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Income-Related Health Transfers Principles and Orderings of Joint Distributions of Income and Health

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Abstract

The objective of this article is to provide the analyst with the necessary tools that allow for a robust ordering of joint distributions of health and income. We contribute to the literature on the measurement and inference of socioeconomic health inequality in three distinct but complementary ways. First, we provide a formalization of the socioeconomic health inequality-specific ethical principle introduced by Erreygers, Clark and Van Ourti, (2012). Second, we propose new graphical tools and dominance tests for the identification of robust orderings of joint distributions of income and health associated with this new ethical principle. Finally, based on both pro-poor and pro-extreme ranks ethical principles we address a very important aspect of dominance literature: the inference. To illustrate the empirical relevance of the proposed approach, we compare joint distributions of income and a health-related behavior in the United States in 1997 and 2014.

Keywords: Health concentration curves, health range curves, socioeconomic health inequality, dominance, inference
JEL Codes: D63, I10

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1 Introduction

Measuring socioeconomic health inequality is important from a social perspective and is critical when evaluating the impact of health policy reforms on the distribution of population's health. There is a large body of literature on the measurement of socioeconomic health inequalities most of which has focused on the properties and issues arising from the use of these indices as well as the ethical principles they should obey. Some argue that the analyst should be concerned with inequalities that occur in the lower part of the distribution of socioeconomic status (Wagstaff, 2002) and others suggest that the analyst may be more concerned with deviations occurring away from the median of the socioeconomic status (Erreygers, Clarke and Van Ourti, 2012). While the desirable ethical principles for these measures may still be on the debate table, this paper adopts a unified approach by including both possibilities. As such, the overarching objective of this paper is to provide the analyst with the necessary tools that allow for a robust ordering of joint distributions of health and income including the associated statistical inference compatible with both ethical principles.

This paper contributes to the literature on socioeconomic health inequality measurement and inference in three distinct but complementary ways. First, it contributes to the literature that formalizes the ethical principles underlying socioeconomic health inequality indices by offering a formalization of Erreygers, Clarke and Van Ourti's (2012) view of what is considered an alternative ethical property for these indices. In doing so, it provides a formal presentation of the ethical principles associated with indices that pass the *upside-down* test and coins these principles as the *symmetry around the median* principle (at the second order) and the *pro-extreme rank* principles (at higher orders). Second, it contributes to the socioeconomic health inequality measurement literature by introducing new graphical tools associated with these principles, a new class of range curves, and by deriving the associated dominance conditions. These range curves have a role analogous to the one played by health concentration curves where the analyst assumes *pro-poor* ethical principles. Developing new dominance conditions (for these new range curves) will help the analyst identify robust orderings under the assumptions of *symmetry around the median* and *pro-extreme rank*. Finally, it contributes to the literature on hypothesis testing for dominance conditions by providing estimators of health concentration curves and health range curves as well as consistent testing methods for dominance compatible with both ethical principles.

The remaining of this paper is organized as follows. In Section 2, we provide a brief review of the literature on measures of socioeconomic health inequality, the basic ethical principles on which they are founded as well as the associated literature on inference. In Section 3, we describe the measurement framework in which we are operating and discuss the associated basic ethical principles. In Section 4, we examine higher order ethical principles for *pro-poor* and *pro-extreme rank* ethical principles then define the sets of indices obeying these principles. In Section 5, we present the health concentration curve, the s-health concentration curves, the health range curve, the s-health range curves and their respective generalized versions. We also develop dominance conditions to identify robust orderings for the sets of indices developed in Section 4. In Section 6, we present the natural estimators for the curves presented in Section 5 and develop the statistical inference to test for dominance. In Section 7, we provide an empirical illustration using information on cigarette consumption and overweightedness from National Health Interview Survey (NHIS) in 1997 and 2014. Finally, Section 8 concludes and presents future research directions.

2 Literature Review

This paper is related to two main strains in the literature, the literature on the measurement of socioeconomic health inequality and the literature on hypothesis testing for dominance in the context of inequality. To highlight our contributions, we provide a brief review of the relevant literature.

The most traditional measure of socioeconomic health inequality is the concentration index proposed by Wagstaff, Paci and van Doorslaer (1991). It has a mathematical structure that assumes a very specific level of aversion to socioeconomic health inequality. Wagstaff (2002) argues that it may be desirable to consider other levels of inequality aversion than the one implicitly assumed in the standard concentration index. He suggests the extended health concentration indices that allow for a wider range of levels for aversion to socioeconomic health inequality. Furthermore, the mathematical structure of the concentration index imposes a specific type of aversion to socioeconomic health inequality. Intuitively, its structure implies that the contribution of an individual's health level to socioeconomic health inequality is increasing with socioeconomic status (Bleichrodt and van Doorslaer, 2004).

Erreygers, Clarke and Van Ourti (2012) highlight that the use of extended health concentration indices imposes a specific ethical view on what constitutes an increase in aversion to socioeconomic health inequality; they label it *pro-poor transfer sensitivity*. In the context of *pro-poor transfer sensitivity*, increasing aversion to socioeconomic health inequality is achieved by increasing the weight of transfers occurring at lower ranks of socioeconomic distribution. This well-known ethical position is based on a concept developed in the (unidimensional) income inequality literature and is adapted to fit the (bi-dimensional) socioeconomic health inequality context. Erreygers, Clarke and Van Ourti (2012) propose alternative ethical principles that we label as the *symmetry around the median* principle and *pro-extreme rank transfer sensitivity* principles. Based on this principle, an increase in socioeconomic health inequality is achieved by increasing the weights on transfers occurring farther away from the median of socioeconomic statuses.

Building on Wagstaff (2002), Makdissi and Yazbeck (2014) formalize the definition of *pro-poor transfer sensitivity* principles and introduce higher orders of health concentration curves, the s-health concentration curves. In their paper, they show how these curves can be used to identify robust orderings of health distributions for indices obeying *pro-poor transfer sensitivity* principles. This being said, they do not consider *pro-extreme rank* principles. This paper fills this gap in the literature by formalizing the *pro-extreme rank* principles introduced by Erreygers, Clarke and Van Ourti (2012) and deriving the corresponding higher order ethical principles. It also contributes to the literature by proposing new graphical tools associated with these principles; the health range curve and the s-health range curves. These curves will be used to derive necessary and sufficient conditions for robust orderings of joint distributions of income and health. From a measurement perspective, this paper is related to Makdissi and Yazbeck (2014)[1] yet is different from it in two distinct ways: the ethical principles it formalizes as well as the graphical and dominance tools it proposes.

Compared to the literature on socioeconomic health inequality measures, the literature on the statistical inference for these measures is scant as most of it focused on income inequality measures (Kakwani, Wagstaff and Van Doorslaer (1997) and O'Donnell, van Dooslaer, Wagstaff and Lindelow (2008)[2] are two exceptions). As for inference regarding various forms of stochastic dominance, it followed the same pattern as statistical inference

[1] Makdissi and Yazbeck's (2014) contribution to the literature is twofold. In a first step, the paper addresses the measurement issues arising in the presence of multiple categorical health variables by proposing a framework that consists of transforming these multiple categorical health variables into one cardinal health measure. In a second step, they provide a formalization of the ethical principles on which this new cardinal health measure is based. So, Makdissi and Yazbeck (2014) starts with categorical variables, proposes a measure to cardinalize the categorical information, then formalizes the ethical principles associated with the proposed (cardinalized) measure.

[2] They consider inference on a grid of points of the concentration curves in Chapter 7.

on inequality measures. It focused on dominance tests in the context of income inequality literature namely in Anderson (1996), Davidson and Duclos (2000), Barrett and Donald (2003), Linton, Maasoumi and Whang (2005), Linton, Song and Whang (2010), Barrett, Donald and Batcharaya (2014) as well as Schechtman, Shelef, Yitzhaki and Zitifkis (2008). While Anderson (1996) test is based on the assumption that observations are drawn from two independent distributions, Davidson and Duclos's asymptotic approach to inference allows for observations to be drawn from a joint distribution. However, as in Anderson (1996), Davidson and Duclos's test uses a fixed number of arbitrary grid points. The use of fixed number of arbitrary grid points is not a desirable feature of the test as the decision will be contingent to the choice of the grid points and thus inconsistent (Barrett and Donald, 2003). To overcome this issue, Barrett and Donald (2003) propose a consistent Kolmogorov-Smirnov (KS) type test. Their approach tests for dominance over all the points of the support, however, their test (as Anderson's) applies in cases where samples are drawn from independent distributions of income. Thus, Barrett and Donald's test does not allow for dominance for bivariate measures of inequality (i.e., for marginal conditional dominance). Schechtman et al. (2008) address this issue and propose a consistent statistical procedure in the context of a bivariate measure of inequality; the absolute concentration curve (a.k.a the generalized concentration curves in the health literature) in the context of portfolio choice in finance.

This paper contributes to this literature by proposing a consistent statistical test akin to the test Schechtman et al. (2008) and Barrett and Donald (2003) for the new dominance conditions introduced in this paper. Given that the dominance conditions we develop are for bivariate distributions, this paper is more akin to the work by Schechtman et al. (2008) than to the work of Barrett Donald (2003) and Linton et al. (2005). Although the hypothesis we are testing is, in some cases, mathematically analogous to the one tested in Schechtman et al. (2008), it remains different for three reasons. First, the framework and ethical principles are different. As a result, many of the welfare foundations and mathematical forms involved are different. Second, all estimators, dominance conditions and inference for indices obeying *pro-extreme rank* ethical principles are new. Finally, all the higher order dominance conditions for indices obeying higher order *pro-poor* ethical principles have no available statistical inference in the literature.

3 Measurement framework

The purpose of this section is to set the measurement framework and elaborate on the ethical principles underlying health achievement and relative socioeconomic health inequality indices. These indices are functionals of the joint distribution of health, H and income, Y. In this paper, the term "income" refers to a measure of socioeconomic status.

Let H and Y be 2 random variables that are absolutely continuous with support on the positive half real line with densities f_H and f_Y respectively, with a joint density $f_{Y,H}$ and with a cumulative distribution of income, $F_Y(y)$.[3] We are interested in measuring health achievement and relative socioeconomic health inequality in a rank-dependent framework where ranks represent an individual's position in the distribution of socioeconomic status. In this context, a socioeconomic health inequality index can be written as

$$I\left(h\right) = \int_0^1 \nu(p)\frac{h(p)}{\mu_h}\mathrm{d}p, \tag{1}$$

where, $\mu_h = \int_0^1 h(p)\mathrm{d}p$ is the expectation of H, $\nu(p)$ represents the social weight of an individual at rank $p \in [0,1]$ in the income distribution, and $h(p)$ is the conditional expectation of health, H, with respect to Y equal to its p-quantile:

$$h(p) = E[H|Y = F_Y^{-1}(p)]. \tag{2}$$

[3] In this paper, we assume that this health measure is a ratio-scale variable. For ease of exposition we also assume that densities exist.

In general, any index of socioeconomic health inequality can be interpreted as the ratio between the cost in health achievement associated with socioeconomic health inequalities and the average health level. This is why it is possible to write rank-dependent health achievement indices as a function of socioeconomic health inequality indices. Thus, a health achievement index $A(h)$ can be directly be computed from a socioeconomic health inequality index as

$$A(h) = \mu_h(1 - I(h)). \tag{3}$$

This health achievement index can also be rewritten as

$$A\left(h\right) = \int_0^1 \omega(p)h(p)\mathrm{d}p, \tag{4}$$

where $\omega(p) = 1 - \nu(p)$. The mathematical properties of the social weight function $\nu(p)$ and $\omega(p)$ are associated with the indices' ethical principles. The following section elaborates on two different ethical principles underlying socioeconomic health inequality and health achievement measures; the principle of *income-related health transfer* and the principle of *symmetry around the median*.

