Optimisation and Design of a Fully Instrumented Mach 12 Nozzle for the X3 Expansion Tube

P. Toniato, D. E. Gildfind, P. A. Jacobs, R. G. Morgan
School of Mechanical and Mining Engineering, University of Queensland, St Lucia, Brisbane, 4072, Queensland, Australia

Corresponding Author's name: Pierpaolo Toniato, p.toniato@uq.edu.au

Abstract This paper describes the optimisation and design of a new Mach 12 hypersonic nozzle to be used in the X3 expansion tube. The contoured nozzle has been designed and built to accommodate large-scale models and reproduce constant Mach 12 flows to allow for scramjet testing. The requirements for this nozzle were a core-flow of at least 300mm and exit flow angles below 2°. A new optimisation process has been developed, using a parallel Nelder-Mead method and a new shape has been calculated where CFD analysis indicates the design objectives were successfully met. Off design performance has been evaluated and the nozzle has been shown to retain good core flow size, Mach number and low flow divergence for different inflow conditions.

1 Introduction

At the Centre for Hypersonics at the University of Queensland (UQ), a variety of scramjet engines have been tested and analysed in the T4 Reflected Shock Tunnel (RST). While this facility (and RSTs at this scale in general) can achieve test times of $\sim 0.7-1.0ms$ [1] at Mach 10 conditions, they are limited by two factors: the flow total pressure that needs to be structurally contained in the facility; and, the amount of dissociation in the test gas generated by the stagnation process, where the gas reaches temperatures of several thousands of degrees.

A 1:2 model of the Mach 12 REST (Rectangular to Elliptical Shape Transition) engine designed by Smart [2] has been tested in T4 at a Mach 10 condition with a Mach 12 equivalent enthalpy, assuming the flow generated is equivalent to the one behind the shock generated from a forebody shock. However, correct pressure length scaling was not achieved, as the total pressure requirements were not attainable by the facility.

The X3 expansion tube facility at the UQ has been recently used to successfully investigate scramjet combustion in Mach 10 pressure-length scaled conditions [3], showing that expansion tubes can be used to examine scramjet performance when correct pressure-length scaled conditions are targeted. Test flows in expansion tubes are never stagnated, thus removing both the total temperature and pressure limitations of RSTs.

To extend the operating envelope of the X3 expansion tube to enable Mach 12 scramjet testing, a variety of upgrades are necessary, amongst which is the design of a new hypersonic nozzle to work at Mach 12.

2 Nozzle design

In an expansion tube the flow is already hypersonic at the entrance of the nozzle, whereas in classical convergent-divergent supersonic nozzles supply region the flow is subsonic, thus hypersonic nozzles for expansion tunnels are characterised by the absence of a throat (where the Mach is unitary) and being only divergent. Previous work on hypersonic nozzles has been undertaken by Craddock [4] and Scott [5], where the latter developed a Mach 10 nozzle for the

X2 expansion tube. This nozzle subsequently has been scaled up and built for X3, where currently it is the only nozzle available for the facility. Although the Mach 10 nozzle has been shown to be capable of producing Mach 10 scramjet conditions, its exit diameter (⌀ 440 mm) and the scramjet sensitivity to inflow Mach numbers and flow divergence makes it unsuitable for operating at higher Mach numbers.

Initial work on a Mach 12 nozzle was started by Wei et al [5], targeting high enthalpy conditions, but it was decided to modify the target to a denser, scramjet condition. The optimisation process has been largely modified and improved. The nozzle profile has been determined by an optimization process whose features are detailed in the rest of this study.

2.1 Contour Description

The nozzle contour in the optimisation process has been defined by means of 11 Bezier nodes defining a 2.8 meter long Bezier curve. In the literature other modelling choices were possible, such as the use of cubic spline polynomials [6], or use of a hyperbolic section profile, or a high-order polynomial [7]. Bezier curves have been chosen as they produce smooth profiles, guaranteeing that the curve is tangent to the control polygon at the endpoints, ensuring smooth transition at the acceleration tube connection. The initial diameter is fixed to that of the acceleration tube, 182.6 mm. The length of the nozzle has been fixed to 2.8m, to integrate with the new test section and to allow for models to be aligned with windows for optical access.

