

Inferring Risk Aversion for Decentralized Investment Portfolios

Jason West

Department of Informatics, Faculty of Business
Bond University
Gold Coast, QLD Australia
jwest@bond.edu.au

Abstract: A shift towards self-managed pension investments has allowed greater transparency, flexibility and control in the way individuals interact with their financial wealth. In contrast to traditional wealth management practices that rely on explicit assessments of individual risk aversion, platform-based investment management services can provide concise metrics that define individual risk aversion, but are computationally-intensive. Using a complete dataset obtained from the interaction of investors with investment management platforms, we provide a detailed insight into risk aversion by age, gender and reaction to investment performance history. We use a MapReduce model to efficiently gauge risk aversion levels in real-time to optimize individual glide paths and investment styles. The use of inferred assessments of risk aversion based on actual investor behavior is undermining the inefficient cohort-based approach to investment management. We anticipate they will eventually replace the need for subjective aversion assessments conducted by financial advisors.

Keywords—*risk aversion; big data; investments; financial advice, MapReduce, Hadoop*

I. INTRODUCTION

For workers approaching retirement, advances in financial system technology have initiated a substantial shift in the practice of DIY pension investment management. Workers, as they approach and then enter retirement, have shifted beyond traditional customer service models of occasional face-to-face meetings with a financial advisor to an on-demand investment management platform. The interaction with the investment platform for DIY investors offers unique insight into the spending and investment behavior of retirees, their level of risk aversion and their investment preferences as they age.

A poorly understood and even more poorly implemented element in the traditional financial advice model is the level of risk aversion attributed to an individual. This characteristic is usually inferred from questionnaires or is simply guessed. The level of risk aversion largely dictates investment strategy so defining a relative metric for it is vital. In traditional advice models, it is an explicit estimate, rather than an implicit observation. Subjective risk aversion assessments skew the asset allocation process out of the investor's favor. Platform-based advice models however are changing this.

Using a complete dataset obtained from the interaction of investors with investment management platforms, we provide an objective assessment of risk aversion through the use of income equivalent outcomes relative to required and desired income inputs. This output therefore subverts more subjective assessments related to age, gender and wealth. The relative risk aversion metric defines the optimal asset allocation strategy for an individual investor. We use a MapReduce programming model on a Hadoop platform to perform the risk aversion analysis inferred by user inputs. The use of big data techniques and data science analytics to devise individual glide paths and investment styles using this approach is undermining the inefficient cohort-based approach to investment management in today's pension funds.

II. OBJECTIVE RISK AVERSION

When observing individual selections via a wealth management platform it is impractical to screen all possible risk selection combinations experimentally due to the exponential increase in the number of outputs. Therefore computational methods can be used for the predictive analysis of risk preferences.

There are three types of computational methods for risk preference analysis. First, stochastic search techniques can be used to solve large-scale combinatorial optimization problems of highly complex systems, such as the multitude of interactions needed for retirement income estimation (based on wealth, required income, age, mortality and asset price volatility). Fast convergence can be achieved using a relatively small number of iterations. Second, median-effect equations can be used where the median score represents the common link between risk preference indicators. Third, a systems approach can be used to examine the effect of various risk preference selection combinations on wealth outcome pathways. We adopt this approach in our analysis.

Efficient prediction methods that are scalable to both the data and the computation process are needed for risk assessment. A MapReduce programming model [1-5] using a Hadoop platform [6,7] was used to process the identification of implied risk aversion levels by integrating user inputs relating to current wealth, current age, expected retirement age, anticipated life expectancy, required income and desired income at retirement. A classification algorithm was developed

using a support vector machine (SVM) approach for calibrating and predicting the preferred risk aversion of individuals as they interacted with the wealth advice platform. Our results were highly efficient relative to traditional algorithms using the same data, and they could be directly related to an asset allocation strategy. As the size of users accessing platform-based wealth management tools expands from around 10,000 currently to over 10 million in the next few years, we believe that a MapReduce-based approach to assess risk aversion based on user inputs related to income needs will derive efficient and objective outcomes. Personal advisory functions will continue to be inefficient and highly subjective, although in some cases bespoke planning needs may be necessary. But platform-based wealth management tools will become prevalent and we anticipate that they will alter the financial advisory landscape forever.