3.1 Principle of income-related health transfer

According to this principle, the contribution of an individual's health level to total health achievement (socioeconomic health inequality) is non-increasing (increasing) with socioeconomic status. This means that *ceteris paribus*, if the rich (poor) are relatively healthier, then the health achievement will be lower (higher), and the socioeconomic health inequality will be higher (lower). This ethical principle can be mathematically translated by the following assumptions on the behavior of the derivatives of the weight functions:

A.1 $\omega^{(1)}(p) \leq 0$

A.1' $\nu^{(1)}(p) > 0$ (i.e. $\omega^{(1)}(p) < 0$), A.2 $\int_0^1 \omega(p)\mathrm{d}p = 1$ (i.e.$\int_0^1 \nu(p)\mathrm{d}p = 0$),

where $\omega^{(i)}(p) = \frac{\partial^i \omega(p)}{\partial p^i}$ and $\nu^{(i)}(p) = \frac{\partial^i \nu(p)}{\partial p^i}$. $A(h)$, as defined in equation (4), is a rank-dependent measure of health achievement when the weight function, $\omega(p)$ satisfies assumptions A.1. and A.2. Similarly, $I(h)$, as defined in equation (1), is a rank-dependent measure of socioeconomic health inequality when the weight function $\nu(p)$ satisfies assumptions A.1' and A.2. The role of assumption A.2 is to guarantee that the weight function $\nu(p)$ sums to zero (i.e., $\int_0^1 \nu(p)\mathrm{d}p = 0$) and thus that inequality indices have two fundamental desirable properties.[4] The first desired property requires that in the absence of health inequality (i.e., when everybody has the same health level), the inequality index $I(h)$ value be equal to zero. The second requires that $I(h)$ remains unchanged if everyone's health increases in the same proportion. The roles of assumptions A.1 and A.1' are embedded in Bleichrodt and van Doorslaer (2006) *principle of income-related health transfer*.[5]

As illustrated in Figure 1, the *principle of income-related health transfer* implies that performing a mean preserving health transfer δ_h from an individual at socioeconomic rank p_2 to a person at a lower socioeconomic rank p_1, increases (decreases) health achievement (socioeconomic health inequality).

Having elaborated on the underlying ethical principle, we define the sets of all rank dependent health achievement and socioeconomic health inequality that we will be considering. In the remaining of this paper, we denote by Ω^2 the set of all rank-dependent health achievement indices obeying assumptions A.1 and A.2, and by Λ^2 the set of all rank-dependent socioeconomic health inequality indices obeying assumptions A.1' and A.2. The formal mathematical definitions of these set of indices as well as all subsequent subsets of indices can be found in the appendix.

[4]Note that $\nu(p) = 1 - \omega(p)$.

[5]It is important to highlight the interpretation of the slight difference between assumptions A.1 and A.1'. Assumption A.1 is less restrictive since it allows for $\omega(p) = 1$ for all p whereas assumption A.1' imposes a strict inequality. When $\omega^{(1)}(p) = 0$ for all p is combined with A.2, there is only one possible weight function, $\omega(p) = 1$, the resulting health achievement is the unweighted average health level μ_h.

3.2 Principle of symmetry around the median

According to Erreygers, Clarke and Van Ourti (2012), one may want to impose an additional ethical constraint on the socioeconomic health inequality indices by focussing on those that pass the *upside-down* test. This test consists in interchanging health levels of individuals at rank p with their "carnival" counterpart at rank $1-p$. If an individual is at the 10th percentile of the income distribution, her health level is compared with the health level of the individual at the 90th percentile, and so on. Imposing this constraint means that inequality is defined by focusing on the range of health levels $r(p) = h(1-p) - h(p)$ (for $p \in [0, 0.5]$) rather than on the level $h(p)$ itself. More formally, let $g(p) = h(1-p)$ be the "carnival" counterpart distribution of $h(p)$, the *upside-down* test consists in verifying if $I(g(p))$ is always positive (negative) when $I(h(p))$ is negative (positive). Erreygers, Clarke and Van Ourti (2012) show that an index passes this test only if its weight functions $\nu(p)$ is symmetric around the median of socioeconomic statuses (i.e., around $p = 0.5$). This leads to the following additional assumptions on the behavior of the social weight function:

A.3 $\nu(1-p) = -\nu(p)$,

A3' $\omega(1-p) = 2 - \omega(p)$.

Assumption A.3 and A.3' implies that $\nu(0.5) = 0$ and $\omega(0.5) = 1$. These additional assumptions allow us to define a subset of indices that pass the *upside-down* test, Λ^2_ρ and propose the associated subset of health achievement indices, Ω^2_ρ.[6]

3.3 Examples of parametric class of indices

As pointed by Erreygers, Clarke and Van Ourti (2012), equation (1) is reminiscent of Mehran (1976) class of rank-dependent income inequality indices with a slight difference;

[6] These proposed health achievement indices passing the *upside-down* test are not yet available in the literature, however, it is always possible to construct the health achievement index underlying each socioeconomic health inequality index.

individual ranks (socioeconomic status) are not determined by the rank of the variable of interest (health). The social weight functions, $\nu(p)$ and $\omega(p)$, may take different functional forms that depend on socioeconomic rank p. As a result, each subset of the class of achievement or inequality indices will depend on the specific form imposed on its weight function. For instance, if the analyst's ethical position is compatible with *sensitivity to poverty*, then a weight function $\omega(p) = \rho(1-p)^{\rho-1}$, where $\rho > 1$ the socioeconomic health inequality aversion parameter, is appropriate. In this case, equation (4) describes Wagstaff's (2002) class of health achievement indices, a subset of Ω^2. For the same specific parametric form of the weight function, equation (1) describes Wagstaff's (2002) class of extended health concentration indices a subset of Λ^2.[7] One may want to impose an additional ethical constraint on socioeconomic health inequality indices by requiring that they pass the *upside-down test* (see, Erreygers, Clarke and van Ourti, 2012). In this case, if the analyst values transfers occurring farther away from the median socioeconomic rank, then, *sensitivity to extremities* is an alternative to *sensitivity to poverty*. A compatible weight function would be $\nu(p) = \beta 2^{\beta-2}\left[\left(p-\frac{1}{2}\right)^2\right]^{\frac{(\beta-2)}{2}}\left(p-\frac{1}{2}\right)$, where $\beta > 1$ is the socioeconomic health inequality aversion parameter. For this specific parametric weight function, equation (1) describes Erreygers, Clarke and Van Ourti's (2012) class of symmetric health socioeconomic inequality indices, which is a subset of Λ^2.[8] It is important to underline that the standard health concentration index (i.e., when $\rho = 2$) passes the *upside-down* test since $1-\rho(1-p)^{\rho-1} = 2p-1$ is by construction symmetric around the median. However, for all values of $\rho \neq 2$, the extended health concentration and health achievement indices do not pass the *upside-down* test.

Wagstaff (2002) class of extended health concentration indices and Erreygers, Clarke and Van Ourti's (2012) class of symmetric health socioeconomic inequality indices are both

[7] Note that for the standard health concentration index $\rho = 2$.

[8] Note that when $\beta = 2$, the symmetric health socioeconomic inequality index collapses to the health concentration index.

subsets of Λ^2. While Wagstaff (2002) and Erreygers, Clarke and Van Ourti (2012) are the most widely known indices in the health economics literature, there are other possible health achievement and socioeconomic health inequality indices.[9]

To conclude, the *income-related health transfer* principle and the *symmetry around the median* principle presented above have intuitive ethical interpretations. The first principle puts more weight on what occurs in the bottom of the socioeconomic distribution based on the level of health, the second principle imposes an additional focus on the range between an individual's health level and the health level of her "carnival" counterpart.

4 Higher order aversion to socioeconomic health inequality

The interpretations of higher order ethical principles are less straightforward than the second order principles discussed in the previous section however; they still have an intuitive interpretation. Generally, a higher order principle can be viewed as a higher degree of aversion to socioeconomic health inequality. This means that, for a given transfer, the magnitude of the social weight depends on the location of the health transfers in the distribution of the socioeconomic status. The higher the degree of aversion to inequality, the larger is the relative magnitude of the social weight given to transfers at the bottom of the income distribution. In this context, higher order ethical principles are defined by comparing health transfers instead of health levels. The appeal in imposing higher order of aversion to socioeconomic health inequality resides in its capacity to increase the power of ordering by narrowing to a subset of indices instead of relying on the arbitrary choice of a specific index.

In the literature, there are two distinct ethical views of higher order principles of aversion to socioeconomic health inequality, one compatible with Wagstaff (2002) and the other with

[9]Erreygers, Clarke and Van Ourti (2012) class of symmetric indices has been included in a new Stata routine. See O'Donnell, O'Neill, Van Ourti and Walsh (2016).

Erreygers, Clarke and Van Ourti (2012). In his paper, Wagstaff (2002) adopts a *pro-poor health transfer sensitivity* approach where health transfers occurring in the lower part of the distribution of socioeconomic ranks are deemed to be more desirable. Subsequently, Makdissi and Yazbeck (2014) present the details of the ethical implication of generalized *pro-poor health transfer sensitivity* principles (i.e. higher order principles). They explain that if one adopts this ethical perspective, an increase in aversion to socioeconomic health inequality is achieved by increasing the weight of transfers occurring at lower ranks in the distribution of socioeconomic status.[10] In the context of this paper, this means that if an index obeys A.1 (or A.1.') and A.2 then, it obeys the sth-order *pro-poor transfer sensitivity* if $(-1)^i\omega^{(i)}(p) \geq 0$ or $(-1)^{i+1}\nu^{(i)}(p) \geq 0$ for all $i = 1$ to $s-1$. We define the sets of indices obeying the principle of income-related health transfer and obeying all *pro-poor transfer sensitivity* principles of order $i = 3$ to s as Ω^s_π and Λ^s_π.

Erreygers, Clarke and Van Ourti (2012) highlight that, in the context of *pro-poor transfer sensitivity*, the weight of a progressive health transfer decreases with socioeconomic status regardless of its distance to the median socioeconomic rank. They introduce a symmetric class of indices that accounts for deviations from the median of socioeconomic rank. However, they do not formalize the associated ethical principles. This is why we propose formal definitions of different transfer-sensitivity principles that are compatible with the *upside-down* test. Given that a class of indices that passes the *upside-down* test is more sensitive to transfers occurring farther away from the median of socioeconomic statuses (i.e., for cases when $\beta > 2$), we label the associated ethical principles as the *pro-extreme rank transfer sensitivity* principles. We also provide a formal presentation of the *pro-extreme*

[10] At this point, it is important to note that some caution has to be taken when interpreting the transfer principles related to Makdissi and Yazbeck (2014). The interpretation of these transfer principles are specific to their cardinalized health variables; a transfer should involve a marginal change that leads to a jump across the threshold. This is not required in the context of this paper. An interesting transfer principle adapted to the case of ordinal (binary) variables are bilateral transfers. These transfers involve a movement of a *mass from one outcome to another* is provided by Sonne-Schmidt et al. (2016)

rank transfer sensitivity principles, starting with the third order then generalizing to the sth order.

Figure 2 illustrates two equivalent 3rd order *pro-extreme rank* favorable combinations of transfers. An index obeying 3rd order *pro-extreme rank transfer sensitivity* reacts favorably to a combination of progressive transfers occurring farther away from the median (p_2 to p_1 or p_6 to p_5) and a regressive one occurring closer to the median (p_3 to p_4). Both progressive transfers have a similar impact on the value of ranges $r(p_1) = h(p_6) - h(p_1) - \delta_h$ and $r(p_2) = h(p_5) - h(p_2) + \delta_h$ when $p_6 = 1 - p_1$ and $p_5 = 1 - p_2$. Since *pro-extreme rank* principles focus on the range, $r(p)$, there is an ethical equivalence between a transfer from p_2 to p_1 and from p_6 to p_5. It is important to note that indices at the third order still weight negatively the transfer from p_3 to p_4. However, since *pro-extreme rank transfer sensitivity* imposes higher ethical weights on transfers affecting ranges in the bottom of socioeconomic statuses, this negative transfer is more than compensated by a progressive transfer occurring further away from the median (from p_2 to p_1 or equivalently, from p_6 to p_5). Following a similar logic, order s *pro-extreme rank transfer sensitivity* principles impose an increasing weight to transfers occurring further away from the median of socioeconomic statuses as s increases. They are defined recursively by combining two groups of transfers of order $s-1$, a favorable one occurring farther from the median and an unfavorable one occurring closer to the median of socioeconomic statuses. Formally, if a rank-dependent socioeconomic health inequality (health achievement) index obeys A.1' (or A.1), A.2 and A.3 (or A.3'), then it obeys the sth-order *pro-extreme rank transfer sensitivity* if $(-1)^{i+1}\nu^{(i)}(p) \geq 0$ $((-1)^{i}\omega^{(i)} \geq 0)$ for $p \in [0, 0.5]$ and for all $i = 1$ to $s - 1$. We define the sets of all rank-dependent indices obeying the principle of *income-related health transfer* and obeying all *pro-extreme rank transfer sensitivity* principles of order $i = 3$ to s as Λ^s_ρ and Ω^s_ρ. It is important to note that since *pro-poor* and *pro-extreme rank* principles are different, an index that belongs to Λ^s_ρ

does not belong to Λ_π^s and an index that belongs to Ω_ρ^s does not belong to Ω_π^s.