2.2 Nozzle inflow

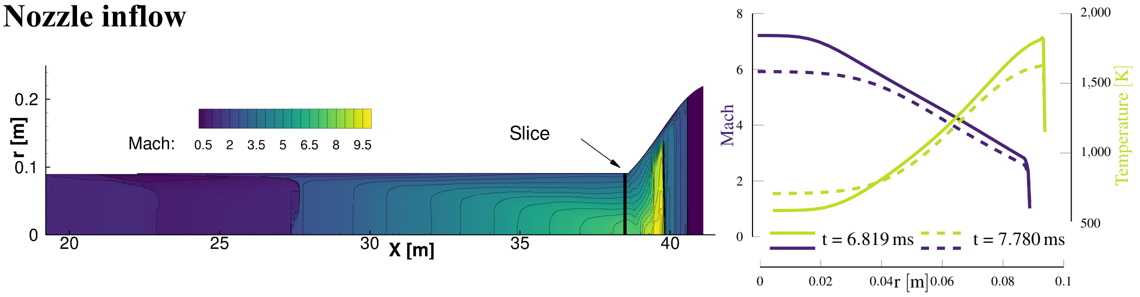

Fig 1. Axisymmetric simulation of Mach 10 scramjet condition in X3 expansion tube (left). Mach number and temperature at the nozzle inlet across the test slug (right) at start and end of test time

The flow in an expansion tube is time-varying and radially non-uniform, and full facility simulations are necessary to provide detailed insight into the nozzle inflow properties. Work towards full facility simulations of X3 are currently being carried out in a separate study, where hybrid simulations of a Mach 10 condition have been used to estimate the flow through the acceleration tube and nozzle. These simulations, while for a different condition, are similar to a future Mach 12 condition. As shown in Fig 1 (right)**,** the boundary layer is significant and varies between $30mm$ and $50mm$ through the test gas, compared to the tube radius, $91.3mm$.

Properties in the test gas are not steady but, due to the computational cost associated with simulating transient flows, it has been assumed that the inflow is steady with averaged properties of the test gas. The nozzle inflow has been computed assuming that the axial velocity follows a 1/7th boundary layer power-law (i.e. fully turbulent) and that the boundary layer thickness is $30mm$ radially. The inflow profile has been determined assuming an inflow pressure of $10kPa$, constant across the radius, velocity of $3650\,m/s$ and temperature of $600\ K$, for a calculated inflow Mach of 7.5, all chosen to closely reproduce the free-stream design condition of the Mach 12 REST engine (dynamic pressure of 50 kPa at 36 km altitude, for a velocity of 3.7 km/s, pressure of 500 Pa and temperature of 240 K).

In previous studies, the nature of the tube boundary layer of the expanded test gas has been found to be directly correlate with the test flow quality observed. It has been noted [8] that laminar boundary layers produce flows of good quality, fully turbulent ones produce still reasonable flows quality but if the boundary layers are transitional, the quality is unacceptable.

Prediction of hypersonic boundary layer transition in expansion tubes has been difficult to establish, with only few correlations being established [8, 9]. The simplest method, often used at the HYPULSE facility, suggest that if the unit Reynolds number, $Re_l, \ \geq \ 1.4 \cdot 10^6 \ 1/m$, the flow will be fully turbulent. Therefore, for the nozzle inflow condition shown above with $Re_l \ \sim 6.5 \cdot 10^6 \ 1/m$, the inflow will be assumed as fully turbulent.

2.3 Contour optimization

The goals of the design are to obtain a nozzle profile that maximises core flow diameter, has a uniform core region of Mach 12 flow across the nozzle exit, which minimizes the flow exit angles and the pressure disturbances in the core flow. This multi-objective problem has been reduced to a single objective problem by means of linear weighting of the different objectives. The optimisation problem is therefore defined as follows:

$$\vec{X} = [x_0, x_1, \dots, xi] \text{ with } 0 \leq i \leq 11 \text{ Bezier points}$$

Objective function:

$$f(\vec{X}) = \left(f_\theta + f_{Mach} + f_{penalty} + f_{pressure}\right)^2$$

With $f_\theta, f_{Mach}, f_{pressure}$ and $f_{penalty}$ given by:

$$f_\theta = \frac{\phi_\theta}{N} \sum_{j=1}^{N} \left(\theta_j(\vec{X}) - \theta_{design}\right)^2$$

$$f_{Mach} = \frac{\phi_{Mach}}{N} \sum_{j=1}^{N} \left(M_j(\vec{X}) - M_{design}\right)^2$$

$$f_{penalty} = \begin{cases} 0 & 0 < y_i < y_{i+1} \\ Z & otherwise \end{cases}$$

$$f_{pressure} = \frac{\phi_{pressure}}{N} \left(\overline{P} - P_{max}\right)$$

Where θ_j, M_j are respectively Mach number and flow angle in the exit plane, with N the number of the radials cells in the core flow. $\phi_\theta, \phi_{Mach}, \phi_{pressure}$ are weights, tuned to balance the relative importance of each terms of the objective function. y_i indicates the evaluated nozzle profile. $f_{penalty}$, in which Z is a large number, used to prevent the optimization algorithm to converge to a solution in which the nozzle profile curves inwards.

