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著者	Narita Kenzo, Nakashima Motomu, Takagi Hideki
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Title:

Developing a methodology for estimating the drag in front-crawl swimming at various velocities

Author:

Kenzo Narita[1)], Motomu Nakashima[2)] and Hideki Takagi[3)]

1) Doctoral Program in Physical Education, Health and Sport Sciences, University of Tsukuba

2) Department of Systems and Control Engineering, Tokyo Institute of Technology

3) Faculty of Health and Sport Sciences, University of Tsukuba

Keywords:

active drag, residual thrust, semi-tethered, passive drag

Abstract

We aimed to develop a new method for evaluating the drag in front-crawl swimming at various velocities and at full stroke. In this study, we introduce the basic principle and apparatus for the new method, which estimates the drag in swimming using measured values of residual thrust (MRT). Furthermore, we applied the MRT to evaluate the active drag (Da) and compared it with the passive drag (Dp) measured for the same swimmers. Da was estimated in five-stages for velocities ranging from 1.0 to 1.4 m s−1; Dp was measured at flow velocities ranging from 0.9 to 1.5 m s−1 at intervals of 0.1 m s−1. The variability in the values of Da at MRT was also investigated for two swimmers. According to the results, Da (Da = 32.2 v3.3, N = 30, R2 = 0.90) was larger than Dp (Dp = 23.5 v2.0, N = 42, R2 = 0.89) and the variability in Da for the two swimmers was 6.5% and 3.0%. MRT can be used to evaluate Da at various velocities and is special in that it can be applied to various swimming styles. Therefore, the evaluation of drag in swimming using MRT is expected to play a role in establishing the fundamental data for swimming.

1. Introduction

Drag has a major influence on swimming performance because swimming is performed in water, which has a greater density than air. Therefore, the evaluation of drag is one of the most important issues in swimming research. However, it is extremely difficult to evaluate the actual drag during swimming (active drag) because the swimmer is continuously moving. To accurately measure the active drag, it is necessary to measure the entire pressure and friction distribution of the swimmer without disturbing their natural swimming movement. Hence, only a few methods have been developed to evaluate the active drag, each with several restrictions. In the measurement of active drag (MAD) approach (Hollander et al., 1986; Toussaint et al., 1988; Toussaint, Roos, & Kolmogorov, 2004; Van der Vaart et al., 1987), only the front-crawl swimming stroke can be assessed owing to the apparatus structure. Furthermore, the method is limited to the use of arms only. This means that swimmers cannot use lower limb actions to maintain streamlined alignments, which they would use during normal swimming.

The velocity perturbation method (Kolmogorov & Duplishcheva, 1992; Toussaint et al., 2004) and the assisted towing method (Formosa, Toussaint, Mason, & Burkett, 2012) require that the swimmers swim with maximal effort. Hence, this method is not suitable for evaluating the active drag at various velocities, i.e., sub-maximal effort. The “energetic approach” proposed by Di Prampero, Pendergast, Wilson, and Rennie (1974) includes the motion of legs but extrapolates the data for active drag by adding (or subtracting) external loads to (or from) a swimmer and measuring the associated energy expenditure. Therefore, the development of a new methodology for estimating the drag in swimming, which can enable upper and lower limb motion (full stroke) at various velocities during front-crawl swimming, would provide swimmers and coaches with beneficial information on improving their swimming performances.

Accordingly, the purpose of this research is to develop a new methodology for evaluating the drag in front-crawl swimming at various velocities and at full stroke. We will introduce the basic principle and apparatus of the new method. We will also apply it to determine the active drag and compared it with the passive drag measured for the same swimmers.