To conclude, it is important to emphasize that the two sets of higher order ethical principles are different, they are developed from (and presented as an extension of) the same ethical view, the principle of *income-related health transfer*. This principle puts more weight on health improvements if they happen further down in the distribution of socioeconomic statuses. This means that a transfer of health to a person with lower income is deemed socially improving. Both ethical views agreed on this principle. The additional ethical constraint that Erreygers, Clarke and Van Ourti (2012) apply on socioeconomic health inequality indices, the *upside-down* test, imposes a view of inequality that focuses on the range of health levels $r(p)$ rather than on the health levels $h(p)$. In this context, it is natural to interpret an increase aversion to socioeconomic health inequality as an increase in the weight of transfers occurring farther away from the median. Since these two normative views co-exist in the health inequality literature, the next section proposes tests that identify robust rankings for each of these views.

5 Identifying robust orderings of health distributions

When an analyst uses indices to perform a comparison between two distributions, one important question surfaces: is the comparison obtained valid for wide spectra of indices obeying the same set of ethical principles? More specifically, is the comparison contingent on the particular mathematical expression of the index? To answer this question, one needs an approach that allows for comparisons that are robust over broad spectra of indices; this is why a dominance approach is necessary.

In this section, we first present some dominance tests developed in Makdissi and Yazbeck (2014). These tests are based on the standard health concentration curves (Wagstaff, Paci and Van Doorslaer, 1991), generalized health concentration curves, s-health concentration curves and generalized s-health concentration curves. The health concentration curve may be used to identify orderings of distributions that are robust for all rank-dependent socioeconomic health inequality indices. Also, generalized health concentration curves may be used to identify robust orderings of health achievement indices. To identify robust orderings for subsets of socioeconomic health inequality and health achievement indices obeying *pro-poor transfer sensitivity* principles, the analyst can rely on s-health concentration curves and generalized s-health concentration curves respectively. Also, in this section, we propose new tests that identify robust orderings of distributions for indices obeying *pro-extreme rank transfer sensitivity* ethical principles. In doing so, we introduce new graphical tools: s-health range and generalized s-health range concentration curves. These curves are akin to the s-health concentration curves and generalized s-health concentration curves but are different as they are designed to obey the *symmetry around the median* principle and *pro-extreme rank* ethical principles.

5.1 Socioeconomic health inequality orderings

Before providing more details about the proposed dominance tests, it is important to provide some background on health concentration curves. Wagstaff, Paci and Van Doorslaer (1991) introduced the health concentration curve in the health economics literature. This curve plots the cumulative proportion of total health in the population against the cumulative proportion of individuals ranked by socioeconomic status. Formally, the health concentration curve, $C_i(p)$, associated with $f^i_{Y,H}$, is defined on the interval $[0, 1]$ as

$$C_i(p) = \frac{1}{\mu_{hi}} \int_0^p h_i(u)\mathrm{d}u \tag{5}$$

When this curve lies above (under) the 45° diagonal, health inequality is pro-poor (pro-rich)[11]. An opposite conclusion may be reached if the analysis is based on an ill-health variable.

[11] In this context pro-poor means that the poor have better health than the rich

Makdissi and Yazbeck (2014) explain how these health concentration curves may be used to identify robust orderings of distributions of income and health.[12]

Theorem 1 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I(h_1) \leq I(h_2)$ for all $I(h) \in \Lambda^2$ if and only if*

$$C_1(p) \geq C_2(p) \text{ for all } p \in [0,1].$$

Theorem 1 is very powerful since it allows for the identification of orderings that are robust for all rank-dependent relative socioeconomic health inequality indices. This robustness comes at a cost, as following such an approach produces an incomplete ordering of socioeconomic health distributions.

When the ranking between two distributions is not robust, two paths may be followed. First, the analyst may decide to rely on a particular index by imposing a specific parametric form on the weight function (as seen in Section 3.3). In this case, depending on whether the analyst's ethical position is compatible with *sensitivity to poverty* or *sensitivity to extremities*, the extended health concentration indices or the symmetric indices may be used. While this solution leads to complete orderings of distributions, these orderings are contingent on the ethical position adopted by the analyst and the specific mathematical structure of the selected index.

An alternative solution is to increase the power of orderings by restricting the set of admissible rank-dependent socioeconomic health inequality indices either via *pro-poor* or via *pro-extreme rank transfer sensitivity* principles. It is important to note that these two sets of principles are based on different ethical views regarding what constitutes an increase in socioeconomic health inequality aversion (i.e., sensitivity to poverty and sensitivity to extremities). As a result, choosing one path or the other leads to different subsets of indices and may potentially lead to different orderings of distributions. In this case, the ordering

[12] For a complete proof see Makdissi and Yazbeck (2014).

will not depend on a specific parametric form of the weight function, and therefore they will be robust. However, as in any dominance tests, the orderings will still be contingent to the ethical position taken by the analyst. In what follows, we focus on the the second path and develop tests that will identify robust orderings for both types of higher order ethical principles.

5.1.1 Pro-poor ethical principles

To test if orderings are robust for a subset of rank-dependent socioeconomic health inequality indices obeying these *pro-poor* ethical principles, Makdissi and Yazbeck (2014) define higher order s-health concentration curves, $C_i^s(p)$.[13] These curves are defined on the interval $[0, 1]$ as:

$$C_i^s(p) = \int_0^p C_i^{s-1}(u)\mathrm{d}u, \tag{6}$$

where $C_i^2(p) = C_i(p)$. It is possible to identify robust rankings of distributions using these higher order health concentration curves.[14]

Theorem 2 *Let $f_{Y,H}^1$ and $f_{Y,H}^2$ represent two joint densities of income and health. $I(h_1) \leq I(h_2)$ for all $I(h) \in \Lambda_\pi^s$ if and only if*

$$C_1^s(p) \geq C_2^s(p) \text{ for all } p \in [0, 1].$$

Theorem 2 proposes a graphical test based on the non-intersection of two curves; the s-health concentration curves associated with the two distributions. If there is an intersection between the two curves at order s, the analyst can impose more restrictions on the subset of rank-dependent relative socioeconomic health inequality indices by imposing the *pro-poor transfer sensitivity* principle of order $s + 1$.[15]

[13] This curves adapt to the health inequality context the concept of s-concentration curves proposed by Makdissi and Mussard (2008) in the context of marginal indirect tax reforms.

[14] For a complete proof, please refer to Makdissi and Yazbeck (2014).

[15] As explained in Davidson and Duclos (2000), when s increases the relative weight assigned at the bottom of the distributions increases and only the very bottom of the distributions determines which dominates the other for very large s. At the limit, when $s \to \infty$, a complete ranking is obtained. In this limit case, the test consists of comparing only $\lim_{p\to 0} \frac{h_1(p)}{\mu_{h1}}$ and $\lim_{p\to 0} \frac{h_2(p)}{\mu_{h2}}$.

5.1.2 Symmetry around the median and pro-extreme rank ethical principles

As mentioned earlier, the analyst can choose to restrict the set of admissible rank-dependent socioeconomic health inequality indices by imposing *symmetry around the median* and *pro-extreme rank transfer sensitivity*. To test if the orderings are robust for a subset of socioeconomic health inequality indices that obey these normative principles, we need to introduce a new graphical tool, the s-health range curves, $R_i^s(p)$, associated with distribution $f_{Y,H}^i$. These curves are defined on the interval $[0, 0.5]$ as:

$$R_i^s(p) = \begin{cases} \frac{1}{\mu_{hi}} \int_0^p r_i(u)\mathrm{d}u & \text{if } s = 2 \\ \int_0^p R_i^{s-1}(u)\mathrm{d}u & \text{if } s \in \{3, 4, \dots\} \end{cases} \tag{7}$$

The health range curve, $R^2(p)$, has as intuitive graphical interpretation that is akin to the interpretation of the health concentration curve, $C^2(p)$. It represents the cumulative relative health range at rank p (relative to the average health level). In this case, the horizontal axis plays a role that is analogous to the 45-degree line for the health concentration curve. If the health range curve, $R^2(p)$, is above this horizontal axis, it indicates that the distribution of health (ill-health) is pro-rich (pro-poor). On the other hand, if it is below this axis, it implies that the distribution is pro-poor (pro-rich). In addition to their intuitive interpretation, it is possible to identify robust orderings using these curves.[16]

Theorem 3 *Let $f_{Y,H}^1$ and $f_{Y,H}^2$ represent two joint densities of income and health. $I(h_1) \leq I(h_2)$ for all $I(h) \in \Lambda_\rho^s$ if and only if*

$$R_2^s(p) \geq R_1^s(p) \textit{ for all } p \in [0, 0.5].$$

Theorem 3 provides a simple graphical test for the identification of robust orderings. Note that since Λ_ρ^s is a subset of indices that belongs to Λ^2, the test based on $R_i^2(p)$ curves has more ordering power (is less general) than the test based on health concentration curves in Theorem 1. This increase in ordering power is obtained by imposing the principle of

[16] For a complete proof, please refer to Appendix A2

symmetry around the median on the indices. Restricting the requirement for a unanimous ordering to a smaller set of indices eases the identification of robust orderings. If the analyst thinks that a socioeconomic health inequality index should pass the *upside-down* test, then she should use $R_i^2(p)$ curves, instead of health concentration curves $C_i(p)$. In this case, the only cost associated with the increase in the ordering power of the test is imposing symmetry of $\nu(p)$.[17] If there is an intersection between two health range curves at order s, the analyst can impose more restriction on the subset of rank-dependent socioeconomic health inequality indices by imposing the *pro-extreme rank transfer sensitivity* principle of order $s+1$.[18]

5.2 Health achievement orderings

Robust rankings of health achievement can be identified using the generalized health concentration curve. At quantile p, the generalized health concentration curve gives the absolute contribution of the p poorest individuals to average health. In other words, its value indicates the average health that would be attained if total health was only the sum of the health of these p poorest individuals. Formally, the generalized health concentration curve, $GC_i(p)$, associated with distribution $f^i_{Y,H}$, is defined on the interval $[0,1]$ as

$$GC_i(p) = \int_0^p h_i(u)\mathrm{d}u \tag{8}$$

Generalized health concentration curves may be used to identify robust orderings of joint distributions of income and health.[19]

[17]As we will see in the empirical examples presented in the Section 7, it is clear that imposing *symmetry around the median* (i.e. using range curves) allows for an increase in the power of orderings of socioeconomic health inequality.

[18]At the limit, when $s \to \infty$, a complete ranking is obtained. In this limit case, the test consists of comparing only $\lim_{p\to 0} \frac{r_1(p)}{\mu_{h1}}$ and $\lim_{p\to 0} \frac{r_2(p)}{\mu_{h2}}$.

[19]For a complete proof, please refer to Makdissi and Yazbeck (2014).

Theorem 4 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $A(h_1) \geq A(h_2)$ for all $A(h) \in \Omega^2$ if and only if*

$$GC_1(p) \geq GC_2(p) \text{ for all } p \in [0, 1].$$

Theorem 4 allows for the identification of health achievement orderings that remain valid for all rank-dependent health achievement indices. However, this robustness comes at the cost of an incomplete order. As earlier, in case there is no dominance, two paths may be followed: choosing a particular index or imposing higher order ethical principles. As in the case of inequality indices, we follow the second path.

5.2.1 Pro-poor ethical principles

Let us first consider *pro-poor transfer sensitivity* principles. Makdissi and Yazbeck (2014) introduce s-generalized health concentration curves, $GC^s_i(p)$, for the identification of these robust orderings. These curves are defined on the interval $[0, 1]$ as

$$GC^s_i(p) = \int_0^p GC^{s-1}_i(u)\mathrm{d}u, \tag{9}$$

where $GC^2_i(p) = GC_i(p)$. Robust rankings of distributions can be identified using these higher order generalized health concentration curves.[20]

Theorem 5 *Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $A(h_1) \geq A(h_2)$ for all $A(h) \in \Omega^s_\pi$ if and only if*

$$GC^s_1(p) \geq GC^s_2(p) \text{ for all } p \in [0, 1].$$

Theorem 5 proposes another graphical test based on the non-intersection of two curves, the s-generalized health concentration curves associated with the two distributions. If there is an intersection between the two curves at order s, the analyst can impose more

[20]For a complete proof, please refer to Makdissi and Yazbeck (2014)

restriction on the subset of rank-dependent achievement indices by imposing the *pro-poor transfer sensitivity* principle of order $s+1$.[21]

5.2.2 Symmetry around the median and pro-extreme rank ethical principles

An alternative path to imposing *pro-poor* ethical principles consists in restricting the set of admissible health achievement indices by imposing *symmetry around the median* and *pro-extreme rank* ethical principles. In this case, the identification of robust orderings is based on a new graphical tool, the s-generalized health range curves, $GR_i^s(p)$, associated with distribution $f_{Y,H}^i$. These curves are defined on the interval $[0, 0.5]$ as:

$$GR_i^s(p) = \begin{cases} \int_0^p r_i(u)\mathrm{d}u & \text{if } s = 2 \\ \int_0^p GR_i^{s-1}(u)\mathrm{d}u & \text{if } s \in \{3, 4, \dots\} \end{cases} \tag{10}$$

As for health range curves, $GR^2(p)$ has an appealing graphical interpretation. It represents the cumulative absolute health range at rank p. In addition to this intuitive graphical interpretation at order 2, these range curves can be used to obtain robust rankings of socioeconomic health distributions.[22]

Theorem 6 *Let $f_{Y,H}^1$ and $f_{Y,H}^2$ represent two joint densities of income and health. $A(h_1) \geq A(h_2)$ for all $A(h) \in \Omega_\rho^s$ if and only if*

$$GR_2^s(p) \geq GR_1^s(p) \text{ for all } p \in [0, 0.5].$$

and,

$$\mu_{h1} \geq \mu_{h2}$$

Theorem 6 offers another graphical test. However, the identification of robust rankings for indices obeying the *symmetry around the median* and *pro-extreme rank* transfer principles has an additional condition on the average of health level when compared to pro-poor transfer principles.[23]

[21] At the limit, when $s \to \infty$, a complete ranking is obtained. In this limit case, the test consists of comparing only $\lim_{p\to 0} h_1(p)$ and $\lim_{p\to 0} h_2(p)$.