Function evaluations are conducted by means of a complete simulation of the nozzle, using UQ's EILMER3 CFD code [10] for 2D and 3D Navier Stokes compressible flow. The simulations have been carried out using NENZFR-R [11], a block-marching code built on top of EILMER3 in order to save computational time. Simulations included viscous effects, and as mentioned above were fully turbulent using the $k - \omega$ turbulence model [12]. The test gas has been modelled as 5-species air, fully reacting, using the Gupta reaction scheme [13]. Alternatives methods, like the method-of-characteristics plus displacement corrections had been considered, but discarded because the presence of a thick boundary layer invalidated their assumptions.

This optimization problem falls into the category of optimization of expensive black-box functions, as each of the function evaluation takes a few hours. The algorithm of choice for this optimization was a modified, parallel Nelder-Mead simplex algorithm [14], in which the evaluation of the different points of the simplex occurs in parallel. This algorithm represents a compromise between the necessity of parallel function evaluations, the computational capability available at the time, simple algorithm implementation, and the empirical observation that the objective function is well behaved. The optimization was considered converged once the variations in the locations Bezier Points were below $0.1 \ mm$ on average.

2.3 Results

The final nozzle profile is shown in Fig 2 (left), showing Mach and pressure contour plots. Attention has been dedicated to ensure that the profile generated is shock-free. Exit flow properties are shown in Fig 2 (right), and they show an excellent profile in term of Mach number, with a core flow diameter in excess of $360mm$, and exit flow angle in average $\sim 0.3°$.

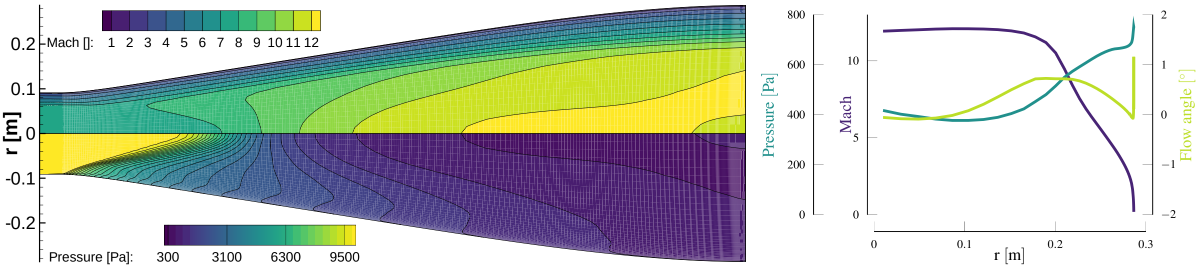

Fig 2. Optimised Mach 12 nozzle Mach number and pressure contours (left). Exit Mach number, flow angle and pressure for optimised Mach 12 nozzle (right)

On the other hand, a less satisfactory pressure profile is present, which is due to re-compression effects that take place in the last $0.3m$, as a side effect of the flow redressing process. This is an intentional result of the weights chosen for the different parts of the objective function, as it has been decided to prioritize flow exit angle against pressure. It has also been observed that an excessive increase of the pressure weights $\boldsymbol{f_p}$, in addition to having a detrimental effect on exit flow angle, would create stronger waves forming at the nozzle exit edge, thereby reducing the total core flow diamond available for model testing. A final comparison with the current Mach 10 nozzle is available in Fig 3.

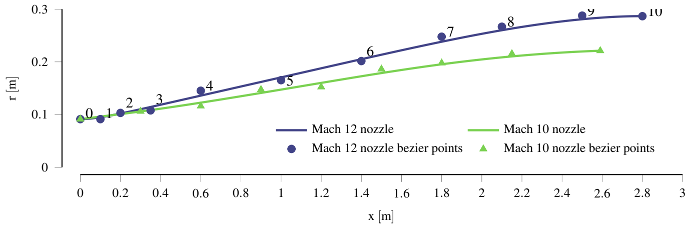

Fig 3. Comparison of Mach 12 nozzle and current Mach 10 nozzle profile

3 Nozzle off-design performances

The nozzle has been designed to target the specific Mach 12 condition (mentioned above), but it will often operate at off-design conditions. A sensitivity study has been undertaken, and a partial summary of the results is shown in Fig 4. Inflow velocity, inflow Mach number and boundary layer thickness have been varied. Exit flow angle was found to be largely invariant to the inflow properties with a variation of ±1°. Exit pressure and exit Mach number exhibit a certain sensitivity to inflow Mach number. If inflow Mach number is >7.5 the gas is over expanded in the nozzle centreline, resulting in a less uniform core flow. The nominal boundary layer thickness was 30 mm, but the analysis has shown that acceptable Mach number exit profiles are maintained above 20 mm. With lower thickness, exit flow quality is decreased.