Nomenclature

k_D: Coefficient of drag

k_P: Coefficient of propulsion

Da: Active drag (N)

Dp: Passive drag (N)

SR: Stroke rate (Hz)

SR_i: Stroke rate when a swimmer propels himself in a water flume at i m s^{-1} (Hz)

T_{re}: Residual thrust (N)
U: Flow velocity in the water flume (m s^{-1})
U_P: Virtual movement velocity of a swimmer's body relative to water (m s^{-1})
U_{Tre0}: Flow velocity at $T_{re} = 0$ (m s^{-1})
V_P: Virtual movement velocity of the swimmer's body relative to the fixed coordinates on the water flume (m s^{-1})
V_{Si}: Targeted swimming velocity for estimating the active drag (m s^{-1})

2. Methods

2.1. Basic principle of a new method for estimating the drag in swimming using measured values of residual thrust (MRT)

Swimming velocity depends on the interaction between two forces: one is generated to propel the swimmer forward (propulsion), whereas the other acts in the direction that prevents propulsion (drag). For instance, when the drag is larger than the propulsion, the swimmer decelerates. In a water flume that can be used to freely adjust the flow velocity (U), the relations between the swimming velocity, propulsion (P), and drag (D) can be formulated from the characteristics of hydrodynamic forces, which are proportional to the square of the velocity (Assumption 1):

$$D = k_D \cdot U^2 \tag{1}$$

$$P = k_P \cdot U_P{}^2 \tag{2}$$

where k_D represents the coefficient of drag and k_P represents the coefficient of propulsion, both of which include the density of water and the representative area. For the sake of simplicity, we do not consider intra-cyclic variations in k_D and k_P, only their average values within a complete swimming cycle. Note that U_P represents the virtual movement velocity of a swimmer's body relative to water, which is needed in order to produce propulsion using the upper and lower limbs. For example, U_P can be considered to be the hand velocity required to push water. However, the hand does not always push water at velocity U_P. Furthermore, other body parts also contribute to producing propulsion. Therefore, U_P can be considered to be a sort of average for the movement in time of body parts through the stroke cycle. When a swimmer swims while maintaining a constant position in a water flume whose flow velocity is U, U_P becomes

$$U_P = V_P - U \tag{3}$$

where V_P represents the virtual movement velocity of the swimmer's body relative to the fixed coordinates of the water flume (not to the water). Therefore, substituting Eq. (3) into Eq. (2), we get

$$P = k_P \cdot (V_P - U)^2 \tag{4}$$

If the swimmer maintains the same technique, body position and kinematics, e.g., stroke rate, when the flow velocity changes, the values of k_P, k_D, and V_P are expected to remain unchanged

(Assumption 2); therefore, the propulsion and drag will only vary depending on the flow velocity (U). Assumption 2 is considered to be valid when the stroke rate is maintained as U is changed. For instance, as a swimmer swims freely in a water flume in which U is set at i m s^{-1}, if the swimmer can keep himself in a given position, the propulsion and drag acting on the swimmer must be balanced. We defined such a swimming velocity condition as a benchmark and termed it the "targeted swimming velocity for estimating the active drag" (V_{Si}). Under the assumption that the swimmer maintains a certain stroke rate at V_{Si}, if U is changed and set to be lower than V_{Si}, resistive forces decrease and propulsive forces increase. In contrast, when $U > V_{Si}$, resistive forces increase and propulsive forces decrease. Therefore, a difference between propulsion and drag occurred from changing U. For these experimental conditions, we named the difference between propulsive and resistive forces the "residual thrust" (T_{re}) and formulated T_{re} as follows:

$$T_{re} = P - D \tag{5}$$

Thus, $T_{re} = 0$ when $U = V_{Si}$ because under this condition, $P = D$; for $U < V_{Si}$, $T_{re} > 0$ because under this condition, $P > D$; finally for $U > V_{Si}$, $T_{re} < 0$ because $P < D$.

Substituting Eqs. (1) and (4) into Eq. (5), we get

$$T_{re} = k_P \cdot (V_P - U)^2 - k_D \cdot U^2 \tag{6}$$

Then, expanding Eq. (6), we get

$$T_{re} = (k_P - k_D)U^2 - 2 \cdot k_P \cdot V_P \cdot U + k_P \cdot V_P^{\,2} \tag{7}$$

In Eq. (7), for a given U, since k_D, k_P, and V_P are assumed to be constant (Assumption 2), T_{re} changes only as a function of U. We derived constant values of k_D, k_P, and V_P by adapting the function of Eq. (7) to approximate the actual measured T_{re} values that occurred in response to U. Accordingly, the active drag when a swimmer swims with a certain stroke at V_{Si} was derived by substituting for k_D in Eq. (1). More information on deriving k_D is described in the appendix.

