

The halo mass function from excursion set theory with a non-Gaussian trispectrum

Michele Maggiore^{1★} and Antonio Riotto^{2,3★}

¹*Département de Physique Théorique, Université de Genève, 24 quai Ansermet, CH-1211 Genève, Switzerland*

²*CERN, PH-TH Division, CH-1211 Genève 23, Switzerland*

³*INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padua, Italy*

Accepted 2010 February 16. Received 2010 February 16; in original form 2009 December 7

ABSTRACT

A sizeable level of non-Gaussianity in the primordial cosmological perturbations may be induced by a large trispectrum, i.e. by a large connected four-point correlation function. We compute the effect of a primordial non-Gaussian trispectrum on the halo mass function, within excursion set theory. We use the formalism that we have developed in a previous series of papers and which allows us to take into account the fact that, in the presence of non-Gaussianity, the stochastic evolution of the smoothed density field, as a function of the smoothing scale, is non-Markovian. In the large mass limit, the leading-order term that we find agrees with the leading-order term of the results found in the literature using a more heuristic Press–Schechter (PS)-type approach. Our approach however also allows us to evaluate consistently the subleading terms, which depend not only on the four-point cumulant but also on derivatives of the four-point correlator, and which cannot be obtained within non-Gaussian extensions of PS theory. We perform explicitly the computation up to next-to-leading order.

Key words: dark matter – large-scale structure of Universe.

1 INTRODUCTION

Over the last decade a great deal of evidence has been accumulated from the cosmic microwave background (CMB) anisotropy and large scale structure (LSS) spectra that the observed structures originated from seed fluctuations generated during a primordial stage of inflation. While standard single-field models of slow-roll inflation predict that these fluctuations are very close to Gaussian (see Acquaviva et al. 2003; Maldacena 2003), non-standard scenarios allow for a larger level of non-Gaussianity (NG; see Bartolo et al. 2004 and references therein). A signal is Gaussian if the information it carries is completely encoded in the two-point correlation function, all higher connected correlators being zero. Deviations from Gaussianity are therefore encoded e.g. in the connected three- and four-point correlation functions which are dubbed the bispectrum and the trispectrum, respectively. A phenomenological way of parametrizing the level of NG is to expand the fully non-linear primordial Bardeen gravitational potential Φ in powers of the linear gravitational potential Φ_L :

$$\Phi = \Phi_L + f_{\text{NL}} (\Phi_L^2 - \langle \Phi_L^2 \rangle) + g_{\text{NL}} \Phi_{\text{NL}}^3. \quad (1.1)$$

The dimensionless quantities f_{NL} and g_{NL} set the magnitude of the three- and four-point correlation functions, respectively (Bartolo et al. 2004). If the process generating the primordial NG is local in space, the parameter f_{NL} and g_{NL} in Fourier space are independent of the momenta entering the corresponding correlation functions; if instead the process which generates the primordial cosmological perturbations is non-local in space, like in models of inflation with non-canonical kinetic terms, f_{NL} and g_{NL} acquire a dependence on the momenta. It is clear that detecting a significant amount of NG and its shape either from the CMB or from the LSS offers the possibility of opening a window into the dynamics of the universe during the very first stages of its evolution. NGs are particularly relevant in the high-mass end of the power spectrum of perturbations, i.e. on the scale of galaxy clusters, since the effect of NG fluctuations becomes especially visible on the tail of the probability distribution. As a result, both the abundance and the clustering properties of very massive haloes are sensitive probes of primordial NGs (Lucchin, Matarrese & Vittorio 1988; Grinstein & Wise 1986; Matarrese, Lucchin & Bonometto 1986; Moscardini et al. 1991; Koyama, Soda & Taruya 1999; Matarrese, Verde & Jimenez 2000; Robinson & Baker 2000; Robinson, Gawiser & Silk 2000), and could be detected or significantly constrained by the various planned large-scale galaxy surveys, both ground based (DES, PanSTARRS and LSST) and in

*E-mail: michele.maggiore@unige.ch (MM); antonio.riotto@cern.ch (AR)

space (such as EUCLID and ADEPT), see e.g. Dalal et al. (2008) and Carbone, Verde & Matarrese (2008). Furthermore, the primordial NG alters the clustering of dark matter (DM) haloes inducing a scale-dependent bias on large scales (Afshordi & Tolley 2008; Dalal et al. 2008; Matarrese & Verde 2008; Slosar et al. 2008) while even for small primordial NG the evolution of perturbations on super-Hubble scales yields extra contributions on smaller scales (Bartolo, Matarrese & Riotto 2005b; Matarrese & Verde 2009). The strongest current limits on the strength of local NG set the f_{NL} parameter to be in the range $-4 < f_{\text{NL}} < 80$ at 95 per cent confidence level (Smith, Senatore & Zaldarriaga 2009).

