential harmonic frequencies. The blade frequencies and disk frequencies overlap each other on the Campbell diagram because of the complex blade geometry and influence of disk flexibility. This makes it difficult to identify the possible resonance.

This paper describes a practical method, which combines 3-D finite element modal analysis, Campbell diagram, Singh's advanced frequency evaluation (SAFE) diagram, and the authors' experience in an effort to prevent excitation of undesirable natural frequencies. Presented within the paper are particular modal analysis steps, which include:

- Techniques to identify possible disk resonance
- Techniques to identify possible blade and bladed disk resonance
- Techniques to improve the design

The method has been successfully used in centrifugal compressor impeller design process and failure cause analyses. A case study in which these methods are applied is given as an example.

INTRODUCTION

The typical arrangement of a centrifugal compressor incorporates a diffuser section immediately downstream of impeller tip. In an effort to obtain additional pressure recovery from the air stream, aerodynamacists often incorporate vanes within the diffuser passage. This arrangement causes impellers to experience fluctuating forces resulting from nonuniform fluid flow through the stationary vanes (diffuser vanes). These fluctuating forces can excite natural frequencies within the impellers, resulting in resonance of the impeller disk and/or blades. This resonance can lead to fatigue of the impeller material and result in failure of the part. Figure 1 shows the typical arrangement of impeller and diffuser in the stage.

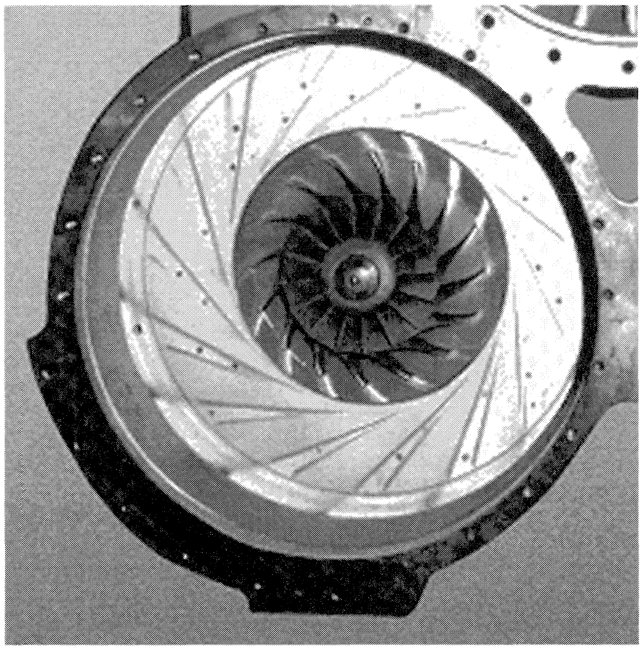

Figure 1. Impeller and Diffuser.

Since fluctuating forces in combination with steady forces cause fatigue failure, the basic design consideration attempts not only to reduce the steady-state stress, but also to avoid or to minimize the dynamic stresses produced by the fluctuating forces. The best way to design an aerodynamic stage is to avoid a resonance condition all together. In troubleshooting the root cause of failure for an existing design, the excitation sources need to be identified and the impeller needs to be redesigned to avoid future resonance.

Since the geometry of the impeller is inherently complex, the natural frequency analysis requires several steps. There are a certain number of blades connected to the disk, and the disk is flexible; therefore, the blades can communicate with each other. When the impeller is excited, many blade frequencies can be excited. These frequencies are close together but the blades vibrate in different phases, subject to the flexibility of the disk.

The analysis process uses the following methods developed by the identified scholars:

- Campbell (1924)—One of the first significant contributions on the subject of disk failure was Campbell's work. In addition to pure disk modes for which attached blades partake in motion, disk flexibility can significantly affect blade vibratory modes.
- Singh (1988) uses SAFE diagram method to identify the possible blade and bladed disk resonance.
- Kushner (1983) also describes a method that can be used to prevent failures.

By combining the identified work of Campbell, Singh, and Kushner along with a forced response study, the potential resonance of the centrifugal compressor impeller can be identified and prevented. To make the analysis process easy to understand, a field case is introduced.

FIELD CASE STUDY

The impeller involved in the field failure is indicated in Figure 2. This impeller has the following data:

- Number of blades: 16
- Disk diameter: 7.6 inch
- Rotor speed: 38,740 rpm
- Number of diffuser vanes: 15

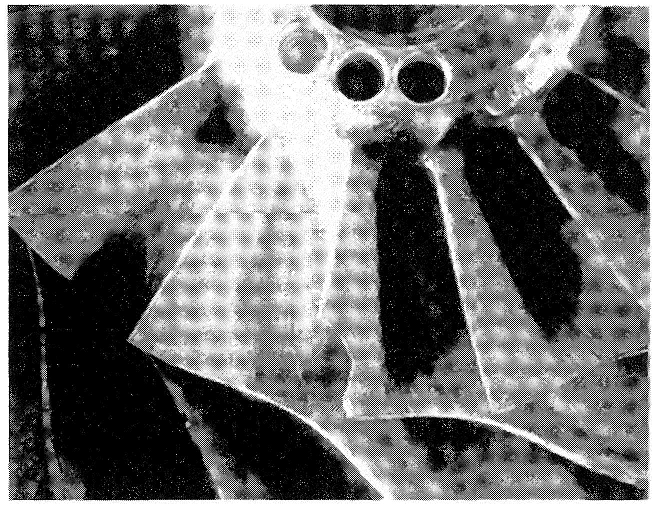

Figure 2. The Failure Pattern.