2.2. Participants

Six male competitive swimmers participated in this study. The anthropometric details and performance level of each swimmer are given in Table 1. The test procedures were approved by the University of Tsukuba Ethics Committee (approval number: 26-69), and all participants signed the informed consent forms.

Table 1. Anthropometric data for swimmers and long-course front-crawl performance.

Swimmer	Age (year)	Height (cm)	Mass (kg)	100mFreestyle (s)
A	21	170	63	51.5
B	19	177	72	52.5
C	21	169	70	52.5
D	19	173	77	52.9
E	21	169	59	53.0
F	19	169	63	53.2
Mean	20.0	171.2	67.3	52.6
SD	1.0	3.0	6.2	0.6

2.3. Data processing

The active and passive drags were measured in a water flume (Igarashi Industrial Works Co. Ltd., Japan) that allowed the flow velocity to be precisely controlled. This channel had a control system for solving heterogeneous systems with respect to an unbalanced flow distribution. We used two load cells (LUX-B-2KN-ID, Kyowa Electronic Instruments Co. Ltd., Japan) (rating capacity: ±2 kN, measurement error: ±0.15%) that were sampled at 50 Hz and used a sensor interface (PCD-330B-F, Kyowa Electronic Instruments Co. Ltd., Japan) linked to a personal computer. To measure only the horizontal component forces, the angles of inclination of each wire, θ_1 and θ_2 (Fig. 1), were considered. In all trials, swimmers used a snorkel to eliminate the influence of the breathing motion, and all of them wore the same type of swimsuit to avoid a difference in the resistance because of the swimsuit type.

Active drag was evaluated for the front-crawl swimming stroke using the new methodology for estimating drag in swimming (MRT). Regarding the targeting swimming velocity for estimating the active drag (V_{Si}), we adopted five-staged velocity from i = 1.0 to 1.4 m s^{-1}. All stages were measured on the same day and the participants were provided with enough rest to deal with the influence of fatigue. Table 2 illustrates an example of a measurement procedure using the stage with i = 1.00 m s^{-1} ($V_{S1.00}$). In deriving a regression equation from Eq. (7), we made the assumption that the swimmer must be able to maintain the same technique, i.e., the same stroke rate, at $V_{S1.00}$ within the range of $0.80 \le U \le 1.20$ m s^{-1}. Thus, each swimmer was instructed to maintain the stroke motion and stroke rate $SR_{1.00}$ at $V_{S1.00}$ even when U was changed. The $SR_{1.00}$ value was measured while each swimmer swam at $V_{S1.00}$. It was then calculated from the reciprocal of the 10-stroke time (defined as the duration of 10 strokes, which was timed from the entry of the right hand on the first stroke to the entry of the same hand after the 10th stroke). To make it easy for the swimmer to maintain the motion and $SR_{1.00}$ at different U values, the swimmer relied not on his own subjective senses but on a sound that had the same interval as $SR_{1.00}$, which was produced using a small waterproof metronome (FINIS Inc., USA). As indicated in Fig. 2, using this procedure, swimmers were indeed able to maintain a constant stroke rate equal to $SR_{1.00}$ (at $V_{S1.00}$) when U was changed; this was true for all investigated speeds (V_{Si}). To measure T_{re} at each value of U ($0.80 \le U \le 1.20$ m s^{-1}), the swimmer was towed in both directions (Fig. 3). The forward (D) and backward (P) forces were measured for 10 s after steady-state conditions were attained (measurements were started from the entry phase of the right hand). Then, T_{re} was calculated from the difference between the forward and backward towing forces (Fig. 4, right panel). The average values of T_{re} over 10 s were calculated and used in the analysis. Best-fit regression curves were derived for the measured values of T_{re} by adjusting the coefficient and constant terms in Eq. (7) (i.e., k_D, k_P, and V_P) using the method of least squares (MATLAB 2014a, Math Works Inc.). Finally, the active drag (Da) was derived by substituting k_D and U_{Tre0} (T_{re} intersecting the x-axis at zero in Eq. (7)) when the swimmer swam at $V_{S1.00}$. Furthermore, Da values at other V_{Si} velocities were derived using the same procedure. As shown in the left panel of Fig. 3, k_D, k_P, and V_P changed with V_{Si}, and the swimming motion since Assumption 2 applied only when the swimmer maintained the same swimming motion. To investigate the variability of MRT, Da at $V_{S1.20}$ was estimated five times over three days for two swimmers. The coefficients of variability for Da, U_{Tre0}, and $SR_{1.20}$ were calculated from the ratio of each standard deviation to each mean value for the five trials at $V_{S1.20}$.