[22] For a complete proof, please refer to Appendix A2.

[23] Since Theorem 6 states necessary and sufficient conditions, the tests on the averages and on the range

6 Estimation and inference

In this section, we show how to estimate the curves presented in the previous section. We then show how one can perform statistical inference and thus identify rankings that are robust to the set of indices selected by the analyst.

6.1 Concentration and range curves estimators

Suppose we have a random sample of N individuals drawn from a joint distribution $f_{H,Y}$. We first show how to construct estimators of C and R, C^s and R^s for $s > 2$ and then show how to test dominance using these curves. We start by showing that C and R can both be written in a form that is directly amenable to non-parametric estimation. Given that $C(p)$ can be re-written as

$$C(p) = \frac{1}{\mu_h} \int_0^1 \mathbb{1}(u < p) h(u) \mathrm{d}u, \tag{11}$$

a simple estimator for $C(p)$ can be written as follows:

$$\widehat{C}(p) = \frac{1}{N\bar{h}} \sum_{i=1}^{N} h_i \mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p)) \tag{12}$$

from a sample (y_i, h_i) for $i = 1, \ldots, n$. Here $\bar{h}$ is the sample average and $\hat{F}_Y^{-1}$ is a non-parametric estimator of the quantile function of Y based on the order statistics of (y_i). Estimators for $C^s(p)$ can be recursively derived from that of $C(p)$ (for details see Appendix A3.1). The resulting estimators are as follows:

$$\widehat{C}^s(p) = \frac{1}{N\bar{h}} \sum_{i=1}^{N} h_i \frac{(p - \hat{F}_Y(y_i))^{s-2}}{(s-1)!} \mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p)) \tag{13}$$

The generalized concentration curve can be therefore written as:

$$\widehat{GC}^s(p) = \bar{h}\widehat{C}^s(p) \tag{14}$$

curves need to be verified. Although $GC_i^2(1) = \int_0^1 h_i(u)\mathrm{d}u = \mu_{hi}$, $GR_i^2(0.5) = \int_0^{0.5} r_i(u)\mathrm{d}u \neq \mu_h$. This is why testing for the first condition at order 2 over [0,0.5] does not imply that the second condition is verified. As a result, the second condition is also necessary at the second order.

In a similar fashion, we can construct an estimator for R. Let us first rewrite $R(p)$ in the same form as $C(p)$:

$$\mu_h R(p) \quad = \quad \int_0^p r(u)\mathrm{d}u \tag{15}$$

Given that $r(u)$ can be written as $h(1-u) - h(u)$ for $u \in [0,1]$, we can re-write this relationship as follows:

$$\mu_h R(p) \quad = \quad \int_{1-p}^1 h(u)\mathrm{d}u - \int_0^p h(u)\mathrm{d}u, \tag{16}$$

which can be re-written as follows:

$$\mu_h R(p) = \int_0^1 [\mathbb{1}(u > 1-p) - \mathbb{1}(u < p)]h(u)\mathrm{d}u \tag{17}$$

A simple estimator for $R(p)$ can be written as follows:

$$\widehat{R}(p) = \frac{1}{N\bar{h}} \left\{ \sum_{i=1}^N h_i[\mathbb{1}(y_i > \hat{F}_Y^{-1}(1-p))] - \sum_{i=1}^N h_i[\mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p))] \right\} \tag{18}$$

Similarly to C^s, it is possible to recursively compute estimators of R^s by first plugging the estimators of $\widehat{R}^{s-1}$, integrating them analytically and then by recursively computing $\widehat{R}^s$ (see details in Appendix A3.2). The resulting estimators are as follows:

$$\begin{aligned} \widehat{R}^s(p) \quad &= \quad \frac{1}{N\bar{h}} \sum_{i=1}^N h_i \frac{1}{(s-1)!} p^{s-2}(p + (s-1)[\hat{F}_Y(y_i) - 1])[\mathbb{1}(y_i > \hat{F}_Y^{-1}(1-p))] \\ &- \quad \frac{1}{N\bar{h}} \sum_{i=1}^N h_i \frac{(p - \hat{F}_Y(y_i))^{s-2}}{(s-1)!} [\mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p))] \end{aligned} \tag{19}$$

The generalized range curve can be written as follows:

$$\widehat{GR}^s(p) = \bar{h}\widehat{R}^s(p) \tag{20}$$

6.2 Dominance tests

Let us denote by L one of the curves from the previous section (e.g., $C(p)$). Let L_1 and L_2 be two different theoretical curves corresponding to two different theoretical populations.

Assume that we have an i.i.d. sample of size n_1 from the random variable corresponding to first theoretical curve L_1 and an i.i.d. sample of size n_2 from the random variable corresponding to the second theoretical curve L_2. Denote those samples by $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively. As we are interested in testing the dominance between two distributions, we define the new function $L_{12}(p) := L_1(p) - L_2(p)$ for $p \in [0, 1]$. The null and alternative hypotheses we are interested in are:

$$\begin{aligned} H_0 &: L_{12}(p) \leqslant 0, \forall p \\ H_1 &: L_{12}(p) > 0 \text{ for some } p \end{aligned}$$

When performing inference, for each pair of distributions we will test a set of inequalities. In this paper, we will test for H_0: $L_{12} \leq 0$ for all p and H_0: $L_{12} \geq 0$ for all p where under the null we assume dominance. If we can reject one of the null hypotheses of dominance for the same pair of distributions, then we have evidence against that null of the dominance of one distribution over the other. While one may think that it is more intuitive to test the null hypothesis of non-dominance and hence establish a case of dominance, such a test requires a strong evidence against the null, which may be difficult to obtain over the entire support (Davidson and Duclos, 2013).

The nonparametric estimators $\hat{L}_1$ and $\hat{L}_2$, of L_1 and L_2 respectively, can be constructed from these two samples and subsequently $\hat{L}_{12}(p) = \hat{L}_1(p) - \hat{L}_2(p)$. Let $\tau = \sup_p L_{12}(p)$, it is straightforward to construct a KS type test statistic $\hat{\tau}$ that is a non-parametric estimator of τ as follows:

$$\hat{\tau} = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \sup_p \hat{L}_{12}(p). \tag{21}$$

The asymptotic distribution of $\hat{\tau}$ will be that of a functional of a two-dimensional Gaussian process that is very complicated to compute. To overcome this issue, we rely on a bootstrap procedure as in Shechtman et al. (2008). For a detailed description of the bootstrap procedure, please refer to the Appendix A4.

As for the indices obeying the *symmetry around the median* principle and *pro-extreme rank* principles (i.e., theorem 6), the associated the joint test H_0^1 and H_0^2 can be defined, using $GR_{12}(p) = GR_1(p) - GR_2(p)$, as follows,

$$H_0^1 \ : \ GR_{12}(p) \leqslant 0, \forall p$$
$$H_1^1 \ : \ GR_{12}(p) > 0 \text{ for some } p,$$

and,

$$H_0^2 \ : \ \mu_1 \geq \mu_2$$
$$H_1^2 \ : \ \mu_1 < \mu_2,$$

where the nonparametric estimators $\widehat{GR}_1$ and $\widehat{GR}_2$ of GR_1 and GR_2 can respectively be constructed from the two samples and subsequently, $\widehat{GR}_{12}(p) = \widehat{GR}_1(p) - \widehat{GR}_2(p)$.

The test statistic, $\hat{\tau}$, for testing H_0 (H_0^1) takes the same form; however, the joint test (i.e., H_0^1 and H_0^2) has an additional condition on the mean that needs to be tested when establishing the dominance results. To account for this additional condition, we adjust the significance level of the joint test by relying on the Holm procedure as described in the Lehmann and Romano (2005) [chapter 9 p. 348]. The purpose of the procedure is to control for the family-wise error rate (FWER), which is the probability of one or more false rejections not exceeding a certain level, by making sure that this error is below a certain threshold α.

Let $I \subset \{1, 2\}$ be the set of indices for which H_0^i is true for $i = 1, 2$, then the objective is to ensure the following condition: $\Pr\left\{\textit{reject any } H_0^i \textit{ with } i \in I\right\} \leqslant \alpha$. Given the two tests H_0^1 and H_0^2 with p-values p_1 and p_2, the Holm procedure works as follows. First, order the p-values $p_A \leqslant p_B$, where $A, B \in \{1, 2\}$ and $A \neq B$, and label correspondingly H_0^A and H_0^B. If $p_A \geqslant \frac{\alpha}{2}$, then we do not reject both hypotheses and stop. However, if $p_A < \frac{\alpha}{2}$ and $p_B \geqslant \alpha$, then we reject H_0^A and do not reject H_0^B. Otherwise, reject both hypotheses. It should be noted that if we reject one of the two hypotheses, then we reject dominance.

7 Empirical illustration

To provide evidence that the differences between various ethical principles adopted by the analyst influence the type of conclusion reached, we conduct an empirical illustration of the methods proposed using National Health Interview Survey data from years 1997 and 2014. We focus on comparisons of two ill-health variables that have been of great interest in the health economics literature: cigarettes consumption (i.e., the number of cigarettes/day) and overweightedness. We follow Bilger, Kruger and Finkelstein (2016) and use max[0,BMI-25] as a measure of overweightedness. Given that the empirical application is mainly for illustration purposes, we will refrain from drawing policy recommendation, but we will indicate potential interesting paths.

The NHIS monitors health outcomes of Americans since 1957. It is a cross-sectional household interview survey representative of American households and non-institutionalized individuals. It contains data on a broad range of health topics that are collected via personal household interviews. For comparison purposes, we focus on 1997 and 2014 public use data and restrict our attention to the adult population. After applying all these restrictions to the data, we end up with a sample size of 34776 for overweightedness and 35667 for cigarette consumption in 1997 and is 35197 for overweightedness and 36363 for cigarette consumption in 2014. We use the sample adult file to extract information on health-related behavior and use family income adjusted for family size to infer the socioeconomic rank of individuals.[24]

We first start the illustration by looking at comparisons from an inequality perspective and then revisit these comparisons from an achievement perspective.

[24] We compute equivalent income by dividing family income by the square root of household size.

7.1 Comparisons of inequalities in health-related behaviors and outcomes

In the first set of inequalities comparisons presented in Table 1 we focus on comparisons (over time) at the national level. These comparisons are complemented by regional comparisons in Table 2. Cigarette consumption seems to display a higher socioeconomic health inequality in 2014 than in 1997 (Figure 3). There is a clear dominance of the concentration curve C^2_{2014} over C^2_{1997} without any intersection on the interval $[0, 1]$. As is shown in Table 1, when the null hypothesis is that of the dominance of C^2_{1997} over C^2_{2014} there is a very weak evidence against the null. However, when the null hypothesis of the dominance of C^2_{2014} over C^2_{1997} is tested, there is strong evidence against the null (p-value=0.0000). As a result, one can conclude that there is more socioeconomic health inequality in smoking in 2014 for all indices obeying the income-related health transfer principle. While deriving any policy conclusion is beyond the scope of this paper, it is important to emphasize that an increase in the disparities at the cigarette consumption level may be a major contributor to the widening disparities in health outcomes.

