NEW ACTIVE MAGNETIC BEARING REQUIREMENTS FOR COMPRESSORS IN API 617 EIGHTH EDITION

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ABSTRACT

Active magnetic bearings (AMBs) are being used for an increasing number of compressors in the oil and gas industry. The applications include cryogenic compressor-expanders, subsea processing, pipeline and other process compressors. The use of AMBs allows totally sealed machines, reduced maintenance, elimination of the lube oil system, enhanced monitoring and diagnostic capability, and provides extremely high levels of reliability. Over the past several decades, these bearings have gone from unique, one-of-a-kind demonstrations to being the bearing of choice in an increasing number of applications.

To address the more widespread use of AMB supported rotors, the eighth edition of API 617 includes a new annex which, for the first time, presents an extensive set of specifications that AMB supported compressors and compressor/expanders must meet for API service. The requirements of this annex cover basic design issues, rotordynamics, testing, and auxiliary bearings.

This tutorial presents an overview of the new requirements

and their rationale, resulting from a joint effort and balance of AMB manufacturers, turbo machinery OEMs and end user experience. Special considerations related to the unique requirements and issues related to rotordynamics are presented. Several examples will be discussed.

INTRODUCTION

One new feature of the Eighth Edition of API 617 (2014) is a new annex (Annex E) containing detailed requirements for Active Magnetic Bearings (AMBs) in compressors which have AMBs instead of fluid film bearings. This new annex is a substantial change from the "informative" annex which first appeared in the Seventh Edition of 617 (API 2009). The new API 617 requirements are a response to the increasing number of AMB supported compressors being used in the oil and gas industry. The applications include cryogenic compressor-expanders, subsea processing, pipeline and other process compressors. The use of AMBs allows totally sealed machines, reduced maintenance, elimination of the lube oil system, enhanced monitoring and diagnostic capability, and provides extremely high levels of reliability. Over the past several decades, these bearings have gone from unique, one-of-a-kind demonstrations to being the bearing of choice in several applications.

As with any API rotating machinery standard, the goal of the new AMB annex is give end-users some assurance that the AMB system in a new compressor meets reasonable minimum design standards, and can be expected to provide reasonable performance over its design life. The new standards also give AMB vendors and compressor OEMs a framework for design and development of AMB systems in new compressors. The requirements of the annex cover basic design issues, rotordynamics, testing, and auxiliary bearings. The annex was developed in 2009/2010 with considerable input from AMB vendors, consultants, compressor OEM's and end users. AMB specific content was also prepared for API 684, and is expected to be included when the next edition of this reference document is finally published. This tutorial presents a practical overview of the new requirements. The next edition of 684 is expected to go into more detail with regards to a number of AMB specific issues, including a number that are not covered in this tutorial.

The goals of this tutorial are twofold. The first goal is to help OEM and end user engineers design, specify and review AMB supported compressors to meet the new API specifications. To meet this goal, the tutorial begins with a considerable amount of background information to help engineers who are less familiar with AMBs understand the systems and some of the major performance and design issues which make them quite different from fluid-film bearings. It then continues with a detailed overview of the new requirements, especially with regards to the rotordynamic analysis and testing requirements.

The second goal is to provide an accessible overview and roadmap for some of the technical details of the analysis for readers who need more background. It is hoped that this more in depth information will be especially valuable to AMB vendor/developer engineers as well as code developers who need to understand the new specifications and analysis requirements.

Finally, two example rotordynamic analyses are presented. The first example is based on a small high speed turbocompressor. This example is intended, in part, to provide a well documented test case for someone new to AMB rotordynamic analysis, as well as for analysis code developers. The second is based on an AMB supported, integral motor driven industrial compressor.

AMB SYSTEM OVERVIEW AND TECHNOLOGY

AMBs support or levitate the rotating assembly (shaft) using an attractive electromagnetic force controlled by a position feedback loop. This position feedback control system uses shaft position measurements to generate forces in electromagnets which pull the shaft as required to keep it centered in the clearance in response to gravity and operating forces on the shaft. The position sensors are usually located adjacent to each actuator.

A typical AMB system has five axes of control. There are two radial axes at each end of the machine (four total axes), and one axial axis. This typical system would have two radial electromagnetic actuator/sensor assemblies (bearings), one on each end of a machine and one thrust actuator. This arrangement is shown in Figure 1. Each radial assembly will have two control axes 90 degrees apart. These are almost always oriented at plus and minus 45 degrees from the vertical in a horizontal machine. Many AMB developers refer to these axes as the "V" and "W" axes.

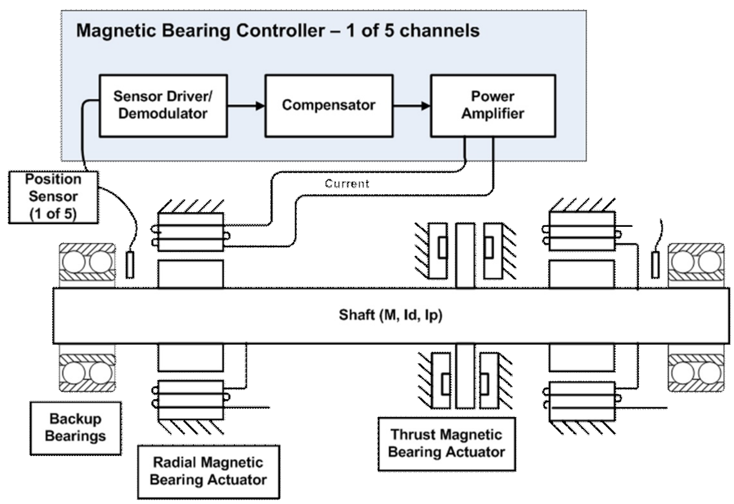


Figure 1. Cross Section of a Rotor Supported on AMBs

A rotor supported on AMBs will also have backup bearings to support the rotor when AMB system is not operating and also in the event of an overload or failure condition of the magnetic bearings. As shown in Figure 1, these are usually located adjacent to the radial actuators.

Physically, a two bearing horizontal machine would appear somewhat similar to familiar fluid-film bearing arrangements. A radial actuator-sensor assembly would be located at each end of the machine. One end would also have a thrust actuator-sensor assembly. There are also multi-bearing AMB machines that are analogous to familiar multi-body trains. Often though, these are located in a single casing, and might have rigid couplings.

The actuators and sensors will be connected to a Magnetic

Bearing Controller (MBC) which contains the power supplies and control hardware needed to operate the AMBs. For most API compressors, this controller will be located in a separate cabinet. Integrated solutions are not unusual for smaller AMB machinery such as chillers. The key components of the MBC are outlined in blue on the single channel control block diagram of Figure 2. A functional block diagram for a typical MBC is shown in Figure 3. The MBC will have sensor drive and demodulation electronics to support the position sensor, a processor or controller that produces a control command signal based on the position error, a power amplifier to convert the command signal into a control current, and necessary power supplies. The Control board can be either digital or analog. However, almost all new AMB systems use digital control due to its flexibility for adding control features and system setup as well as for diagnostics and monitoring. A digital control board will have processor (often a Digital Signal Processor or DSP) and associated peripherals to store and run the MBC control program. The control board may also have a sensor electronics section to produce a high frequency drive signal for the machine mounted position sensors and to demodulate the return sensor signal. In addition to executing a control algorithm, the MBC control program in the DSP also handles levitation logic, fault and trend monitoring and diagnostic functions. A typical power amplifier used with commercial AMBs would use Pulse Width Modulated (PWM) control of a half or full MOSFET H-bridge and a current feedback loop to regulate the control current through the AMB actuator coils. For a five axis system, five power amplifier channels are needed and they can be combined into one integrated board or implemented as separate modules.

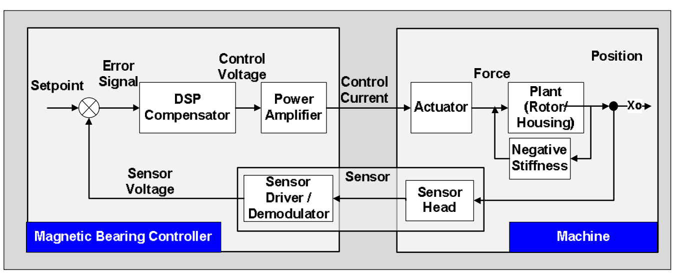


Figure 2. Single Axis Control Loop for AMB

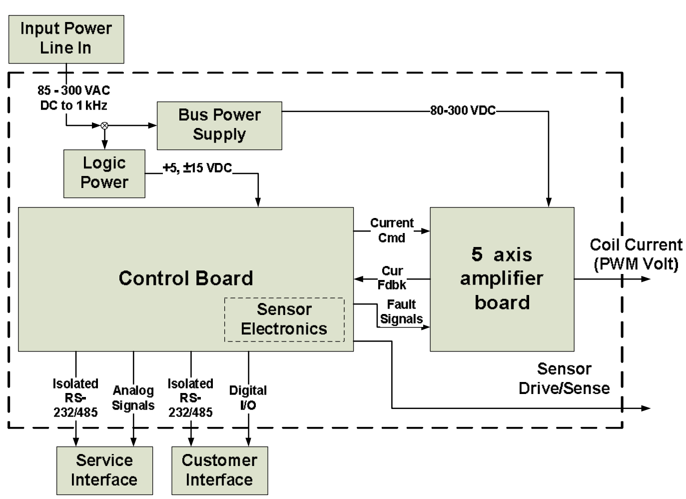


Figure 3. MBC Functional Block Diagram

FORCE GENERATION IN A MAGNETIC BEARING

A cross-section of a typical radial magnetic bearing actuator consists of a set of electromagnets arranged around the shaft as shown in Figure 4. A typical axial actuator consists of a single pair of electromagnets and a large thrust disk as shown in Figure 5. Each electromagnet in an AMB actuator produces an attractive force on the shaft and works together with the opposing electromagnet to form one control axis. In many ways, the basic model of an actuator is much simpler than for a hydrodynamic bearing. This section summarizes the fundamental equations and develops the linearized actuator model which is used for rotordynamic analysis.

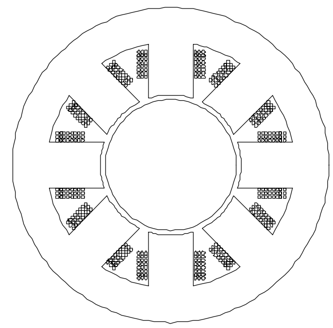


Figure 4. Typical Radial Actuator Arrangement

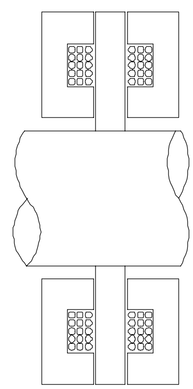


Figure 5. Typical Axial Actuator Arrangement

From the basic physics of electromagnetic actuators, the attractive magnetic pressure, P_{mag}, in the air gap of a magnetic bearing is:

$$P_{mag} = \frac{f_r}{A} = \frac{B^2}{2\mu_0} \tag{1}$$

Where
f_r = attractive air gap force, normal to surface
B = magnetic flux density, T
A = pole area, m2
μ_0 = permeability of free space, 4π E-7 m-kg/s2

This is the magnetic pressure normal to the pole surface. For a radial magnetic bearing, the force in the direction of pull must be calculated from the projected area of the pole in the axis direction, which is less than the pole face area. The force along the axis for one quadrant – the quadrant force – is obtained if the projected area in the axis direction for all poles in a quadrant, A_{proj}, is used:

$$F_q = P_{mag} A_{proj} = \frac{B^2 A_{proj}}{2\mu_0} \tag{2}$$

Note that for an axial magnetic bearing, the pole face area and projected area of poles are the same, since the pole surface area of an axial magnetic bearing is entirely normal to axis direction.

The attractive force is controlled by varying the flux using an electromagnetic coil. Assuming that the reluctance (magnetic resistance) of the air gap is significantly larger than the iron pole pieces in the AMB rotor and stator, it can be shown via Ampere's Circuit Law that the flux density in a magnetic bearing air gap, g, is related to coil current, I, by:

$$B = \frac{NI\mu_0}{g} \tag{3}$$

where N is the number of turns per pole. This assumption is valid as long as the AMB is operated such that the flux density is below the onset of saturation of the pole material. That is, less than about 1.4T for silicon iron and less than about 2.0 T for vanadium cobalt iron (Hiperco, for example).

Equation 3 indicates that the flux density, and therefore the force, in a magnetic bearing can be controlled by controlling the coil current. Combining Equations 2 and 3, gives the expression for force in terms of current for a single actuator quadrant (pole):

$$F_q = \frac{\mu_0 N^2 A_{proj}}{2} \frac{i^2}{g^2} \tag{4}$$

This equation shows that the quadrant force for a magnetic bearing is proportional to the square of the current. This quadratic relationship is inconvenient for control purposes – it is much more desirable to have a linear relation between the control current and force. Additionally, the quadrant force is always attractive; the rotor is pulled toward the stator, whereas it is necessary to pull the rotor in both directions to make a general purpose bearing. Both problems can be solved by using a pair of opposing electromagnets with equal opposing bias forces. The biasing is most often created by operating each coil with a bias (steady) current near 50% of the saturation current of the iron pole pieces. Some vendors use lower bias current levels in some situations to reduce power consumption. At least one vendor uses permanent magnets to bias the air gap.

Using Equation 4 and assuming two opposing electromagnets pulling against each other, the net force is:

$$F = F_{q1} - F_{q2} = \frac{\mu_o N^2 A_{proj}}{2} \left(\frac{i_1^2}{g_1^2} - \frac{i_2^2}{g_2^2} \right) \tag{5}$$

Where I_1 and g_1 are the coil current and air gap of the one electromagnet and I_2 and g_2 are coil current and air gap for the opposing electromagnet. Now if I_1 and I_2 are composed of a steady bias current, I_b, and a control current, I_c:

$$I_1 = I_b + I_c\,, \quad I_2 = I_b - I_c \tag{6}$$

With the rotor centered, the gap for both electromagnets is g_0, and if Equation 6 is combined with Equation 5:

$$F = \frac{2\mu_o N^2 A_{proj}}{g_o} I_b I_c \tag{7}$$

Equation 7 illustrates that the bias flux linearizes the force current relationship, a key motivation for using bias flux. The actuator gain or force constant can be obtained by taking the partial derivative of force with respect to current:

$$K_i = \frac{\partial F}{\partial I} = \frac{2\mu_o N^2 A_{proj}}{g_o} I_b \tag{8}$$

With the rotor centered, the gap for both electromagnets is g_0, and if the rotor moves a distance +x toward the first electromagnet, then the gaps can be defined:

$$g_1 = g_0 + x\,, \quad g_2 = g_0 - x \tag{9}$$

Another important relationship can be obtained by substituting Equation 6 and Equation 8 into Equation 5 and taking the partial derivative with respect to displacement:

$$K_x = \frac{\partial F}{\partial x} = \frac{2\mu_o N^2 A_{proj}}{g_o^3} I_b^2 \tag{10}$$

Now the standard linearized force equation describing an AMB actuator can be written:

$$F = -\frac{\partial F}{\partial I} I - \frac{\partial F}{\partial x} x = -K_i I - K_x x \tag{11}$$

Equation 11 shows that the linearized force model for an AMB actuator pair is composed of two components:

1) A force proportional to *control current*. This is the force that the AMB system controls to move the shaft.

2) A force proportional to *shaft displacement*. This force acts similar to a spring stiffness, but the sign is negative, indicating that it is a force that pulls the shaft away from the centered location. Thus it is often referred as a "negative stiffness." It is due to the fact that magnetic attraction gets stronger the closer the shaft gets to the magnetic actuator.

The negative stiffness term (K_x) is the physical reason that there must be a feedback control system present for the magnetic bearing system to work. This also makes intuitive sense. Without a feedback control system, the shaft would simply be pulled into contact with the auxiliary bearings and remain there.

From the perspective of an API 617 analysis, it is very important to realize that both of these components must be included in the model. It is very easy to forget to include the actuator negative stiffness, since it is usually modeled as a separate bearing-like stiffness at the actuator axial centerline.

AMB SYSTEM FORCE LIMITATIONS

As shown in Figure 6 there are several limits on the force that is available from the AMB actuator. Unlike fluid-film bearings, where substantial short-term overload capacity exists, these are hard limits. An AMB system does not have reserve capacity beyond what is designed into it. If this force capacity is exceeded, the rotor will move until it contacts the auxiliary bearings.

Because of these limits, it is quite important for the machine designer to accurately assess bearing loads early in the design process before the final sizing of the AMB system. The potential for non-symmetric diffuser loads, for example, should be examined. Obviously, it is undesirable to undersize the AMB system. However, extremely conservative estimates are also undesirable, since the bearing size is proportion to the load capacity.

These limits are described briefly in the sections below. More detail is given in Alban (2009). These limits are frequency dependent, and must be taken into account in the design and performance analysis of AMB systems.

Saturation and Maximum Load Capacity

The ultimate limit on actuator load capacity is magnetic saturation of the actuator pole laminations. All of the materials used for industrial AMBs have an upper limit on flux density (saturation flux) and therefore a practical upper limit on the amount of magnetic force they can produce for a given pole face area. This limit is an inherent material characteristic and varies depending on what materials are used. The most

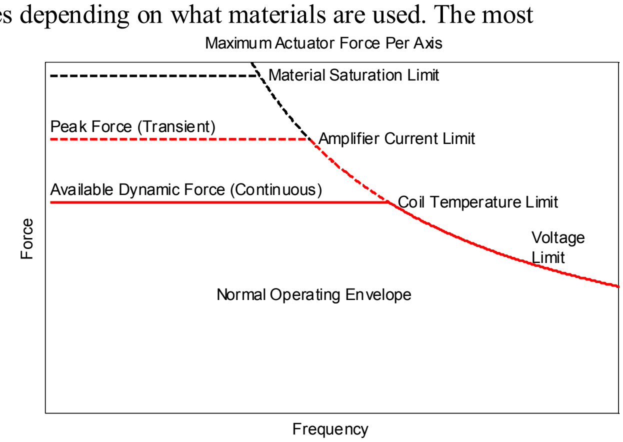


Figure 6. AMB Force Limitations

common magnetic material used for laminations is silicon iron (for example M-19) which saturates at about 1.5 Tesla. More costly cobalt alloys such as Hiperco 50 or 27 can raise this limit to about 2.1 Tesla. The maximum magnetic pressure available in a magnetic bearing can be calculated from Equation 1 by using the saturation flux of the lamination material. The resulting equation is:

$$P_{max} = \frac{B_{sat}^2}{2\mu_0} \tag{12}$$

Using Equation 12, the maximum magnetic pressure in a magnetic bearing can be calculated to be 130 psi (89 kPa) for M-19 and 250 psi (1700 kPa) for Hiperco. A more practical figure of merit can be obtained by accounting for the projected area (in radial actuators) and the slots for the coil/windings. Considering these factors, the maximum specific load capacity ranges from 40 to 70 psi (275 480 kPa) to for silicon steel and 85 to 135 psi (590 to 930 kPa) for Hiperco.

A related upper bound is the current available from the power amplifier. As shown in Equation 4, the actuator force is proportional to the applied current. Thus, the amplifier current capability provides another absolute upper bound on the amount of force a given actuator can apply to the shaft. This limit is often about the same or slightly less than the saturation limit.

Thermal Limits

In most actuators, another set of limits which are important are actuator cooling and the winding insulation temperature limits. Copper losses in the windings are proportional to the square of the current and the coil resistance. These losses generate heat within the actuator. If the system design includes sufficient conduction or convection cooling to remove heat generated by the copper losses then there are no thermal limits on the bearing. In some cases though, design constraints limit actuator size and/or how much cooling is available. This can lead to a design where there is a steady-state force limit which is less than the transient, dynamic load capacity to avoid overheating the actuator.

Slew Rate

The dynamic force capacity of an AMB actuator is limited at higher frequencies. This is because the voltage required to drive current through the actuator coils increases as the frequency increases due to the actuator's substantial inductance. The relationship is expressed by:

$$V = L\, I\, \omega \tag{13}$$

Where V is the available amplifier voltage, L is coil inductance, I is coil current, and ω is the required frequency. The available voltage is fixed by the power supply and power amplifier design. Thus, at some frequency – the slew rate limit – the voltage required to drive maximum design current through the coils will just equal the available voltage. Above this frequency there is not enough voltage available to force the design current through the actuators. As a result, the available dynamic load capacity falls with any increase in frequency.