Passive drag (Dp) was measured by towing the swimmers, who maintained streamlined positions. The swimmers were towed forward and the forces were measured for 5 s at values of U ranging from 0.9 to 1.5 m s^{-1} in 0.1 m s^{-1} intervals. To compare Dp and Da for the same swimmers, the average measured values of Dp over 5 s were calculated for each value of U.

Table 2. Measurement procedure using the stages of i = 1.00 m s^{-1} ($V_{S1.00}$). Initially, the stroke rate ($SR_{1.00}$) and swimming motion at U = 1.00 m s^{-1} ($V_{S1.00}$) are examined. The swimmer is then instructed to maintain the stroke motion and stroke rate $SR_{1.00}$ at different flow velocities ($0.80 \leq U \leq 1.20$ m s^{-1}).

Trial no.	Flow velocity : U (m s^{-1})	Notes
1	1.00	Measured stroke rate (= $SR_{1.00}$)
↓ Maintain the specified stroke technique and $SR_{1.00}$ ↓		
2	0.80	
3	0.85	
4	0.90	
5	0.95	
6	1.00	
7	1.05	
8	1.10	
9	1.15	
10	1.20	

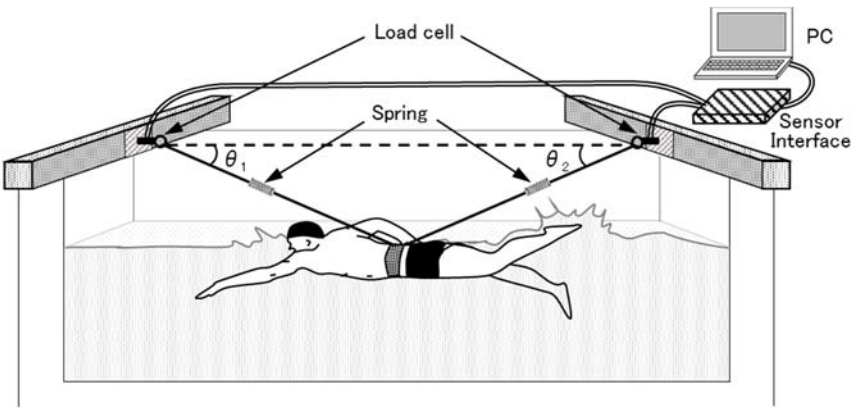

Fig. 1. Bird's eye view of the measurement process. A swimmer is towed in both directions and is connected to each load cell through non-elastic wires. The forces measured in both directions are processed through a sensor interface, which is linked to a load cell sampled at 50 Hz, and is inputted to a personal computer. Furthermore, springs are used to prevent slack in the wire, caused by fluctuations in the longitudinal direction, and the effects of tension caused when a swimmer alternates acceleration and deceleration in a stroke cycle.