In addition to testing dominance at the second order (i.e., C^2) we provide a test for order 3 dominance, C^3, and order 2 dominance for indices meeting the *upside-down* test criteria (i.e., ranges curves R^2). We know from our theoretical results that if dominance is obtained at the second order for the C^2, it follows that dominance will be obtained for both higher order dominance C^s and second order range curves R^2. To show the empirical validity of these theoretical results we conduct this additional test. Test results presented in the lower panel of Table 1 confirm what was theoretically expected; there is more socioeconomic health inequality in smoking in 2014 than 1997. This is true for all indices obeying the income-related health transfer principle and for the subset of these indices obeying the *pro-poor transfer sensitivity* principle. Similarly, as there is more socioeconomic health inequality in smoking in 2014 for all indices obeying the income-related health transfer principle, this result applies to the subset of these indices passing the *upside-down* test.

Another health variable that one may want to consider in the analysis of socioeconomic health inequality is overweightedness (defined as $\max[0, BMI - 25]$). Overweightedness is defined as any positive deviation (in BMI units) from the maximal threshold of the normal (healthy) weight category. Since deviations from below and above the healthy weight category are deemed negative, BMI is not an ill-health ratio-scale variable. However, Bilger, Kruger and Finkelstein's (2016) transformation, for a given threshold, is a cardinal variable with a well defined 0. Looking at the top left panel in Figure 4, one can notice that the two overweightedness concentration curves (C^2) intersect. There is strong evidence against the null when dominance of C^2_{1997} over C^2_{2014} and dominance of C^2_{2014} over C^2_{1997} are tested at the 1% level (p-values are respectively 0.0060 and 0.0000 in Table-1).[25] As a result, we cannot assess whether socioeconomic health inequality in overweightedness has increased or decreased when we consider all indices obeying the income-related health transfer principle. As mentioned earlier in the paper, when there is no clear dominance in the context of concentration curves (i.e., C^2), one can consider the subset of indices obeying a higher order ethical such as the *pro-poor transfer sensitivity* principle (i.e. considering, C^3) so we follow this path. The results shown in the top right panel of Figure 4 allow for the conclusion that socioeconomic health inequality in overweightedness decreases from 1997 to 2014. In other words, there is more socioeconomic health inequality in overweightedness in 1997 for all indices obeying the income-related health transfer principle as well as the *pro-poor transfer sensitivity* principle. An alternative path may be taken in the absence of dominance at the second order if one is willing to focus on the subset of indices passing

[25]It is important to note that if one decreases the level to 0.5%, the dominance conclusions reached at order 2 will not hold at third order. While this may seem in contradiction with the theory at first, it is not the case in this application. In reality this "incoherence" between the second and third order dominance is due to the magnitude of the distance just before $p = 0.8$ in the first panel of Figure 4. Given that this distance is quite large, integrating the second order curves results apparently flipped around result at the third order.

the *upside-down* test (i.e., R^2 curves). As shown in the lower panel of Figure 4, following these paths lead to the same conclusion as the one reached when exploring higher order dominance in the case of concentration curves. Indeed, there is more socioeconomic health inequality in overweightedness in 1997 for all indices obeying the income-related health transfer principle and the *upside-down* test and these results are supported by the associated p-values displayed in Table 1. It is important to note that the subset of indices obeying higher order *pro-poor* principles and the subset of indices that pass the *upside-down* test are disjoint. So while the conclusions reached in this part of the empirical application are the same, there is no theoretical reason for this to be always the case.

As for regional comparisons, we focus our attention on the most recent year, 2014 and compare the Northeast, the West, the Midwest and the South.[26] When we focus on cigarette consumption, we notice there are no clear patterns of dominance and thus no complete order.[27] More specifically, at 5% significance level, the West dominates the Northeast for all indices obeying *pro-poor transfer sensitivity* (i.e., Λ^3_π), and dominates the South for all indices (i.e., Λ^2). Also, for the subset passing the *upside-down* test, the West dominates the Northeast, the Midwest at the second order (i.e., Λ^2_ρ). If we decrease the significance level to 1%, then we only have one dominance result: the West dominates the South at the second order for all indices (i.e., Λ^2).

Turning our attention to regional dominance in the case of overweightedness, we notice that the Northeast is dominated by the West, the Midwest as well as the South. While the significance level and the order at which this dominance occur vary by region, one can safely conclude that this dominance occurs at the 5% significance level. If we were to decrease the significance level to 1%, then the order and subset of indices at which this dominance occurs changes. For instance, the Northeast is dominated by the Midwest at the *pro-poor*

[26]It is important to note that we chose the most recent year to save on space.

[27]It is important to note that the rows have lower socioeconomic inequality than the columns.

third order instead of the second order. Also, the Northeast is dominated by the South and the Midwest for all subset of indices passing the *upside-down* test.

7.2 Comparisons of health achievements in health-related behaviors and outcomes

In this section, we compare health achievements between 1997 and 2014. To save on space, Table 3 will not report the p-values but rather dominance results along with the standard notation to indicate the significance level of the dominance tests. When reading Table 3, one has to keep in mind that the columns dominate the rows; this means that when there is dominance, the year that dominates has lower "ill health" level and hence higher health achievement. Given that we are dealing with an "ill-health" variable it is more sensible to talk about health failures rather than health achievements (see Makdissi, Sylla and Yazbeck, 2013).[28]

Comparisons of generalized health concentration curve reveal that GC^2_{2014} dominates GC^2_{1997} with strong evidence against the null hypothesis. This means that, as far as cigarette consumption is concerned, there is more health failure in 1997 than in 2014 for all health achievement(/failure) indices obeying the principle of *income-related health transfer*. As in the case of inequality, we reach the same conclusion if we test for a higher order dominance (i.e., GC^3). This, once again, confirms what was theoretically expected. In other words, since there is more health failure (when smoking is considered) in 1997 for all indices obeying the income-related health transfer principle, it is expected this is true for the subset of these indices that are obeying the *pro-poor transfer sensitivity* principle. As mentioned earlier, the analyst may argue that the principle of *pro-poor transfers sensitivity* is debatable and focus on the set of indices that pass the *upside-down* test. To account for this possibility, we test for dominance using the generalized health range curves. Empirical results show

[28]It is important to note that the rows have lower failure than the columns in the tables which means that the rows have a higher achievement.

that there is more failure in smoking in 1997 for all indices obeying the *income-related health transfer* principle and the *upside-down* test. Given that the set of indices that pass the *upside-down* test are subsets of the indices belonging to Ω^2, we can re-write the results concisely by saying that there is a dominance at the second order for all rank-dependent indices that is all indices in Ω^2. As for overweightedness, we have the mirror picture of the cigarette consumption comparison as there is more failure in 2014 than in 1997. These second order dominance results are statistically significant at the 1% level and hold for all rank-dependent indices (see Table 3).

Before turning to the regional comparisons, it is important to compare the results obtained from the inequality analysis with the results obtained from the achievement analysis to emphasize the policy relevance of developing and using both approaches in an inequality analysis. While the inequality analysis revealed that there is more inequality in cigarette consumption in 2014, the analysis on health achievement (or failure) shows that there is a lower failure in health in 2014 than 1997. So while the inequality analysis may show that there are concerns regarding socioeconomic inequality in smoking behavior, this same behavior seems to be less prevalent when assessed by a measure that puts higher weight for smoking behavior when it occurs in the lower part of the income distribution. The same logic applies to the results obtained in overweightedness. The socioeconomic inequalities are lower in 2014 than in 1997 but the failure is higher in 2014 than in 1997. The results discussed in this section indicate that focusing on inequality alone provides an incomplete picture of the situation.

As for regional comparisons for health failures, it is clear that we have more results than in the inequality section. The first panel of Table 4 focuses on cigarette consumption in 2014 and shows that we have dominance results for all regions at the 1% level. This allows for a complete ordering of regions in ascending inequalities as follows West, Northeast, South and Midwest. The second panel in Table 4 shows second order dominance results for overweightedness at mixed significance levels (i.e., some are at the 5% level and others are at the 1% level). Unlike the case of cigarette consumption, we do not have a complete ordering of regions for overweightedness. All we can say is that the West dominates the Northeast, the Midwest and the South and that the Northeast dominates the Midwest and South. To assess whether we can have a dominance result at a higher significance level (i.e., 1% level instead of 5% level) for the Northeast and West, we focus on indices that pass the *upside-down* text. In doing so, one needs to remember that testing for achievement (/failure) for these subsets of indices requires a joint test on the range curves and the average value of the health variable. Figure 6 displays GR^2 curves for the two regions where GR^2_W seem to be everywhere above (or equal) to GR^2_{NE}. The results of the associated statistical tests displayed in Table 5 suggest that we cannot reject dominance and that the West has less failure in overweightedness than the Northeast if we focus our attention on indices that pass the *upside-down* test and obey the *principle of income-related health transfer*.[29]

8 Conclusion

In this paper, we adopt a unified approach to indices that obey *pro-poor ethical* principles as well as the *symmetry around the median* ethical principle and *pro-extreme rank* principles. To do so, we first fill the gap in the literature by formalizing the ethical principles associated with the symmetric indices (i.e., the set of indices that pass the *upside-down* test). We coin these ethical principles as the *symmetry around the median* ethical principle and *pro-extreme rank* ethical principles. We then develop the curves associated with these principles, the health range curve and the s-health range curves, and derive the dominance conditions that allow us to identify robust orderings of joint distributions of income and health. Having filled the gap in the inequality measurement literature, we proceed to the literature on

[29]Note that, if we consider the p-values in Table 5, we cannot reject equality between μ_{NE} and μ_W.

the statistical inference and provide the natural estimators for the indices obeying both *pro-poor* and *pro-extreme rank* ethical principles. Based on these estimators and on the work of Schechtman et al. (2008) we develop KS-type statistical tests associated with the dominance tests for indices obeying both ethical principles. Finally, to illustrate the applicability of the methods proposed we provide an empirical illustration using information on overweightedness and cigarette consumption from the NHIS 1997 and 2014.

Throughout this paper, we have assumed that the information on the level of health (ill-health) takes the form of a ratio-scale cardinal variable. This means that we allow the social weight functions to take different mathematical forms assuming that we know the value of the level of health. This last assumption does not hold for categorical health variables such as self-reported health statuses as any increasing cardinal scale is a valid representation of a categorical variable. To address this issue, one may cardinalize the categorical health measure (Makdissi and Yazbeck, 2014) or use a dominance approach that is invariant to the numerical scale (Allison and Foster, 2004; Sonne-Schmidt, Tarp and Østerdal, 2016; and Makdissi and Yazbeck, 2017). Since many of health variables available in surveys are categorical, future research is needed to allow the researcher to use cardinal variables while dealing simultaneously with the multiplicity of social weight functions and of cardinal scales.

In addition, it may be argued that the normative views presented in this paper (rank-dependent views) are debatable. One could extend this paper to account for this possibility by following Karsu (2016) and Karsu, Morton and Argyris (2012) and imposing other normative restrictions on the decision maker's preferences. This can be achieved by proposing a class of indices displaying aversion to pure health inequality (by transforming the health level using an s-concave function) in addition to aversion to socioeconomic health inequality as in Makdissi and Yazbeck (2016).

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Table 1: Dominance tests for C^s and R^s comparisons for cigarette consumption and overweightedness

	p-value			
	cigarette cons.		overweightedness	
	s=2	s=3	s=2	s=3
$H_0 : C^s_{1997}(p) \leqslant C^s_{2014}(p), \forall p$ $H_1 : C^s_{1997}(p) > C^s_{2014}(p)$ for some p	0.9970	0.8248	0.0060	0.0010
$H_0 : C^s_{2014}(p) \leqslant C^s_{1997}(p), \forall p$ $H_1 : C^s_{2014}(p) > C^s_{1997}(p)$ for some p	0.0000	0.0000	0.0000	0.8658
$H_0 : R^s_{1997}(p) \leqslant R^s_{2014}(p), \forall p$ $H_1 : R^s_{1997}(p) > R^s_{2014}(p)$ for some p	0.0000		0.5305	
$H_0 : R^s_{2014}(p) \leqslant R^s_{1997}(p), \forall p$ $H_1 : R^s_{2014}(p) > R^s_{1997}(p)$ for some p	0.9670		0.0020	

Table 2: Regional dominance tests: cigarette consumption and overweightedness

Cigarette consumption				
	Northeast	West	Midwest	South
Northeast	-		ND	ND
West	Λ^3_π ** and Λ^2_ρ **	-	Λ^2_ρ **	Λ^2 ***
Midwest	ND		-	ND
South	ND		ND	-
overweightedness				
	Northeast	West	Midwest	South
Northeast	-			
West	Λ^2 ***	-	ND	ND
Midwest	Λ^2 ** and Λ^3_π *** and Λ^2_ρ ***	ND	-	ND
South	Λ^2 ** and Λ^2_ρ ***	ND	ND	-

Significance levels ** 5%; *** 1%

Table 3: Evolution of health failures: 1997-2014

	1997	2014
1997	-	overweightedness: Ω^2 ***
2014	Cigarette: Ω^2 ***	-

Significance levels: ** 5%; *** 1%.