Eddy current losses

Another limitation on AMB actuator bandwidth is set by eddy current losses. The AMB controller must drive an alternating (dynamic) control flux through the actuator poles in order to respond to a dynamic load. This alternating flux creates eddy currents in the actuator that result in resistive losses. These losses use power from the power supply to create heat. Eddy currents also limit actuator bandwidth by causing a frequency dependent reduction in control flux magnitude and a phase lag of the control flux relative to the control current. Radial actuators can (and are) laminated, which substantially reduces the effect of eddy currents within the bandwidth of the control system. Unfortunately, it is quite difficult to build laminated axial actuators to reduce the effect of eddy currents. Therefore, most commercial axial actuators are solid (although at least one vendor does offer laminated construction as an option in some cases). Solid axial actuators normally have fairly severe limits on available bandwidth and also can consume substantial power if called on to produce significant dynamic load. For the purposes of this tutorial, the key observation is it is usually very important for the axial transfer function model of the AMB system to consider the effects of eddy currents.

Other Limitations

From a practical perspective, there are several other limitations and constraints on actuator force which are worth noting. These include:

- *Material Stress Limits*: Centrifugal forces and interference fit stresses limit the maximum diameter of the rotating component(s) in higher speed applications.
- *Machine Integration Constraints*: There are usually limits on how large the actuator components can be due to aerodynamic, rotordynamic or other considerations.
- *Environmental Considerations*: In some applications, canned configurations which have a thin metal barrier between the actuator components and process fluids are required. This construction can increase the effective gap in the magnetic circuit. It can also degrade sensor performance and reduce bandwidth.

ANALYSIS OF AMB SYSTEMS

Overview of Transfer Functions

One key concept to understanding analysis and modeling of AMB systems is the "transfer function." In the context of feedback control system, a "transfer function," is simply a frequency dependent relationship between some input and some output. It is usually expressed as a ratio of polynomials that are a function of frequency (or the complex variable "s") and is plotted as amplitude and phase in a Bode plot (see, for example Nise 2011, ISO 2006, and Schweitzer and Maslen 2009).

Although the term is not usually used, "transfer functions" are actually quite widely used in rotordynamic analysis, for example, when looking at steady-state unbalance response. Unbalance response analysis is used to evaluate the response (output) of the system to synchronous unbalance force (input). The relationship is often plotted as a Bode plot, which is a pair of plots showing amplitude and phase lag of the response versus frequency.

As another example, the input to the AMB system is the shaft displacement measured at the position sensor. The output of the system is the reaction force applied by the actuator. The relationship between the force and displacement can be described using a transfer function. Figure 7 shows an example magnetic bearing force displacement transfer function. At any given frequency the transfer function defines the ratio of reaction force to measured displacement (the dynamic stiffness) and the phase defines the phase lead of the force relative to the displacement. For example, at 10 Hz, the actuator force is 10,000 lbf per in of sensor displacement, with a 3 degree phase lead. Both gain and phase are important. Gain is related to stiffness, while phase is related to damping.

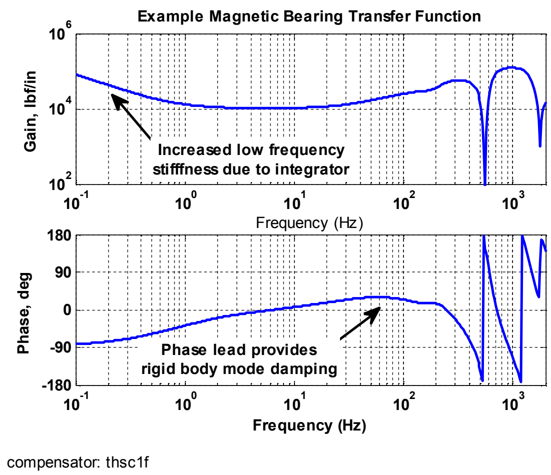


Figure 7. Example Magnetic Bearing Transfer Function

The magnetic bearing transfer function is always highly frequency dependent. The specific transfer function shown in Figure 7 represents the direct x1 axis transfer function for an example machine on magnetic bearings. This can be stated mathematically as:

$$fx_1 = G_{x1,fx1}(s) * x_1 \qquad (14)$$

Where

fx1 = the force applied to the rotor at the actuator location (output)

x1 = physical rotor displacement at the x1 position sensor location (input)

Gi = transfer function of Figure 7

In general, the transfer function will be represented by a ratio of polynomials:

$$G_{i,i}(s) = Force\ \frac{b_n s^n + \cdots + b_2 s^2 + b_1 s^1 + b_0}{a_n s^n + \cdots + a_2 s^2 + a_1 s^1 + a_0} \qquad (15)$$

Where $s = j\,\omega$ and a_i and b_i are the coefficients that determine the frequency response. For example, the specific transfer function shown in Figure 7 can be represented exactly by a numerator polynomial of 14^{th} order and a 20^{th} order denominator polynomial function of frequency. The denominator of a practical AMB system will always be of higher order than the numerator as there are multiple elements that act as low pass filters and will have polynomial terms in the denominator and only constant terms in the numerator.

The simplest control approach for a magnetic bearing system is single input, single output (SISO) in which the force output of an actuator depends only on the signal from the adjacent sensor. SISO is often used for control of the radial bearings and is always used for the control of axial bearings. For a five axis SISO controller, the full set of transfer functions between the control system inputs (displacements) and outputs (actuator forces) can be represented as a matrix of transfer functions:

$$\begin{Bmatrix} fx_1 \\ fy_1 \\ fx_2 \\ fy_2 \\ fz_1 \end{Bmatrix} = \begin{bmatrix} G_{1,1}(s) & 0 & 0 & 0 & 0 \\ 0 & G_{2,2}(s) & 0 & 0 & 0 \\ 0 & 0 & G_{3,3}(s) & 0 & 0 \\ 0 & 0 & 0 & G_{4,4}(s) & 0 \\ 0 & 0 & 0 & 0 & G_{5,5}(s) \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ z_1 \end{Bmatrix} \quad (16)$$

The controller can also be multi input, multi output (MIMO) in which the output of each actuator depends on some combination of signals from two or more of the sensors. MIMO control is used in many industrial AMBs, specifically in the form of Center-of-Gravity control (also called tilt and translation control). The matrix of transfer functions used for Center-of-Gravity control can be represented as:

$$\begin{Bmatrix} fx_1 \\ fy_1 \\ fx_2 \\ fy_2 \\ fz_1 \end{Bmatrix} = \begin{bmatrix} G_{1,1}(s) & 0 & G_{1,3}(s) & 0 & 0 \\ 0 & G_{2,2}(s) & 0 & G_{2,4}(s) & 0 \\ G_{3,1}(s)0 & 0 & G_{3,3}(s) & 0 & 0 \\ 0 & G_{4,2}(s) & 0 & G_{4,4}(s) & 0 \\ 0 & 0 & 0 & 0 & G_{5,5}(s) \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ z_1 \end{Bmatrix} \quad (17)$$

In this case there are an additional 4 transfer functions that must be included in the analysis.

There has also been substantial development work in the academic and controls community for control synthesis methods that produced highly coupled MIMO control systems for the lateral axes. The transfer function matrix for these controllers would include 16 transfer functions, one between each axis (x_1, x2, y_1, ands y_2), and each force (f_{x1}, f_{x2}, f_{y2}, f_{y2}). However, these techniques have not yet been widely adopted by industrial AMB vendors. The discussion in this tutorial will be focused on SISO control as this is the easiest to understand and the closest to conventional bearings.

Each transfer function is created by the series combination of the magnetic bearing components in Figure 2. The control component of the magnetic bearing is the compensator. The compensator is created by the control designer to provide the best combination of stability, robustness to variations, and forced response for a particular system. Prior to the mid 1990's, the compensator for most AMB systems was realized as a series of op-amp based analog filters. Most AMB systems now use digital control, wherein the compensator is realized as a series of digital filters executed in a Digital Signal Processor (DSP) or other suitable digital electronics. Common sample rates in AMB digital controllers range from 5 kHz to 15 kHz.

The most common AMB compensator structure is enhanced Proportional-Integral-Derivative (PID) (Schweitzer and Maslen 2009) which combines a classic PID controller with additional elements to further shape the frequency response. The proportional gain, P, is represented by the units $Volts_{command}/Volts_{error}$ sets the basic stiffness of the magnetic bearing. The derivative term, D, is created using a lead-lag filter that provides phase lead over a particular frequency range with the byproduct of increasing gain over a related frequency range. The integrator, I, is used to provide a high static stiffness at the expense of causing a phase loss at very low frequencies. Other filters are usually added in series with the lead-lag filter to further shape the frequency response. Some of the common filters are:

1) Notch filters to reduce gain in a particular frequency band,
2) Phase bump filters used to significantly increase or decrease phase in a particular frequency band,
3) Low pass filters used to decrease gain above a certain frequency.

In a digital controller, there is also a conversion delay, a calculation delay and a sampling delay that cause additional phase lag of the controller output command relative to the input signal. These are commonly modeled using a second or third order Pade approximation when analyzing as a continuous system.

The other components of the magnetic bearing system also change the gain (stiffness) of the magnetic bearing. For example, some of frequency dependence of the magnetic bearing comes about because both the sensor and the amplifier have frequency responses similar to a low pass filter. These elements can often be characterized by a DC gain and a bandwidth. The sensor gain can be represented by the units $Volts_{error}/in$ and the amplifier gain by the units $Amps/Volts_{command}$. There may also be an additional anti-alias (low pass) filter after the sensor. The actuators can also have a limited bandwidth due to eddy current effects that are generated when responding to a dynamic load. The radial actuators are laminated to reduce eddy current effects so the bandwidth is usually high enough such that this effect isn't considered in the analysis. The axial actuator, which cannot be easily laminated, does have a low bandwidth and a useful stability analysis must generally include a model to represent the frequency response (whether based on analysis or measurement).

The details of control system design are covered in more depth in Schweitzer and Maslen (2009). Within the context of API 617 compressor analysis, it is the responsibility of the AMB vendor to develop this magnetic bearing transfer function (control algorithm). Indeed, Annex E does not require the vendor to supply the details of each individual element of the control. If data for an independent audit are specified, the vendor need only supply the overall transfer function(s).

State Space vs. Transfer Function AMB Models

Annex E requires that the AMB vendor supply the frequency dependent displacement to force characteristics of the AMB system in transfer function form when data for independent analysis are specified. Thus, much of the control system discussion in this tutorial is written from the perspective of describing the AMB system dynamics using a matrix of magnetic bearing transfer functions. Each of the transfer functions in the matrix describes the frequency dependent relationship between a single sensed displacement and resulting applied force. The transfer function model often gives the best physical insight into how a particular control algorithm functions and can be optimized. However, an overall magnetic bearing transfer function is generally of very high order which can cause numerical difficulties. Annex E addresses this concern by giving the option of describing the control system characteristics in state-space form, which will often have better numerical characteristics.

A state space model is an alternative way of describing frequency dependent dynamics. This model is a set of four matrices that describe the control system frequency dependent characteristics as a coupled set of first order differential equations in matrix form. A transfer function model can always be converted to an equivalent state space model, and vice-versa, using standard algorithms. A state space model is much more convenient to use in analysis and design. It is the form usually used to couple the AMB system dynamics with the rotor and structural system dynamics and perform the various analyses (thus, transfer function models are almost always converted to state-space anyway). The state space model is not unique (Nise 2011). Some forms will be numerically better behaved than others. Thus, it is often a good idea to use some type of balancing and scaling algorithm after conversion from transfer function form to state space form.

Magnetic Bearing and System Transfer Functions

The transfer function introduced above is the AMB control (or compensator) transfer function which relates the measured shaft displacement to the actuator force. However, considering the entire AMB/rotor system, there are several other transfer functions which are very useful from the perspective of evaluating performance, model validation, and troubleshooting. These system transfer functions can be generated both analytically and experimentally during system testing. Annex E requires evaluation and measurement of several of these.

Generating the transfer functions analytically involves relatively straightforward mathematical manipulations very similar to the traditional calculation of unbalance response. Measuring them experimentally requires injecting a known (measured) excitation signal at one point in the AMB system control loop, then measuring the response at another. A key advantage of magnetic bearing systems is the built in ability to use the magnetic bearing actuators as electromagnetic shakers at the same time that they are also being controlled to levitate the rotating assembly of a machine. They can be supplied a random noise signal, a sinusoidal sweep or any other type of excitation signal that might be used in an external shaker to make modal measurements. The magnetic bearing position sensors are then used as vibration transducers to detect shaft motion for the measurement.

For many modern AMB digital controller architectures, the dynamic signal analyzer functionalities are embedded on the control electronics and software to facilitate transfer function measurement by the commissioning engineer or when remote diagnostics functions must be present on the system. Alternatively, the AMB vendor can include provisions for analog measurements of the position and amplifier command and a summing point to allow the injection of the excitation signal into the control loop. In this case a multichannel spectrum analyzer with signal generation capabilities can be used to inject sweeping disturbing signals on the feedback loop and measure the corresponding output(s).

Referring to Figure 8, most vendors supply the capability to inject a signal both before (EXC1) and after (EXC2) the compensator. Important and useful transfer functions that thus can be measured on any magnetic bearing system are:

1) Open Loop System- see below for some important comments
2) Compensator (controller)
3) The Plant (rotor plus sensor and amplifier)
4) Closed Loop System (sometimes referred to as Dynamic Compliance)
5) Input and Output Sensitivity

Each of these transfer functions are described briefly below.

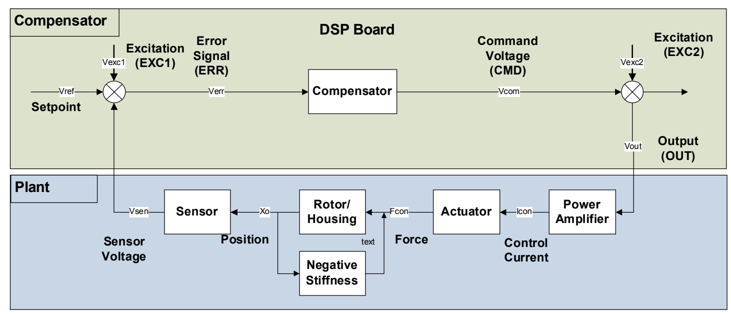


Figure 8. Typical AMB System Inputs and Output Measurement Points

Open Loop Transfer Function (CMD/EXC2)

The open loop transfer function is the transfer function that would be measured if the feedback control loop were cut. Conceptually, one approach would be to unplug the output of the position sensor (plant) from the input to the compensator, then make the transfer function measurements. Analytically, this is very simple to do. However, for a physical AMB system in a compressor, this measurement is not possible. The AMB system would be unstable, and the rotor would be pulled into contact with the auxiliary bearings. The problem is the negative stiffness of the actuator. It is not possible to have the shaft levitated with a position sensor disconnected.

However, referring to Figure 8, it is possible to measure the transfer function between EXC2 and the command signal CMD. Even for a levitated rotor, this pseudo open loop transfer function is very nearly the open loop transfer function. The main differences are related to the effects that the other controller axes have on the shaft modes (ISO 2006, Schweitzer and Maslen 2009).

1) For a radial axis, the other controlled axes will likely add some damping to the free-free shaft modes. The frequencies, however, would generally be expected to be accurate.
2) The rigid body shaft modes, which should be at 0 Hz for a true open loop transfer function if the shaft is not rotating, will generally appear at higher frequencies.

Since the rotor free-free natural frequencies are generally easily indentified in the (pseudo) open loop transfer function, it can be extremely valuable for rotor model validation and tuning. Likewise, it also gives useful information related to the rest of the AMB system for debugging and model tuning. This transfer function is also discussed at length in ISO 14839-3 (ISO 2002).

Plant Transfer Function (Vsens/Vout)

Referring to Figure 8, the Plant Transfer Function is the ratio of the displacement response of the plant (Vsen) to the amplifier current command signal (Vout). It can be measured using excitation applied at EXC2. This is a similar to receptance transfer function of the rotor but it includes the dynamics of the sensor, actuator and power amplifier. This is the plant that must be controlled by the magnetic bearing compensator. Note that this is not the free-free rotor transfer function as would be measured by hanging the rotor and performing a ring test. The power amplifier and sensor dynamics, as well as the actuator negative stiffness are also included. For a levitated rotor, there may also be effects in radial axis measurements due to coupling with control systems for the other axes.

Compensator Transfer Function (CMD/ERR)

Referring to Figure 8, the Compensator Transfer Function is the ratio of the compensator output (CMD) to the compensator input (ERR).). It can be measured using excitation applied at EXC1. With a digital control system, this transfer function would rarely be explicitly measured during testing, since it is known exactly from the digital control algorithm being used (assuming that the algorithm has been correctly programmed).

Closed Loop Transfer Function (CMD/EXC2)

The closed loop transfer function (CLTF) is the ratio of output response to input excitation signal for an actively controlled system, including the effects of the feedback loop. Annex E indicates that the closed loop transfer function should be measured with an excitation applied to the plant (EXC2 in Figure 8), and the response as measured at the output of the compensator (CMD in Figure 8). The CLTF measurement can optionally be used for model validation instead of an unbalance response test. This is discussed more fully below. This transfer function is also discussed at length in ISO 14839-3 (ISO 2006).

Conceptually, this transfer function has some similarities to the traditional imbalance influence coefficients, but there are some important differences. Both provide the ratio of machine response to a particular excitation as a function of frequency. The CLTF is the transfer function (ratio) of rotor position from rotor excitation by the AMB, while the influence coefficients are the ratio of rotor position response to rotor excitation by rotating imbalance. The excitation frequency for the imbalance response is typically shaft speed, thus gyroscopic effects are present. The frequency for the CLTF is independent of shaft speed. Indeed, the measurements can be performed both at zero speed and with the machine running. A zero speed measurement will generally be a higher quality measurement, since it will not be contaminated by effects such as unbalance, aerodynamic noise, etc. On the other hand, measurements with the machine running are required to see the effects of gyroscopic effects, aerodynamic cross-coupling, etc.

Sensitivity Transfer Function (ERR/EXC1)

The sensitivity transfer function is the ratio of the excitation signal plus response to the excitation signal. It could be measured at either the input or output of the compensator. Annex E makes reference to ISO 14839-3 for the details of this measurement and evaluation of the results. Referring to Figure 8, the ISO reference indicates that the sensitivity transfer function of interest is the input sensitivity function. It is measured or calculated as the ratio of the error signal (ERR) to excitation at EXC1. Note that this is not the same input location as indicated in Annex E for the closed loop transfer function.

The ISO specification defines four quality zones, identified as A to D, related to the peak value of the sensitivity transfer function, ranging from 3 to 5. Despite the ISO14839-3 referring to those limits as stability limits, it is intended that the sensitivity margins represents a robustness index other than a stability index.

The sensitivity transfer function is an easy and practical way to evaluate the stability robustness of the overall AMB system to changes in the gain or phase lag of the elements of the AMB system control loop. Very high peak values of the sensitivity transfer function indicate a high "sensitivity" to changes. It is worth noting that there are almost always tradeoffs between a low sensitivity transfer function and good unbalance response. Both the ISO specification (ISO 2006) and Schweitzer and Maslen (2009) have considerably more discussion related to this transfer function. These references also note that the intent of the ISO specification is that the transfer function measurements be made in physical sensor/actuator coordinates. Use of transformed coordinates (i.e., transformed to center-of-gravity control system coordinates), is useful for debugging, but does not meet the ISO requirements. For those readers more familiar with traditional single-input, single-output control theory, there is a very close connection between the sensitivity transfer function and other stability indexes like the gain and phase margin.

Magnetic Bearing Transfer Function

The actual overall Magnetic Bearing Transfer Function (Force/Displacement transfer function) obviously would be of considerable interest. However, it cannot normally be directly measured during machine testing, since measurements of the actuator force are usually not available. An exception to this general rule is systems with magnetic flux sensors, which can be used to generate a signal proportional to the applied force.

Axis by Axis versus Full

One important question that must be addressed when doing transfer function measurements is whether to measure cross-axis transfer functions. The most basic approach is to simply measure the transfer functions of each axis individually for each actuator and the adjacent sensor. This is the default approach for Annex E and ISO 14839-3. Thus, for a two radial bearing machine, four radial closed loop transfer function measurements/analyses would be made (two for each actuator). Likewise, for a three bearing machine, six transfer functions would be measured/calculated.