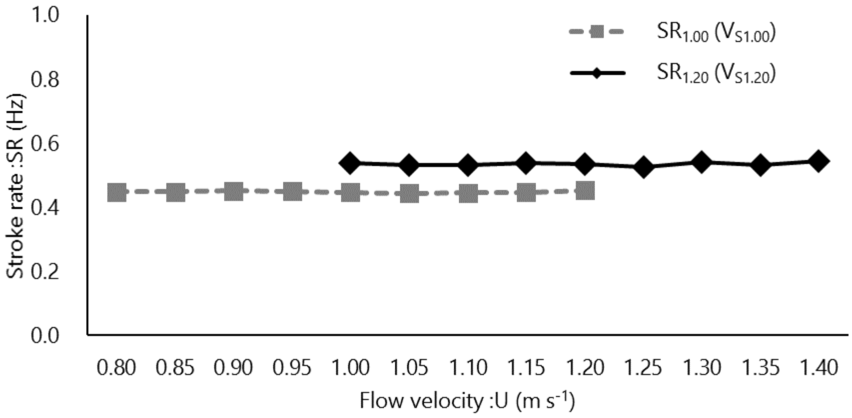

Fig. 2. The results of the stroke rate at $V_{S1.00}$ and $V_{S1.20}$ (corresponding to Fig. 3) over several flow velocities (U) for swimmer A. The results of $SR_{1.00}$ and $SR_{1.20}$ evaluated from motion analysis were 0.45 ± 0.01 Hz (coefficient of variation: CV = 0.6%) and 0.53 ± 0.01 Hz (CV = 1.0%), respectively. Therefore, we considered that Assumption 2 was valid from the standpoint of the near-constant SR.

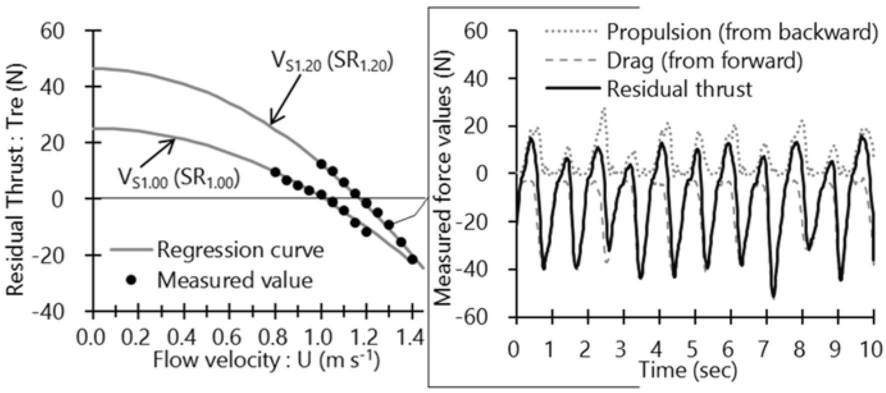

Fig. 3. Relation between flow velocity U and residual thrust T_{re} at $V_{S1.00}$ and $V_{S1.20}$ (left panel) and measured force values at $U = 1.30$ m s^{-1} (right panel) for swimmer A. In the right panel, T_{re} (the solid line) is calculated by subtracting the forward towing forces (defined as drag; dashed line) from the backward towing forces (defined as propulsion; dotted line). Then, in the left panel, the regression curve (gray line) is derived from the best fit for the measured T_{re} values (black dots). It is assumed that the values of k_D, k_P, and V_P on the line for $V_{S1.00}$ are constant (from Assumption 2). Conversely, for the line of $V_{S1.20}$, these values are different from the values of $V_{S1.00}$ because Assumption 2 is valid only when the swimmer maintains the same swimming motion.

3. Results

The active drag (*Da*), velocity at T_{re} = 0 (U_{Tre0}), coefficient of drag (k_D), and stroke rate (*SR*) for each swimmer over five stages are shown in Table 3. In addition, *Da* and *Dp* for each swimmer is shown in Fig. 4. Moreover, the *D* vs. *v* relations in passive and active conditions were calculated for each subject and are also shown in Fig. 4. When pooling together data from all subjects, the relations can be expressed as $Da = 32.3\ v^{3.3}$, $N = 30$, $R^2 = 0.90$, and $Dp = 23.5\ v^{2.0}$, $N = 42$, $R^2 = 0.89$. We would report, in this Fig. 4, only the regression lines (both for *Da* and *Dp*) not the individual equations.

In the results on the variability in *Da* at $V_{S1.20}$ for two swimmers, Swimmer A had a variability of 6.5% (48.1 ± 3.1 N), whereas Swimmer B had a variability of 3.0% (48.7 ± 1.4 N). Furthermore, the variability in U_{Tre0} was 2.2% (1.15 ± 0.03 m s^{-1}) and 2.3% (1.15 ± 0.03 m s^{-1}), and that of $SR_{1.20}$ was 2.1% (0.52 ± 0.01 Hz) and 1.1% (0.48 ± 0.01 Hz) for Swimmers A and B, respectively.