The compressor studied was operated in a highly humid and corrosive environment. The air streams were studied and found to contain high humidity and/or condensate rich in polluting substances such as chlorine, sulfur, potassium, calcium, and aluminum. The sulfur, as SO_2, creates a shape on the bladed surface, which is similar to a small spot of erosion. The same stream, however, caused corrosion over the entire air side. A metallographic sample was studied. The result showed that the blade rupture was caused by a combination of the corrosive environment and a fatigue phenomenon that originated from the corroded surface.

A second field failure occurred on the replaced impeller. The chemical analysis revealed the impeller met all requirements except the nickel requirement. The lower than specification nickel content may slightly affect the general corrosion resistance, but was not a factor contributing to the fracture. A metallographic sample showed that even with the severe corrosive environment, the cause of the failure was fatigue resulting from disk resonance. Fatigue cracks were visible along the outer edge of the impeller blades.

To understand the fatigue cracks formation principle, we must first study:

- The natural frequencies of this kind of impeller.
- The mode shapes of this kind of impeller.
- The excitation forces in this kind of compressor.

The impeller finite element analysis (FEA) model is built by the 3-D brick elements. To verify the reliability of the FEA studies, ANSYS® package is used to perform the modal analysis without the stress stiffening (centrifugal stress) effects. Ring tests are conducted to identify the impeller natural frequencies. The correlation studies are performed to identify the model adequation. The correlation coefficient for each nodal diameter can range from 98 percent to 100 percent, depending on the dimension precision of models and impellers and impeller manufacturing quality.

MODE SHAPES OF OPEN BLADED IMPELLER

Each impeller has numerous mode shapes. Some of the mode shapes will have strong effects on the impeller failures and some are not harmful. To help the reader understand the mechanism of resonance and the effect of mode shapes, each of the typical mode shapes is presented and discussed below. The mode shapes were calculated and created by ANSYS® with the stress stiffening effect.

Mode Shape

For an open bladed impeller, the mode shape is characterized by either diametrical nodal lines or by circular nodal lines, and combinations of these two. The former has much greater importance, as there have not been reported failures due to circular modes to the authors' knowledge. The first nodal diameter mode shape of this 16-blade impeller is illustrated in Figure 3. There are two phase changes in the disk, which are noted by the radial lines forming the first nodal diameter mode shape. Each nodal diameter mode shape occurs in a pair. It is easy to see that even in the very first pair of the first nodal diameter mode, there is deflection at the blade tip at inducer side. More precisely, these are blade combined disk modes, not pure disk modes.

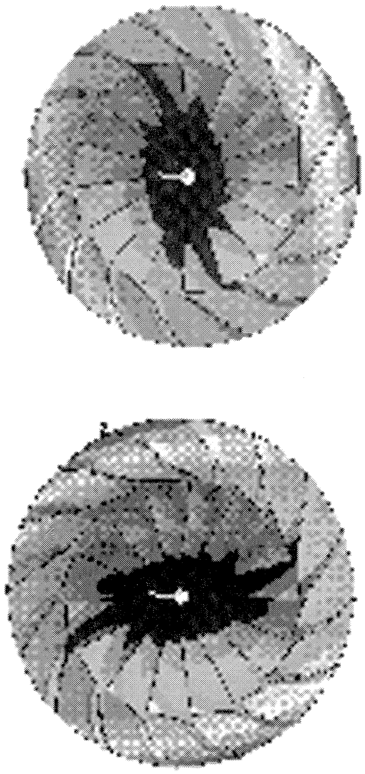

Figure 3. The First Nodal Diameter Mode.

The second nodal diameter mode shape of the same bladed disk is shown in Figure 4. The blade modes are also presented in these very first second nodal diameter modes. They are also blade combined disk modes.

Figure 5 shows the circular disk mode (or umbrella mode) and Figure 6 shows the pure blade modes. In the pure blade mode shape, there is only the blade deflection and no disk deflection.

Figure 7 shows the nodal diameter mode combined with the second circular mode shape.

The maximum number of nodal diameters in an open bladed impeller is half the number of blades for an even number of blades. For a disk having an odd number of blades, the maximum nodal diameter is half the total number of blades minus one (Singh, 1988).