While the literature on NG has vastly focussed on the corrections to the observables induced by a non-vanishing bispectrum, only recently attention has been devoted to those corrections induced by a non-vanishing trispectrum of cosmological perturbations (Okamoto & Hu 2002; Bartolo, Matarrese & Riotto 2005a; Kogo & Komatsu 2006; D’Amico et al. 2008; Munshi et al. 2009). This has been computed in several models, such as multifield slow-roll inflation model (Byrnes, Sasaki & Wands 2006; Seery & Lidsey 2007; Seery, Lidsey & Sloth 2007; Seery, Sloth & Vernizzi 2009), the curvaton model (Sasaki et al. 2006), theories with non-canonical kinetic terms both in single field (Chen, Huang & Shiu 2006; Arroya & Koyama 2008; Arroya et al. 2009; Chen et al. 2009) and in the multifield case (Gao & Hu 2009) and in the case in which the cosmological perturbations are induced by vector perturbations of some non-abelian gauge field (Dimastrogiovanni et al. 2009). While the most natural value of the g_{NL} parameter is $\mathcal{O}(f_{\text{NL}}^2)$, there are cases in which $|g_{\text{NL}}| \gg 1$ even if f_{NL} is tiny (Enqvist & Takahashi 2008). The effects of a cubic correction to the primordial gravitational potential on to the mass function and bias of DM haloes have been recently analysed in Desjacques & Seljak (2010) where the theoretical predictions have been compared to the results extracted from a series of large N -body simulations. The limit $-3.5 \times 10^5 < g_{\text{NL}} < +8.2 \times 10^5$ has been obtained at 95 per cent confidence level in the case in which the NG is of the local type.

The goal of this paper is to present the computation of the DM halo mass function from the excursion set theory in the presence of a trispectrum, thus extending our previous computation of the DM halo mass function when the NG is induced by a bispectrum (Maggiore & Riotto 2009b). The halo mass function can be written as

$$\frac{dn(M)}{dM} = f(\sigma) \frac{\bar{\rho}}{M^2} \frac{d \ln \sigma^{-1}(M)}{d \ln M}, \quad (1.2)$$

where $n(M)$ is the number density of DM haloes of mass M , $\sigma(M)$ is the variance of the linear density field smoothed on a scale R corresponding to a mass M and $\bar{\rho}$ is the average density of the universe. Analytical derivations of the halo mass function are typically based on Press–Schechter (PS) theory (Press & Schechter 1974) and its extension (Peacock & Heavens 1990; Bond et al. 1991) known as excursion set theory (see Zentner 2007 for a recent review). In excursion set theory the density perturbation evolves stochastically with the smoothing scale, and the problem of computing the probability of halo formation is mapped into the so-called first-passage time problem in the presence of a barrier.

The computation of the effect of a primordial trispectrum on the mass function has been performed in Matarrese et al. (2000), LoVerde et al. (2008), Desjacques & Seljak (2010), and is based on NG extensions of PS theory (Press & Schechter 1974). Performing the computation using the two different approximations proposed in Matarrese et al. (2000) and LoVerde et al. (2008), respectively, to evaluate the impact of a non-vanishing bispectrum on the DM halo mass functions, one finds that, to leading order in the large mass limit, the predicted halo mass functions are the same for the two methods, but differ in the subleading terms, i.e. in the intermediate-mass regime. Apart from understanding what is the correct result for the subleading terms, there is yet a fundamental reason why we wish to apply the excursion set theory to compute the DM mass function with a non-vanishing trispectrum. The derivation of the PS mass function in Bond et al. (1991) requires that the density field δ evolves with the smoothing scale R (or more precisely with the variance $S(R)$ of the smoothed density field) in a Markovian way. Only under this assumption one can derive the correct factor of 2 that Press and Schechter were forced to introduce by hand. As we have discussed at length in Maggiore & Riotto (2010), this Markovian assumption is broken by the use of a filter function different from a sharp filter in momentum space and, of course, it is further violated by the inclusion of NG corrections. The non-Markovianity induced by the NG introduces memory effects which have to be appropriately accounted for in the excursion set. As we will see, the mass function indeed gets ‘memory’ corrections in the intermediate-mass regime, which depend on derivatives of the correlators, and therefore cannot be computed with the NG extensions of PS theory studied in Matarrese et al. (2000) and LoVerde et al. (2008).