4 Nozzle manufacturing

The mechanical design has been carried out and the nozzle has been built. The nozzle will have a new connection system to the acceleration tube, which allows for a Pitot rake to be inserted at the nozzle inlet to measure radial variation in impact pressures at nozzle inflow. The nozzle

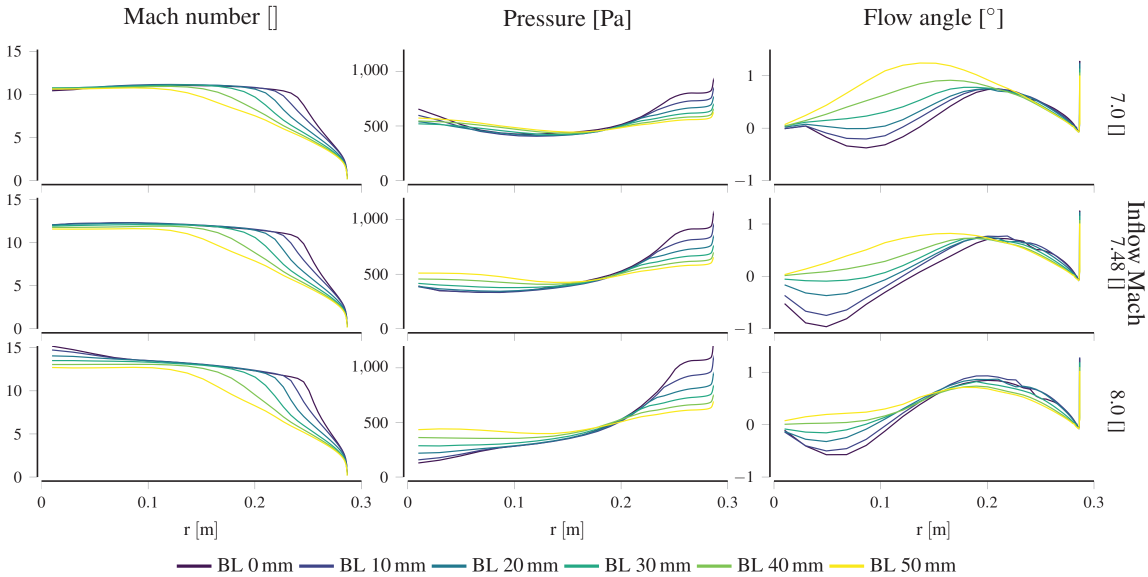

Fig 4. Nozzle performance with varying inflow Mach number and boundary layer (BL) thickness for an inflow velocity of $\mathbf{3650}\ \boldsymbol{m/s}$

wall will host six wall sensor mounts, able to simultaneously house pressure transducers and heat transfer gauges.

The nozzle was filament-wound from fibre glass around a foam mandrel. A steel insert, which contains the initial part of the nozzle profile has been machined is inserted on the mandrel, and fiberglass was wound around the mandrel and the steel part. After the winding process, the nozzle has been removed and polished. The nozzle sits inside the dump tank, and is connected to the acceleration tube by means of the steel insert. Adequate sealing is provided by O-rings around the polished external surface of the steel part. A CAD representation of nozzle installed in the test section is shown in Fig 5.

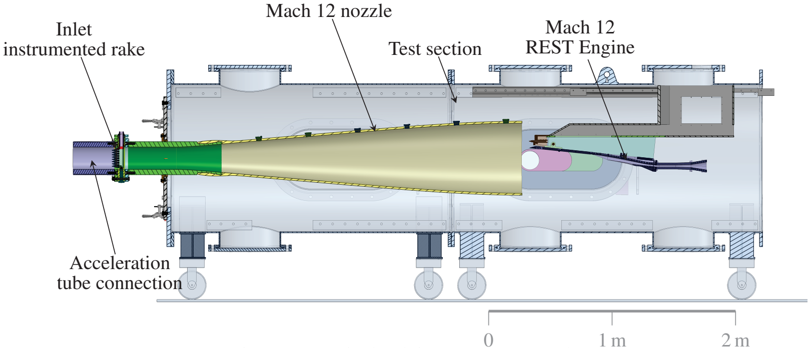

Fig 5. Mach 12 nozzle inside X3 test section.