EXCITATION FORCES

The excitation force in centrifugal compressors can be divided into three types.

The first type is the pressure pulse caused by the stationary vanes. That is, the forces imposed on a blade in one complete revolution emanate from the circumferential distortion in the flow. The pressure variation in the fluid flow causes the forces to vary circumferentially. These forces depend on the relative position of blading with respect to the diffuser vanes and any other interruptions such as inlet guide vanes.

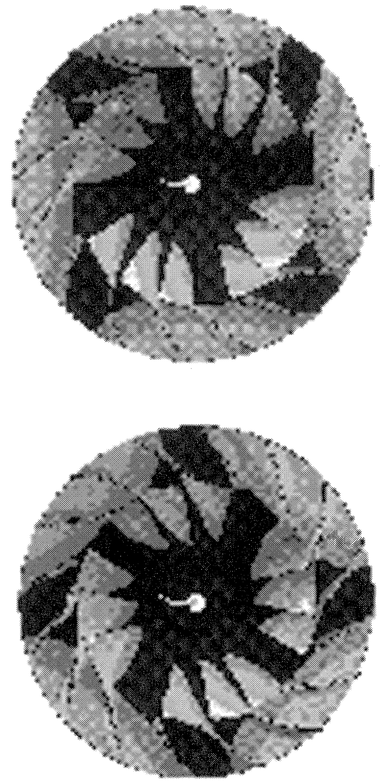

Figure 4. The Second Nodal Diameter Mode.

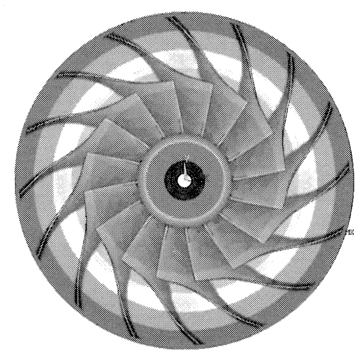

Figure 5. Circular Disk Mode.

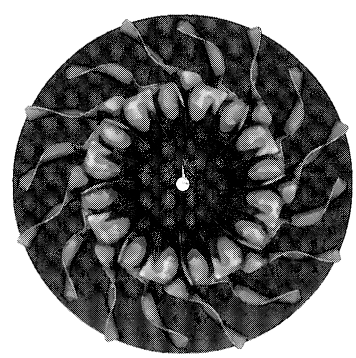

Figure 6. The Pure Blade Mode.

The second type is aerodynamic instability or excitation, such as surge or rotation stall, which can happen with or without the diffuser vane. This can cause the fundamental mode of the blade to vibrate.

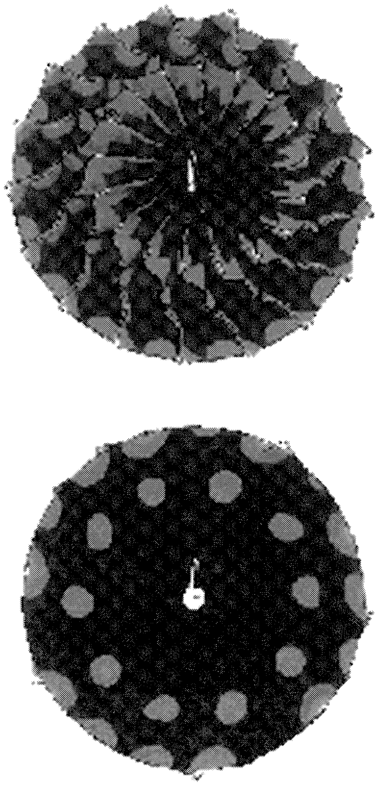

Figure 7. The Nodal Diameter Mode Combined with Circular Mode.

The third type is mechanical excitation. The system torsional vibration can be transferred to the impeller shaft causing impeller vibration.

This paper will discuss the solution for resonance caused by stationary vanes. The frequency of the harmonics of this kind of force depends on the speed of the impeller and number of interruptions in each rotation (e.g., the number of diffuser vanes) and is expressed as:

$$w = (K)\,(N)\,/60 \text{ Hz} \tag{1}$$

where:
w = Frequency
K = Number of diffuser vanes
N = Rotor speed, rpm

PROCEDURE TO CHECK THE POTENTIAL RESONANCE

Based on visual observation, the failure is a blade fracture that was caused by resonance, more precisely by the blade resonance. As we know, the pure blade modes (Figure 6) and pure disk modes, such as umbrella modes (Figure 5), cannot cause resonance because they cannot absorb the external excitation energy. The blade failure was caused by blade combined nodal disk mode resonance.

The complex geometry of this kind of impeller, even at the first several natural frequencies (Figures 3 and 4), does not have the pure nodal disk mode. It is always blade combined nodal disk mode or, in other words, the disk mode dominates blade combined disk modes. In this paper, we will only study blade combined nodal disk mode or simplify as blade combined disk mode or only as disk mode.