However for a two radial bearing machine, since there are four radial displacement sensors, and four radial actuators, there are a total of 16 possible radial transfer functions between excitations and response which could be measured (coupling between the radial and axial axes is rarely considered, and is not required by Annex E). Measurement of all of the transfer functions, including cross-axis coupling, is an if-specified option. Measuring the cross-axis coupling can provide additional insight into the system dynamics and more checks on the rotordynamic model. It is especially relevant in the case of machines with large gyroscopic effects (overhung compressors, for example), and multi-input, multi-output control systems such as center-of-gravity control. In principle, all AMB systems do have coupling between the lateral axes as a result of the shaft dynamics, even if the control system is not internally coupled. Applying a force at one actuator location almost always causes motion at all of the sensors, especially if the shaft is rotating.

SOME OTHER IMPORTANT AMB SPECIFIC SYSTEM ISSUES

Non-collocation

For most practical AMB configurations, there is an axial separation between the actuator and sensor centerlines. Thus, AMB systems almost always have to deal with the issue of non-collocation. For a non-collocated feedback control system, the motion at the sensor is different from the motion at the actuator. This has the potential to cause a problem if a given mode has a node point between the sensor and actuator centerline because it causes a phase reversal of 180° between the sensor motion and the actuator force. For experienced AMB control system designers, this is not normally an issue because:

1) The phase reversal can often be used to advantage in the compensator design, allowing the gain to be rolled off more quickly at higher frequencies.
2) If there is a node between the sensor and actuator, the modal displacement at either or both the sensor and actuator will almost always be low, which means the mode will have low *observability* and/or low *controllability*. This generally makes the mode easier to *gain stabilize* – meaning that the compensator gain is set low enough such that the AMB force will be less than rotor internal damping force regardless of the phase of the AMB force relative to sensed displacement. Gain stabilization is only possible for modes above the operating speed.

It should be noted that if the rotordynamic analysis places the node of a mode very close to the sensor or actuator, care would need to be taken in design of the control system. Given the limits of normal modeling accuracy, the node may not actually be between the sensor and actuator in the real machine. A good control system design would maintain acceptable performance for a range of node locations.

Synchronous cancellation

Since AMB systems apply actively controlled dynamic forces to the rotor, they can do more than just supply stiffness and damping. One of the most important and unique features of AMBs is synchronous cancellation. This allows the AMB to significantly reduce the effect of mass imbalance on the system. There are many different methods and approaches used by various vendors and researchers, but they are all generally aimed at suppressing either synchronous forces or synchronous displacements.

In industrial machines it is most common to use cancellation to reduce synchronous forces since this reduces transmitted vibration to the machine housing, reduces actuator power and reduces dynamic load requirements. This type of cancellation also goes by names such as Automatic Balancing Control (ABC), Adaptive Vibration Control (AVC), Adaptive Vibration Rejection (AVR), and Unbalance Force Rejection Control (UFRC, is the generic term used by ISO 14839-1 (ISO 2002)). The cancellation signal for UFRC can be determined by adaptively multiplying the synchronous signal by a suitable influence coefficient matrix (developed either analytically or by measurement). This approach is required for the traverse of the rotor rigid body modes and can be used for higher speeds as well. For speeds above the rigid body mode frequencies, UFRC can be as simple as adaptively subtracting the measured synchronous displacement from the sensor signal before presenting it to the controller. With no synchronous component in the control signal, the controller produces no synchronous current and therefore the rotor spins about its inertial axis.

In some types of machines, such as machine tool spindles, it is important to reduce synchronous displacements. Here a rotating force is added by the magnetic bearing with an amplitude and phase that will reduce the synchronous displacement to near zero at up to two axial locations on the rotor. For a machine tool, one of these points will be at the tool tip. This type of cancellation also goes by names such as peak of gain control, Adaptive Vibration Control (AVC), and Unbalance Force Counteracting Control (UFCC is the generic term used by ISO 14839-1).

There are also approaches to supply synchronous damping to assist the traverse of a bending mode in a supercritical machine. This is has been called Synchronous Damping control (SDC) and Optimum Damping Control (ODC).

Gain Scheduling

In some AMB machines, it is necessary (or desirable) to change the AMB control system parameters as a function of speed or some other parameter. This is known as gain scheduling. The most common reason to use gain scheduling in an AMB controller is for machines with substantial gyroscopic effects. For example, an overhung machine with a large wheel will typically have forward and backward modes that vary significantly with spin speed. For some optimized control algorithms, it would be necessary to vary the AMB control system parameters as a function of speed to maintain optimized performance. Another reason for using gain scheduling is when rotordynamic coefficients associated with a compressor or turbine stage change significantly with inlet pressure or spin speed. These effects can change the characteristics of the plant enough to require gain scheduling of the AMB controller. Typically, this variation is done in discrete steps over multiple speed ranges, rather than continuously. If the AMB control system uses gain scheduling, it must be considered during any rotordynamics analysis.

Auxiliary Bearings

AMB systems for API 617 compressors are required to have an auxiliary bearing system. The auxiliary bearings are a separate mechanical bearing system which provides:

- Full shaft support if the AMB system is powered down,
- Load sharing in some systems for short term transient AMB system overload,
- Full shaft support in most systems for continuous AMB system overload,
- Full shaft support in the event of AMB system or component failure.

This bearing system is referred to by a variety of names, including: auxiliary bearings, backup bearings, touchdown bearings, catcher bearings, emergency bearings, retainer bearings and coast-down bearings.

In most AMB systems, the auxiliary bearing system consists of one backup bearing adjacent to each actuator as shown in Figure 1. Usually one of the backup bearings will support both radial and thrust loads and the other will support radial loads only. Both rolling element and solid lubricated bearings have been used. During normal operation, there is a gap between the backup bearing and the rotating shaft, ensuring that there is no contact. The auxiliary bearings become active only when the shaft moves radially or axially through the clearance gap and comes into contact with one or more auxiliary bearings.

To help ensure that the shaft motion remains controlled during operation on the auxiliary bearings, Annex E requires that the backup bearing system include a damped mount. This mount damping provides a way to remove vibrational energy from the system when the backup bearing is in contact with the shaft.

Some Comments on AMB Rotor Balancing

API 617 contains a number of paragraphs related to balancing requirements and procedures. Annex E does not include any special balancing requirements. Balancing of rotor supported by AMB can be achieved by using conventional low speed or high speed balancing techniques, but some specific considerations are required.

When performing low speed balancing of the rotor, provision for dedicated areas to support the rotor on rollers must be included. Supporting and spinning the rotor on soft material laminations can result with damage of laminations. This is especially detrimental if sensor target areas are involved. When rotordynamic constraints dictate to limit or avoid presence of permanent supporting sleeves, temporary shaft extension can be added the rotor to support it during balancing. Whether permanent or removable, the support surface must be located close to the AMB sensor and with a total run-out between the sensor surface and the balancing support area lower than 0.0002 inches (5 microns). This tight tolerance is required since the AMB system center of rotation is governed by the sensor surface. To minimize run-out error between the sensor surface and support sleeve, machining of the parts is usually done at the same time.

When high speed balancing of rotors is required, as in the case of flexible rotors of multistage compressors, this operation is generally performed in dedicated bunker balancing facilities. Because the added shaft length of a separate support sleeve(s) could adversely affect the rotordynamic of the machine, it is common practice to support the rotor with oil film bearings located at the rotor laminations. Again, the rotor laminations and sensor target area should have a total run-out of 0.0002 inches (5 microns) or lower.

The obvious alternative is to do high speed balancing of the rotors with the job AMBs providing the rotor support. This could be done in a balancing facility or on the final machine once it is assembled. In addition to traditional vibration measurements, bearing control currents can also be used to provide an approximate measurement of bearing dynamic forces. Despite both solutions offering clear advantages in terms of final balancing quality and performance checks with direct measurement of currents and vibration margins, some physical problems limit the practicality of this approach.

Using the AMB job bearings for high speed balancing in a bunker requires dedicated pedestals to install the AMBs in a balancing bunker facility. More significantly, it also requires cooling and sealing systems, due to presence of vacuum inside the bunker (which can cause actuator cooling problems), with significant time and cost impacts. The second option of rotor balancing on the final machine has practical problems due to difficulties on accessing to balancing planes on the rotor once the machine is assembled. Thus, high speed balancing on oil film bearings is the first choice for most OEMs.

API 617 EIGHTH EDITION AMB REQUIREMENTS

Annex E covers issues ranging from materials to balancing to man-machine interfaces. Some of the more notable paragraphs include:

- Requirements on bearing load capacity and testing.
- Lateral and axial rotordynamic requirements.
- Auxiliary bearing design, performance and testing requirements.
- Cabinets and electrical interconnections.
- A set of AMB specific report data requirements and AMB related data to be provided for independent audits (when this is specified).

The AMB annex indicates that most of the annex major paragraph numbers are aligned to the paragraphs in the main body of chapter 1 for convenience. With the exception of the axial analysis requirements, which overlaps the paragraph numbering used for as the torsional analysis in the main body of API 617, this is generally true.

It is worth noting that many of the AMB specific requirements include the typical API specification disclaimer indicating that if there is no practical way to meet a specific requirement for a particular machine, the purchaser and vender need to reach an agreement as to what is acceptable.

The next few section of this tutorial will take a detailed look at some of the major items in Annex E, especially with regards to rotordynamics analysis and testing.

ROTORDYNAMIC ANALYSIS FOR AMB SUPPORTED ROTORS PER API 617 EIGHTH EDITION

Introduction and Overview

Much of the Annex E lateral rotordynamic analysis of an AMB supported compressor will be quite familiar to anyone who has evaluated fluid-film bearing machinery under previous editions of API 617. As with fluid-film bearing supported machinery, the two primary issues which need to be examined are unbalance response and overall rotordynamic stability. However, there are some significant changes. These changes are required to address the unique characteristics of an AMB system. They are primarily related to four issues which have been discussed previously:

1. There is a finite, maximum force that any given AMB system can apply. If this force is exceeded, the shaft will move until it contacts the backup bearings, which must then support the overload force.
2. An AMB system can theoretically go unstable at any frequency within the control system bandwidth, thus supersynchronous instability is possible.
3. The dynamics of the axial bearing must be evaluated.
4. An AMB system can apply an excitation force to the system, which allows several key transfer functions to readily be evaluated on the test stand to verify the overall system behavior.

Annex E specifies how to apply the traditional section 4.8 rotordynamic requirements to an AMB supported machine. The annex requires some modifications to the section 4.8 analyses, as well as several new analyses. There are also corresponding changes to some of the reporting requirements outlined in the new Annex C.

This section will describe the elements of an API 617 Eight Edition rotordynamic analysis for a machine with an AMB system in detail. It will be assumed that the reader has at least a basic familiarity with rotordynamics analysis, although not necessarily with the specific requirements of previous editions of API 617. There are a number of analysis and modeling issues covered by API 617 and Annex E. These will not be discussed in this tutorial. The reader should refer to the new standard for this information. It is also expected that there will be a detailed discussion of many of the AMB specific issues in the next edition of API 684.

Two important issues which this tutorial does not address are rotor modeling and support structure dynamics. Because AMB systems can excite supersynchronous modes, the stability analysis must consider these modes. Additional care may be required to ensure that that the rotor model accurately predicts the first few bending modes. Likewise, accurately predicting stability can require a better support structure dynamic model than would be the case for a fluid-film bearing supported machine.

New System/Modeling Data Vendor Requirements

The new edition of 617 includes a new set of specific report requirements, including data for independent audit. Annex E adds several data items related to the AMB system. For a standard report, these include general dimensions, dynamic force capacities and plots of the AMB system transfer functions. These are expected to be provided by the AMB vendor. If data for an independent audit are specified as being required, some additional data are to be provided. These data include the specific coefficients for the AMB system transfer functions. These data are analogous to the geometry requirements for fluid-film bearings in fluid-film bearing supported compressors.

AMB Rotordynamic Analysis Details

The sections below discuss the rotordynamic analysis of an AMB supported compressor per API 617 Eighth edition in some detail. Note, however, that API 617 Eighth Edition Chapter 1, section 4.8, machinery specific chapters, and Annexes C and E would need to be consulted for additional details on the specific limits and requirements required by the specifications. This tutorial is not intended to replace these references. Annex E also makes reference to part 3 of the ISO active magnetic bearing specification ISO 14839 (ISO 2006).

Also, as with any API specification, these specifications are understood to be a minimum set of requirements. Many purchasers will have additional requirements to address their particular needs.

Outline of the Analysis

A high level outline of the general analysis flow for an AMB supported machine per API 617 Eighth Edition is as follows:

1. Lateral undamped critical speed map and representative modes
2. Lateral free-free map and (optionally) representative modes
3. Unbalance response analysis and evaluation
    a. For specified unbalance distributions
    b. (Optionally) for model verification test unbalance distributions
4. (Optionally) lateral closed loop transfer function analysis
5. (Suggested, but not required by API) lateral open loop transfer function analysis
6. Lateral stability and sensitivity function analysis
    a. Level I
    b. Level II (if required)
7. Torsional analysis
8. Axial stability and sensitivity function analysis
9. As installed analysis
10. Auxiliary bearing performance evaluation

Undamped Critical Speed Map

As with most any rotordynamic analysis, the first set of calculations to be performed and presented in a report are the undamped critical speed map and the corresponding mode shapes. For a two radial bearing AMB machine, these plots would be essentially the same as for a fluid-film bearing machine. In most cases, the analyses would effectively assume decentralized (uncoupled) bearings with collocated sensors at the actuator locations. The synchronous bearing dynamic stiffnesses would most likely be assumed for the stiffness curves overlaid on the plot. As with a fluid-film bearing, the undamped critical speed map and corresponding undamped modes are not intended to be accurate predictions of the actual machine critical speeds. Instead, they are intended to give some insight into the general dynamic characteristics of the machine.

To generate representative bearing dynamic stiffness curves typically shown on the undamped critical speed map, equivalent synchronous AMB stiffness and damping characteristics could be derived with the following relationships:

$$\text{Stiffness} = Gain\ \cdot \cos\,(phase) \tag{18}$$

$$\text{Damping} = \frac{Gain\ \cdot \sin(phase)}{\omega}$$

Note that these representative equivalent stiffness and damping values in general <u>should not be used</u> for stability or unbalance response calculations!

Free-Free Map

The first major addition to the report requirements for AMB machinery is the addition of a free-free map. This map is a plot of the free-free (zero bearing support stiffness) modes versus operating speed. Several example plots are shown in the examples below (Figures 11 and 18). This plot is similar to the lateral Campbell diagram which is sometimes generated to assist in determining damped critical speeds (this is also sometimes called a "whirl-speed" plot). However, the free-free map is generated with the assumption that the bearing stiffness and damping are both zero. This map would typically include both forward and backward precessing modes. Note that some analysis codes have numerical difficulties generating a true zero support stiffness free-free map. In this case, a very soft support stiffness could be used.

This analysis step provides an overview of the dynamic properties of the rotor, independently from AMB characteristics. Because the rotordynamic properties of only the rotor are involved, rotor free-free analysis can easily be performed by OEMs with commercial rotordynamic codes. Thus, this analysis step is usually run both by the OEM and the AMB vendor.

Like with the undamped critical speed map, the purpose of including this plot is insight into the dynamic characteristics of the system. Some of the questions that this plots helps address include:

- How strong are the gyroscopic effects on the first few free-free modes? Strong gyroscopic effects may complicate the control system design.
- Is there a free-free mode below running speed? Some AMB machines operate supercritically above the first free-free mode. However, supercritical operation generally requires much more care in the control system design. Operation above a free-free critical has also been suggested as being much more demanding of the backup bearing system.
- Where are the free-free modes relative to running speed and how tightly are they spaced? This also gives some insight into the challenges that the control system designer faced.

Finally, if the free-free mode shapes are also presented, they can give some indication as to whether there might be a node point between a sensor and an actuator for certain modes at some point over the operating speed range. As discussed previously with regards to noncollocation, the presence of a mode with a node between a sensor and the corresponding actuator within the control system bandwidth must be considered in the control system design.

During the early rotor design phases the OEM can optimize the rotor design and AMB/sensor position to meet preliminary requirements on free-free modes, operating speed range separation margins and node positions, based on AMB vendor recommendations. During detailed design, the AMB vendor can use the free-free mode shapes and frequencies to correct mode interlacing violations due to AMB and sensor non collocation effects or determine the operating range of applicable speed tracking filters.

The specification suggests that all modes below three times the maximum operating speed need to be included on the plot. This limit is expected to cover most systems. However, some engineering judgment is required. Any mode that the control system can reasonably be expected to excite should be included in the plot. In particular, an unusually wide bandwidth AMB system should have more modes shown on the free-free map.

Unbalance Response

All API turbomachinery has to meet a number of requirements with regards to unbalance response. The basic idea is that lower vibration tends to be correlated with longer machine life. For AMB supported machinery, there is also the issue of finite actuator force capability to be considered.

The same general unbalance response analysis procedure used for fluid-film bearing supported machines is also used for AMB supported machines. The same quasi-modal unbalance distributions and magnitudes specified for fluid-film supported machines are used. These cases are described in detail in the specification (note that the some of the details of the unbalance response calculation and evaluation in 617 Eighth edition have changed relative to previous editions). As with fluid-film bearing compressors, the response evaluation is conducted on a probe by probe basis. For most AMB compressors, the AMB sensors are treated as the "probes." As noted above, the full set of coupled dynamics are used for each analysis. The unbalance response analysis is specified as being performed without any unbalance force cancellation algorithm active. For most machinery, this should be a more conservative approach than including unbalance force cancellation in the analysis.

The first requirements that must be met for the unbalance response are separation margins. As with fluid-film bearing supported compressors, critical speed response peaks with an amplification factor greater than 2.5 must be adequately separated from the running speed range. The same evaluation and acceptance criteria as for fluid-film supported compressors are used.

The second requirement that must be met is a limit on amplitudes over the running speed range. Historically, the peak to peak amplitudes have been limited for API machinery as in Equation 19. This level is based on long experience with fluid-film bearings. It is usually achievable on the test stand for new compressors, and is generally accepted as being a level of test stand vibration that will contribute to reliable, long running machinery (alarm and trip limits for installed machinery are almost always larger than this value though).

$$Peak\ to\ Peak\ Amp(mils) \leq \min\left(\sqrt{\frac{12000}{N_{mc}}}, 1.0\right) \quad (19)$$

AMB compressors can often be designed to meet this amplitude limit. However, there are several strong arguments in favor of increasing this limit for AMB supported compressors:

- The minimum clearance is almost always the auxiliary bearings. An unexpected excursion due to a minor process upset will result in contact at the auxiliary bearing(s), rather than at a seal. Thus, the likelihood of performance degradation due to increased seal clearances resulting from a rub is reduced. This reduces the risk of operating with larger shaft orbits.
- AMB systems tend to be dynamically softer than fluid-film bearings. Thus the transmitted force for a given shaft vibration level tends to be much lower and less likely to cause fatigue damage to other machine components.
- Most importantly, AMB systems have a finite force capacity. It is usually worthwhile to reserve more of this capacity for transient forces than using it to cause the synchronous vibration levels to be extremely low.

Because of these considerations, the unbalance response limits for AMB compressors were raised to the limit shown in Equation 20, where C_{min} is the minimum diametral clearance, which is typically at the auxiliary bearings.

$$Peak\ to\ Peak\ Amp(mils) \leq \min\left(3\sqrt{\frac{12000}{N_{mc}}}, 0.3C_{min}\right) \quad (20)$$

The third item to be checked for the unbalance response is that the amplitudes at all close clearance locations are acceptable for a degraded balance state. The approach is presented slightly different in 617 Eighth edition than in the previous editions, although the net result is essentially the same. The amplitudes are scaled such that the maximum probe response over the operating speed range is at the vibration limit (up to a specified maximum scaling factor). The scaled response amplitudes at each close clearance are then examined to ensure that they are acceptable from zero to trip speed. This procedure is used unchanged for AMB machinery.

The final issue to be examined for an AMB machine is the bearing force limit. Since AMBs have a finite force capability, it is important to ensure that there is some margin between the available force and expected dynamic force. One of difficulties that the task force faced when writing this part of the new AMB annex was how to determine the available force at the actuator for comparison to the predicted dynamic forces due to unbalance. As described above, a number of factors can be involved including (Alban 2009):

- Actuator/shaft geometry and materials,
- Actuator windings (including temperature limits on insulation),
- Actuator thermal environment and cooling,
- Amplifier voltage and current capability versus actuator resistance and inductance,
- Bias current,
- Bearing steady radial load.

Ultimately, the task force decided the most practical approach was to require the AMB vendor to supply a machine specific allowable force envelope as a function of frequency. This allowable envelope is required to have a factor of safety of at least 1.5. If data for data for an independent audit are specified as being provided, the vendor also is expected to indicate what specific issues were considered and supply enough information to confirm that the envelope is reasonable.