This paper is organized as follows. In Section 2, we recall the basic points of the formalism developed in Maggiore & Riotto (2010) for Gaussian fluctuations, and extended in Maggiore & Riotto (2009b) to the NG case. In Section 3, we compute the NG corrections with the excursion set method induced by a large trispectrum, and we present our results for the halo mass function. Finally, in Section 4 we present our conclusions.

2 THE BASIC PRINCIPLES OF THE COMPUTATION

2.1 Excursion set theory

In excursion set theory one considers the density field δ smoothed over a radius R with a top-hat filter in coordinate space, and studies its stochastic evolution as a function of the smoothing scale R . As it was found in the classical paper (Bond et al. 1991), when the density $\delta(R)$ is smoothed with a sharp filter in momentum space, and the density fluctuations have Gaussian statistics, the smoothed density field satisfies

the equation

$$\frac{\partial \delta(S)}{\partial S} = \eta(S), \quad (2.1)$$

where $S = \sigma^2(R)$ is the variance of the linear density field smoothed on the scale R and computed with a sharp filter in momentum space, while $\eta(S)$ is a stochastic variable that satisfies

$$\langle \eta(S_1)\eta(S_2) \rangle = \delta_D(S_1 - S_2), \quad (2.2)$$

where δ_D denotes the Dirac delta function. Equations (2.1) and (2.2) are the same as a Langevin equation with a Dirac delta noise $\eta(S)$, with the variance S formally playing the role of time. Let us denote by $\Pi(\delta, S) d\delta$ the probability density that the variable $\delta(S)$ reaches a value between δ and $\delta + d\delta$ by time S . A textbook result in statistical physics is that, if a variable $\delta(S)$ satisfies a Langevin equation with a Dirac delta noise, the probability density $\Pi(\delta, S)$ satisfies the Fokker–Planck (FP) equation

$$\frac{\partial \Pi}{\partial S} = \frac{1}{2} \frac{\partial^2 \Pi}{\partial \delta^2}. \quad (2.3)$$

The solution of this equation over the whole real axis $-\infty < \delta < \infty$, with the boundary condition that it vanishes at $\delta = \pm\infty$, is

$$\Pi^0(\delta, S) = \frac{1}{\sqrt{2\pi S}} e^{-\delta^2/(2S)}, \quad (2.4)$$

and is nothing but the distribution function of PS theory. Since large R , i.e. large halo masses, correspond to small values of the variance S , in Bond et al. (1991) it was realized that we are actually interested in the stochastic evolution of δ against S only until the ‘trajectory’ crosses for the first time the threshold δ_c for collapse. All the subsequent stochastic evolution of δ as a function of S , which in general results in trajectories going multiple times above and below the threshold, is irrelevant, since it corresponds to smaller-scale structures that will be erased and engulfed by the collapse and virialization of the halo corresponding to the largest value of R , i.e. the smallest value of S , for which the threshold has been crossed. In other words, trajectories should be eliminated from further consideration once they have reached the threshold for the first time. In Bond et al. (1991) this is implemented by imposing the boundary condition

$$\Pi(\delta, S)|_{\delta=\delta_c} = 0. \quad (2.5)$$

The solution of the FP equation with this boundary condition is

$$\Pi(\delta, S) = \frac{1}{\sqrt{2\pi S}} \left[e^{-\delta^2/(2S)} - e^{-(2\delta_c - \delta)^2/(2S)} \right], \quad (2.6)$$

and gives the distribution function of excursion set theory. The first term is the PS result, while the second term in equation (2.6) is an ‘image’ Gaussian centred in $\delta = 2\delta_c$. Integrating this $\Pi(\delta, S)$ over $d\delta$ from $-\infty$ to δ_c gives the probability that a trajectory, at ‘time’ S , has always been below the threshold. Increasing S this integral decreases because more and more trajectories cross the threshold for the first time, so the probability of first crossing the threshold between ‘time’ S and $S + dS$ is given by $\mathcal{F}(S) dS$, with

$$\mathcal{F}(S) = -\frac{\partial}{\partial S} \int_{-\infty}^{\delta_c} d\delta \Pi(\delta; S). \quad (2.7)$$