The purpose of establishing this procedure is to use analytical methods to identify possible resonance and reduce both blades and disk vibratory response in any general cases. The IMPRO® package was used to create the Campbell diagram, the SAFE diagram, and other figures. In the general applications, the Campbell diagram and SAFE diagram can be created by Excel® or any other graphic generating software. Any FEA package similar to ANSYS® can calculate and create the stress contour.

Step 1—Check Possible Disk Resonance

The frequencies of a single blade fixed at the root can be studied and calculated to understand the disk flexibility effect on the impeller disk and the blade natural frequencies. This assumes the disk is rigid. The results are plotted on the Campbell diagram and later compared with the natural frequencies of the complete impeller (plotted on the Campbell diagram), in order to assess the effect of a flexible disk. In this paper, we did not make this comparison.

Traditionally, the Campbell diagram is used to illustrate interference between impeller natural frequencies and the common exciting forces. Specifically, in the case of a compressor, the impeller natural frequency is plotted against the rotor running speed (Figure 8). The diagonal lines represent sources of excitation (e.g., one times diffuser vane passing frequency (1×VPF), and two times diffuser vane passing frequency (2×VPF)). This diagram predicts where the impeller natural frequencies coincide with the excitation frequencies. The Campbell diagram is a good tool to identify the disk resonance. The word "disk" here does not mean only the disk resonance, but also a combination of disk and blade resonance. As mentioned earlier, there is no pure nodal disk mode. The blade combined nodal disk mode can cause both disk failures and blade failures.

The natural frequency of a single blade of an impeller is shown as a single line on the Campbell diagram. The complete impeller has multiple lines as dark bands of frequencies representing blade modes, as shown in Figure 8.

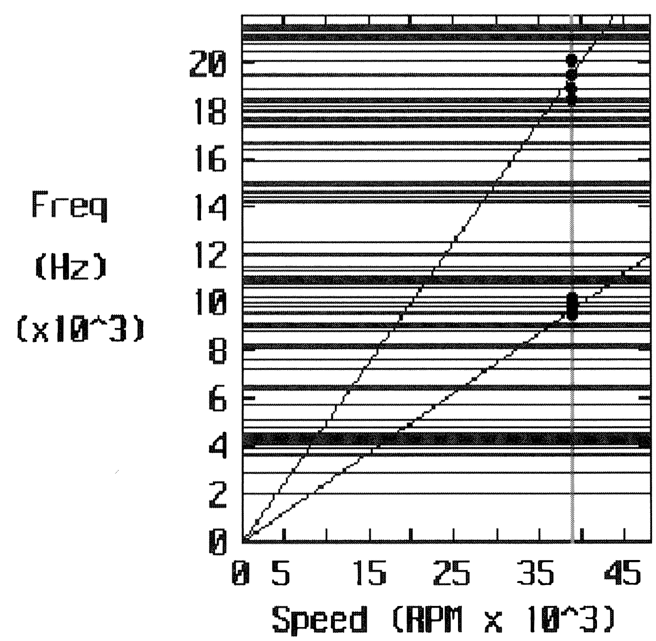

Figure 8. Example Impeller Campbell Diagram.

For this particular type of impeller, the first and second disk modes can be very harmful, due to the diffuser vanes being immediately downstream of the impeller tip. Based on Kushner (1983) and the authors' experience, the possible disk resonance can be predicted using the following method combined with the Campbell diagram.

Excitation of the first and second disk modes can be checked by comparing whether the absolute value of the difference between rotating blades and stationary vanes is equal to the number of diameter nodes and the frequency of that particular mode's interference with the exciting frequencies on the Campbell diagram.

Table 1 illustrates the technique used to identify possible disk resonance based on impeller blade number and vane diffuser number combinations.

Table 1. Potential Disk Resonance Identification.

No. of Impeller Blades	16
Rotating Speed	38,740
No. of Diffuser Vanes	15

Multiple Vane Passing Frequency	**Vane Multiplier**	**Nodal Diameter**			
		1	**2**	**3**	**4**
(1×) 9685	(1×15) 15	**−1**	−17	−33	−49
(2×) 19,370	(2×15) 30	14	**−2**	−18	−34
(3×) 29,055	(3×15) 45	29	13	**−3**	−19
(4×) 38,740	(4×15) 60	44	28	12	**−4**

Table 1 layout: the first column indicates the excitation frequency based on the multiple of the rotor rotating speed calculated by Equation (1). The second column gives the diffuser vane corresponding to each multiplier. The next group of four columns shows the nodal diameter for each multiple. The header numbers 1, 2, 3, and 4 in these columns indicate the first four nodal diameters.

To complete Table 1, use the following technique: use the vane number in column 2 and subtract the number of impeller blades from that number. For instance: 15 (vanes) minus 16 (number of impeller blades) gives the value of −1. This number is put in the first nodal diameter column.