Thus, to ensure that the machine will not overload the AMB system, the final evaluation for unbalance responses is to compare the predicted bearing (actuator) forces at each actuator to this available force envelope.

Unbalance Rotor Response Verification Test (If Specified Item)

The closed loop transfer function measurement described below is generally a much more convenient approach for an AMB supported rotor. However, the AMB annex leaves open the possibility of performing the conventional optional if specified unbalance response test for model verification. If the unbalance response model verification is used, an additional unbalance response case using the same unbalance distribution expected to be used for the test needs to be analyzed and the plot included for comparison to test data.

Closed Loop Transfer Functions (If Specified Item)

One of the unique features of an AMB system is its ability to apply an excitation and measure the response. This capability makes it very easy to measure transfer functions. Indeed, transfer functions are almost always measured as part of the commissioning process to ensure that the system is performing as expected. In the case of a sealed, integrally driven compressor, the ability to measure transfer functions is perhaps the only practical way to perform model validation, since there is generally no way to perform a traditional unbalance response based model verification test without opening the case. Opening the case is usually very undesirable.

The Annex includes an "if specified" item to use a closed-loop transfer function measurement for model validation. If this option is selected, the corresponding closed-loop transfer function analyses needs to be performed and the results included in the report for use in the comparison described later in this tutorial. The excitation and measurement points for this transfer function are specified as described previously. The default option is to make these measurements with the rotor levitated, but not spinning. Measurements at other operation speeds are an optional "if-specified" item. The Annex notes that measurements with the shaft spinning may not be as useful for model validation due to the presence of additional forces due to unbalance, aerodynamic effects and so on. However, measurements with the shaft spinning are the only practical way to evaluate any gyroscopic and/or aerodynamic effects.

The basic validation option in the specifications is to perform the measurements/analyses on an "axis by axis" basis. This is intended to mean making the measurement between each actuator and the adjacent sensor. Thus, for a two radial bearing machine, four radial closed loop transfer function measurements/analyses would be made (two for each actuator). Likewise, for a three bearing machine, six transfer functions would be measured/calculated. Optionally ("if specified"), the closed loop transfer functions between each radial sensor and each radial actuator would be measured/calculated. For a two radial bearing machine, this would be 16 transfer functions.

For the case of decentralized (SISO) control, where there is no coupling between axes within the control system, axis-by-axis measurements provide a very good way to check and validate the analysis model. For MIMO control (such as tilt-translate/center of gravity control), it is possible for axis-by-axis validation to miss errors in the model or hardware, since it does not look at the coupling between the axes. In these cases, the optional "if specified" full measurement for each lateral axis combination is probably worthwhile.

If specified, the axial bearing actuator-sensor closed loop transfer function also needs to be validated. Thus, it must also be analyzed and the corresponding closed loop transfer function plots generated. The intent of the specification is that cross-talk between the axial and radial axes be neglected, so transfer functions between the axial and radial axes would not be measured or calculated.

Open Loop Transfer Functions (If Specified Item)

Annex E includes an if specified measurement of the (pseudo) open loop transfer function. The excitation is specified as being applied as shown previously. The expectation is that the input to the power amplifier will be used. There is no corresponding analysis requirement.

The intent of this optional measurement is to provide additional data to assist with model tuning should it be required. The (pseudo) open loop transfer function generally allows very good estimates to be made of rotor free-free mode frequencies. It can also help pinpoint structural resonance problems if they exist. The specification does not require that the predicted open loop transfer functions be included in the rotordynamics report. However, there is considerable benefit in documenting the open loop transfer function to help troubleshoot any problems during testing, commissioning and/or during the life of the machine.

Level I Stability Analysis, Including Sensitivity Function Analysis

For an AMB supported rotor, a thorough stability analysis is essential. Unlike fluid-film bearings, AMB systems can theoretically go unstable at any natural frequency (mode) within the system bandwidth at any operating speed. Indeed, without a feedback control system, the inherent negative stiffness of most practical industrial AMB actuators would result in an instability where in the rotor is immediately pulled away from centerline and into contact with the auxiliary bearings when the AMB system is first activated. As with any compressor, the effects of aerodynamic cross-coupling from seals and/or impellers must be considered as part of the stability analysis. It may also be necessary to have a more detailed model of the support structure than is typically used for fluid-film bearings, since AMB systems can be much more sensitive to structural resonances.

Annex E uses the same basic Level I/Level II approach as for fluid-film bearing supported machines, with a few changes to account for AMB specific issues. As described in Nicholas and Kocur (2005), the API Level I stability screening analysis is intended to be a conservative screening analysis. The basis of the Level I analysis is a parametric study of the effect of added cross-coupling on stability. The results of this study establish how much cross-coupling is required to drive the machine unstable. The amount required to drive the machine unstable is then compared to the anticipated cross-coupling as estimated using a modified Alford/Wachel equation.

A Level I stability analysis for an AMB supported machine is performed in the same way as for a fluid-film bearing compressor. No changes are specified for the required margin between the anticipated cross coupling and the amount required to drive one of the modes unstable. However, the analysis includes all of the modes up to twice maximum continuous speed to account for the wider bandwidth of typical AMB systems and the potential for unstable supersynchronous modes. For modes below maximum continuous running speed, the standard Level I requirement that log. dec. is greater than 0.1 applies. For modes above 1.25 times the maximum continuous running speed (N_{mc}), the requirement is simply that the modes be stable (i.e., have a log. dec. greater than or equal to zero).

Between these two speeds, a simple linear transition in required minimum log. dec. is specified. This transition was not based on rigorous analysis. It may need to be modified in future editions of the standard. Note also that some engineering judgment is required. Any mode that the control system can reasonably be expected to excite should be considered. Thus, modes higher than 2x N_{mc} might need to be considered for an unusually wide bandwidth AMB system.

An important distinction between conventional bearing and magnetic bearing supported rotors is importance of backward modes. In turbomachinery on conventional bearings, the cross-coupled stiffness characteristics generated by rotor rotation serve to de-stabilize forward modes and increase the stability of backward modes. Thus, backward modes are normally safely ignored. With magnetic bearings, spin speed dependent bearing cross-coupling doesn't exist, however, there will generally be frequency bands in the magnetic bearing transfer function where negative damping is produced and can de-stabilize either a forward or backward mode if it falls in that band. So even though a backward mode does not respond to unbalance, attention must be paid to the backward modes to ensure stability.

The standard also requires that the sensitivity transfer function defined in ISO14839 part 3 be evaluated as part of the Level I stability analysis. As described previously, the sensitivity transfer function is a powerful tool to evaluate how the stability is affected by changes within the AMB system. The lateral sensitivity transfer function peaks must fall within zone "A" as defined in the ISO specification. To account for the effects of aerodynamic cross-coupling, the sensitivity function calculations are to be performed with two times the anticipated cross-coupling. If any of the stability or sensitivity function requirements are not met, a full Level II analysis must be performed.

Although not explicitly stated in the Annex, the intent is that these criteria should be applied to modes with significant rotor motion. A good system model for AMB stability analysis may have a fairly detailed structural model with multiple structural modes. Structural modes typically have very little damping, and would not meet the log. dec. greater than 0.1 criterion. Some amount of engineering judgment can be required to determine which modes should be considered, and which should be ignored. Obviously though, any mode which is significantly affected by cross-coupling needs to be considered - especially if it can be driven unstable by adding cross-coupling.

Level II Stability Analysis, Including Sensitivity Function Analysis

For AMB supported compressors that do not pass the Level I screening test, a full Level II stability analysis is required. As with fluid-film bearing supported compressors, the intent of the Level II analysis is to use a model which includes more accurate modeling of the actual stabilizing and destabilizing forces. For AMB machinery, the same progressive set of calculations from the basic rotor/bearing system without seals, up to the complete system is to be used.

Similar to the Level I analysis, the standard Level II requirement that the final log. dec. be greater than 0.1 applies to modes below maximum continuous running speed. For modes from 1.25 to 2.00 times maximum continuous running speed, the requirement is simply that the modes be stable (i.e., have a log. dec. greater than or equal to zero). Between these two speeds, a simple linear transition in required minimum log. dec. is specified. As noted above, this transition is not based on rigorous analysis. It may need to be modified in future editions of the standard. As noted for Level I, any mode that the control system can reasonably be expected to excite should be considered. Thus, modes higher than twice maximum continuous running speed would need to be considered for an unusually wide bandwidth AMB system. However, support structure modes that the AMB system cannot excite usually can be neglected.

A corresponding set of sensitivity function analyses are also required to be performed. These are performed progressively as stabilizing and destabilizing sources are added. The peak values of the sensitivity function for the final, complete model must fall within zone A as defined in the ISO specification. It is important to note that the sensitivity function analysis does not replace the stability analysis. It only can be used to evaluate the robustness of a stable system to changes in the AMB system parameters. The ISO sensitivity function analysis also does not directly address the destabilizing effects of aerodynamic cross-coupling (Li, et al., 2006). Thus, it is still very important that the Level II model include accurate estimates of any aerodynamic cross-coupling.

Torsional Analysis

The new AMB annex does not contain any additional requirements related to torsional analysis. The requirements are the same as for a fluid-film bearing supported compressor train. Torsional analysis will not be discussed in this tutorial. It should be noted that there are some changes in the torsional analysis requirements from 617 Seventh Edition to 617 Eighth Edition.

Axial Analysis

In contrast to most fluid-film bearing supported compressors, it crucial that that the rotordynamic evaluation for an AMB supported compressor also consider the axial dynamics of the machine train. The main concern for active magnetic thrust bearings is stability. Thus, Annex E requires that a train axial stability analysis be performed. Like a train torsional analysis, a simplified lumped mass model of the major train elements (including couplings) is considered to be adequate. The standard lists a number of items that should be considered when developing the axial model. Particular attention is required to ensure that all of the relevant dynamics within the axial AMB system's bandwidth are included. Frequently, eddy-current effects limit this bandwidth substantially as compared to the lateral system.

Once the model is developed, the damped natural frequencies and corresponding amplification factors are to be calculated. The specification indicates that "all modes with an amplification factor greater than 2.5" are to be calculated and reported. From a practical perspective, all of the modes would be calculated, and the ones with an amplification factor greater than 2.5 would be reported.

Assuming that a lumped mass model is being used, there would be a limited number of damped natural frequencies, so reporting all of them would be practical. Obviously, if a detailed model is used, some engineering judgment would be required to identify which ones are within the AMB system's bandwidth.

Assuming that the axial system is stable, the ISO 14839 part 3 sensitivity function analysis is also performed on the axial system. The sensitivity functions for the rotor modes must fall within zones A or B from 0 to 1000 Hz. Since the AMB system generally has limited control authority over coupling modes, modes dominated by motion in the coupling are excluded from this requirement. These axial coupling modes also exist in fluid-film bearing supported compressors, and are rarely a problem.

As Installed Analysis

One significant difference between the API 617 rotordynamics analysis and report requirements for an AMB machine versus a fluid-film bearing machine is the requirement for an as-installed analysis. It is not uncommon to adjust the AMB control system parameters during commissioning to achieve the highest levels of overall performance. These adjustments are often the result of unmodeled structural dynamics or uncertainties in predicted process conditions.

The final rotordynamics report is required to include a full set of results which reflect any parameter tuning that was performed during field commissioning. Thus, if an AMB supported compressor is fine-tuned during commissioning, a corresponding set of analyses need to be generated and included in the report. There are no acceptance requirements for these results, they are primarily for reference. Obviously though, it would be hoped that the as-tuned machine still meets all of the normal analysis acceptance requirements.

This requirement means that the rotordynamics report cannot be finalized until after the machine is commissioned. This requirement is in contrast to a fluid-film bearing machine report, which can generally be finalized after the mechanical run test/factory acceptance test.

Auxiliary Bearing Analysis

Annex E states that auxiliary bearings are required for all AMB supported compressors and that they must have a damping mechanism. It also notes that they are considered to "be a consumable machinery protective device." It indicates that they must prevent rotor-stator contact at any location other than at the auxiliary bearings under all design conditions when the machine is operating or shut-down. The specification requires that the rotordynamics report include a section describing why the auxiliary bearing system is expected to meet these requirements. However, the specification is intentionally vague as to what is required. The approach is likely to be quite vendor specific. This approach reflects the lack of a consensus approach to auxiliary bearing analysis when the specification was developed.

The specification suggests that both analytical predictions and experimental data might be used to "show" adequate performance. A purely analytical prediction that covers both bearing dynamics behavior and life currently seems to be beyond the state of the art. On the other hand, a purely experimental demonstration seems likely to be very machine specific. Thus, from a practical perspective, it seems likely that most vendors and OEMs will take a hybrid approach to meeting this requirement. For example, an empirically tuned model of the bearing damped mount system might be used as part of a linear and/or nonlinear rotordynamics analysis. This analysis might also be anchored with system level test data for a similar machine which shows that the analysis is valid for some class of machines. These results might form the basis for concluding that the auxiliary bearing system would be expected to prevent rotor-stator contact under the agreed upon set of conditions. Accurately predicting the precise behavior and life of the auxiliary bearing system under all operating conditions is believed to be beyond the current state of the art. Testing is almost always required. Thus, adequate bearing life might be established almost purely based on scaled test data.

From an independent audit perspective, verifying the predicted performance of the auxiliary bearings is likely to be challenging. Auxiliary bearings and many damped support systems are very nonlinear. There are some basic sanity check analyses which can be performed. These include sag checks and simplified unbalance response analyses (Swanson, et al. 2008). In some cases, a more complex time transient analysis may even be appropriate (Ransom, et al. 2009 and Hawkins, et al., 2006). However, it is likely that at least some subcomponent test data would be required to perform an adequate analysis.

As discussed in the testing section below, Annex E also contains several if-specified items that cover auxiliary bearing testing on the test stand. Drop testing can be used to validate the performance of the backup bearing system. However, there are risks associated with drop testing. Drop testing during the mechanical run test also does not guarantee that acceptable performance will be obtained under full load conditions in the field.

Note that Annex E also contains several system requirements intended to help users monitor bearing usage so that some indication of bearing life can be obtained.

Rotordynamics Analysis Code Requirements

When planning to perform an AMB analysis per the new specifications, it is important to be aware that performing the analyses described above will require capabilities not found in many of the non-AMB oriented rotordynamic codes widely used at the present time. This issue was recognized when the new standards were developed. However, it was widely agreed that these capabilities are absolutely crucial to ensure that an adequate analysis of an AMB supported compressor is performed. It is expected that code developers will add the required features in the future. Alternatively, these capabilities are available in general purpose codes such as Matlab, Octave or SciLab. The mathematical basis for implementation of the required capabilities is well described in the literature and in standard textbooks on control systems, as well as AMB specific texts such as Schweitzer and Maslen (2009). Some of the details are also presented in the discussion for example 1 below. It is expected that there will be a detailed discussion of many of the AMB specific issues in the next edition of API 684

It is also worth noting that some currently available codes intended for analysis and design of AMB supported machinery require a specific structure for the transfer function(s). A common choice is a Proportional-Integral-Derivative (PID) controller in series with certain pre-defined filter structures (phase lead, notch, etc). This approach is very convenient for control system development. However, AMB vendors are not required to provide the displacement to force transfer functions in this format. This approach allows them to avoid revealing what might be a proprietary arrangement. Thus, a code which does not allow an arbitrary transfer function may or may not be useable for a given machine.

In practice, many AMB system developers, and some of the independent consultants, currently rely on a general purpose analysis tool such as Matlab for control system development and overall system analysis. Several of these general purpose packages have a very mature, robust, well developed set of control system design and analysis tools. Assuming that the rotor model can be generated using a traditional rotordynamics code or a custom tool, leveraging the capabilities of general purpose codes can be a very effective way to do AMB system design and analysis. The main drawback to this approach is that a high level of controls and AMB system understanding is required to build the model and perform the analysis.

Rotordynamic Analysis Summary

An analysis for an AMB supported compressor is a little bit more complicated than for a fluid-film bearing. The presence of a feedback control system and the finite bearing force capacity require that new issues be considered during the analysis. Assuming that the rotordynamic software being used has the required capabilities, the additional requirements are not too difficult to evaluate. As with any rotordynamic analysis of a compressor, some level of engineering understanding is required to correctly perform the analyses and interpret the results. However, knowing that a machine will meet these minimum requirements should give a less knowledgeable end-user/purchaser some assurance that the AMB supported compressor would be expected to have adequate rotordynamic characteristics.

ACTIVE MAGNETIC BEARING TESTING

Overview

The peculiarities of AMB dynamic performances and limitations discussed above, along with opportunities for system testing and diagnostics, were translated into specific testing requirements for AMB equipped compressors and turbo expanders in the new Annex E. As noted previously, these requirements were generated from the existing ISO 14839 Standards and contributions from AMB vendors, OEMs and end users with experience on oil and gas turbomachinery applications.

Similar to other mechanical or electrical components, two consecutive and complementary levels of test are required to achieve a full validation of an Active Magnetic Bearing system. The first validation step is component testing. This testing is performed before the AMB hardware, electronics and software are assembled together. The second step, system testing, is performed after the AMBs are finally integrated on the final turbomachinery for mechanical running test or full load test at an OEM workshop. Further testing and validation activity is also performed in field, when the machine is installed in the production plant and is commissioned for operation.

AMB Component Testing

Component testing of magnetic bearings is typically performed by AMB vendors according testing procedures and acceptance criteria derived by manufacturer experience. Typical tests include dimensional checks and electrical tests on the AMB hardware, environmental stress tests on the electronics and functional tests on the control system software. Annex E specifically mentions:

- A 24-hour burn-in test and a functional test of the control system prior to shipment,
- Optional wet insulation test,
- Optional AMB load capacity tests.

No details on procedures and acceptance criteria are given for these three tests. The procedure and acceptance criteria are left to AMB vendor and customer agreement.

The wet insulation test would be performed on those components that are more susceptible to failure or improper operation in presence of water or heavy hydrocarbon condensates during operation. Examples might be the AMB position sensors and the actuator winding insulation. Depending on the probability and amount of liquid on the process gas, the customer and vendor would need to agree on the requirements for AMB design and possible wet insulation tests. A typical procedure is to submerging the AMB actuator and sensor in water for a reasonable amount of time, typically 12 hours or more. Then check that the coil insulation resistance does not fall below given acceptance values while the component is submerged, and that it recovers to the original values upon component drying.

The AMB load capacity test would be used to validate the capacity of the AMB system to meet the requirements of static and dynamic load capacity of the AMBs as defined during the design phase. Because of the high level of accuracy on AMB flux density and force prediction with commercial magnetic FEM codes, testing of the AMB actuator static load capacity is performed only in case of special AMB designs or where very low bearing load capacity margin is expected. Load capacity testing requires a dedicated test rig or a means to lift the rotor with a jacking system. In-machine testing is generally limited to verifying the static load capacity of the AMBs.

No requirements for AMB dynamic load capacity validation were introduced by the new Annex E, despite that this is more susceptible to uncertainties and dynamic effects that reduce the bandwidth of the AMB actuator and electronics. When this type of test is required, AMB vendor and customer would need to agree on testing procedure and acceptance results.

AMB system testing

AMB system testing procedures and requirements identified in Annex E are intended to verify the performance of the magnetic bearings when installed on the final machine and levitating the rotor at standstill or under running conditions during mechanical running test or string test of the machine at partial or full load conditions. The following tasks are identified as part of AMB system validation:

1. Validating AMB and rotor dynamic performance in terms of vibration amplitudes and system stability when the machine is operating at different speeds and operating conditions,
2. Validating adherence of the models and simulations used during the design phase to the test results,
3. Testing the dynamic performance and robustness of the auxiliary bearing system during a partial or full landing event.

Mechanical Running Test

The compressor mechanical running test (MRT) is usually performed at the OEM workshop to demonstrate the capability of the assembled machine to operate within specified vibration limits and prove proper operation of sealing and lubricating systems over a continuous run of 4 hours. When active magnetic bearings are used, the MRT represents also an important step for validating the performance of the magnetic bearings and specific procedures and acceptance criteria were defined in Annex E for this purpose.

No special requirements for MRT test stand configuration and testing procedure are given in Annex E; hence it is implicit that the same general requisites defined for fluid-film bearing machinery would apply. It is worth noting that the mechanical running test is performed at no load conditions and in many cases, vacuum is kept inside the machine to limit the power request to the driver. Because this operating mode can represent an off-design condition for the AMB, especially in regards to

AMB cooling, provision for an external supply of cooling can be required. Such considerations are even more important for integrated or seal-less motorcompressors, where the AMBs are immerged and cooled by the process gas during operation. In this case operation at low gas pressure and density may represent an unacceptable off-design condition of the compressor and AMB system because proper cooling of the AMBs cannot be guaranteed. When such limitations are expected, the mechanical running test would need to be replaced by a partial or full load test.