III. PENSION PLANS

A. Defined Contribution (DC) Plans

DC plans offer retirees pension benefits depending on contributions made to the portfolio coupled with the investment performance of the portfolio's assets over the working life of the member. Much like a savings account, a worker's DC account balance is equivalent to the market value of assets accumulated in the account. Unlike defined benefit (DB) plans however, employees have substantial control over how the contributions to their plan are invested and can therefore freely choose from a number of asset classes (stocks, fixed income assets, real estate, etc.).

B. Investor Competency

The range and complexity of choices used in retirement planning, particularly around risk tolerance and asset allocation, has increased without an equally commensurate rise in the underlying level of financial literacy of workers [8]. Consumer decisions surrounding retirement planning are very sensitive to risk tolerance, investment styles and economic assumptions. Importantly, the responsibility for nominating an optimal asset allocation rests unfairly on the worker to make investment decisions. These decisions are often framed around the maximization of wealth at retirement rather than on the more measured need to cater for the income of the worker through both the working and retirement phases.

C. Wealth Management Platforms

Demand for greater transparency, flexibility and control has transformed the way individuals interact with their financial wealth. The appetite of consumers for internet and online services is increasing at ever faster rates, with huge implications for financial advisors. Consumers conduct more of their business over the internet, using more devices and accessing more online services. Growth in these areas is posing challenges to traditional industry revenue streams.

In line with legislative changes, the provision of transparent and flexible interactions has enabled individuals the freedom of choice to shift their wealth as they please. Individuals demand flexible engagement through multiple channels which has led

to a form of direct engagement and a DIY attitude, manifest in the growth of 401(k) funds in the US and self-managed superannuation funds (SMSF) in Australia. This trend in self-management is suggested by the volume of searches related to DIY investments initiated by investors in Fig. 1.

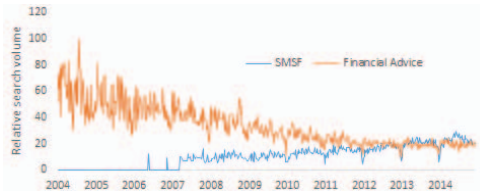


Fig 1. Australian Google search volumes of direct investing methods against a downward trend in searches for financial advice.

Despite the growth in self-managing wealth, few wealth management businesses are capable of re-engineering their advisory offering to connected customers. Those that are can capitalize on the demand for direct advice. Presently, the vast majority of workers who have a pension fund generally withdraw the proceeds from the pension fund on, or soon after, retirement. Today's pension funds resemble wealth accumulation engines that lose access to customers and their wealth at each retirement date. Insurers and private wealth managers subsequently capture this business by appealing to the low level of risk aversion of retirees. This unusual characteristic of the industry is unsustainable and pension funds will gradually diminish in size, influence and capability. To alter their business strategy they will need to cater for the income needs, wealth management and level of risk aversion of retirees.

In the UK traditional advice models are being disrupted by online aggregators. Some of these have captured 50% of some insurance markets. They also capture the customer relationship. There has been a proliferation of online wealth management start-up companies in the US (Future Adviser, Wealthfront and Betterment). Threats to the wealth management industry also come from other sectors (airlines, utilities, supermarkets) that can leverage their data capacity to offer financial advice.

IV. ADVICE MODELS

Automated wealth management approaches use some form of simulation to help generate an investment strategy for an individual investor. We have focused on one platform (WealthEd) under development to test the level of risk aversion of investors. User inputs are adapted to the simulation process to derive a prediction of wealth paths.

A. Simulation

A Monte Carlo simulation process models portfolio performance in the presence of uncertain returns, contributions, withdrawals, inflation and tax. The linear return of each asset is simulated over an investment horizon. Simulations of the price $P(t)$ are mapped to a generic horizon τ . Superimposing the rescaled histogram from the simulations of $P(t+\tau)$ is used to

confirm that they coincide. The risk-free rate is assumed to evolve deterministically while risky assets follow a geometric Brownian motion with drift μ and volatility σ

$$\ln P(t+\Delta t) = \ln P(t) + (\mu - \sigma^2/2)\Delta t + \sigma\sqrt{\Delta t}Z(t) \quad (1)$$

where $Z(t) \sim N(0,1)$ are independent across non-overlapping time steps. To simulate the allocation of assets for a generic market of n asset classes, we apply the following process:

- Estimate the series $P(t), P(t+1), \dots$ at a yearly frequency for n asset classes over the full time horizon covering both the accumulation (pre-retirement) and retirement phase;
- Estimate the $n \times n$ covariance matrix Σ of the linear returns;
- Estimate the n means μ of the linear returns through using historical analysis (with the option of inserting other forms of forward projections or analyst expectations);
- Define the actual investment horizon (assuming a mortality) and then project the means and covariances to the horizon by

$$\mu^k \equiv k\mu, \Sigma^k \equiv k\Sigma. \quad (2)$$

This is the multivariate version of the square-root rule. This rule only applies under the assumption that compounded returns are invariant (iid). While this may be approximately true for stocks it may not necessarily be true for other asset classes;

- Compute the mean-variance efficient allocations (the efficient frontier). This is posed as

$$w_\lambda \equiv \text{argmax}_{w \text{ sat } C} \{w'\mu^k - \lambda w'\Sigma^k w\}, \quad (3)$$

where w' denotes the transpose of w and 'w sat C' denotes that the n weights w that must satisfy a set of investment constraints C including long-only asset allocations and a maximum number of asset classes. Here we define $w'\mu^k$ as the mean of the portfolio linear return over the horizon and $w'\Sigma^k w$ as the associated variance.

To estimate μ^k and Σ^k we search suitable market invariants, estimate their distribution, project this distribution to the horizon, map this horizon distribution into the linear returns at the horizon and then extract from the whole distribution the means μ^k and the covariances Σ^k . In the case of stocks, the monthly compounded returns $C(t)$ can be estimated and thus a monthly distribution is estimated, assuming as a first approximation that the compounded returns are iid. The mapping from iid to linear returns is formulated as $L(t) = \exp(C(t)) - 1$. The estimates for μ^k and Σ^k can then be extracted either analytically or numerically from the distribution of $L(t)$.

This process is performed each time the user alters any of the user inputs.

B. Mean variance portfolio optimization

To improve computation speed asset class simulation invites the use of closed-form solutions of the mean variance

portfolio problem by minimizing $w'\Sigma^k w$ subject to $w'\mu^k = r_0$ and $w'e=1$, where $e=(1,1,\dots,1)'$, Σ^k is the covariance matrix of the assets, μ^k is the vector of expected returns and r_0 the desired level of expected return of the portfolio. A complete derivation of the closed form solution to the mean-variance problem can be found in [9,10].

The model assumes that an individual begins retirement with an initial withdrawal from their retirement portfolio and the post-withdrawal portfolio remainder is invested in stocks, bonds and cash. The portfolio earns an inflation-adjusted rate of return, weighted initially by constant asset allocation, until the next annual withdrawal.

C. Inferring Risk Aversion

We use a stochastic optimization model to identify the optimal withdrawal rate for a set of asset allocations and a known investment horizon that minimizes the probability of portfolio ruin, for each user input. Prior to retirement we incorporate annual cash flows into the accumulation account up to the nominated date of retirement as well as initial portfolio conditions. The portfolio value $V(t)$ at time t is defined as

$$V(t) = (V(t-1) + CF(t))(1 + X(t)) - LS(\tau) + 1_E(SSP(\tau < T)) \quad (4)$$

where $t, \tau < T$, $CF(t)$ is the after-tax cash inflow (positive) or outflow (negative), $X(t)$ is the weighted average portfolio return $w'\mu^k$ at time t , $LS(\tau)$ is the random lump sum payment withdrawn at retirement date τ and $1_E(SSP(\tau < T))$ is an indicator function where 1_E is equal to one if the investor qualifies for social security payments (SSP) during retirement $t > \tau$ and zero if the investor does not qualify for such payments.

We derive the stochastic present value at either the date of retirement (which assumes a deterministic terminal portfolio value) or at any point before retirement as

$$PV = \sum_i \prod_j (1 + r_j)^{-1}, \quad (5)$$

where r_j is the random investment return in year j . As $t \rightarrow \infty$ the stochastic PV simply reduces to the infinitely-lived endowment [11]. The simulation process in this model assumes t is fixed and is explicitly estimated by the investor. This greatly simplifies the simulation and then optimization process.

The asset values and projections are simulated 10,000 times and the key percentiles at each time t are estimated from the simulation. A range of percentiles are extracted from the simulated terminal values (at time T) for the investor's portfolio and then used as the future value to iterate backwards to retirement date τ .

To conduct the search we use a simple generalized reduced gradient search algorithm [12] to solve for the annual withdrawal over the withdrawal period ($\tau \rightarrow T$), which is also simulated 10,000 times to achieve convergence. This method is sufficiently robust to find at least a local optimum where the function is continuously differentiable. This approach is also known to be robust relative to other nonlinear optimization methods.