Table 3. Results of active drag (*Da*: N), calculated velocities from the regression curve (U_{Tre0}: m s^{-1}), coefficient of drag (k_D), and stroke rate (*SR*: Hz) for each swimmer at five-stage velocities ranging from 1.0 to 1.4 m s^{-1}.

Stage	A				B				C			
	Da (N)	U_{Tre0} (m s^{-1})	k_D	*SR* (Hz)	*Da* (N)	U_{Tre0} (m s^{-1})	k_D	*SR* (Hz)	*Da* (N)	U_{Tre0} (m s^{-1})	k_D	*SR* (Hz)
1	25.1	1.02	24.3	0.45	28.6	0.95	31.5	0.41	28.4	0.98	29.4	0.40
2	42.9	1.09	36.4	0.46	40.8	1.13	32.2	0.44	50.3	1.11	40.8	0.41
3	46.4	1.17	33.7	0.53	48.6	1.19	34.6	0.49	62.7	1.16	46.4	0.45
4	54.3	1.21	37.2	0.57	55.0	1.23	36.2	0.56	73.9	1.22	49.6	0.50
5	84.2	1.34	47.0	0.70	73.2	1.37	39.0	0.69	95.3	1.37	51.0	0.59

Stage	D				E				F			
	Da (N)	U_{Tre0} (m s^{-1})	k_D	*SR* (Hz)	*Da* (N)	U_{Tre0} (m s^{-1})	k_D	*SR* (Hz)	*Da* (N)	U_{Tre0} (m s^{-1})	k_D	*SR* (Hz)
1	26.8	0.95	29.5	0.43	49.7	1.09	41.8	0.41	31.8	0.99	32.6	0.41
2	43.7	1.09	36.9	0.46	54.0	1.12	42.9	0.42	45.9	1.06	40.5	0.43
3	54.7	1.19	38.8	0.49	66.3	1.18	47.7	0.45	53.5	1.13	41.8	0.49
4	69.9	1.27	43.5	0.56	89.0	1.30	53.0	0.51	68.0	1.24	44.2	0.56
5	100.4	1.39	51.8	0.64	103.3	1.38	54.2	0.59	84.0	1.34	46.6	0.65