Vibration Response limits

Specific testing acceptance limits are described in Annex E for AMB equipped machines. The acceptance limits are set in terms of vibration amplitudes and Peak Sensitivity Transfer Function, both measured during the mechanical running test.

As with fluid-film bearings, the rotor vibration limit shown previously in Equation 20 is used for the test stand limit. It is worth noting that this limit is for valid for an unfiltered rotor vibration whereas the acceptance limits on rotor unbalance analysis were referred to the synchronous vibration component only.

At first glance such testing requirement may appear more conservative and somehow conflicting with the design requirements, since unfiltered vibration amplitude are used during the mechanical running test and only synchronous peak to peak vibration amplitudes at design stage. It must be noted that because the limits set on design phase are relevant to an unbalance level which is double the maximum allowable residual unbalance, U_r, specified on paragraph 4.8.2.7 of API 617 Eighth Edition, a safety factor of two was implicitly introduced on the design phase to meet the mechanical running test vibration limits.

When Unbalance Force Rejection Control is applied during the mechanical running test, the margin on vibration amplitude is even further increased, compared to the design verification, where this feature was disabled during unbalance calculation.

The residual unbalance level and possibility to implement Unbalance Force Rejection Control algorithms during test, overall provide a reasonable margin to meet the vibration amplitude requirements during mechanical running test also when unfiltered rotor displacement on the real machine are considered. In case less stringent residual balancing quality were achieved and agreed by OEM and end users, different rotor vibration amplitude limits can be agreed between AMB vendor, OEM and end-user.

In general the limits set for mechanical running test are not applicable when the machine is tested in a full speed-full load configuration. In such operating conditions, low frequency excitation phenomena from the gas may lead to vibration amplitudes higher than the limits specified and efforts to reduce the vibration amplitudes to within those limits may degrade the overall dynamic performance of the system. In this case different acceptance vibration limits can be agreed, on the base of a more general set of performance indicators, including static and dynamic coil currents, stability margins and power amplifier saturation. In any case, it is reasonable good practice to limit the overall rotor peak to peak displacement of the rotor to 0.5 C_{min}, which corresponds to the Zone-C threshold in ISO 14839-2.

One more point to note, which is not addressed in Annex E, is the value of minimum clearance, C_{min}, to be used as reference to determine the acceptance limit. During the design phase, C_{min} can usually be unequivocally considered as the minimum clearance (on auxiliary bearings), including its tolerance range. During the testing phase the actual minimum clearance can differ from this value.

The actual clearances on auxiliary bearings are determined during the commissioning phase when the rotor center position is set, and these can differ from nominal values due to manufacturing and measurement tolerances or because of the alignment configuration when more than two bearings are present on a rigid shaft line. In this case the measured minimum clearances may differ from the nominal values and the AMB vendor and customer would need to agree on whether the nominal or measured minimum clearances are to be used in Equation 20.

Sensitivity function measurement

Annex E requires a sensitivity transfer function test as part of the mechanical running test. The annex makes reference to ISO14839-3 for the details of the evaluation. Briefly though, the signal injection and measurement points are as described previously with reference to Figure 8. The sweeping frequency should range from 0 Hz up to a maximum frequency f_{max}, set according ISO14839-3 to be the maximum between 2 kHz or three times the operating speed range. As with the ISO specification, the peak value must fall within zone A for the radial system. The axial system has a less stringent requirement to fall in Zone-B. Testing and field measurements of the sensitivity transfer function on new or existing turbomachinery equipped by AMB has confirmed these limits are a reasonable indicator of AMB system robustness and stability.

It is worth noting that such test requirements are based on measurements performed at zero speed. Further measurement when the machinery is running at higher speed can be agreed between AMB vendor and customers. But, because of the higher noise to signal ratio when the rotor is rotating, these measurements are often of lower quality, and no requirements were specified.

Model validation - Unbalance Response (If Specified Item)

API 617 provides the option of performing an unbalance response verification test to validate the model by showing good agreement between calculated and measured responses. This option remains for AMB supported compressors. The test unbalance is typically placed at a readily accessible location such as a coupling flange. Since AMB machinery can be levitated without spinning, some OEMs to deviate from this requirement by using an external magnetic shaker or impact test on the rotor at stand-still condition.

Once the test stand measurements have been made, it is compared to the analysis. The frequencies and amplitudes of the resonance peaks must agree within limits presented in the standard. If there is disagreement, the model is to be corrected, and all of the analyses repeated with the tuned model.

Model validation - Closed Loop Transfer Functions (If Specified Item)

On AMB equipped machines the requirement for model validation is very much simplified by the possibility to inject disturbance signals on the control loop and measure a response. For example, it is quite easy to measure the open loop, closed loop and sensitivity transfer functions. For this reason in Annex E for AMB equipped machines, the requirement for an unbalance verification test was optionally replaced with an "if specified" comparison between predicted and measured closed loop transfer function as described below. The use of the close-loop transfer function model verification test is especially valuable in the case of a sealed, integrally driven compressor design. For these machines, there is no easy way to perform a traditional unbalance response based model verification test without opening the case.

If this option is selected, the closed-loop transfer function analyses needs to be performed and the results plotted. The corresponding measurements also need to made and reported during the mechanical running test. The default option is to make these measurements at 0 rpm. This means that the model validation can, in fact, be performed prior to the first run of the machine. This is a major advantage of this approach versus the unbalance response test. It offers a significant risk reduction by confirming that the AMB system and rotor are behaving as predicted before spinning the rotor for the first time.

Measurements at agreed-upon operation speeds are an optional "if-specified" item. The Annex notes that measurements with shaft rotation may not be as useful for model validation due to the presence of additional forces due to unbalance, aerodynamic effects and so on. However, measurements with the shaft spinning are required to allow any gyroscopic effects to be evaluated.

The axial bearing actuator-sensor closed loop transfer function also needs to be measured/analyzed. The intent of the specification is that cross-talk between the axial and radial axes be neglected, so transfer functions between the axial and radial axes would not be measured or estimated.

Once the test stand measurements have been made, they are compared to the analysis. The frequencies and amplitudes of the resonance peaks must agree within limits presented in the standard. If there is disagreement, the model is to be corrected, and all of the analyses repeated with the tuned model. A typical comparison is shown in Figure 9. In this case the measured peaks of the closed loop transfer function match predicted ones either in amplitude and frequency, for frequencies below 1.25 Nmc. Discrepancies higher than 5% on the peak frequency were present only at frequencies higher than 1.25 Nmc. This machine would have passed the API acceptance criteria for closed loop transfer function verification with no need for further model correction.

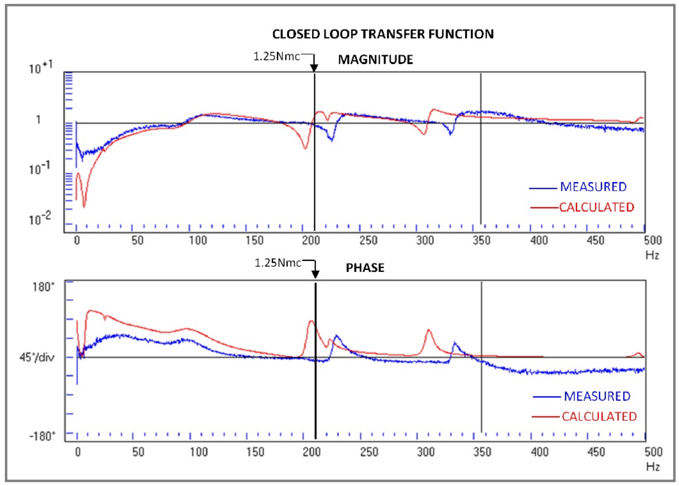


Figure 9. Typical Model Validation Plot

Rotor drop onto auxiliary bearings test (If Specified Item)

Annex E provides for if-specified rotor drop tests onto auxiliary bearings to be performed as part of the mechanical running test or full/partial load test of the machine. The intent is to verify the performance of the auxiliary bearings, including the complete rotor dynamics and residual clearance checks in case of an emergency landing. The worst case scenario for a rotor drop event onto auxiliary bearings is typically associated with a simultaneous delevitation of all AMB control axis. Even though this full axis delevitation event is a rare occurrence in the real life of the machine, it represents the baseline for performing the rotor drop test.

Some modification on the rotor drop test requirements were introduced on the new Annex E of API 617 Eight Edition, compared to the previous edition of the Standard. In the eighth edition, the requirement for a repetition of a three second delevitation test, was replaced by a more generic optional requirement for a “few second" delevitation test and/or a full speed coast-down test, depending on customer requirements. Drop test conditions, repetitions, coast-down time along with all procedural details of the test to be "agreed" between AMB vendor and customers.

The modification resulted from a shared awareness among AMB vendors, OEMs and end-users, that a rotor drop test of three seconds might not be sufficiently representative of a full speed landing event to standstill conditions and more appropriate event and procedure must be devised, case by case, depending on project requirements, available braking torque, representativeness of the testing conditions with real operating conditions and, least but not last, specific constraints on further machine inspection and auxiliary bearings replacement following a landing test.

It is also intended that because of the risks and possible costs associated with such delevitation test, in particular when a number of full speed landings higher than five and with long coastdown time must be achieved, a share of responsibility and mutual understanding on objectives, acceptance limits and risks, to be agreed case by case, is more appropriate than a fixed procedure and set of requirements not applicable to the specific application or test stand capabilities. The next edition of API 684 is expected to address the issues surrounding these tests at greater length.

EXAMPLE CASES

This tutorial concludes with two examples. The first example is based on a small 60,000 rpm turbo compressor. This example has very simple rotordynamic characteristics. In part, this example is being used to show many of the details of how the magnetic bearing transfer functions can be combined with the rotor model. One of the goals of this example is to give readers who have never evaluated an AMB system a realistic test case to use for validating the results of their rotordynamic/AMB modeling process and tools. This example includes the complete details of the rotor model, magnetic bearing transfer functions and basic analysis results. The rotor model has been simplified to reduce complexity

The second example is used to show the full application of Annex E requirements for AMB rotordynamic analysis, testing and model validation. This example is based on a real industrial high speed motor compressor with a solid shaft including motor and compressor, supported by two radial AMBs and one axial AMB.

Small AMB Turbocompressor Example

As described above, the intent of this example is to provide an example which can be re-created by the reader. To help with those who are working on analysis details, or do not have access to a suitable rotordynamics code, we have also included some of the Matlab commands that are typically used in this popular general purpose modeling environment. The commands and process for open source (i.e., free) tools such as Octave or SciLab are similar.

Rotor Model

The rotordynamic structural model geometry of a high speed turbocompressor is shown in Figure 10. The model is based on two-node, Timoshenko beam elements with mass and inertia properties lumped at the nodes (or stations). For lateral analysis, four degrees-of-freedom (dofs) are used per station (two translations, and two rotations). The model was developed using widely recognized rotordynamic modeling techniques (API 2005). The magnetic bearing actuator and sensor laminations are assumed to add mass to the model but no significant bending stiffness. The complete shaft input table for the model is given in Appendix A to allow users to recreate the results of this example. The radial bearing sensor stations are shown in Figure 10 at stations 5 and 29. The actuator stations are shown in Figure 10 at stations 7 and 27. The actuator location is represented in the figure by a spring and in fact, the magnetic bearing negative stiffness term will be added to the model at the actuator location in the same way a conventional bearing stiffness is added.

When a rotordynamic modeling program builds a rotordynamic model from the beam input table, it will create a stiffness matrix, and a mass matrix. Then the second order equations of motion for the rotor can be represented as usual:

$$\mathbf{M}_\mathrm{R}\ddot{\mathbf{q}}_\mathrm{R} + [\,\mathbf{D}_\mathrm{R} + \mathbf{G}_\mathrm{R}]\dot{\mathbf{q}}_\mathrm{R} + \mathbf{K}_\mathrm{R}\mathbf{q}_\mathrm{R} = \mathbf{f}_\mathrm{R,ext} \qquad (21)$$

Where $\mathbf{M}_\mathrm{R}$, $\mathbf{D}_\mathrm{R}$ and $\mathbf{K}_\mathrm{R}$ are rotor mass, damping and stiffness matrices respectively, $\mathbf{G}_\mathrm{R}$ is the gyroscopic matrix (containing skew symmetric products of polar inertia multiplied by spin speed), $\mathbf{q}$ represents a physical displacement vector, and $\mathbf{f}_\mathrm{R,ext}$ represents an external force vector. The model in Figure 10 has 37 stations and the resulting model, with 4 dofs per station, has 148 dofs or equations.

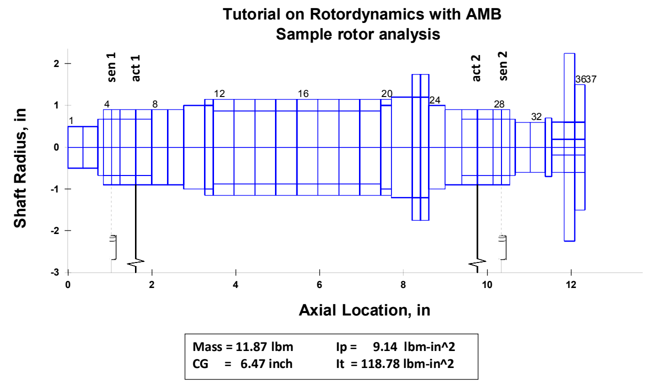


Figure 10. Rotor Model Geometry

Figure 11 shows a rotordynamic free/free natural frequency map created from the rotor model of Figure 10. Corresponding mode shapes are shown in

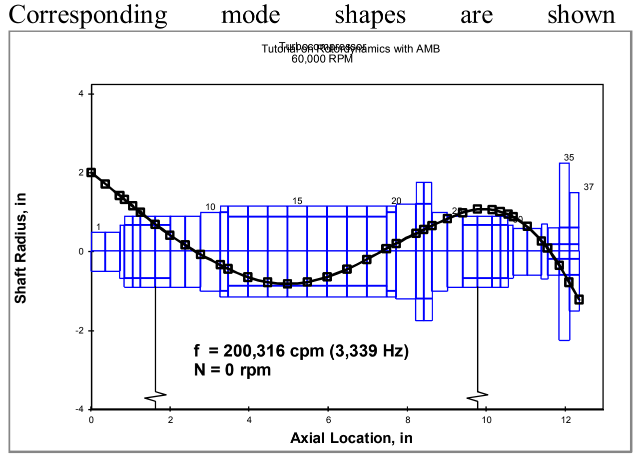


Figure 12. This rotor is well subcritical and exhibits moderate gyroscopic effects.

An important distinction between conventional bearing and magnetic bearing supported rotors is importance of backward modes. In turbomachinery on conventional bearings, the cross-coupled stiffness bearing/seal characteristics generated by rotor rotation serves to de-stabilize forward modes and increase the stability of backward modes. Backward modes are normally safely ignored. With magnetic bearings, spin speed dependent cross-coupling doesn't exist. There will generally be frequency bands in the magnetic bearing transfer function where negative damping is produced and can de-stabilize either a forward or backward mode if it falls in that band. So even though a backward mode does not respond to unbalance, attention must be paid to the backward modes to ensure stability.

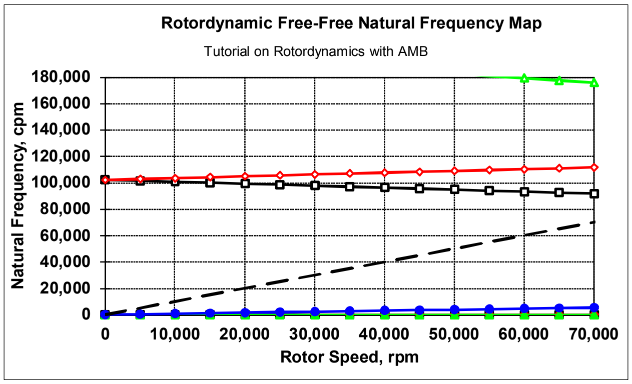


Figure 11. Free-Free Natural Frequency Map

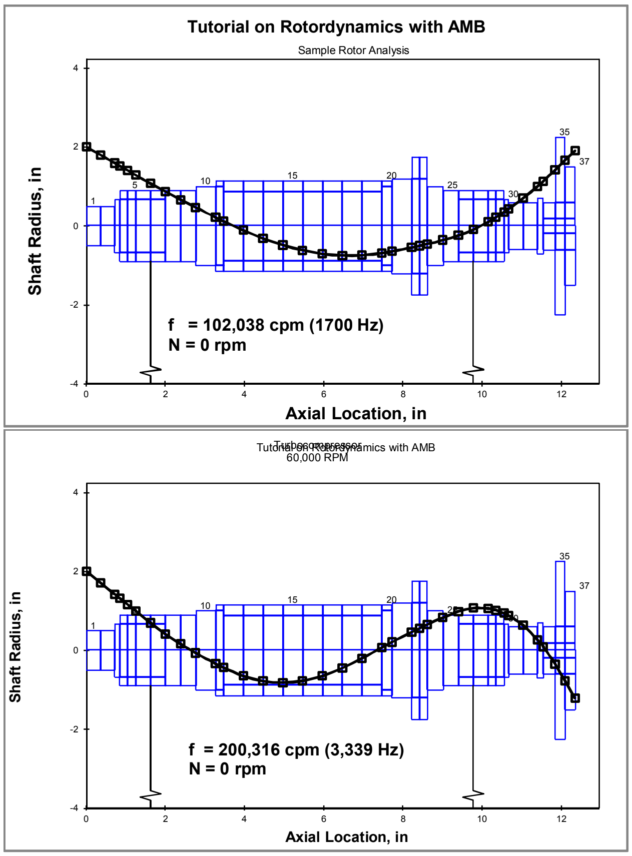


Figure 12. Free-Free Mode Shapes at 0 rpm

Complete Coupled Model

Since forces generated from conventional bearing rotordynamic coefficients depend only on rotor motion (or relative rotor/housing motion), these coefficients can be added directly to the rotor model. However, to model a magnetic bearing, each control channel must be represented by a separate set of equations that is coupled into the rotor model at two locations: 1) the position sensor and 2) the magnetic actuator. A rotor model that includes both rotordynamic coefficients and a place to couple the magnetic bearing forces looks like this:

$$[\mathbf{M}_{\mathrm{R}} + \mathbf{M}_{\mathrm{Brg}}]\ddot{\mathbf{q}}_{\mathrm{R}} + [\mathbf{D}_{\mathrm{R}} + \mathbf{G}_{\mathrm{R}} + \mathbf{D}_{\mathrm{Brg}}]\,\dot{\mathbf{q}}_{\mathrm{R}} + [\mathbf{K}_{\mathrm{R}} + \mathbf{K}_{\mathrm{Brg}}]\mathbf{q}_{\mathrm{R}} = \mathbf{f}_{\mathrm{R,ext}} + \mathbf{f}_{\mathrm{R,mb}} \tag{22}$$

where $\mathbf{M}_{\mathrm{Brg}}$, $\mathbf{D}_{\mathrm{Brg}}$, and $\mathbf{K}_{\mathrm{Brg}}$ are sparse matrices containing the (speed dependent) rotordynamic coefficients, and $\mathbf{f}_{\mathrm{R,mb}}$ is a force vector for applying the magnetic bearing forces. The equations for the magnetic bearing control will be added later. The magnetic bearing negative stiffness values should be added to $\mathbf{K}_{\mathrm{Brg}}$ on the main diagonal in the row and column associated with each actuator degree-of-freedom (dof).

For convenience going forward, the rotor and bearing matrices are combined to give:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}_{\mathrm{mb}} + \mathbf{f}_{\mathrm{ext}} \tag{23}$$

In many cases, a housing model or at least a pedestal model must be included to represent foundation effects. The housing model can often be constructed using similar tools and represented by a system of second order equations of motion in which case additional equations of motion, and interconnections for the rotordynamic coefficients will be added to Equation 23. To keep this example simple, only a rotor model is used.