The stochastic optimization process selects a constant withdrawal rate through the retirement phase that yields an expected terminal wealth of zero at the 10% and 25%

confidence level coinciding with the investor’s ‘expiry’ date (death or other nominated future date). This is equivalent to the Value at Risk (VaR) calculation commonly conducted for financial portfolios. The Box Method iteratively searches possible input values for withdrawal amounts to reduce the simulated probability of ruin at a 10% and a 25% confidence level, to find a global minimum solution (if one exists). The optimal withdrawal values are then used in a second set of Monte Carlo simulations to estimate the probability of portfolio ruin.

V. INCOME FORECASTING AND RISK AVERSION

A. Risk Aversion Inferred from Income Preferences

User inputs are limited to simplify the interaction between the investor and the platform. Inputs are limited to current wealth, current age, expected retirement age, anticipated life expectancy (users anticipate this value better using personal health and family history compared with assumed population mortality rates), required income and desired income at retirement.

Normally, risk aversion is measured via a questionnaire that indicates the capacity for an investor to suffer a loss in portfolio value in 1 out of a 5-, 7- or 10-year horizon. The result will provide an indication of the willingness of an investor to allocate a greater proportion of their portfolio to risky assets. Self-assessments of risk aversion have been shown to be correlated with age, wealth, gender and race [13]. But the actual risk aversion levels of individuals differs from their own self-assessment and it also varies through time and in response to external events (economic recession, family composition, work prospects, etc.). Instead of relying on a subjective assessment of risk aversion made by an advisor or by the investor themselves, our algorithm objectively quantifies the level of risk aversion via the income selections investors make relative to their required and desired incomes during retirement.

The objective function of the financial advice model is to maximize ACI subject to the constraint that the probability of portfolio ruin is not greater than 10%. Simultaneously, a ‘likely’ income level is equated as the constant annual withdrawal amount that yields a 75% chance of avoiding portfolio ruin (i.e., 25% probability of portfolio ruin). The model communicates this to an investor seeking advice that they can spend above their likely income level but that doing so increases the risk that they will outlive their wealth. The model also communicates the magnitude of the difference between expected ACI and desired ACI which allows users to iteratively re-adjust their future retirement planning objectives and current investment decisions. The model allows for a direct feedback of expected income during retirement given a set of initial conditions, which can be easily altered to refine the investment decision. This is where the actual level of risk aversion is estimated.

The model is able to answer the basic question: at what level should investors set their retirement income expectations and expenditure levels? It motivates the investor to focus on both a ‘likely’ income and an ‘almost certain’ income relative

to a ‘required’ and a ‘desired’ income. Only where the capital market is particularly adverse will the likely level and consequent expenditure need to be adjusted, towards but not coinciding with the ACI level (unless markets were extremely poor). Investors however can spend above the likely level but to ensure that spending does not reach unsustainable levels the model intervenes and advertises this to the investor.

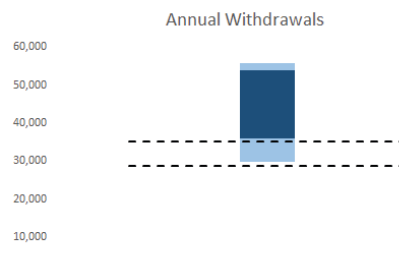
The model uses a candlestick diagram to communicate the preferred income to the user as illustrated in Fig 2. The lowermost end of the candle represents the ACI (10th percentile), the next segment begins at the 25th percentile (likely income), the same segment ends at the 75th percentile and the upper most end of the candle represents the 95th percentile. The required and desired income are represented by the lower and upper dashed lines respectively.

B. Construction

To encode the risk aversion parameters we focus on the investment inputs selected by investors. The risk profiles of 1800 platform users were downloaded from the proprietary wealth management platform (beta version). The total size of the data was 210 GB.

There are three inputs to the risk aversion algorithm. The first is the portfolio simulation profile (PSP) that represents the simulated wealth paths for each user input. The second is the initial income selection (IIS) by the user, matching required income with either almost certain income (ACI) or likely income with required or desired income given initial inputs such as retirement age and contribution rate. The third is the final income selection (FIS) by the user which detects whether ACI or likely income is then adjusted to match desired income, rather than required income. The user inputs for both the IIS and the FIS are captured and aggregated to form a complete set of input parameters. For simplicity the required income is set to 60% of final investor income (assuming income growth at 2% pa) and desired income is set to 85% of final income. When the user is adjusting ACI or likely income to match either required or desired income, the simulation is reconstructing the allocation of assets iteratively such that the need for greater income forces the portfolio to allocate a greater proportion to risky assets, in line with the formulation in (4) and (5).