To continue filling out the table, take the −1 from the previous calculation and subtract the number of blades (16) to obtain the −17 shown in the nodal diameter, column 2. Continue using the same method until all the values are complete.

Moving on to the second row, the calculation continues. Use the vane multiplier of 30 and subtract the number of blades on the impeller (16) to obtain a value of 14 in the first nodal diameter column. Use this number (14) and subtract the number of impeller blades (16) to obtain the value of −2 in the second nodal diameter column. Continue in the same way until the entire table is populated.

After the table is complete, change all the values to the absolute value. As stated earlier, excitation of the first and second disk modes can be checked by comparing whether the absolute value of the difference between rotating blades and stationary vanes is equal to the number of diameter nodes and the frequency of that particular modes' interference with the exciting frequencies on the Campbell diagram. This means the impeller blade and diffuser vane combination could cause first disk mode shape resonance and, consequently, any blade modes combined with the first disk mode.

Table 1 shows that the first, second, third, and fourth disk modes, and any blade modes combined with these disk modes, could be excited based on the impeller blade number and diffuser vane number combinations. These numbers are shown in bold font in Table 1. This is not a desirable situation because the lower modes have high vibratory energy. Designers should never use any combination of diffuser vanes and impeller blades that differs by one.

The results in Table 1 need to be verified with the Campbell diagram. Our experience is to use ±5 percent safety margin on the Campbell diagram to accommodate the impeller casting, machining, power instability, and other tolerance factors.

In Figure 8, there is interference between the first disk mode (first nodal diameter) and first diffuser vane passing frequency within ±5 percent safety range. All the frequencies that fall into this range are listed in Table 2. As we understand from Table 1, any first disk mode that falls into this range will be dangerous. But in this case there is one frequency with 10,134 Hz, which is 4.6 percent from the excitation frequency and is a blade combined disk mode.

Table 2. Frequencies Interference with the First Diffuser Vane Passing Frequency.

Frequencies (Hz)	**Mode Type**
9423	Third disk mode combined with blade modes
9452	Sixth disk mode combined with blade modes
9550	Fourth disk mode combined with blade modes
9560	Fifth disk mode combined with blade modes
9772	Blade modes
9992	Eighth disk mode combined with blade modes
10,105	Seventh disk mode combined with blade modes
* 10,134	First disk mode combined with blade modes
10,184	Second disk mode combined with blade modes
10,519	Sixth disk mode combined with blade modes

All the frequencies within ±5 percent of the second diffuser vane passing frequency are listed in Table 3. One of these frequencies is the second disk mode (second nodal diameter), which is 19,451 Hz and is 0.42 percent from the excitation frequency, and it is also a blade combined disk mode.

Table 3. Frequencies Interference with the Second Diffuser Vane Passing Frequency.

Frequencies (Hz)	**Mode Type**
19,443	Blade mode
18,495	First disk mode combined with blade modes
18,870	Third disk mode combined with blade modes
* 19,451	Second disk mode combined with blade modes
20,039	Blade mode
20,021	Fourth disk mode combined with blade modes

In conclusion, the procedure can be performed as:

- Create Table 1 to find out what disk modes (nodal diameter modes) can be excited by the diffuser vane passing frequencies.
- Verify this on the Campbell diagram.
- Do the disk (nodal diameter) modes, identified by Table 1, fall into the ± 5 percent range of both the first and second vane passing frequencies? If they do, there is possible disk resonance.

A similar table can be used to check possible resonance caused by inlet guide vanes.

Step 2—Check Blade and Bladed Disk Possible Resonance

Among the impeller field failures, the blade failures are not less than the disk failures. A Campbell diagram combined with a SAFE diagram is a good tool to identify the possible blade resonance.

The blades are integrated with a disk, which can influence the dynamic behavior of the blades. When the bladed disk gets into a state of vibration, there must be two simultaneous conditions present for the energy built up per cycle of vibration to reach a maximum amplitude (Singh, 1988). These two conditions are:

- The frequency of the exciting force equals the natural frequency of vibration, and
- The exciting force profile has the same shape as the associated mode shape of vibration.

The Singh's advanced frequency evaluation (SAFE) diagram, introduced by Singh (1988), shares the same vertical (frequency) axis of the Campbell diagram. But instead of the running speed on the horizontal axis of the Campbell, the SAFE diagram uses nodal diameter on the horizontal axis. Nodal diameters describe the mode shapes of the disk and also the shape of the varying force.

Figure 9 is a SAFE diagram of the example impeller. The SAFE diagram further confirms the interference with the 15 and 30 nodal diameter modes within ±5 percent at safety range. Since resonance was identified, the next step is to perform a forced response analysis.