It is very desirable to work with the model in state space form when including magnetic bearings. The model of Equation 23 can be represented in state space form as follows:

$$\begin{bmatrix}\dot{\mathbf{q}}\\ \ddot{\mathbf{q}}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D}\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{B}_{\mathrm{S}}^{*}\end{bmatrix}\mathbf{u}_{\mathrm{mb}} + \begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\end{bmatrix}\mathbf{f}_{\mathrm{ext}}$$

Or

$$\begin{aligned}\dot{\mathbf{x}}_{\mathrm{S}} &= \mathbf{A}_{\mathrm{S}}\mathbf{x}_{\mathrm{S}} + \mathbf{B}_{\mathrm{S,act}}\mathbf{u}_{\mathrm{mb}} + \mathbf{B}_{\mathrm{S,ext}}\mathbf{f}_{\mathrm{ext}}\\ \mathbf{y}_{\mathbf{sen}} &= \mathbf{C}_{\mathrm{S,sen}}\mathbf{x}_{\mathrm{S}}\end{aligned} \tag{24}$$

where the subscript S indicates the combined structural dynamic models and $\mathbf{u}_{\mathrm{mb}}$ is a vector of actuator forces and has one row for each actuator. Additionally the following definitions apply:

$$\mathbf{x}_{\mathrm{S}} = \begin{Bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{Bmatrix}, \qquad \mathbf{A}_{\mathrm{S}} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D}\end{bmatrix}$$

$$\mathbf{B}_{\mathrm{S,act}} = \begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{B}_{\mathrm{S}}^{*}\end{bmatrix}, \qquad \mathbf{B}_{\mathrm{S,ext}} = \begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\end{bmatrix}$$

$$\mathbf{C}_{\mathrm{S,sen}} = [\mathbf{C}_{\mathrm{S}}^{*} \quad \mathbf{0}]$$

The matrix $\mathbf{B}_{\mathrm{S}}^{*}$ is a selection matrix to connect the actuator forces to the correct rotor degrees-of-freedom:

$$B_S^* = ndof\ x\ nact$$
$$= \begin{array}{c}\left\{\begin{matrix}0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{matrix}\right\}\end{array}\begin{array}{l}\\ \\ dof\ of\ actuator\ channel\ 1\\ dof\ of\ actuator\ channel\ 2\\ \\ dof\ of\ actuator\ channel\ 3\\ dof\ of\ actuator\ channel\ 4\\ \\ \end{array}$$

and $\mathbf{C}_{\mathrm{S}}^{*}$ is a selection matrix to retrieve the rotor position sensor signals $\mathbf{y}_{\mathrm{sen}}$ for connection to the magnetic bearing transfer function model:

$$C_S^*: nsen \; x \; ndof = \begin{Bmatrix} 0 & \ldots & 1 & 0 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 1 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 1 & 0 & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 1 & 0 \end{Bmatrix}$$

$$dofs \; of \; sensor \; channel = \begin{matrix} & & \uparrow & \uparrow & & \uparrow & \uparrow & \\ \cdots & \cdots & 1 & 2 & \cdots & 3 & 4 & \cdots \end{matrix}$$

So $\mathbf{y}_{sen}$ has one row for each position sensor. The abbreviations that have been used to indicate the dimension of the matrices are:

$ndof =$ number of rotor degrees-of-freedom
$nact =$ number of magnetic bearing actuator locations
$nsen =$ number of position sensor locations

Use of the input and output matrices $\mathbf{B}_{S,act}$ and $\mathbf{C}_{S,sen}$ simplify the task of connecting the plant and magnetic bearing models at the sensor and actuator locations. The separate force vector, $\mathbf{f}_{ext}$, allows the user to add unbalance or other forces.

As mentioned previously the magnetic bearing force/displacement transfer functions for a SISO five axis system can be represented by:

$$\begin{Bmatrix} fx_1 \\ fy_1 \\ fx_2 \\ fy_2 \\ fz_1 \end{Bmatrix} = \begin{bmatrix} G_{1,1}(s) & 0 & 0 & 0 & 0 \\ 0 & G_{2,2}(s) & 0 & 0 & 0 \\ 0 & 0 & G_{3,3}(s) & 0 & 0 \\ 0 & 0 & 0 & G_{4,4}(s) & 0 \\ 0 & 0 & 0 & 0 & G_{5,5}(s) \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ z_1 \end{Bmatrix} \quad (25)$$

Generally the control for the x and y axes of the same bearing will be identical so the $G_{1,1} = G_{2,2}$ and $G_{3,3} = G_{4,4}$. The transfer functions developed for this rotor are shown in Figures 13 through 15. The numerator coefficients, b_n and denominator coefficients, a_n of the three transfer function models are given in Appendix A, where the transfer function is defined as shown previously:

$$G_{i,i}(s) = \frac{b_n s^n + \cdots + b_2 s^2 + b_1 s^1 + b_0}{a_n s^n + \cdots + a_2 s^2 + a_1 s^1 + a_0} \quad (26)$$

The axial magnetic bearing transfer function has more states than the radial transfer functions because it includes a number of additional states to model eddy current losses that are unavoidable in an axial magnetic bearing.

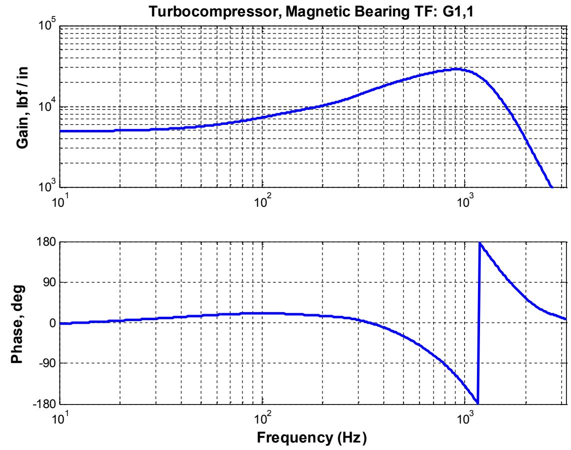


Figure 13. Magnetic Bearing Transfer Function for Bearing 1, G_{11} and G_{22}

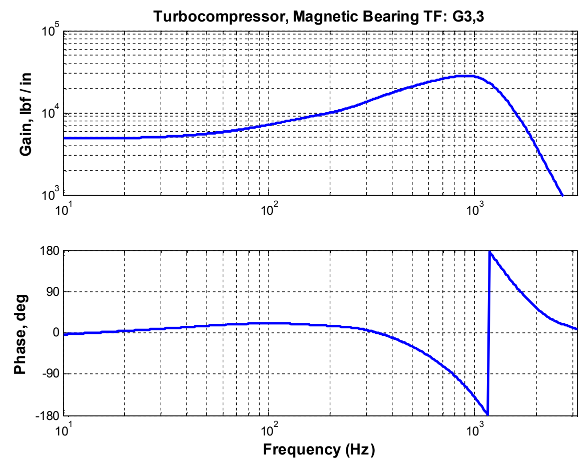


Figure 14. Magnetic Bearing Transfer Function for Bearing 1, G_{33} and G_{44}

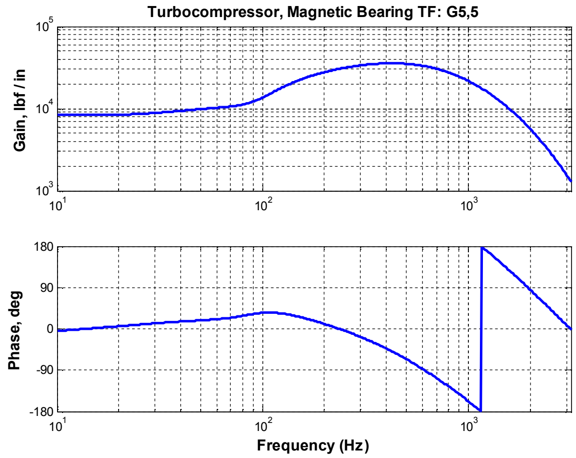


Figure 15. Magnetic Bearing Transfer Function for Axial Bearing, G_{55}

As mentioned previously, the most convenient method for modeling a coupled rotor/ housing/ magnetic bearing system is to express the system dynamics as state space models. Equation 24 gives the rotor or rotor and housing model in state space form. The magnetic bearing transfer functions can be converted to state space form using standard techniques.

Matlab Commands for Model Creation

As a reference, commands for creating this model in Matlab, using the Control Systems Toolbox, are as follows (Octave and SciLab are similar).

A magnetic bearing state space model can be created from the numerator and denominator coefficients, b_n and a_n in Appendix A as follows for the x1 axis, G_{11}:

```
num = [b1 b9 b8 b7 b6 b5 b4 b3 b2 b1 b0];

den = [a14 a13 a12 a11 a10 a9 a8 a7 a6 a5
a4 a3 a2 a1 a0];

G11 = ss(tf(num,den));
```

The transfer functions for the other axes can be created in a similar manner. Then the magnetic bearing state space matrix for the radial axes can be assembled as follows:

```
sys_mbrg = append(G11, G22, G33, G44);
```

The axial axis can also be included in the same model, but many practitioners find it more convenient to handle the axial axis separately. This matrix, `sys_mbrg`, has four unconnected transfer functions since this is the SISO case. The form of the individual transfer functions, G_{ii}, and the form of `sys_mbrg` is a Matlab LTI object. If Matlab is used to perform all of the analysis then the model should stay as an LTI object because this form is much more convenient to manipulate. However, programmers writing their own code will find it convenient to work with the usual state space *a, b, c* matrices. These matrices can be extracted from the LTI object as follows:

```
Amb = sys_mbrg.a;
Bmb = sys_mbrg.b;
Cmb = sys_mbrg.c;
```

And the magnetic bearing model is expressed as a state space model:

$$\begin{aligned}\dot{\mathbf{x}}_{\mathrm{mb}} &= \mathbf{A}_{\mathrm{mb}}\,\mathbf{x}_{\mathrm{mb}} + \mathbf{B}_{\mathrm{mb}}\,\mathbf{y}_{\mathrm{sen}} \\ \mathbf{u}_{\mathrm{mb}} &= \mathbf{C}_{\mathrm{mb}}\,\mathbf{x}_{\mathrm{mb}}\end{aligned} \tag{27}$$

Which can now be easily coupled to the rotor/bearing model as follows:

$$\begin{Bmatrix}\dot{\mathbf{x}}_{\mathrm{S}}\\ \dot{\mathbf{x}}_{\mathrm{mb}}\end{Bmatrix} = \begin{bmatrix}\mathbf{A}_{\mathrm{S}} & \mathbf{B}_{\mathrm{S,act}}\mathbf{C}_{\mathrm{mb}}\\ \mathbf{B}_{\mathrm{mb}}\mathbf{C}_{\mathrm{S,sen}} & \mathbf{A}_{\mathrm{mb}}\end{bmatrix}\begin{Bmatrix}\mathbf{x}_{\mathrm{S}}\\ \mathbf{x}_{\mathrm{mb}}\end{Bmatrix} + \begin{bmatrix}\mathbf{B}_{\mathrm{S,ext}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{Bmatrix}\mathbf{f}_{\mathrm{ext}}\\ \mathbf{0}\end{Bmatrix}$$

$$\begin{Bmatrix}\mathbf{y}_{\mathrm{sen}}\\ \mathbf{u}_{\mathrm{mb}}\end{Bmatrix} = \begin{bmatrix}\mathbf{C}_{\mathrm{S,sen}} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_{\mathrm{mb}}\end{bmatrix}\begin{Bmatrix}\mathbf{x}_{\mathrm{S}}\\ \mathbf{x}_{\mathrm{mb}}\end{Bmatrix}$$

Or

$$\begin{aligned}\dot{\mathbf{x}}_{\mathrm{sys}} &= \mathbf{A}_{\mathrm{sys}}\mathbf{x}_{\mathrm{sys}} + \mathbf{B}_{\mathrm{sys,ext}}\,\mathbf{f}_{\mathrm{ext}} \\ \mathbf{y} &= \mathbf{C}_{\mathrm{sys}}\mathbf{x}_{\mathrm{sys}}\end{aligned} \tag{28}$$

where the output vector y contains the sensor displacements and actuator forces. The coupling between the magnetic bearing outputs (actuator forces) and the rotor inputs (the dofs at the actuator locations) is created by the matrix multiplication $\mathbf{B}_{\mathrm{S}}\mathbf{C}_{\mathrm{mb}}$. The coupling between the rotor outputs (the rotor dofs at the sensor locations) and the magnetic bearing inputs (sensor displacements) is created by the matrix multiplication $\mathbf{B}_{\mathrm{mb}}\mathbf{C}_{\mathrm{S}}$. The sources of the various terms in the coupled system matrix are shown graphically in Figure 16. As can be seen, each block has a physical basis in the system.

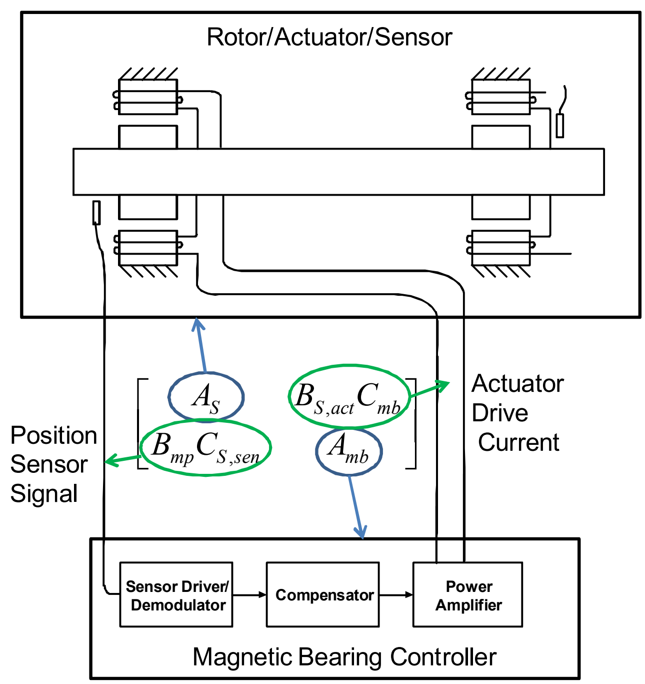


Figure 16. System Matrix Terms

If working in Matlab, the coupling can be accomplished alternatively by first converting the plant model to an LTI object.

```
sys_plant = ss(As,Bs,Cs,Ds);
```

then using the feedback command:

```
sys_CL=feedback(sys_plant,sys_mbrg);
```

It is also worth noting that the model, especially the part derived from the transfer functions, may not be very well conditioned numerically. Thus, it is often worthwhile to rescale the matrix coefficients. For instance, in Matlab, the command `ssbal` will balance a state space model to help to equalize the row and column norms. This scaling effectively reduces the spread in magnitudes of the elements of the model, which should improve numerical behavior in the overall analysis.

Eigenvalues (Whirlspeeds and Stability)

The eigenvalues calculated for the Example 1 rotor at zero rpm, and 60,000 rpm are presented in Appendix B. All of the modes are stable and meet API stability requirements. As discussed previously, no distinction is made between forward or backwards modes since there will generally be frequency bands in the magnetic bearing transfer function where negative damping is produced and can de-stabilize either a forward or backward mode if it falls in that band. Also note that both sub and supersynchronous modes are considered at each speed point used for calculation.

The remaining data items and plots required for an API 617 rotordynamics report would be generated as outlined previously. The next example focuses on these items.

Industrial Compressor Example

Overview

A second example of application of Annex E requirements for AMB rotordynamic analysis, testing and model validation is discussed in this section. This example is based on a real, high speed-high power motorcompressor for gas boosting and reinjection service. The rotor is a 1 .5 metric ton solid shaft that includes the motor and compressor bundle and is supported by two radial AMBs and one axial AMB. Some details on rotor modeling, preliminary and detailed analysis steps specific to AMB rotors are given in this example. Analysis steps that may fall into AMB vendor and OEM responsibility will be discussed

This machine was designed and tested prior to the new API 617 standard being released. Thus, while the plots and analysis address most of the same issues as the new standard, there are some unavoidable differences. These are highlighted in the discussion.

Rotordynamic model

A sketch of the rotordynamic model is presented in Figure 17. The rotor model was generated by the OEMs but as normal and good practice, was reviewed by AMB vendors to check consistency and sensitivity of results on modeling assumptions.

The centerline section of AMB actuator and positions sensor were included in the model, along with auxiliary bearing stations, impellers, seals etc. AMB rotor laminations effect was accounted as distributed masses along rotor shaft, with no added contribution to shaft stiffness. Impeller, thrust disc etc were modeled as lumped inertias applied at relevant center of gravity positions on the shaft. The AMB negative stiffnesses acting at relevant AMB actuator station were also included in the model.

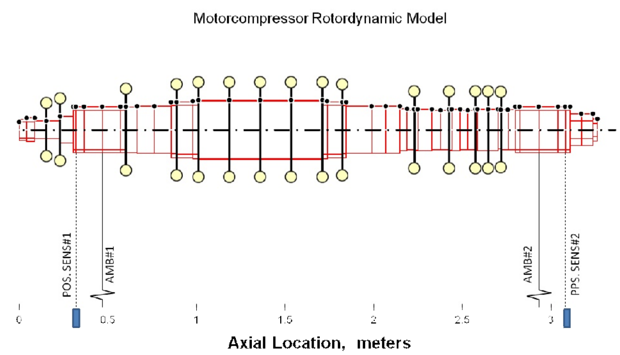


Figure 17. Rotor Model

Free-Free Map and Mode Shapes

Per Annex E, a rotor free-free map from standstill up to 150% N_{mc} was generated with the natural frequencies up to 3 times Nmc. This map is presented in Figure 18. The rotor free-free modes at different speeds (standstill and maximum continuous speed) were generated, and are shown in Figure 19, to identify deflected modes nodes about sensor and actuator.

From free-free map presented in Figure 18, crossing of the synchronous line (1X) with the 1st bending mode inside the operating speed range was identified. Proper separation of the second bending mode from the operating speed range could also be envisaged.

Rotor free-free deflected mode shapes at standstill and maximum continuous speed revealed presence of deflected mode shape nodes close to AMB actuator station for both the 2nd and 3rd bending modes, at the motor side and compressor side respectively. Higher frequency modes, not included here, were also considered by the AMB vendor to check the high frequency dynamics of the rotor and during the synthesis of the AMB controller.

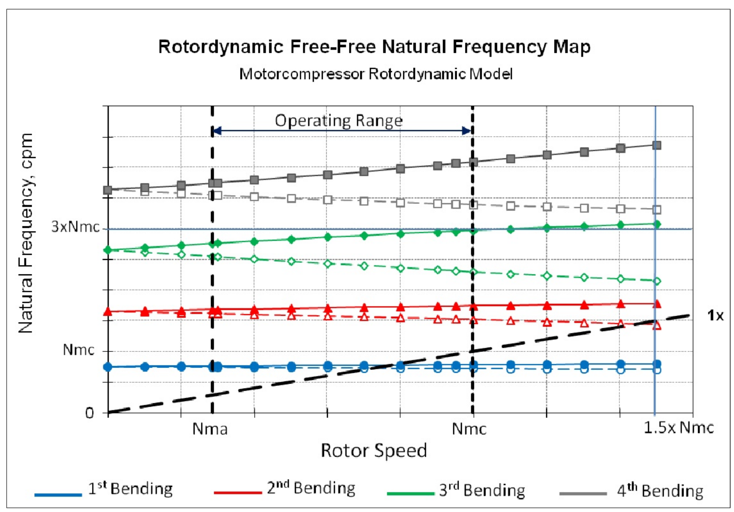


Figure 18. Rotor Free-Free Map

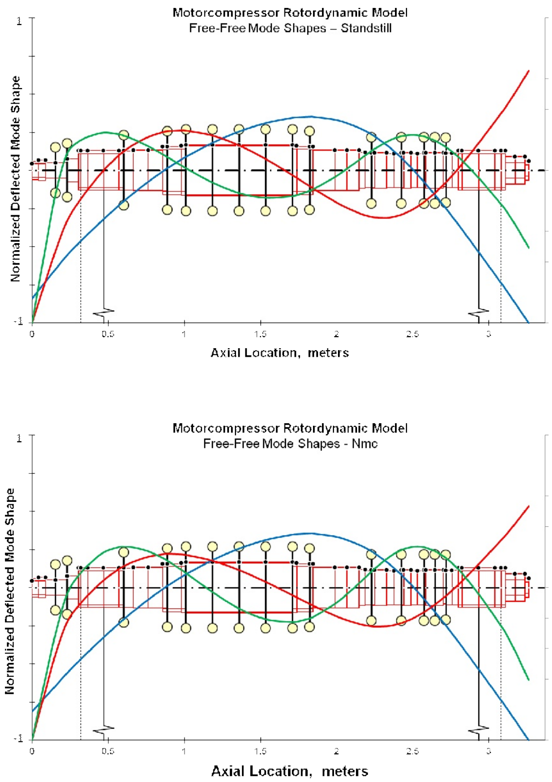


Figure 19. Free-Free Rotor Mode Shapes

Undamped Critical Speed Map

Similarly to rotor on oil film bearings, an Undamped Critical Speed (UCS) Map was generated to identify possible stiffness ranges of the AMB to target specific position of the rotor critical speed across the operating speed range. Once the AMB synthesis process was completed, the UCS map can be used to represent critical speeds and relevant mode shapes.