Table 4: Regional dominance tests: cigarette consumption and overweightedness

Cigarette consumption				
	Northeast	West	Midwest	South
Northeast	-		Ω^2 ***	Ω^2 ***
West	Ω^2 ***	-	Ω^2 ***	Ω^2 ***
Midwest			-	
South			Ω^2 ***	-
overweightedness				
	Northeast	West	Midwest	South
Northeast	-		Ω^2 ***	Ω^2 ***
West	Ω^2 ** and Ω^2_ρ ***	-	Ω^2 ***	Ω^2 ***
Midwest			-	ND
South			ND	-

Significance levels ** 5%; *** 1%

Table 5: Dominance tests for failure in overweightedness between the Northeast and the West for indices belonging to Ω^2_ρ

	p-value
$H_0 : GR^2_W(p) \leqslant GR^2_{NE}(p), \forall p$ $H_1 : GR^2_W(p) > GR^2_{NE}(p)$ for some p	0.0040
$H_0 : GR^2_{NE}(p) \leqslant GR^2_W(p), \forall p$ $H_1 : GR^2_{NE}(p) > GR^2_W(p)$ for some p	0.9219
$H_0 : \mu_W \geqslant \mu_{NE}$ $H_1 : \mu_W < \mu_{NE}$	0.2843
$H_0 : \mu_{NE} \geqslant \mu_W$ $H_1 : \mu_{NE} < \mu_W$	0.7157
$H_0 : \mu_W = \mu_{NE}$ $H_1 : \mu_W \neq \mu_{NE}$	0.5876

Figure 1: Second order ethical principle

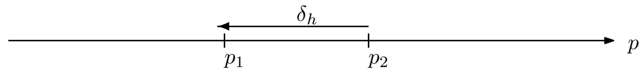


Figure 2: Third order pro-extreme rank transfer sensitivity

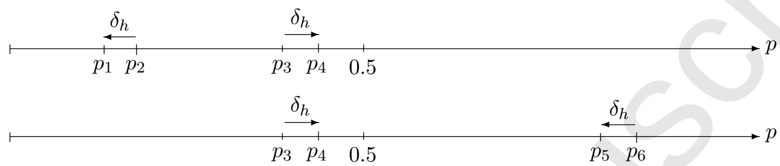


Figure 3: C^2 comparison: cigarette consumption

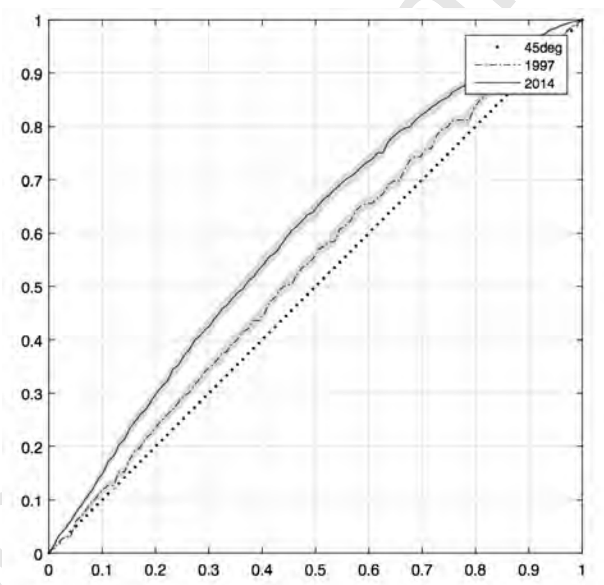
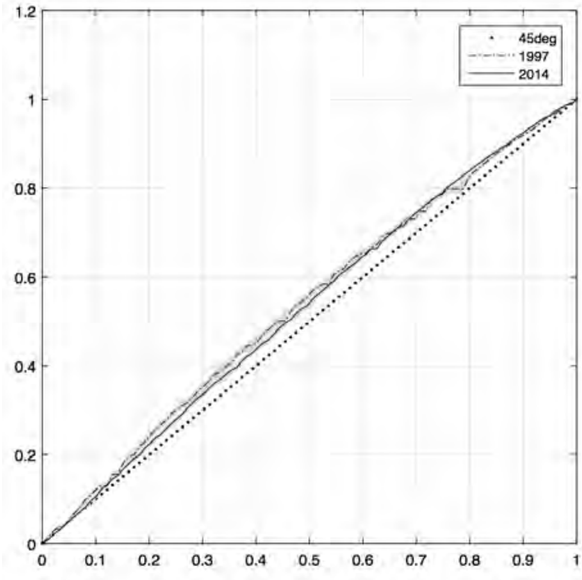
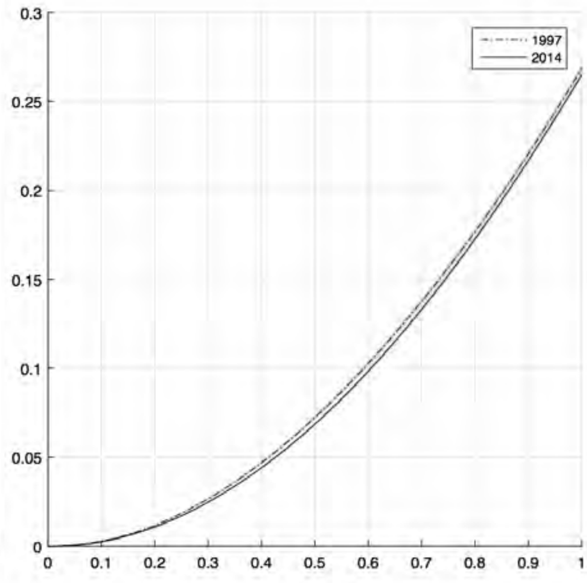


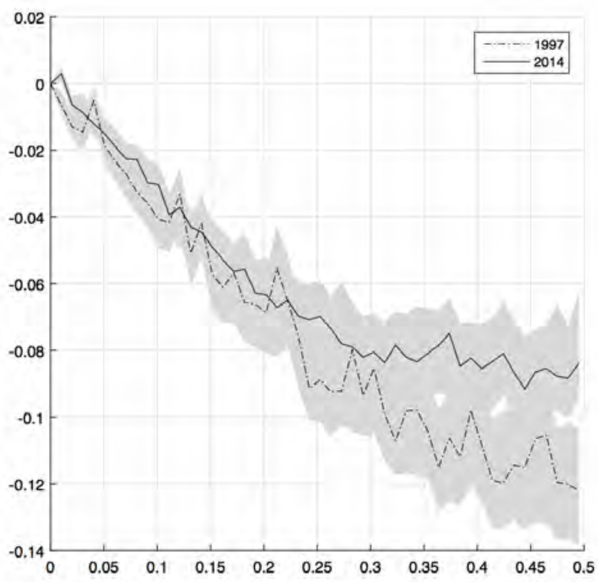
Figure 4: Socioeconomic inequality in Obesity/Overweight



$C^2(p)$

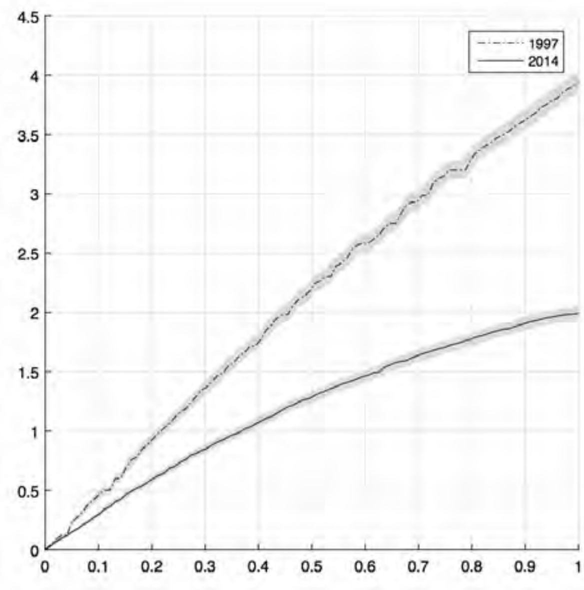


$C^3(p)$

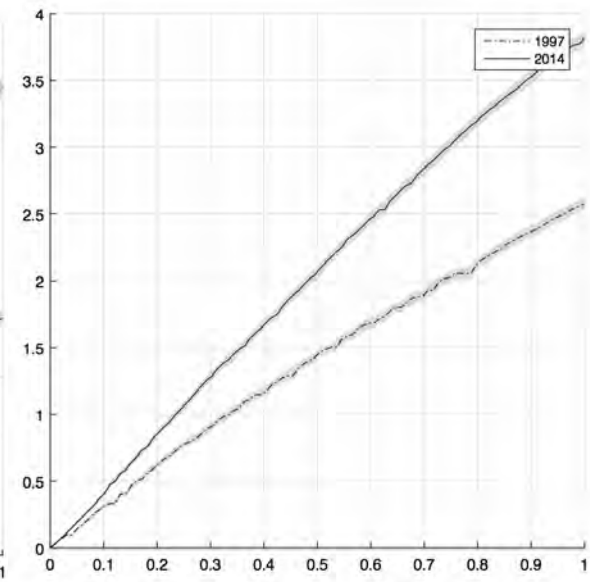


$R^2(p)$

Figure 5: Second order health failures comparisons

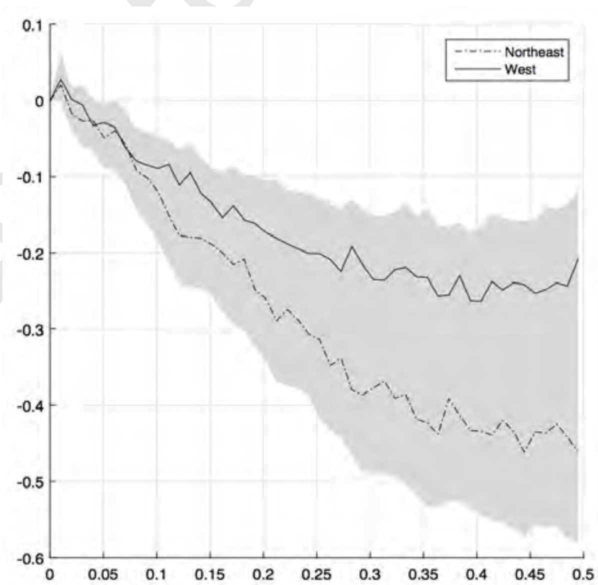


Cigarette



overweightedness

Figure 6: GR^2 comparison: overweightedness



Appendix

A1 Set of indices

The proofs are based on the following mathematical definition of the set of indices.

$$\begin{aligned}
\Omega^2 &:= \left\{ A(h) \,\middle|\, \begin{array}{c} \omega(p) \text{ is continuous and differentiable almost} \\ \text{everywhere over } [0,1], \int_0^1 \omega(p)\mathrm{d}p = 1, \\ \omega(1) = 0, \omega^{(1)}(p) \leq 0, \ \forall p \in [0,1] \end{array} \right\}. \\
\Lambda^2 &:= \left\{ I(h) \,\middle|\, \begin{array}{c} \nu(p) \text{ is continuous and differentiable almost} \\ \text{everywhere over } [0,1], \int_0^1 \nu(p)\mathrm{d}p = 0, \\ \nu(1) = 1, \nu^{(1)}(p) > 0, \ \forall p \in [0,1] \end{array} \right\}. \\
\Lambda^2_\rho &:= \left\{ I(h) \in \Lambda^2 \mid \nu(1-p) = -\nu(p) \ \forall p \in [0,1] \right\}. \\
\Omega^2_\rho &:= \left\{ A_A(h) \in \Omega^2 \mid \omega(1-p) = 2 - \omega(p) \ \forall p \in [0,1] \right\}. \\
\Omega^s_\pi &:= \left\{ A(h) \in \Omega^2 \,\middle|\, \begin{array}{c} \omega(p) \text{ is continuous and } (s-1)\text{-time differentiable almost} \\ \text{everywhere over } [0,1], \omega^{(i)}(1) = 0, (-1)^i \omega^{(i)}(p) \geq 0, \ \forall p \in [0,1], \\ \forall i = 1, 2, \ldots, s-1 \end{array} \right\}. \\
\Lambda^s_\pi &:= \left\{ I(h) \in \Lambda^2 \,\middle|\, \begin{array}{c} \nu(p) \text{ is continuous and } (s-1)\text{-time differentiable almost} \\ \text{everywhere over } [0,1], \nu^{(i)}(1) = 0, (-1)^{i+1} \nu^{(i)}(p) \geq 0, \ \forall p \in [0,1], \\ \forall i = 1, 2, \ldots, s-1 \end{array} \right\}. \\
\Lambda^s_\rho &:= \left\{ I(h) \in \Lambda^2_\rho \,\middle|\, \begin{array}{c} \nu(p) \text{is continuous and } (s-1)\text{-time differentiable almost} \\ \text{everywhere over } [0,1], \nu^{(i)}(0.5) = 0, (-1)^{i+1} \nu^{(i)}(p) \geq 0, \\ \forall p \in [0,0.5], \ \forall i = 1, 2, \ldots, s-1 \end{array} \right\}. \\
\Omega^s_\rho &:= \left\{ A(h) \in \Omega^2_\rho \,\middle|\, \begin{array}{c} \omega(p) \text{is continuous and } (s-1)\text{-time differentiable almost} \\ \text{everywhere over } [0,1], \omega^{(i)}(0.5) = 0, (-1)^i \omega^{(i)}(p) \geq 0, \\ \forall p \in [0,0.5], \ \forall i = 1, 2, \ldots, s-1 \end{array} \right\}.
\end{aligned}$$

Note that increasing s means imposing more ethical structure on indices. This in turns implies that $\Omega^s_\pi \subset \Omega^{s-1}_\pi \subset \cdots \subset \Omega^3_\pi \subset \Omega^2$, $\Lambda^s_\pi \subset \Lambda^{s-1}_\pi \subset \cdots \subset \Lambda^3_\pi \subset \Lambda^2$, $\Lambda^s_\rho \subset \Lambda^{s-1}_\rho \subset \cdots \subset \Lambda^3_\rho \subset \Lambda^2_\rho \subset \Lambda^2$ and $\Omega^s_\rho \subset \Omega^{s-1}_\rho \subset \cdots \subset \Omega^3_\rho \subset \Omega^2_\rho \subset \Omega^2$.