In conclusion, the blade resonance identification procedure can be performed as:

- Create the Campbell diagram.
- Identify all impeller natural frequencies that fall into the ±5 percent range of both the first and second vane passing frequencies on the Campbell diagram.
- Create the SAFE diagram.
- Verify whether the frequencies identified by the Campbell diagram are still interfering with both the first and second vane passing frequencies on the SAFE diagram. If they are, there is possible blade or bladed disk resonance.

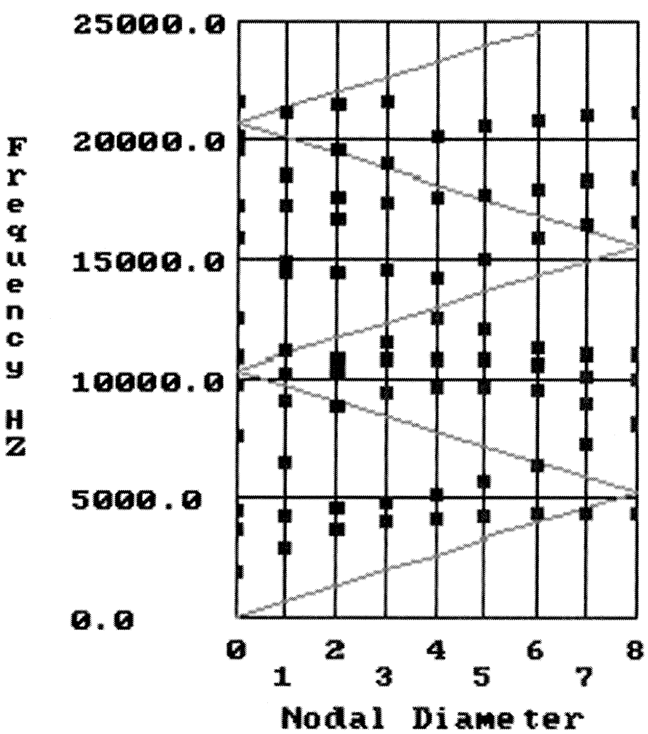

Figure 9. Example Impeller SAFE Diagram.

Step 3—Forced Response Study and Dynamic Stresses

Modal analysis results show the deflection of the disk and blades. To understand the stress distribution, the forced response study must be performed. The purpose of the forced response study is to determine the possible vibration amplitude, the alternating stress contours, and impeller failure patterns; that is, a disk failure or a blade failure.

The basic equation of motion used in the process is solved for displacement. Both disk and blade stress can then be calculated from the displacement solution.

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{2}$$

When resonance occurs, the dynamic stress amplitudes may be calculated. Dynamic stress amplitude is a function of dynamic loading, material damping, and proximity to resonance.

Dynamic loading is specific to the ratio of dynamic aerodynamic forces to steady forces, known as the stimulus ratio. Typically stimulus ratio values for low-pressure steam turbine stages are between 0.01 and 0.05 for low-pressure stages. This value can be determined experimentally through radio telemetry testing. These values are applicable to the study of centrifugal compressor impellers.

Damping helps to reduce vibration amplitude by absorbing energy. Material damping or gas dynamic damping is included in the critical damping ratio that ranges between 0.001 and 0.002. The transmissibility function is used to illustrate the effect on blade dynamic stress amplitude as the excitation and natural frequencies get closer to each other.

The formula used for transmissibility function is:

$$\sigma_d = \frac{2\zeta\,\sigma_r}{\sqrt{(1-\eta^2)^2 + (2\zeta\eta)^2}} \tag{3}$$

where:

σ_d = Dynamic stress
σ_r = Resonance stress
η = Frequency ratio ω_f/ω_n
ζ = Critical damping ratio (0.12% to 0.20%)

For this analysis, a unit force of 1.0 lb was applied uniformly at the blade pressure side. Two calculations were performed at the first and second nodal diameter interference frequencies. The results are shown in Figure 10 and Figure 11.

Figure 10 shows the dynamic stress contour of the frequency 10,134 Hz, when this mode is excited. Figure 11 shows the dynamic stress contour of the frequency 19,451 Hz, when this mode is excited.

The dynamic stress contour shown in Figures 10 and 11 indicates the highest stress locations on both leading and top edges of the blades. These dynamic stresses coincide with the origin of the cracks as shown in Figure 2. The cracks can propagate to catastrophic separation of blade section.

SOLUTION

Once the resonance is identified, either the structure must be changed or the forcing mechanism must be removed. Several techniques can be used to solve this problem. Any of the following methods can be used either alone or in combination:

- For blade resonance: any modification of the disk will not raise the blade frequencies above those calculated assuming a rigid disk. Therefore, to avoid the blade's interference with downstream diffuser vanes, something would need to be done other than a disk face modification. Changing the number of diffuser vanes, changing impeller blade geometry, or changing material may be possible remedies dependent upon rotordynamics, stress levels, aerodynamic performance, and other considerations.