The AMB force to displacement transfer functions for the two bearings are represented in Figure 20 in terms of gain and phase from 0 cpm to 150% Nmc.

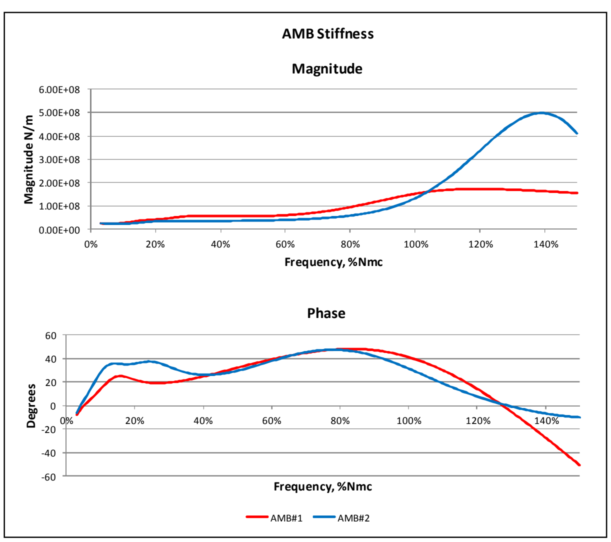


Figure 20. Typical Force to Displacement TF

An UCS map for the rotor discussed in this example is shown in Figure 21. The AMB stiffnesses are shown superimposed on the plot similar to conventional fluid film bearings.

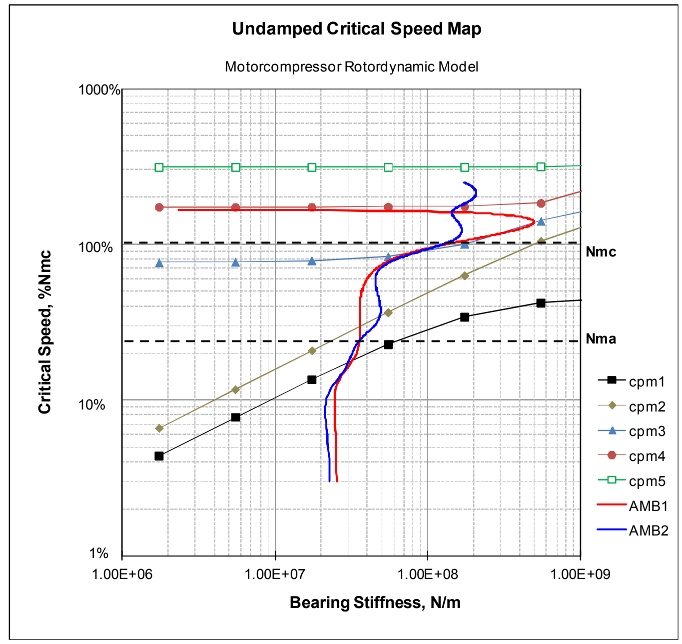


Figure 21. Undamped Critical Speed Map

Unbalance Response displacements and forces

An unbalanced rotor response was performed to determine rotor critical speeds, separation margins and vibration amplitudes resulting from different unbalance distributions on the rotor. For each unbalance case, AMB dynamic loads applied were determined and compared to the dynamic load capacity of the AMBs to verify that required load capacity margin was available.

The overall unbalance load, calculated according API617 standard paragraph 4.8.2.7 was:

$$2 \text{ x } Ur = 2 \text{ x } 6350 \text{ W/Nmc} = 2100 \text{ g-mm} \qquad (29)$$

As a first step, no Unbalance Force Rejection Control (UFRC) algorithms were considered for critical speed frequency, amplification factor, vibration amplitudes and loads identification according Annex E requirements. For the sake of completeness, an analysis with UFRC filters activated would also typically be included in the final rotordynamic analysis report.

The peak-to-peak vibration limit for this machine was calculated using Equation 20. As generally expected for high speed machines, the vibration limit resulting from rotor speed criteria overruled the one resulting from minimum clearance.

Typical rotor unbalance response results are presented in Figure 22.

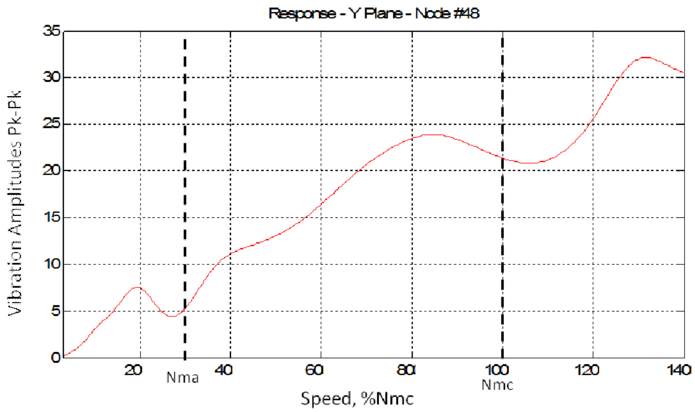


Figure 22. Rotor Unbalance Response (Typical)

Finally, the actuator dynamic forces were evaluated. Typical results of this analysis are shown in Figure 23. This analysis predicted a margin well in excess of the 50% margin required by Annex E. For the worst case unbalance load scenario, the dynamic force at maximum continuous speed at the most excited bearing was 13% of the maximum static force and 15% of the dynamic load capacity at N_{mc} frequency.

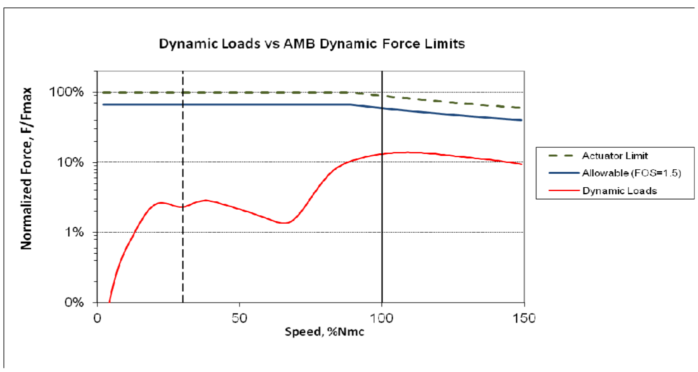


Figure 23. Dynamic Loads and Allowable Limits

Closed Loop Transfer Functions

Since this machine was tested prior to the new Annex E requirements being published, the closed-loop transfer functions were not required to be reported. They were, however, calculated for internal use and are represented in Figure 24 for each bearing.

Figure 24. Calculated Closed Loop Transfer Function

Stability Analysis

The stability analysis was a joint effort of the AMB vendor and the compressor OEM. The OEM provided AMB vendors with all necessary inputs regarding machine operating conditions and destabilizing effects, such as seals, external excitation etc. The AMB vendor was responsible for modeling the AMB system dynamics and adjusting the controller algorithms and parameters to meet acceptance criteria essentially the same as those specified in Annex E, along with some proprietary stability acceptance criteria on the feedback control loop. A share of responsibility and mutual understanding on governing phenomena and stability criteria was hence required on this task.

The stability analysis included a Level I stability analysis, a Level II stability analysis and a final assessment of the Sensitivity Transfer functions as defined in ISO14839-3. For this example, only the final Level II results, including seals effects, will be presented. The overall system included the dynamics of the rotor and the AMB system (i.e. controller, power amplifiers etc). Thus many more natural frequencies than what expected from the rotor only system were present in the results for the frequency range of interest. These results are shown in the natural frequency/logarithmic decrement plot in Figure 25. All natural frequencies and relevant log-decrements up to 200% rotor N_{mc} stay above the stability threshold defined by Annex E, and would meet the new stability requirements.

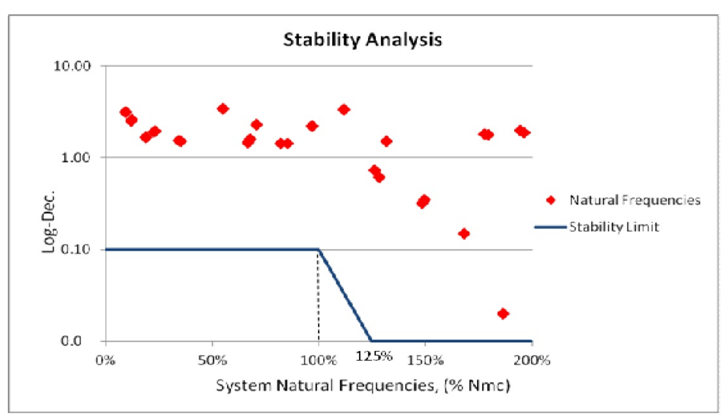


Figure 25. Stability Analysis Eigenvalues

Sensitivity Transfer Function

As the final step of the stability performance assessment, the sensitivity transfer functions for both bearings were calculated for the rotor. This analysis traditionally has been an AMB vendor responsibility. Because of the relative newness of such stability assessment criteria recently introduced on ISO 14839-3 Standard and the new Annex E Standard, commercial rotordynamic codes generally do not have Sensitivity Transfer function assessment capability. In the case of this machine, the OEM used a custom developed code to complete this assessment step.

The sensitivity analysis was performed for rotor at standstill condition and while operating at maximum continuous speed and with seals coefficients included. A typical result at design point conditions for one of the two bearings is depicted in Figure 26. In all cases peak sensitivity transfer function gains meet the Annex E requirements.

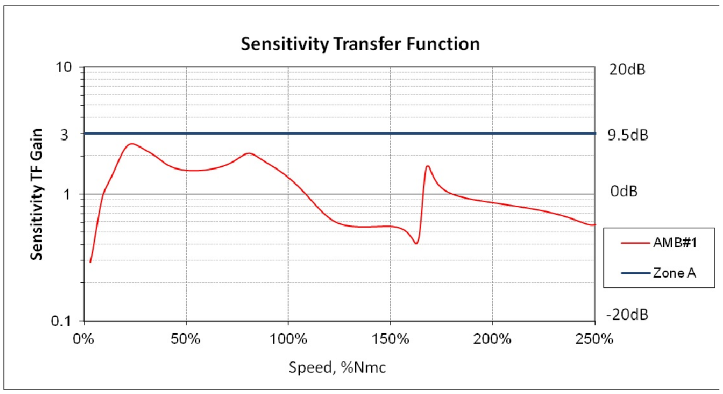


Figure 26. Calculated Sensitivity Transfer function

Axial stability and sensitivity function analysis

This machine was modeled as a single lumped mass for the axial analysis. All of the predicted eigenvalues had an amplification factor less than 2.5. Therefore, these values are not required to be included in the rotordynamics report per the new Annex E standard. A complete report would include the sensitivity transfer function plot for the axial system.

As Installed Analysis

The results presented above are for the as-designed control system. For a machine governed by the new API 617 specifications, an additional set of analyses would be required if any field tuning was performed.

Testing

As part of the factory acceptance test, the machine in this example underwent an extensive testing campaign that included full load/ full speed testing and rotor drop testing. This section presents selected results which highlights on new API617 test requirements and results

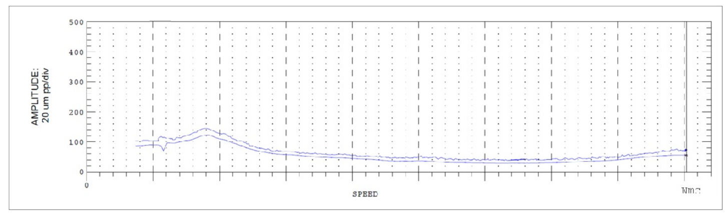
Vibration

Vibration amplitudes and waterfall trends for a typical acceleration curve of the rotor from zero speed up to full speed when the machine was operating at reduced inlet pressure and load are represented in Figures 27 and 28.

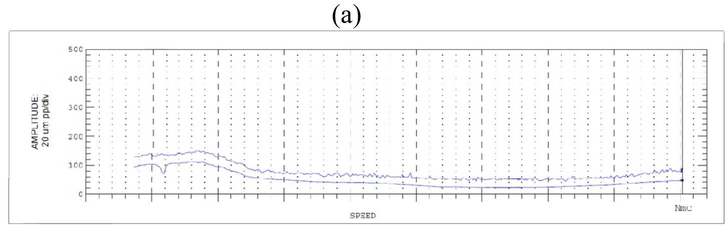
Figure 28 shows that the vibration amplitude was dominated by the synchronous vibration component as expected. No major sub-synchronous component was evident. Peak-peak radial vibration amplitudes values were as low as 80 µm pk-pk and well below acceptance value of 170 µm pk-pk determined according Annex E.

As normally expected for AMB equipped compressors, the sub-synchronous vibration component increased as the gas inlet pressure and compressor head increased up to nominal values. In such condition some sub synchronous vibration activity was generated by the gas low frequency aerodynamic excitation of the first rigid mode of the rotor.

Vibration and currents waterfall plots at full load operating conditions are represented in Figure 29. A good margin against Annex E acceptance requirements for mechanical running test was widely satisfied. Overall the new Annex E requirements on vibration amplitudes were demonstrated to be a reasonable reference for vibration limit also for machines undergoing a full load acceptance test.

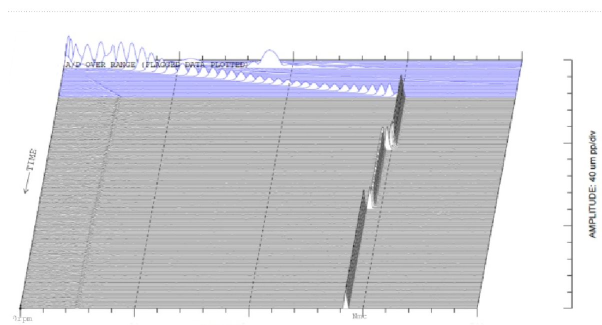


(a)

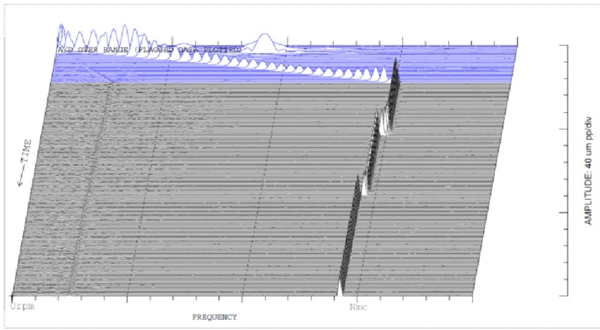


(b)

Figure 27. Vibration Amplitude Trends - AMB1 (a) and AMB2 (b)

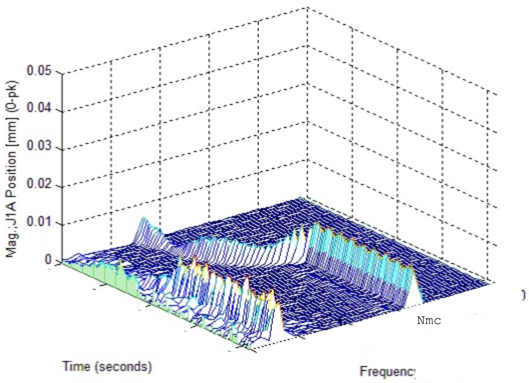


(a)

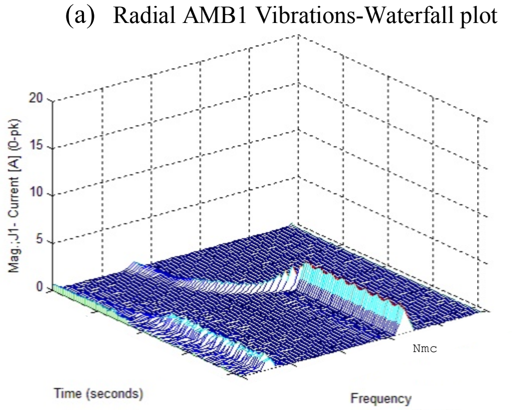


(b)

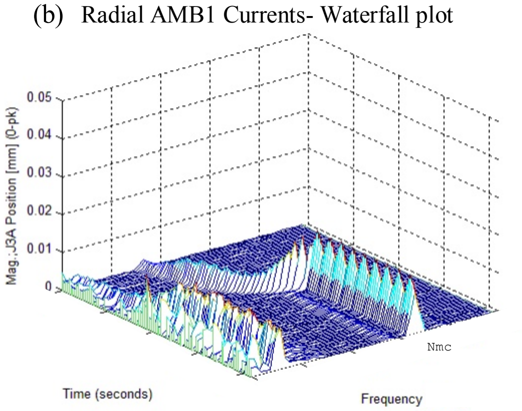
Figure 28. Vibration Amplitude Waterfall, AMB1 (a) and AMB2 (b)



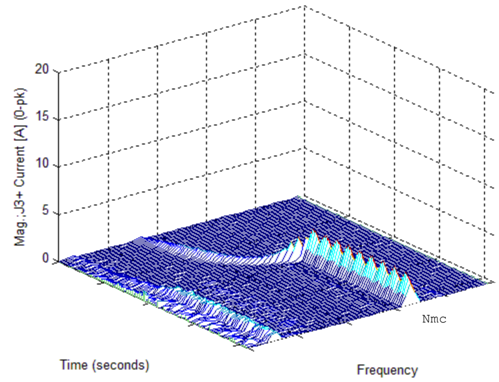
(a) Radial AMB1 Vibrations-Waterfall plot



(b) Radial AMB1 Currents- Waterfall plot



(c) Radial AMB2 Vibrations Waterfall plot



(d) Radial AMB2 Currents- Waterfall plot

Figure 29. Radial Vibration and Currents when Operating at Full Load

Sensitivity Transfer Function

As part of the factory acceptance test, the machine underwent an extensive stability analysis campaign to evaluate machine stability at different operating conditions. Some reference operating points considered for this purpose are identified on the compressor map of the compressor for a given gas suction pressure shown in Figure 30. Point P0 on the map represents rotor standstill condition (where the Annex E requirements apply) and point P1, P3, P3 some operating points at increasing speed up to N_{mc} located at the center of the performance map. Points P3S and P3C represent the surge and choking limit of the performance curve at maximum continuous speed.

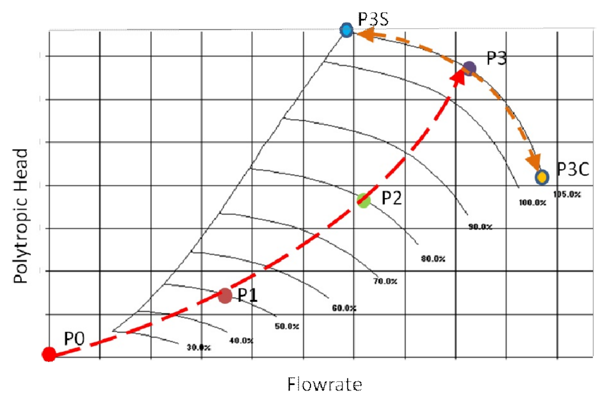


Figure 30. Operating Points on Compressor Map

Open Loop and Sensitivity Transfer Functions at different operating points were measured and evaluated as stability performance indexes.

The Sensitivity Transfer Functions gains for the two radial and one axial bearing at standstill conditions are represented in Figure 31. Peaks of Sensitivity Transfer function were verified to be lower than 3 (9.5 dB) for all AMBs, thus meeting the new Annex E requirements.

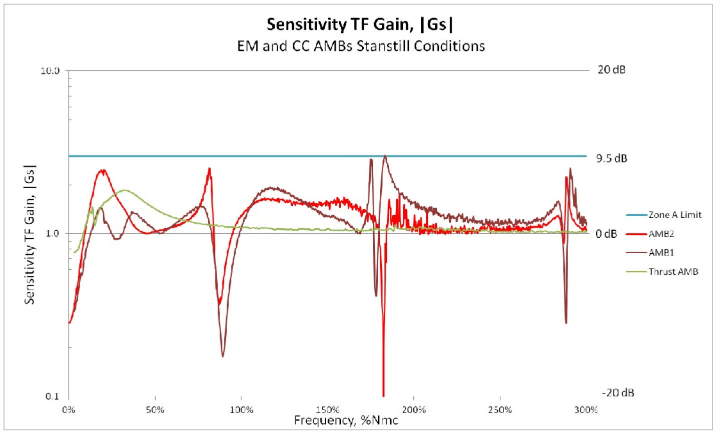


Figure 31. Sensitivity Transfer Function Gains at Standstill

Additional sensitivity transfer function measurements while the machine was in operation are shown in Figure 32 (Points P1, P2, P3). These measurements highlight that sensitivity transfer function measurements at higher speed was strongly affected by synchronous vibration and external excitation that became predominant over the external excitation injected into the system to measure the transfer functions.