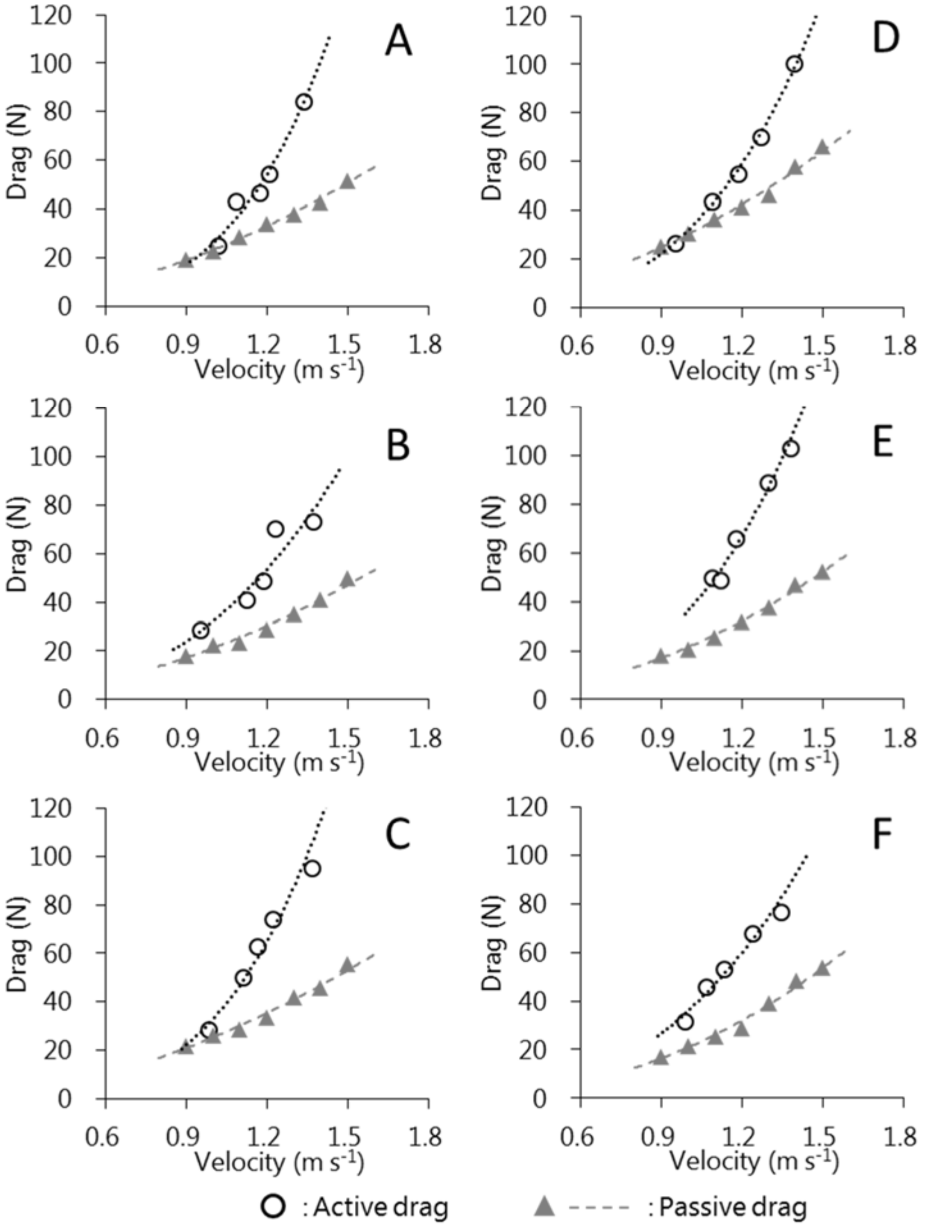

Fig. 4. Results of active and passive drags for each swimmer. We would report, in this figure, only the regression lines (both for *Da* and *Dp*) not the individual equations.

4. Discussion

The variability in *Da* evaluated using MRT indicated differences for both swimmers (Swimmer A: 6.5%, Swimmer B: 3.0%). As for the cause, we believe that in reality, the variability in *SR* influenced the variability in *Da*. Thus, it can be considered that *Da* as estimated using MRT was also impacted by the difference in *SR*, which was influenced by various factors such as the swimmers' conditions and fatigue level. From a previous study on undulatory underwater swimming which reported on the variability in human movement (Connaboy, Coleman, Moir, & Sanders, 2010), we know that differences in motion are a common occurrence when the same swimmers repeatedly perform the same trial. Thus, since Assumption 2 is technically valid only when a swimmer maintains the same exact swimming motion and stroke rate, in reality, the change in the kinematics influenced the variability in *Da* estimated using the MRT. For this reason, we consider the variability in this study to be caused by the influence of human error rather than any systematic error. Hence, this new methodology can be considered to be capable of evaluating a correct *Da* value that reflects a swimmer's condition during a given day.

Additionally, concerning the prerequisite for the MRT (Assumption 2: a swimmer maintains a certain stroke while T_{re} is measured at different values of U), the maintenance of a certain stroke by the swimmers was also confirmed by measuring the *SR* for each trial using a stopwatch and using visual confirmation, not just by setting the required frequency on the waterproof metronome. As indicated in Fig. 2, using this procedure, swimmers were indeed able to maintain a fairly constant stroke rate when U was changed. As an example, the coefficients of variation for data reported in this figure ($SR_{1.00}$ and $SR_{1.20}$ at $V_{S1.00}$ and $V_{S1.20}$, respectively, for one swimmer) were of 0.6% and 1.0%, respectively. Therefore, it can be assumed that Assumption 2 was valid experimentally because the swimmers maintained a near-constant stroke in different flow velocities, with the small variations in *SR* resulting in small variations in *Da* because of human error. Furthermore, in a preliminary experiment, we confirmed that the times of each stroke phase and movement pattern were not altered for different values of U.