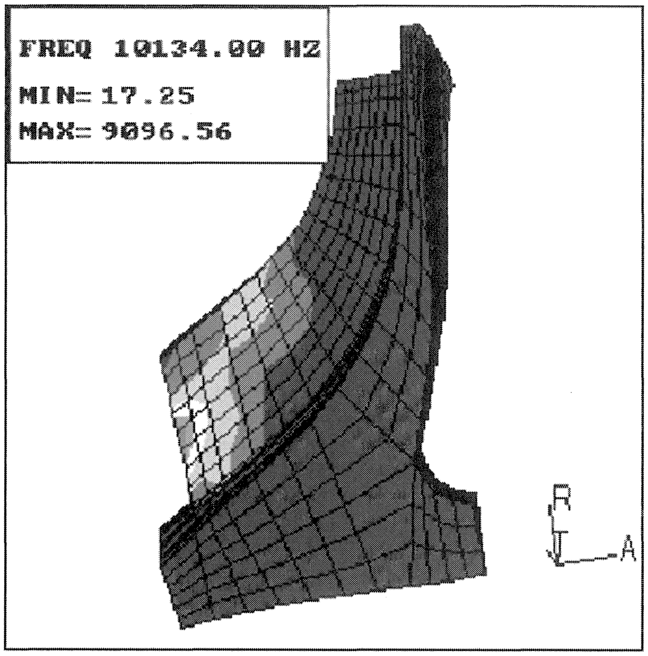

Figure 10. Dynamic Stress Contour of 10,134 Hz.

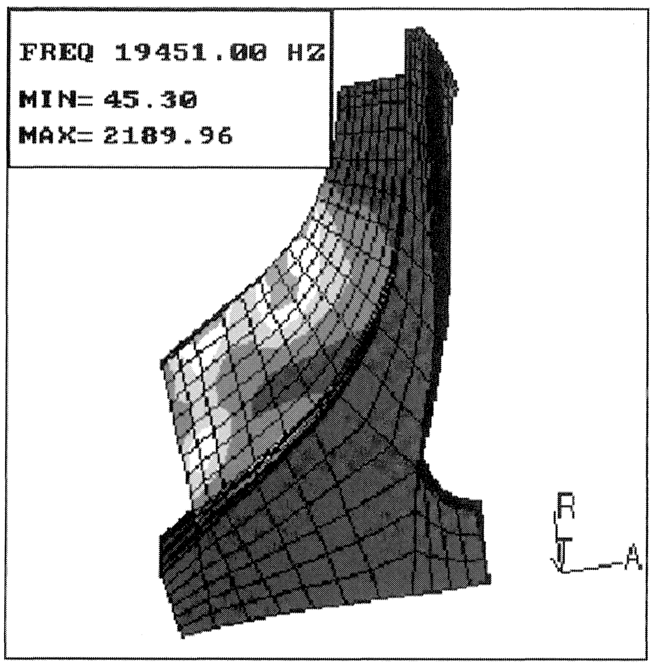

Figure 11. Dynamic Stress Contour of 19,451 Hz.

- For disk resonance: modify disk flexibility through changes to disk thickness or back face design.

- Changing material can benefit to solve both blade and disk resonance problems. The new material can have higher yield strength so as to improve the fatigue life. With the different density, the impeller natural frequencies will be totally changed; but the rotordynamic behaviors need to be studied. And the new material cost will be another key factor to be considered.

- Shot peening is also a proven process that can be applied to improve the impeller fatigue life.

- Scarfing of the impeller blade is a practice whereby the geometry of the blade may be altered to change the resonant characteristic without adversely affecting the aerodynamic values. Usually a triangle piece of the blade is cut from either the inducer or exducer side. Scarfing at the inducer side could alter the capacity, while scarfing at the exducer side could reduce the pressure ratio of the stage.

In the field failure described here, a 10-degree scarfing was applied on this impeller at the blade inducer side. This change has two results, it increases the overall stiffness of the impeller and raises the natural frequencies, and it rearranges the nodal diameter pattern of the modes. The modification of the blade geometry can also eliminate the interference out of the ±5 percent interference range.

Figure 12 is the Campbell diagram of the scarfed impeller. Comparing Figure 12 with Figure 8, note that the blade geometry modification moved the dense blade modes out of the interference areas, especially the 1× vane passing frequency area.