4 Discussion

The validity of the parallel Nelder-Mead method has been proven by means of multiple restarts from different initial profiles, which showed (local) convergence to the same profile. Alternative global optimization methods have been considered, like evolutionary/genetic algorithms, or global optimization using surrogate modelling/response surfaces [15], but they have been shown to converge slower, requiring a higher number of function evaluations. In addition, they have shown to be sensitive to tuning parameters, and the high cost of function evaluations resulted in a reduce capability to modify the parameters specific to each algorithm. Black-box optimisation is an active field of research and further work could investigate the capability of newer algorithms to converge to a solution reducing the number of total function evaluations.

A number of simplifying hypotheses have been made to complete this optimization, amongst which the most significant is the steady state assumption for the inflow, however no feasible alternatives are currently available to optimise for a fully transient inflow condition.

5 Conclusions

An expansion tube hypersonic nozzle has been designed to target Mach 12 scramjet conditions. The optimized nozzle profile has shown to produce excellent results for on-design condition and good results in off-design condition if the entry Mach number of the test flow can be maintained around 7 – 7.5. Exit flow angle has proven to be insensitive to all inflow parameters.

The nozzle has been manufactured and its nozzle performance will be analysed experimentally once it is commissioned. New instrumentation will allow for a full characterization of the performance of this hypersonic nozzle, with new hardware built to obtain experimental measurements of nozzle transient inflow, allowing to determine the test gas free stream properties and significantly reducing uncertainties on them.

Acknowledgments

This research is supported by an Australian Government Research Training Program (RTP) Scholarship and the Cooperative Research Centre for Space Environment Management (SERC Limited) through the Australian Government's Cooperative Research Centre Programme. This research was undertaken with the assistance of resources from the National Computational Infrastructure (NCI), which is supported by the Australian Government. The authors finally wish to thank F. De Beurs and N. Duncan for technical assistance with the X3 hardware.

References

[1] Doherty, L.J., Smart, M.K., and Mee, D.J. *Experimental Testing of an Airframe Integrated 3-D Scramjet at True Mach 10 flight Conditions*. in *19th AIAA International Space Planes and Hypersonic Systems and Technologies Conference*. American Institute of Aeronautics and Astronautics (AIAA).

[2] Wise, D., *Experimental Investigation of a Three Dimensional Scramjet Engine at Hypervelocity Conditions*. 2015, University of Queensland. p. 197-197.

[3] Sancho Ponce, J., *Scramjet Testing at High Total Pressure*. 2016, University of Queensland.

[4] Craddock, C.S., *Computational Optimization of Scramjets and Shock Tunnel Nozzles*. 1999, Citeseer.

[5] Scott, M.P., *Development and Modelling of Expansion Tubes*. 2007, University of Queensland.

[6] Shope, F. *Contour Design Techniques for Super/Hypersonic Wind Tunnel Nozzles*. in *24th AIAA Applied Aerodynamics Conference*. American Institute of Aeronautics and Astronautics (AIAA).

[7] Chue, R.S.M., et al., *Design of a Shock-Free Expansion Tunnel Nozzle in Hypulse.* Shock Waves, 2003. **13**(4): p. 261-270.

[8] Erdos, J.I. and Bakos, R.J. *Prospects for a Quiet Hypervelocity Shock-Expansion Tunnel*. in *25th Plasmadynamics and Lasers Conference*. American Institute of Aeronautics and Astronautics.

[9] McGilvray, M., *Scramjet Testing at High Enthalpies in Expansion Tube Facilities*. 2008, University of Queensland.

[10]Gollan, R.J. and Jacobs, P.A., *About the Formulation, Verification and Validation of the Hypersonic Flow Solver Eilmer.* Int. J. Numer. Meth. Fluids, 2013. **73**(1): p. 19-57.

[11]Doherty, L., et al., *Nenzf-R: Non-Equilibrium Nozzle Flow, Reloaded. A User Guide*. 2011.

[12]Wilcox, D. *Formulation of the K-Omega Turbulence Model Revisited.* in *45th AIAA Aerospace Sciences Meeting and Exhibit*. AIAA.

[13]Gupta, R.N., et al., *A Review of Reaction Rates and Thermodynamic and Transport Properties for an 11-Species Air Model for Chemical and Thermal Nonequilibrium Calculations to 30000 K.* 1990.

[14]Lee, D. and Wiswall, M., *A Parallel Implementation of the Simplex Function Minimization Routine.* Comput Econ, 2007. **30**(2): p. 171-187.

[15]Regis, R.G. and Shoemaker, C.A., *Constrained Global Optimization of Expensive Black Box Functions Using Radial Basis Functions.* Journal of Global Optimization, 2005. **31**(1): p. 153-171.