For *Dp*, our values are similar to those reported by others (e.g., Zamparo, Gatta, Pendergast and Capelli (2009); Chatard, Lavoie, Bourgoin and Lacour (1990)); indeed, as reported by Havriluk (2007), the *Dp* data were very similar across different experimental procedures. Therefore, the validity of the apparatus and conditions in this study was confirmed. For *Da*, previous studies reported quite different results because of the differences in the anthropometric and technical characteristics of the swimmers observed as well as because of the differences in the adopted methodologies. All these adopted methodologies have their pros and cons (for a discussion on this point, the reader is referred to papers by Sacilotto, Ball, and Mason (2014), Toussaint et al. (2004), and Zamparo et al. (2009)). For these same reasons, despite the difficulty in directly comparing our data with that reported in the literature, our results indicate that *Da* values obtained using MRT are larger than *Dp*; this agrees with the findings of some previous studies (e.g., Di Prampero et al., 1974; Formosa et al., 2012; Gatta, Cortesi, Fantozzi, & Zamparo, 2015; Zamparo et al., 2009) but not with others in which *Da* was reported to be equal (or even lower) than *Dp* (e.g., Hollander et al., 1986; Toussaint et al., 1988; Toussaint et al., 2004; Van der Vaart et al., 1987). As recently pointed out by Gatta et al. (2015), *Da* should be expected to be larger than *Dp*. This is because the frontal projected area of *Da* when the swimmers perform a swimming motion to propel themselves forward is larger than the area of *Dp* in the streamlined position (the smallest frontal projected area against the traveling direction). The lack of a common finding in the determination of *Da* in swimming studies was indeed the reason why we wanted to establish a new methodology for evaluating drag in swimming (MRT) by gathering findings about drag in swimming, i.e., active drag in various situations (swimming styles and velocities). Furthermore, the results in this study indicate that the values of *Da* obtained using the MRT were approximately proportional to the cube of the velocity and not the square. This is thought to be the cause of the changes in the swimming motion and stroke rate when increasing V_{Si}, contrary to *Dp*. Therefore, it is necessary to gain a further understanding of factors that influence *Da* by analyzing the relation between the kinematics and *Da*.

5. Future prospects

In this study, we evaluated active drag at various velocities for front-crawl swimming with lower limb motion. However, the new methodology (measured values of residual thrust; MRT) is special in that it is applicable to various swimming styles. Therefore, the evaluation of the active drag (*Da*) using MRT would allow us to compare *Da* for different swimming styles and motions (techniques) as well as to evaluate swimming efficiency by estimating physiological indices, e.g., oxygen uptake. Hence, in the future, the evaluation of drag in swimming using MRT is expected to play a role in establishing fundamental swimming data. It has the potential to provide swimmers and coaches with beneficial information for improving swimming performance.

Conflicts of interest

None.

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Appendix

The coefficient and constant terms in Eq. (7) are replaced with α, β, and γ:

$$T_{\mathrm{re}} = \alpha \cdot U^2 - \beta \cdot U + \gamma \tag{8}$$

That is,

$$\alpha = k_{\mathrm{P}} - k_{\mathrm{D}} \tag{9}$$

$$\beta = 2 \cdot k_{\mathrm{P}} \cdot V_{\mathrm{P}} \tag{10}$$

$$\gamma = k_{\mathrm{P}} \cdot {V_{\mathrm{P}}}^2 \tag{11}$$

From Eq. (9),

$$k_{\mathrm{D}} = k_{\mathrm{P}} - \alpha \tag{12}$$

where k_{P} is derived from Eqs. (10) and (11);

$$k_{\mathrm{P}} = \frac{\beta^2}{4 \cdot \gamma} \tag{13}$$

When Eq. (13) is substituted into Eq. (12), k_{D} can be expressed in terms of α, β, and γ.

$$k_{\mathrm{D}} = \frac{\beta^2}{4 \cdot \gamma} - \alpha \tag{14}$$

We derived the coefficient and constant terms in Eq. (8) (i.e., α, β, and γ) by best-fitting the measured values of T_{re} using the method of least squares (Fig. 3, left panel). Accordingly, the active drag (Da) was derived by substituting k_{D} and U_{Tre0} (T_{re} intersecting the x-axis at zero in Eq. (8)) into Eq. (1) when the swimmer swam with a certain stroke at $V_{\mathrm{S}i}$.