With standard manipulations (see e.g. Zentner 2007 or Maggiore & Riotto 2010) one then finds that the function $f(\sigma)$ which appears in equation (1.2) is given by

$$f(\sigma) = 2\sigma^2 \mathcal{F}(\sigma^2), \quad (2.8)$$

where we wrote $S = \sigma^2$. Using equation (2.6) one finds

$$f(\sigma) = \left(\frac{2}{\pi}\right)^{1/2} \frac{\delta_c}{\sigma} e^{-\delta_c^2/(2\sigma^2)}. \quad (2.9)$$

Observe that, when computing the first-crossing rate, the contribution of the Gaussian centred in $\delta = 0$ and of the image Gaussian in equation (2.6) add up, giving the factor of 2 that was missed in the original PS theory.

2.2 Refinements of excursion set theory

While excursion set theory is quite elegant, and gives a first analytic understanding of the halo mass function, it suffers of two important set of problems. First, it is based on the spherical (or ellipsoidal) collapse model, which is a significant oversimplification of the actual complex dynamics of halo formation. We have discussed these limitations in detail in Maggiore & Riotto (2009a), where we also proposed that some of the physical complications inherent to a realistic description of halo formation can be included in the excursion set theory framework, at least at an effective level, by taking into account that the critical value for collapse is not a fixed constant δ_c , as in the spherical collapse model, nor a fixed function of the variance σ of the smoothed density field, as in the ellipsoidal collapse model, but rather is itself a stochastic variable, whose scatter reflects a number of complicated aspects of the underlying dynamics. The simplest implementation of this idea consists in solving the first-passage time problem in the presence of a barrier that performs a random walk, with diffusion coefficient D_B , around an average value given by the constant barrier of the spherical collapse model (more generally, one should consider a barrier that fluctuates over an average value given by the ellipsoidal collapse model). In this simple case, we found in Maggiore & Riotto (2009a) that the exponential factor in the PS mass function changes from $\exp\{-\delta_c^2/2\sigma^2\}$ to $\exp\{-a\delta_c^2/2\sigma^2\}$, where $a = 1/(1 + D_B)$. In this approach all our ignorance

on the details of halo formation is buried into D_B . The numerical value of D_B , and therefore the corresponding value of a , depends among other things also on the details of the algorithm used for identifying haloes (e.g. on the link length in a friends-of-friends algorithm). Observe that the replacement of $\exp\{-\delta_c^2/2\sigma^2\}$ with $\exp\{-a\delta_c^2/2\sigma^2\}$ (with a taken however as a fitting parameter) is just the replacement that was made in Sheth, Mo & Tormen (2001) and Sheth & Tormen (1999), in order to fit the results of N -body simulations.

The second set of problems of excursion set theory is of a more technical nature, and is due to the fact that the Langevin equation with Dirac delta noise, which is at the basis of the whole construction of Bond et al. (1991), can only be derived if one works with a sharp filter in momentum space, and if the fluctuations are Gaussian. However, as it is well known (Bond et al. 1991), and as we have discussed at length in Maggiore & Riotto (2010), with such a filter it is not possible to associate a halo mass to the smoothing scale R . A unambiguous relation between M and R is rather obtained with a sharp filter in *coordinate* space, in which case one simply has $M = (4/3)\pi R^3 \rho$, where ρ is the density. When one uses a sharp filter in coordinate space, the evolution of the density with the smoothing scale however becomes non-Markovian, and therefore the problem becomes technically much more difficult. In particular, the distribution function $\Pi(\delta, S)$ no longer satisfies a local differential equation such as the FP equation. The issue is particularly relevant when one wants to include NGs in the formalism, since again the inclusion of NGs renders the dynamics non-Markovian.

In Maggiore & Riotto (2009b, 2010) we have developed a formalism that allows us to generalize excursion set theory to the case of a non-Markovian dynamics, either generated by the filter function or by primordial NGs. The basic idea is the following. Rather than trying to derive a simple, local, differential equation for $\Pi(\delta, S)$ [which, as we have shown in Maggiore & Riotto (2010), is impossible; in the non-Markovian case $\Pi(\delta, S)$ rather satisfies a very complicated equation which is non-local with respect to ‘time’ S], we construct the probability distribution $\Pi(\delta, S)$ directly by summing over all paths that never exceeded the threshold δ_c , i.e. by writing $\Pi(\delta, S)$ as a path integral with boundaries. To obtain such a representation, we consider an ensemble of trajectories all starting at $S_0 = 0$ from an initial position $\delta(0) = \delta_0$ and we follow them for a ‘time’ S . We discretize the interval $[0, S]$ in steps $\Delta S = \epsilon$, so $S_k = k\epsilon$ with $k = 1, \dots, n$ and $S_n \equiv S$. A trajectory is then defined by the collection of values $\{\delta_1, \dots, \delta_n\}$, such that $\delta(S_k) = \delta_k$. The probability density in the space of trajectories is