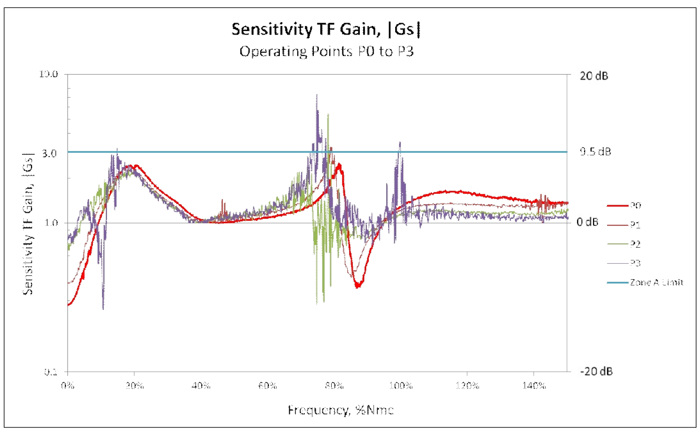


Figure 32. AMB2 Sensitivity Transfer Function at P0, P1, P2 and P3 operating points

Finally, the sensitivity transfer functions were measured while moving the operating point of the compressor from surge to choking limits along the maximum continuous speed curve. These measurements are shown in Figure 33 Some effects on sensitivity TF gain were measured but overall coherence of sensitivity transfer function was within acceptable range only for a limited frequency range. Sensitivity transfer function measurements while machine was in operation was demonstrated to be a valid indicator of stability performances but Annex E requirements could not be strictly applied.

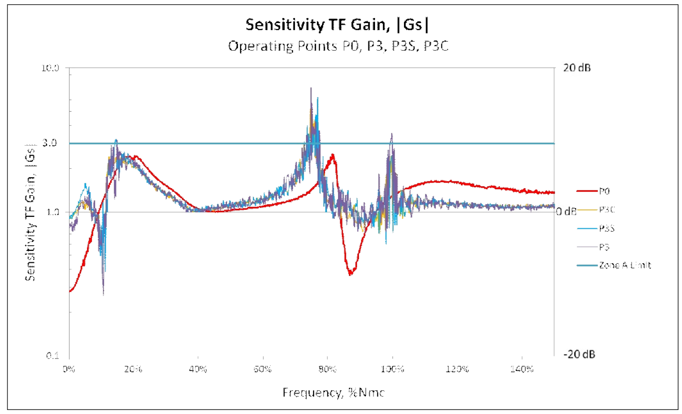


Figure 33. AMB2 Sensitivity Transfer Function at P0, P3, P3S and P3C operating points

Auxiliary Bearing Testing

As a concluding part of the factory acceptance test, a rotor drop test from full speed condition was performed. The number of drops, procedure and acceptance criteria were agreed between AMB vendor and OEM. The detailed results will not be presented; however, the system met all agreed-upon test requirements.

Summary

Overall the new Annex E acceptance criteria for AMB equipped rotor were verified to identify a sound envelope of requirements, procedures and acceptance criteria resulting with a robust AMB system and machine design.

CLOSING COMMENTS

We believe that the new API 617 Eighth Edition Annex E requirements for AMB supported compressors are a significant step forward to providing a framework to help users ensure that these machines will provide years of reliable service. The new standard draws from a substantial body of AMB developer, OEM and academic knowledge. It also leverages the extensive work done by the ISO Active Magnetic Bearing Committee. It takes into account both the unique limitations and extra capabilities of AMB systems.

In this tutorial, we have attempted to cover most of the major issues to help engineers tasked with reading reports and evaluating machinery. We have also tried to assemble some of the major mathematical details into a single document for readers who need a deeper understanding and for future rotordynamic code developers.

NOMENCLATURE

AMB	Active Magnetic Bearing
API	American Petroleum Institute
DSP	Digital Signal Processor
OEM	Original Equipment Manufacturer
MBC	Magnetic Bearing Controller
MRT	Mechanical Running Test
MIMO	Multi-input, Multi-Output
PID	Proportional-Integral-Derivative
PWM	Pulse Width Modulated
SISO	Single-Input, Single-Output

UCS Undamped Critical Speed

A	pole area, m^2
A_s	Rotor/structure system matrix in first order form
B	Magnetic flux density, T
B_s	Rotor/structure input matrix in first order form
C	Damping matrix
C_{min}	Minimum clearance, m
C_p	Plant output matrix in first order form
C_R	Rotor damping matrix
C_{Brg}	Fluid film bearing damping matrix
CMD	Command signal
D	Derivative gain
D_p	Plant feedthrough matrix in first order form
ERR	Error signal
EXC	Excitation input
F	Force
G	Transfer function
G_R	Rotor gyroscopic matrix at a given speed
I	Current, Integral Gain
Id	Transverse mass moment of inertia
Ip	Polar mass moment of inertia
K	Stiffness, Stiffness matrix
K_{Brg}	Fluid film bearing stiffness matrix or actuator negative stiffness
K_i	Control current force constant
K_R	Rotor stiffness matrix
K_x	Displacement force constant
L	Inductance
M	Mass, Mass matrix
M_{Brg}	Fluid film bearing mass matrix
M_R	Rotor mass matrix
N	Number of turns per pole
N_{mc}	Maximum continuous operating speed
OUT	Compensator output
P	Proportional gain
Pmag	Magnetic "pressure"
T	Tesla
V	Volts, Voltage

a,b	Transfer function coefficients
fr	Attractive air gap force, normal to surface
f	Force vector
g	Gap, m
q	Physical displacement vector
s	Complex frequency, $j\omega$
u	Actuator forces
x,y	Displacements

μ0	Permeability of free space, 4π E-7 m-kg/s2
x_p	State vector
ω	frequency (rad/s)

APPENDIX A

Model Input Table for Example 1

Station	Length	OD	ID	Weight Density	Elastic Modulus	Shear Modulus	Added Weight	Added Ip	Added It	Speed Factor	Material	
#	in	in	in	lb/in^3	psi	psi	lb	$lb\text{-}in^2$	$lb\text{-}in^2$			
stnum	length	oda	ida	rhoa	ea	ga	awt	aip	ait			
1	0.360	0.994	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
2	0.360	0.994	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	bbrg1
3	0.125	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
4	0.200	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
5	0.200	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	sen1
6	0.375	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
7	0.375	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	act1
8	0.384	1.800	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
9	0.384	1.800	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
10	0.500	2.000	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
11	0.200	2.000	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
12	0.500	1.750	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	motor start
13	0.500	1.750	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
14	0.500	1.750	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
15	0.500	1.750	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
16	0.500	1.750	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	motor center
17	0.500	1.750	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
18	0.500	1.750	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
19	0.500	1.750	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
20	0.250	2.000	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	motor end
21	0.500	2.400	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
22	0.200	2.400	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
23	0.200	2.400	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	thrust disk
24	0.390	2.000	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
25	0.390	1.800	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
26	0.375	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
27	0.375	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	act2
28	0.200	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
29	0.200	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	sen2
30	0.125	1.350	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
31	0.360	1.191	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
32	0.360	1.191	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
33	0.150	1.400	0.0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
34	0.300	0.375	0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
35	0.250	0.375	0	0.283	30,000,000	11,500,000	0	0	0	1	steel	
36	0.250	0.375	0	0.283	30,000,000	11,500,000	0	0	0	1	steel	Impeller CG
37	0.000	0.000	0.0	0.000	-	-	0	0	0	1		
4	0.200	1.800	1.350	0.290	-	-	0	0	0	1	laminations	
5	0.200	1.800	1.350	0.290	-	-	0	0	0	1	laminations	
6	0.375	1.800	1.350	0.290	-	-	0	0	0	1	laminations	
7	0.375	1.800	1.350	0.290	-	-	0	0	0	1	laminations	
11	0.200	2.300	2.000	0.296	-	-	0	0	0	1	Inconel_sleeve	
12	0.500	2.300	1.750	0.296	-	-	0	0	0	1	Inconel_sleeve	
13	0.500	2.300	1.750	0.296	-	-	0	0	0	1	Inconel_sleeve	
14	0.500	2.300	1.750	0.296	-	-	0	0	0	1	Inconel_sleeve	
15	0.500	2.300	1.750	0.296	-	-	0	0	0	1	Inconel_sleeve	
16	0.500	2.300	1.750	0.296	-	-	0	0	0	1	Inconel_sleeve	
17	0.500	2.300	1.750	0.296	-	-	0	0	0	1	Inconel_sleeve	
18	0.500	2.300	1.750	0.296	-	-	0	0	0	1	Inconel_sleeve	
19	0.500	2.300	1.750	0.296	-	-	0	0	0	1	Inconel_sleeve	
20	0.250	2.300	2.000	0.296	-	-	0	0	0	1	Inconel_sleeve	
22	0.200	3.500	2.4	0.283	-	-	0	0	0	1	steel_sleeve	
23	0.200	3.500	2.4	0.283	-	-	0	0	0	1	steel_sleeve	thrust disk
26	0.375	1.800	1.350	0.290	-	-	0	0	0	1	laminations	
27	0.375	1.800	1.350	0.290	-	-	0	0	0	1	laminations	
28	0.200	1.800	1.350	0.290	-	-	0	0	0	1	laminations	
29	0.200	1.800	1.350	0.290	-	-	0	0	0	1	laminations	
34	0.300	1.200	0.375	0.160	16,500,000	6,600,000	0	0	0	1	Ti	
35	0.250	1.200	0.375	0.160	16,500,000	6,600,000	0	0	0	1	Ti	
36	0.250	1.200	0.375	0.160	16,500,000	6,600,000	0	0	0	1	Ti	
35	0.250	4.500	1.200	0.160	-	-	0	0	0	1	Ti_mass	
36	0.250	3.000	1.200	0.160	-	-	0	0	0	1	Ti_mass	

Actuator (negative stiffness) for Example 1

Brg 1 (x1, y1): 2,500 lbf/in
Brg 2 (x2, y2): 2,500 lbf/in
Axial Brg (z): 4,000 lbf/in

Rotor Damping Matrix for Example 1

The rotor damping matrix used in Example 1 is a proportional damping matrix obtained by multiplying the rotor stiffness matrix by a constant value:

$$C_R = K_R\left(\frac{2\varepsilon}{\omega_1}\right)$$

Where ω1 = 10,580 rad/s is the frequency of the first bending mode at 0 rpm and ε = 0.002 is the damping ratio of the first bending mode at 0 rpm. With this simple approach, the higher bending modes become more heavily damped. As a standard practice, the authors normally apply damping ratios to factor to each bending mode individually, usually from 0.002 to 0.005. However, the simple approach used in this tutorial keeps the example simpler so that it can be more easily reproduced by a wider audience.

Magnetic Bearing Force Displacement Transfer Functions for Example 1

	G11, G22				G33, G44				G55	
n	num, bn	den, an		n	num, bn	den, an		n	num, bn	den, an
14	0	1.000E+00		14	0	1.000E+00		18	0	1.00E+00
13	0	4.146E+05		13	0	4.146E+05		17	0	4.09E+05
12	0	7.393E+10		12	0	7.393E+10		16	0	7.15E+10
11	0	6.889E+15		11	0	6.889E+15		15	0	6.45E+15
10	2.078E+20	3.665E+20		10	2.053E+20	3.665E+20		14	1.84E+20	3.24E+20
9	-7.222E+25	1.137E+25		9	-7.132E+25	1.137E+25		13	-6.12E+25	9.07E+24
8	1.011E+31	2.266E+29		8	9.980E+30	2.266E+29		12	7.90E+30	1.54E+29
7	-6.056E+35	3.005E+33		7	-5.980E+35	3.005E+33		11	-3.71E+35	1.62E+33
6	1.376E+40	2.723E+37		6	1.359E+40	2.723E+37		10	2.34E+36	1.04E+37
5	7.672E+42	1.678E+41		5	7.578E+42	1.678E+41		9	4.22E+44	3.98E+40
4	5.932E+48	6.983E+44		4	5.858E+48	6.983E+44		8	1.39E+48	8.87E+43
3	1.169E+52	1.653E+48		3	1.155E+52	1.653E+48		7	1.59E+51	1.12E+47
2	1.718E+55	2.017E+51		2	1.697E+55	2.017E+51		6	1.06E+54	8.41E+49
1	6.576E+57	1.329E+54		1	6.505E+57	1.329E+54		5	4.42E+56	3.92E+52
0	4.592E+58	0		0	5.427E+58	0		4	1.00E+59	1.01E+55
								3	1.10E+61	1.16E+57
								2	5.00E+62	4.75E+58
								1	8.22E+63	5.55E+59
								0	3.73E+64	0

APPENDIX B

The first 25 eigenvalues up to 20,000 Hz, including overdamped and real eigenvalues, at 0 and 60,000 rpm for the turbocompressor example are presented in the two tables below.

Rotor spin speed:		0 rpm				
Mode	Real (rad/s)	Imag (rad/s)	Nat (rpm)	Freq (Hz)	Zeta	Pole Freq (Hz)
Complex Eigenvalues with Zeta <= 0.7070 (Positive freq listed)						
1	-219.13	441.73	4218.5	70.3	0.4444	78.48
2	-219.13	441.73	4218.5	70.3	0.4444	78.48
3	-280.21	967.48	9239.45	153.98	0.2782	160.31
4	-280.21	967.48	9239.45	153.98	0.2782	160.31
5	-1097.13	1163.17	11108.31	185.12	0.6862	254.48
6	-1097.13	1163.17	11108.31	185.12	0.6862	254.48
7	-1061.95	1575.67	15047.67	250.78	0.5589	302.41
8	-1061.95	1575.67	15047.67	250.78	0.5589	302.41
9	-2971.73	6840.61	65327.78	1088.72	0.3985	1187.01
10	-2971.73	6840.61	65327.78	1088.72	0.3985	1187.01
11	-3019.48	7031.86	67154.27	1119.16	0.3946	1217.97
12	-3019.48	7031.86	67154.27	1119.16	0.3946	1217.97
13	-47.07	10684.37	102035.7	1700.47	0.0044	1700.49
14	-47.07	10684.37	102035.7	1700.47	0.0044	1700.49
15	-82.42	20980.84	200367.1	3339.2	0.0039	3339.23
16	-82.42	20980.84	200367.1	3339.2	0.0039	3339.23
17	-236.68	35567.11	339665.9	5660.68	0.0067	5660.81
18	-236.68	35567.11	339665.9	5660.68	0.0067	5660.81
19	-454.19	49265.66	470487.1	7840.87	0.0092	7841.21
20	-454.19	49265.66	470487.1	7840.87	0.0092	7841.21
21	-887.58	68867.1	657680.8	10960.54	0.0129	10961.45
22	-887.58	68867.1	657680.8	10960.54	0.0129	10961.45
23	-1508.28	89769.45	857298.3	14287.25	0.0168	14289.27
24	-1508.28	89769.45	857298.3	14287.25	0.0168	14289.27
25	-2120.65	106438.9	1016492	16940.28	0.0199	16943.64
26	-2120.65	106438.9	1016492	16940.28	0.0199	16943.64
27	-2597.38	117791.9	1124913	18747.17	0.022	18751.73
28	-2597.38	117791.9	1124913	18747.17	0.022	18751.73
Complex Eigenvalues with Zeta > 0.7070 (Positive freq listed)						
1	-11141.5	8098.84	77343.88	1288.97	0.8089	2192.2
2	-11141.5	8098.84	77343.88	1288.97	0.8089	2192.2
3	-11725.5	8111.75	77467.24	1291.03	0.8224	2269.21
4	-11725.5	8111.75	77467.24	1291.03	0.8224	2269.21
5	-10971.5	10750.23	102664.7	1710.95	0.7143	2444.68
6	-10971.5	10750.23	102664.7	1710.95	0.7143	2444.68
7	-11229.2	10662.48	101826.7	1696.99	0.7252	2464.5
8	-11229.2	10662.48	101826.7	1696.99	0.7252	2464.5
9	-60000.7	34640.6	330817.7	5513.22	0.866	11026.64
10	-60000.7	34640.6	330817.7	5513.22	0.866	11026.64
11	-60000.6	34640.78	330819.4	5513.25	0.866	11026.65
12	-60000.6	34640.78	330819.4	5513.25	0.866	11026.65
Real Eigenvalues						
1	-13.72	0	0	0	1	2.18
2	-13.72	0	0	0	1	2.18
3	-17.4	0	0	0	1	2.77
4	-17.4	0	0	0	1	2.77
5	-470.49	0	0	0	1	74.88
6	-470.49	0	0	0	1	74.88
7	-1285.84	0	0	0	1	204.65
8	-1285.84	0	0	0	1	204.65

Rotor spin speed:		60000 rpm				
Mode	Real (rad/s)	Imag (rad/s)	Nat (rpm)	Freq (Hz)	Zeta	Pole Freq (Hz)
Complex Eigenvalues with Zeta <= 0.7070 (Positive freq listed)						
1	-218.47	376.4	3594.59	59.91	0.502	69.26
2	-226.58	454.59	4341.32	72.35	0.4461	80.84
3	-437.15	701.68	6701.05	111.68	0.5288	131.58
4	-224.33	1286.7	12288	204.78	0.1718	207.87
5	-1098.19	1152.3	11004.48	183.39	0.6899	253.34
6	-1098.21	1170.32	11176.58	186.26	0.6843	255.43
7	-884.15	1576.21	15052.77	250.86	0.4892	287.63
8	-1195.77	1513.25	14451.57	240.84	0.62	306.96
9	-2965.02	6835.25	65276.63	1087.86	0.398	1185.81
10	-2982.25	6839.4	65316.29	1088.52	0.3997	1187.51
11	-2987.31	7031.11	67147.13	1119.04	0.391	1215.85
12	-3044.67	7051.55	67342.26	1122.29	0.3964	1222.43
13	-53.87	9742.25	93038.51	1550.53	0.0055	1550.55
14	-40.92	11539.51	110202.35	1836.57	0.0035	1836.58
15	-73.49	18753	179091.12	2984.63	0.0039	2984.66
16	-91.58	23227.78	221825.29	3696.82	0.0039	3696.84
17	-220.51	33442.26	319373.56	5322.5	0.0066	5322.62
18	-253.28	38396.83	366689.68	6111.04	0.0066	6111.18
19	-436.32	47475.32	453389.33	7555.93	0.0092	7556.25
20	-471.67	51535.38	492162.89	8202.11	0.0092	8202.45
21	-870.24	67544.02	645045.36	10749.96	0.0129	10750.86
22	-904.8	70266.09	671041.2	11183.2	0.0129	11184.12
23	-1489.85	88615.18	846274.97	14103.54	0.0168	14105.54
24	-1526.75	90852.48	867641.15	14459.62	0.0168	14461.66
25	-2062.26	103237.5	985918.02	16430.76	0.02	16434.03
26	-2177.97	109033.7	1041272.05	17353.26	0.02	17356.72
27	-2503.2	113961.5	1088332.36	18137.54	0.022	18141.91
28	-2693.12	122486.7	1169747.99	19494.36	0.022	19499.08
Complex Eigenvalues with Zeta > 0.7070 (Positive freq listed)						
1	-13.71	0.23	2.15	0.04	0.9999	2.18
2	-17.39	0.11	1.03	0.02	1	2.77
3	-420.3	285.22	2723.88	45.39	0.8275	80.84
4	-1272.97	51.98	496.43	8.27	0.9992	202.77
5	-11133.2	8104	77393.2	1289.79	0.8085	2191.62
6	-11151.9	8091.17	77270.71	1287.75	0.8094	2192.83
7	-11716.3	8087.11	77231.94	1287.1	0.823	2265.78
8	-11732.4	8137.01	77708.48	1295.05	0.8217	2272.41
9	-10962.3	10741.12	102577.67	1709.5	0.7143	2442.62
10	-10981.5	10757.7	102736.05	1712.14	0.7143	2446.65
11	-11221.9	10667.61	101875.64	1697.8	0.7248	2464.22
12	-11235.1	10660.43	101807.12	1696.66	0.7254	2464.96
13	-60000.7	34640.6	330817.7	5513.22	0.866	11026.64
14	-60000.7	34640.6	330817.71	5513.22	0.866	11026.64
15	-60000.6	34640.78	330819.41	5513.25	0.866	11026.64
16	-60000.6	34640.78	330819.45	5513.25	0.866	11026.65

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