Fig. 2. ACI and likely income candlestick.



Simplified pseudo-code for the algorithm is provided in Fig. 3.

```

Input : PSP, IIS, FIS
Output : Asset Allocation  $w$ 
for each Trial P in PSP do
  for each Income in IIS do
    for each Final Income in FIS do
      Lookup Age, Gender, Wealth,
      Contribution, Retirement Age and Final
      Age;
      Apply Asset Allocation;
       $w^k \mu^k = r_0$ ;
    end
    Apply Initial Asset Allocation;
     $w^k \mu^k = r_i$ ;
  end
  Aggregate all inputs;
  Match inputs to user characteristics;
end
Aggregate user inputs for FIS;
Populate Asset Allocation  $w$  to each user;

```

Fig 3. Pseudo-code for risk aversion analysis.

In the analysis of risk aversion leading to asset allocation, the Trials contained in the Portfolio are independent of each other, the Income outputs contained in each Trial are independent of each other and the Final Income outputs in each Trial are also independent of each other. This process requires a large number of computations which can be performed as independent parallel problems.

The MapReduce model readily caters for large data processing to derive the level of risk aversion for each user input. For an advisory portfolio of 100 users (combined to form a fund), a Trial would contain 300 million simulations if each user accessed the platform once per week. This equates to around 200 GB. The MapReduce implementation using Hadoop allows for dynamic job scheduling based on the availability of cluster resources and distributed file system fault tolerance, so that the batch process can take place on a weekly basis which is a sufficient frequency for portfolio rebalancing.

To validate the model to existing data and known regression results, we used a support vector machine approach to train a classifier for predicting risk aversion, given user characteristics (current age, gender, wealth). The SVM classifiers were implemented by using LibSVM package [3]. Two key parameters for training the SVM classifier is the cost factor (for outliers) and gamma. The optimal parameters were derived using a standard grid search. The search range of the parameters (cost and gamma) was 0.05-65, with each step set to 0.001.

C. The Big Data Platform

To allow for the process to become scalable we used a machine virtualization to build the Hadoop (Apache) cluster. The master virtual machines included 6 Intel core i5 processor cores and 4 GB RAM and the two slave virtual machines with 2 Intel core i5 processor cores and 2 GB RAM. The software environment included Hadoop-2.4.0, Hive-0.11.0 and R.Hadoop. We used the Hadoop distributed file system (HDFS)

to store the raw data and used Hive as a data ETL tool to interact with a relational database to process local files.

We further constructed a scalable version of the mining tool to identify risk aversion relationships among individuals and compared the efficiency to that of a sequential implementation. Pre-processing steps were parallelized by a chain of mappers. The algorithm was implemented by a series of MapReduce jobs tested with a number of batches of ten simultaneous users processing income simulations in real-time.

In the first MapReduce sequence superior processing outcomes were achieved using 6 Mappers and 6 Reducers which lasted, on average, around 300 seconds (240 seconds for the Mapper and 60 seconds for the Reducer). This equates to nearly 80% efficiency in each simulation sequence using multiple worker nodes compared to a single worker. The greatest increase in efficiency occurred with 3 workers and performance deteriorated thereafter. In the second MapReduce sequence the superior processing outcome was again achieved using 6 Mappers and 6 Reducers which lasted, on average, around 92 seconds (62 seconds for the Mapper and 30 seconds for the Reducer).

A scalable version of the above process could achieve marginally higher efficiency in most of the processing steps such as user input inference and eventually the asset allocation outputs, especially when used for a large number (>100) of simultaneous users.

VI. FINDINGS

Several outputs from the risk aversion calibration process were trivial while others were somewhat contrary to standard assumptions on the relationship between risk aversion and user characteristics.

A. Gender

One of the main differences often cited is that the level of risk aversion is distinctly different when categorized by gender. Males are often attributed with having a higher tolerance for risk than females. Our results suggest that females used ACI to a greater extent to define their needs than males, but this was not always true. In over half of the cases, females were twice as likely to test the sensitivity of their risk tolerance as males. This suggests that females focus more on lowering risk without sacrificing unnecessary opportunity costs.