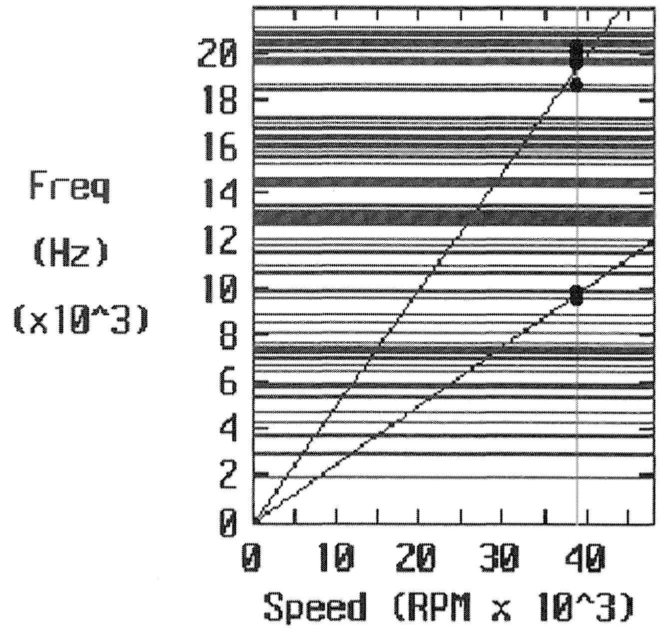

Figure 12. Campbell Diagram of 10-Degree Scarfed Impeller.

Two natural frequencies, 10,134 Hz and 19,451 Hz, which caused the interference with diffuser vane passing frequencies, are raised up and moved out from the ±5 percent safety area.

Because of the complexity of the geometry, other natural frequencies moved into the interference area. There is the interference between the first nodal diameter mode, 9,799 Hz, and first diffuser vane passing frequency, and the second nodal diameter mode, 19,696 Hz, with the second diffuser passing frequency.

The force response study was performed on these new interference frequencies. The results are listed in Table 4. The stress level is greatly reduced from that of the original design. The dynamic stress contours indicated in Figure 13 and Figure 14 are also changed.

RESULTS

The scarfing increased the reliability of the design. A chemical filter was also added to the inlet air system. The impeller is now operating without failure.

CONCLUSION

Based on the preceding researcher's work and the authors' experience, a process was developed to identify open front (impellers without shrouds) impeller blade and disk resonance. The process was used successfully in the design of new components and failure analysis of impellers, as shown in this field failure case study.

Table 4. The Dynamic Stress Level.

Original Impeller	
Frequency	**Stress psi**
10,134	9096
19,451	2189

Scarfed Impeller	
Frequency	**Stress psi**
9799	443
19,696	293

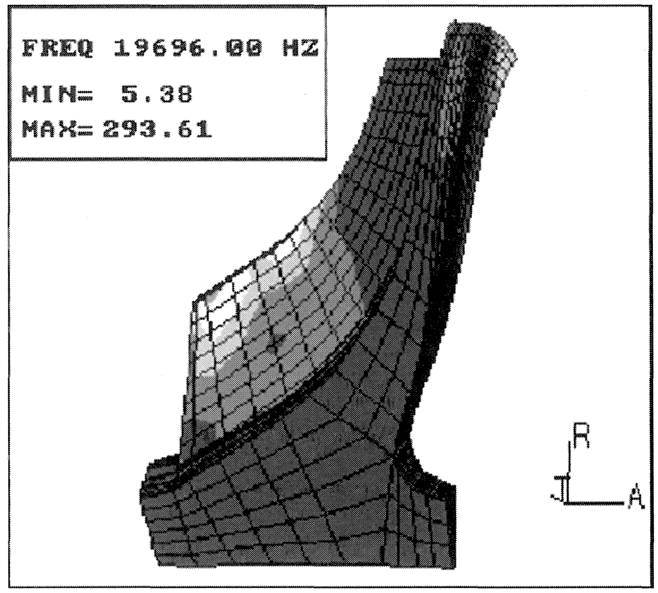

Figure 14. Dynamic Stress Contour of 10-Degree Scarfed Impeller at 19,696 Hz.

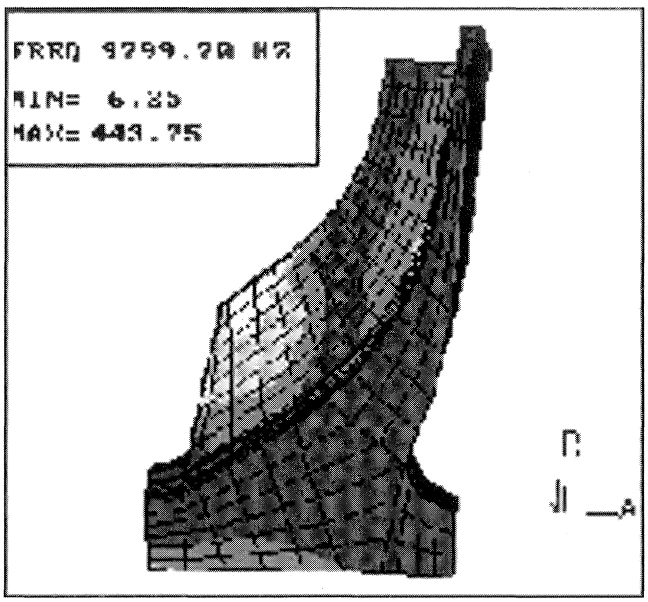

Figure 13. Dynamic Stress Contour of 10-Degree Scarfed Impeller at 9,799 Hz.

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