A2 Proofs for section 4

Proofs of Theorems 1, 2, 4, 5 are provided by Makdissi and Yazbeck (2014).

Proof of Theorem 3. First note that for $I(h) \in \Lambda_\rho^s$, equation (1) can be rewritten as

$$I(h) = -\frac{1}{\mu_h} \int_0^{0.5} \nu(p) r(p) \mathrm{d}p \tag{A1}$$

Integrating by parts equation (A1), we get

$$I(h) = -\nu(p) R^2(p)\Big|_0^{0.5} + \int_0^{0.5} \nu^{(1)}(p) R^2(p) \mathrm{d}p. \tag{A2}$$

Since by definition $R^2(0) = 0$ and $\nu(0.5) = 0$ for all indices $I(h) \in \Lambda_\rho^s$, the first term on the right hand side of the equation is nil. This yields to

$$I(h) = \int_0^{0.5} \nu^{(1)}(p) R^2(p) \mathrm{d}p. \tag{A3}$$

Now assume that for $s-1$, we have

$$I(h) = (-1)^{s-1} \int_0^{0.5} \nu^{(s-2)}(p) R^{s-1}(p) \mathrm{d}p. \tag{A4}$$

Integrating by parts equation (A4) yields

$$I(h) = (-1)^{s-1} \left\{ \nu^{(s-2)}(p) R^s(p)\Big|_0^{0.5} - \int_0^{0.5} \nu^{(s-1)} R^s(p) \mathrm{d}p \right\}. \tag{A5}$$

Since by definition $R^s(0) = 0$ and $\nu^{(s-2)}(0.5) = 0$ for all indices $I(h) \in \Lambda_\rho^s$, the first term in the braces on the right hand side of the equation is nil. This yield

$$I(h) = (-1)^s \int_0^{0.5} \nu^{(s-1)}(p) R^s(p) \mathrm{d}p. \tag{A6}$$

Given that equations (A3) and (A6) both conform to the relation depicted in equation (A4), it follows that equation (A6) holds for all $s \in \{2, 3, \dots\}$. Let $\Delta I_{12} = I(h_2) - I(h_1)$. From equation (A6), we get

$$\Delta I_{12} = (-1)^s \int_0^{0.5} \nu^{(s-1)}(p) \left[R_2^s(p) - R_1^s(p)\right] \mathrm{d}p. \tag{A7}$$

Note that $(-1)^s\nu^{(s-1)}(p)$ is non negative. This implies that if $R_2^s(p) \geq R_1^s(p)$ for all $p \in [0, 0.5]$, then $\Delta I_{12} \geq 0$. This proves for sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. Consider now the set of indices $I(h) \in \Lambda_\rho^s$ for which $\nu^{(s-2)}(p)$ takes the following form:

$$\nu^{(s-2)}(p) = \begin{cases} (-1)^{s-1}\varepsilon & 0 \leq p_c \\ (-1)^{s-1}\left[p_c + \varepsilon - p\right] & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{A8}$$

where $p_c \in [0, 0.5]$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of Λ_ρ^s. Differentiating equation (A8) yields

$$\nu^{(s-1)}(p) = \begin{cases} 0 & 0 \leq p_c \\ (-1)^s & p_c \leq p \leq p_c + \varepsilon \\ 0 & p \geq p_c + \varepsilon \end{cases} \tag{A9}$$

Imagine now that $R_2^s(p) < R_1^s(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (A8), the expression in equation (A7) is negative. Hence it cannot be that $R_2^s(p) < R_1^s(p)$ for $p \in [p_c, p_c + \varepsilon]$. This proves the necessity of the condition. ■

Proof of Theorem 6. First note that for $A(h) \in \Omega_\rho^s$, equation (4) can be rewritten as

$$A(h) = \int_0^1 (1 - \nu(p))h(p)\mathrm{d}p \tag{A10}$$

$$A(h) = \mu_h - \int_0^1 \nu(p)h(p)\mathrm{d}p \tag{A11}$$

$$A(h) = \mu_h - \int_0^{0.5} \nu(p)h(p)\mathrm{d}p - \int_{0.5}^1 \nu(p)h(p)\mathrm{d}p \tag{A12}$$

Since $r(p) = h(1-p) - h(p)$ and $\nu(p) = -\nu(1-p)$, we can rewrite equation (A12) as

$$A(h) = \mu_h + \int_0^{0.5} \nu(p)r(p)\mathrm{d}p \tag{A13}$$

Integrating by parts equation (A13), we get

$$A(h) = \mu_h + \nu(p)GR^2(p)\Big|_0^{0.5} - \int_0^{0.5} \nu^{(1)}(p)GR^2(p)\mathrm{d}p. \tag{A14}$$

Since by definition $GR^2(0) = 0$ and $\nu(0.5) = 0$ for all indices $A(h) \in \Omega_\rho^s$, the second term on the right hand side of the equation is nil. This yields to

$$A(h) = \mu_h - \int_0^{0.5} \nu^{(1)}(p) GR^2(p) \mathrm{d}p. \tag{A15}$$

Now assume that for $s-1$, we have

$$A(h) = \mu_h + (-1)^{s-2} \int_0^{0.5} \nu^{(s-2)}(p) GR^{s-1}(p) \mathrm{d}p. \tag{A16}$$

Integrating by parts the second term of the r.h.s. of equation (A16) yields

$$A(h) = \mu_h + (-1)^{s-2} \left\{ \nu^{(s-2)}(p) GR^s(p) \Big|_0^{0.5} - \int_0^{0.5} \nu^{(s-1)} GR^s(p) \mathrm{d}p \right\}. \tag{A17}$$

Since by definition $GR^s(0) = 0$ and $\nu^{(s-2)}(0.5) = 0$ for all indices $A(h) \in \Omega_\rho$, the first term in the braces on the right hand side of the equation is nil. This yield

$$A(h) = \mu_h + (-1)^{s-1} \int_0^{0.5} \nu(s-1)(p) GR^s(p) \mathrm{d}p. \tag{A18}$$

Given that equations (A15) and (A18) both conform to the relation depicted in equation (A16), it follows that equation (A18) holds for all $s \in \{2, 3, \dots\}$. Let $\Delta A_{12} = A(h_2) - A(h_1)$. From equation (A18), we get

$$\Delta A_{12} = \mu_{h2} = \mu_{h1} + (-1)^{s-1} \int_0^{0.5} \nu^{(s-1)}(p) \left[GR_2^s(p) - GR_1^s(p) \right] \mathrm{d}p. \tag{A19}$$

Note that $(-1)^{s-1}\nu^{(s-1)}(p)$ is non positive. This implies that if $GR_2^s(p) \geq GR_1^s(p)$ for all $p \in [0, 0.5]$, then $(-1)^{s-1} \int_0^{0.5} \nu^{(s-1)}(p) \left[GR_2^s(p) - GR_1^s(p) \right] \mathrm{d}p \geq 0$. If in addition, $\mu_{h2} \leq \mu_{h1}$, then $\Delta A_{12} \leq 0$. This proves for sufficiency of the condition.

Having provided a sufficiency condition let us now prove for the necessity of the condition. In order to prove necessity, we need to consider three cases:

1. $\mu_{h1} < \mu_{h2}$ together with $GR_2^s(p) \geq GR_1^s(p)$ for all $p \in [0, 0.5]$

2. $GR_2^s(p) < GR_1^s(p)$ on some arbitrary small interval $[p_c, p_c + \varepsilon]$ together with $\mu_{h1} = \mu_{h2}$

3. $GR_2^s(p) < GR_1^s(p)$ on some arbitrary small interval $[p_c, p_c+\varepsilon]$ together with $\mu_{h1} > \mu_{h2}$

Case 1: Consider the set of indices $A(h) \in \Omega_\rho^s$ for which $\nu^{(s-2)}(p)$ is constant for all $p \in [0, 0.5]$. This weight function $\nu(p)$ satisfies the conditions in the definition of Ω_ρ^s. Since $\nu^{(s-1)}(p) = 0$ for all $p \in [0, 0.5]$, $(-1)^{s-1} \int_0^{0.5} \nu^{(s-1)}(p) \left[GR_2^s(p) - GR_1^s(p)\right] \mathrm{d}p = 0$. From equation (A19) this implies that $\Delta A_{12} > 0$. Hence it cannot be that $\mu_{h1} < \mu_{h2}$.

Case 2: Consider the set of indices $A(h) \in \Omega_\rho^s$ for which $\nu^{(s-2)}(p)$ takes the following form:

$$\nu^{(s-2)}(p) = \begin{cases} (-1)^{s-1}\varepsilon & 0 \le p_c \\ (-1)^{s-1}\left[p_c + \varepsilon - p\right] & p_c \le p \le p_c + \varepsilon \\ 0 & p \ge p_c + \varepsilon \end{cases} \tag{A20}$$

where $p_c \in [0, 0.5]$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of Ω_ρ^s. Differentiating equation (A20) yields

$$\nu^{(s-1)}(p) = \begin{cases} 0 & 0 \le p_c \\ (-1)^s & p_c \le p \le p_c + \varepsilon \\ 0 & p \ge p_c + \varepsilon \end{cases} \tag{A21}$$

Imagine now that $GR_2^s(p) < GR_1^s(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (A20), the expression in equation (A19) is negative. Hence it cannot be that $GR_2^s(p) < GR_1^s(p)$ for $p \in [p_c, p_c+\varepsilon]$ if $\mu_{h1} = \mu_{h2}$.

Case 3: Consider the set of indices $A(h) \in \Omega_\rho^s$ for which $\nu^{(s-2)}(p)$ takes the following form:

$$\nu^{(s-2)}(p) = \begin{cases} (-1)^{s-1}\kappa & 0 \le p_c \\ (-1)^{s-1}\kappa\left[p_c + \varepsilon - p\right] & p_c \le p \le p_c + \varepsilon \\ 0 & p \ge p_c + \varepsilon \end{cases} \tag{A22}$$

where $\kappa > \left(\frac{\mu_{h1} - \mu_{h2}}{\varepsilon}\right)$ and $p_c \in [0, 0.5]$. Since $\nu(p)$ is differentiable almost everywhere, it satisfies the conditions in the definition of Ω_ρ^s. Differentiating equation (A22) yields

$$\nu^{(s-1)}(p) = \begin{cases} 0 & 0 \le p_c \\ (-1)^s\kappa & p_c \le p \le p_c + \varepsilon \\ 0 & p \ge p_c + \varepsilon \end{cases} \tag{A23}$$

Imagine now that $GR_2^s(p) < GR_1^s(p)$ on an interval $[p_c, p_c + \varepsilon]$ for ε that can be arbitrarily close to 0. For any $\nu(p)$ obeying the relation in equation (A22), the expression in equation (A20) is negative. Hence it cannot be that $GR_2^s(p) < GR_1^s(p)$ for $p \in [p_c, p_c+\varepsilon]$ if $\mu_{h1} > \mu_{h2}$.