B. Age

Most financial advisors and researchers hypothesize that age and risk tolerance are negatively related. The results from our algorithm suggest otherwise. Age is an important descriptive variable as it relates to expected retirement age, rather than expected 'expiry' age. That is, users were less sensitive to longevity than they were to the number of years left of working. While many users tested the sensitivity of working longer to fund retirement with greater income, most reverted to their initial inputs when declaring their interaction complete.

C. Income and Wealth

Scholars have shown that relative risk tolerance increases with income and wealth. Our results confirmed this conclusion. While workers with low account balances tested the sensitivity of all variables to maximize retirement income, workers with high initial balances were much less concerned with retirement age and focused more on minimizing risk (i.e., reducing the variability implied by the length of the candlestick, Fig. 2).

VII. SUMMARY

The growth in self-managed pension investments will continue to allow greater transparency, flexibility and control for individuals to regularly interact with their financial wealth. Traditional wealth management practices that rely on explicit assessments of individual risk aversion are inefficient and in some cases, wrong. Platform-based investment management services can provide concise metrics that define individual risk aversion and can automate the asset allocation process, which lowers costs and errors. They also provide inexpensive pension portfolio advice to a wide audience.

The volume of computational processing needed to accurately define risk aversion will increasingly look to techniques such as MapReduce models to assess the needs of investors in real-time. This approach will eventually replace inefficient cohort-based methods of investment management and avoid errors induced via subjective risk aversion assessments.

While a number of interesting results were produced from our approach which contrasted with accepted wisdom, we leave a detailed analysis of this to future research.

ACKNOWLEDGMENT

Jason West thanks the Faculty of Business at Bond University for supporting this project as well as Professor Michael E. Drew for technical advice on the outputs.

REFERENCES

- [1] J. Dean, S. Ghemawat, Mapreduce: simplified data processing on large clusters, *Communications of the ACM* 51(1) 2008, pp. 107-113.
- [2] T. Condie, N. Conway, P. Alvaro, J. M. Hellerstein, K. Elmeleegy, R. Sears, Mapreduce online, Tech. Rep. UCB/EECS-2009-136, EECS Department, University of California, Berkeley, Oct 2009.
- [3] K.-H. Lee, Y.-J. Lee, H. Choi, Y. D. Chung, B. Moon, Parallel data processing with mapreduce: A survey, *SIGMOD Record* 40 (4) (2011) 11-20. H. Nordberg, K. Bhatia, K. Wang, and Z. Wang, "BioPig: a Hadoop-based analytic toolkit for large-scale sequence data," *Bioinformatics* 29(23), 2013, pp. 3014-3019.
- [4] R. C. Taylor, "An overview of the Hadoop/MapReduce/HBase framework and its current applications in bioinformatics," *BMC Bioinformatics* 11(12-S1), 2010.
- [5] C. Chang and C. Lin, "LIBSVM: a library for support vector machines," LIBSVM software website, 2001.
- [6] T. White, *Hadoop: The Definitive Guide*, 1st Edition, O'Reilly Media, Inc., 2009.
- [7] K. Shvachko, K. Hairong, S. Radia, R. Chansler, The hadoop distributed file system, in: 26th IEEE Symposium on Mass Storage Systems and Technologies, 2010, pp. 1 -10.
- [8] A.C. Lyons, L. Palmer, K.S.U. Jayaratne, and E. Scherpf, "Are we making the grade? A national overview of financial education and program evaluation," *Journal of Consumer Affairs* 40(2), 2006, pp. 208-235.
- [9] E.J. Elton, M.J. Gruber and C.R. Blake, "Fundamental variables, APT, and bond fund performance," *Journal of Finance* 50, 1995, pp. 1229-1256.
- [10] R.C. Merton, "An analytic derivation of the efficient portfolio frontier," *Journal of Financial and Quantitative Analysis* 7(4), 1972, pp. 1851-1872.
- [11] M. Milevsky, *The Calculus of Retirement Income: Financial Models for Pension Annuities and Life Insurance*. New York: Cambridge University Press, 2006.
- [12] L.S. Lasdon, A.D. Waren, A. Jain and M. Ratner, "Design and testing of a generalized reduced gradient code for nonlinear programming," *ACM Transactions on Mathematical Software* 4(1), 1978, 34-50.
- [13] C.A. Holt and S.K. Laury, "Risk aversion and incentive effects," *American Economic Review* 92(5) December 2002, pp 1644-1655.