Cases 1 to 3 prove the necessity of the condition. ■

A3 Construction of $C^s(p)$ and $R^s(p)$ estimators

A3.1 Estimator for $C^s(p)$

As seen earlier, the health concentration curve $C(p)$ is defined as follows

$$C(p) = \frac{1}{\mu_h} \int_0^p h(u)\mathrm{d}u. \tag{A24}$$

It can be re-written as

$$C(p) = \frac{1}{\mu_h} \int_0^1 \mathbb{1}(u < p)h(u)\mathrm{d}u. \tag{A25}$$

Apply the transformation $y = F_Y^{-1}(u)$ (with jacobian term $f_Y(y)$)

$$C(p) = \frac{1}{\mu_h} \int_0^\infty \mathbb{1}(y < F_Y^{-1}(p))h(F_Y(y))f_Y(y)\mathrm{d}y \tag{A26}$$

Let $f_{H|Y}$ be the conditional density of H on Y insert the following definition of the conditional expectation

$$E[H|Y = y] = \int_0^\infty h f_{H|Y}(h|y)\mathrm{d}h \tag{A27}$$

in equation A26 and using the definition for the joint density $f_{H,Y} = f_{H|Y} f_Y$, we get

$$C(p) = \frac{1}{\mu_h} \int_0^\infty \int_0^\infty h\mathbb{1}(y < F_Y^{-1}(p))f_{H,Y}(h, y)\mathrm{d}h\mathrm{d}y, \tag{A28}$$

which gives the simple estimator for $C(p)$ from a sample (y_i, h_i) for $i = 1, \dots, n$:

$$\hat{C}(p) = \frac{1}{N\bar{h}} \sum_{i=1}^N h_i \mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p)). \tag{A29}$$

Here $\bar{h}$ is the sample average and $\hat{F}_Y^{-1}$ is a non-parametric estimator of the quantile function of Y based on the order statistics of (y_i).

Estimators for $C^s(p)$ could be recursively derived from that of $C(p)$ (derivation of this result is in section A3.3).

$$\hat{C}^s(p) = \frac{1}{N\bar{h}} \sum_{i=1}^N h_i \frac{(p - \hat{F}_Y(y_i))^{s-2}}{(s-1)!} \mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p)) \tag{A30}$$

■

A3.2 Estimator for $R^s(p)$

In a similar fashion we can construct an estimator for R. Let us first rewrite $R(p)$ in the same form as $C(p)$:

$$\mu_h R(p) = \int_0^p r(u)\mathrm{d}u \tag{A31}$$

Given that Define $r(u)$ can be written as $h(1-u) - h(u)$ for $u \in [0,1]$, we can re-write this relationship as follows:

$$\mu_h R(p) = \int_{1-p}^1 h(u)\mathrm{d}u - \int_0^p h(u)\mathrm{d}u, \tag{A32}$$

which can be re-written as follows:

$$\mu_h R(p) = \int_0^1 [\mathbb{1}(u > 1-p) - \mathbb{1}(u < p)]h(u)\mathrm{d}u \tag{A33}$$

If ones defines a new variable $t = 1-u$, then $u = \phi(t) = 1-t$. In this framework,

$$\begin{aligned}
\int_0^p h(1-u)\mathrm{d}u &= \int_1^{1-p} h\left(1-\phi(t)\right)\phi'(t)\mathrm{d}t && \text{(A34)}\\
&= \int_1^{1-p} h\left(t\right)(-1)\mathrm{d}t && \text{(A35)}\\
&= \int_{1-p}^1 h(t)\mathrm{d}t && \text{(A36)}
\end{aligned}$$

The above sequence you have written should be

$$\begin{aligned}
\mu_h R(p) &= \int_0^p r(u)\mathrm{d}u && \text{(A37)}\\
&= \int_0^p h(1-u)\mathrm{d}u - \int_0^p h(u)\mathrm{d}u && \text{(A38)}\\
&= -\int_1^{1-p} h(u)\mathrm{d}u - \int_0^p h(u)\mathrm{d}u && \text{(A39)}\\
&= \int_{1-p}^1 h(u)\mathrm{d}u - \int_0^p h(u)\mathrm{d}u && \text{(A40)}
\end{aligned}$$

Furthermore, we could deduce that

$$\mu_h R(p) = \int_0^1 [\mathbb{1}(u > 1-p) - \mathbb{1}(u < p)]h(u)\mathrm{d}u \tag{A41}$$

This expression, upon applying a transformation $y = F_Y^{-1}(u)$, expanding the formula for h, yields

$$R(p) \times \mu_h = \int_0^\infty \int_0^\infty h[\mathbb{1}(y > F_Y^{-1}(1-p))] f_{H,Y}(h,y) \mathrm{d}h \mathrm{d}y \tag{A42}$$

$$- \int_0^\infty \int_0^\infty h[\mathbb{1}(y < F_Y^{-1}(p))] f_{H,Y}(h,y) \mathrm{d}h \mathrm{d}y \tag{A43}$$

which yields the estimator of $R(p)$

$$\hat{R}(p) = \frac{1}{N\bar{h}} \left\{ \sum_{i=1}^N h_i [\mathbb{1}(y_i > \hat{F}_Y^{-1}(1-p))] - \sum_{i=1}^N h_i [\mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p))] \right\} \tag{A44}$$

As for C^s, it is possible to recursively compute estimators of R^s by first plugging the estimators of $\hat{R}$ and then by recursively computing (derivation of this result is in section A3.3)

$$\hat{R}^s(p) = \int_0^p \hat{R}^{s-1}(u) \mathrm{d}u, \tag{A45}$$

and

$$\widehat{R}^s(p) = \frac{1}{N\bar{h}} \sum_{i=1}^N h_i \frac{1}{(s-1)!} p^{s-2} (p + (s-1)[\hat{F}_Y(y_i) - 1]) [\mathbb{1}(y_i > \hat{F}_Y^{-1}(1-p))]$$

$$- \frac{1}{N\bar{h}} \sum_{i=1}^N h_i \frac{(p - \hat{F}_Y(y_i))^{s-2}}{(s-1)!} [\mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(p))] \tag{A46}$$

are the resulting estimators. ■

A3.3 Computation of integrals containing indicator variables involving inverse of $\hat{F}_Y$

Even though $\hat{F}_Y$ is a step function, the following standard result holds: $y_i \leqslant \hat{F}_Y^{-1}(p)$ if and only if $\hat{F}_Y(y_i) \leqslant p$. In what follows, We will check the formula for the estimator by induction.

First set $I_1(p) = \int_0^p \mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(u))\mathrm{d}u$ and compute

$$\begin{aligned} \int_0^p \mathbb{1}(y_i \leqslant \hat{F}_Y^{-1}(u))\mathrm{d}u &= \int_0^p \mathbb{1}(\hat{F}_Y(y_i) \leqslant u)\mathrm{d}u && \text{(A47)} \\ &= (p - \hat{F}_Y(y_i))\mathbb{1}(\hat{F}_Y(y_i) \leqslant p). && \text{(A48)} \end{aligned}$$

Then recursively compute

$$\begin{aligned} I_k(p) &= \int_0^p I_{k-1}(u)\mathrm{d}u && \text{(A49)} \\ &= \int_0^p \frac{(u - \hat{F}_Y(y_i))^{k-1}}{k!}\mathbb{1}(\hat{F}_Y(y_i) \leqslant u)\mathrm{d}u && \text{(A50)} \\ &= \frac{(p - \hat{F}_Y(y_i))^k}{(k+1)!}\mathbb{1}(\hat{F}_Y(y_i) \leqslant p). && \text{(A51)} \end{aligned}$$

By making the change of variable $s = k + 2$, the result follows. ■

In order to compute integrals containing indicator variables involving the (quantile) inverse of $\hat{F}_Y$, it is important to make a previous argument more explicit. In fact because $\hat{F}_Y$ is non-decreasing $\{y_i : \hat{F}_Y(y_i) \geqslant p\}$ is unbounded from above and because $\hat{F}_Y$ is right-continuous, $\{y_i : \hat{F}_Y(y_i) \geqslant p\}$ is closed to the left, thus it is closed at its infimum. However, by the definition of the quantile function,

$$\hat{F}_Y^{-1}(p) = \inf_{y_i}\{y_i : \hat{F}_Y(p) \geqslant p\}, \tag{A52}$$

we get the set equality

$$\{y_i : \hat{F}_Y(y_i) \geqslant p\} = [\hat{F}_Y^{-1}(p), \infty) \tag{A53}$$

This set inequality shows that $\hat{F}_Y^{-1}(p) \leqslant y_i$ if and only if $p \leqslant \hat{F}_Y(y_i)$. Taking complements of the set equality in $[0, \infty)$ yields the equality

$$\{y_i : \hat{F}_Y(y_i) < p\} = [0, \hat{F}_Y^{-1}(p))], \tag{A54}$$

which implies $y_i < \hat{F}_Y^{-1}(p)$ if and only if $\hat{F}_Y(y_i) < p$.This allows us to compute the following integral,

$$
\begin{aligned}
\int_0^p \mathbb{1}(y_i > \hat{F}_Y^{-1}(1-u))\mathrm{d}u &= \int_0^p \mathbb{1}(\hat{F}_Y(y_i) > (1-u))\mathrm{d}u && \text{(A55)}\\
&= \int_{1-p}^1 \mathbb{1}(\hat{F}_Y(y_i) > u)\mathrm{d}u && \text{(A56)}\\
&= \mathbb{1}(\hat{F}_Y(y_i) > (1-p))(\hat{F}_Y(y_i) - 1 + p). && \text{(A57)}
\end{aligned}
$$

From equation (A57), it is clear that integrating recursively, we should obtain at step k an integrand of the form

$$
\frac{1}{k!}p^{k-1}(p + k[\hat{F}_Y(y_i) - 1])\mathbb{1}(\hat{F}_Y(y_i) > (1-p)), \qquad \text{(A58)}
$$

resulting at step $k+1$ in an integrand of the form

$$
\frac{1}{(k+1)!}p^k(p + (k+1)[\hat{F}_Y(y_i) - 1])\mathbb{1}(\hat{F}_Y(y_i) > (1-p)). \qquad \text{(A59)}
$$

We could verify that by induction. Since we checked for $k = 1$, what remains to do is to check for an arbitrary k and see if we get the correct form for $k+1$.

Set $J_1(p) = \mathbb{1}(y_i > \hat{F}_Y^{-1}(1-u))\mathrm{d}u$ and recursively compute

$$
\begin{aligned}
J_k(p) &= \int_0^p J_{k-1}(u)\mathrm{d}u && \text{(A60)}\\
&= \int_0^p \mathbb{1}(\hat{F}_Y(y_i) > (1-u))\frac{1}{k!}u^{k-1}(u + k[\hat{F}_Y(y_i) - 1])\mathrm{d}u && \text{(A61)}\\
&= \mathbb{1}(\hat{F}_Y(y_i) > (1-p))\frac{1}{k!}\int_0^p u^{k-1}(u + k[\hat{F}_Y(y_i) - 1])\mathrm{d}u && \text{(A62)}\\
&= \mathbb{1}(\hat{F}_Y(y_i) > (1-p))\frac{1}{k!}\left[\frac{p^{k+1}}{k+1} + \frac{p^k}{k}k[\hat{F}_Y(y_i) - 1]\right] && \text{(A63)}\\
&= \mathbb{1}(\hat{F}_Y(y_i) > (1-p))\frac{p^k}{(k+1)!}[p + (k+1)[\hat{F}_Y(y_i) - 1]] && \text{(A64)}
\end{aligned}
$$

By making the change of variable $s = k + 2$, the result follows. ■

A4 Bootstrap procedure

As suggest by Linton et al. (2005) and Shechtman et al. (2008), we used a recentered bootstrap procedure. The bootstrap algorithm for B repetitions is constructed as follows:

1. Repeat for $b = 1, \ldots, B$

   - Draw a sample of size n_1 from $\mathcal{S}_1$. Compute the nonparametric estimator $\hat{L}_{1b}$.
   - Draw a sample of size n_2 from $\mathcal{S}_2$. Compute the nonparametric estimator $\hat{L}_{2b}$.
   - Compute $\hat{L}_{12b}(p) = \hat{L}_{1b}(p) - \hat{L}_{2b}(p)$.
   - Compute $\hat{\tau}_b = \sup_p \sqrt{\frac{n_1 n_2}{n_1+n_2}}[\hat{L}_{12b}(p) - \hat{L}_{12}(p)]$.

2. Using the sample $\hat{\tau}_1, \ldots, \hat{\tau}_B$, compute the bootstrap p-value

$$\frac{1}{B}\sum_{b=1}^{B} \mathbb{1}(\hat{\tau}_b > \hat{\tau}).$$