$$W(\delta_0; \delta_1, \dots, \delta_n; S_n) \equiv \langle \delta_D(\delta(S_1) - \delta_1) \cdots \delta_D(\delta(S_n) - \delta_n) \rangle, \quad (2.10)$$

where δ_D denotes the Dirac delta. Then the probability of arriving in δ_n in a ‘time’ S_n , starting from an initial value δ_0 , without ever going above the threshold, is¹

$$\Pi_\epsilon(\delta_0; \delta_n; S_n) \equiv \int_{-\infty}^{\delta_c} d\delta_1 \cdots \int_{-\infty}^{\delta_c} d\delta_{n-1} W(\delta_0; \delta_1, \dots, \delta_{n-1}, \delta_n; S_n). \quad (2.11)$$

The label ϵ in Π_ϵ reminds us that this quantity is defined with a finite spacing ϵ , and we are finally interested in the continuum limit $\epsilon \rightarrow 0$. As we discussed in Maggiore & Riotto (2009b, 2010), $W(\delta_0; \delta_1, \dots, \delta_{n-1}, \delta_n; S_n)$ can be expressed in terms of the connected correlators of the theory:

$$W(\delta_0; \delta_1, \dots, \delta_n; S_n) = \int \mathcal{D}\lambda \times \exp \left\{ i \sum_{i=1}^n \lambda_i \delta_i + \sum_{p=2}^{\infty} \frac{(-i)^p}{p!} \sum_{i_1=1}^n \cdots \sum_{i_p=1}^n \lambda_{i_1} \cdots \lambda_{i_p} \langle \delta_{i_1} \cdots \delta_{i_p} \rangle_c \right\}, \quad (2.12)$$

where

$$\int \mathcal{D}\lambda \equiv \int_{-\infty}^{\infty} \frac{d\lambda_1}{2\pi} \cdots \frac{d\lambda_n}{2\pi}, \quad (2.13)$$

$\delta_i = \delta(S_i)$, and $\langle \delta_1 \cdots \delta_n \rangle_c$ denotes the connected n -point correlator. So

$$\Pi_\epsilon(\delta_0; \delta_n; S_n) = \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \int \mathcal{D}\lambda \times \exp \left\{ i \sum_{i=1}^n \lambda_i \delta_i + \sum_{p=2}^{\infty} \frac{(-i)^p}{p!} \sum_{i_1=1}^n \cdots \sum_{i_p=1}^n \lambda_{i_1} \cdots \lambda_{i_p} \langle \delta_{i_1} \cdots \delta_{i_p} \rangle_c \right\}. \quad (2.14)$$

When $\delta(S)$ satisfies equations (2.1) and (2.2) (which is the case for sharp filter in momentum space) the two-point function can be easily computed, and is given by

$$\langle \delta(S_i) \delta(S_j) \rangle = \min(S_i, S_j). \quad (2.15)$$

If furthermore we consider Gaussian fluctuations, all n -point connected correlators with $n \geq 3$ vanish, and the probability density W can be computed explicitly:

$$W^{\text{gm}}(\delta_0; \delta_1, \dots, \delta_n; S_n) = \frac{1}{(2\pi\epsilon)^{n/2}} \exp \left\{ -\frac{1}{2\epsilon} \sum_{i=0}^{n-1} (\delta_{i+1} - \delta_i)^2 \right\}, \quad (2.16)$$

where the superscript ‘gm’ (Gaussian–Markovian) reminds that this value of W is computed for Gaussian fluctuations, whose dynamics with respect to the smoothing scale is Markovian. Using this result, in Maggiore & Riotto (2010) we have shown that, in the continuum limit,

¹In equations (2.4) and (2.6) we implicitly assumed $\delta_0 = 0$. In the following however it will be necessary to keep track also of the initial position δ_0 .

the distribution function $\Pi_{\epsilon=0}(\delta; S)$, computed with a sharp filter in momentum space, satisfies a Fokker–Planck equation with the boundary condition $\Pi_{\epsilon=0}(\delta_c, S) = 0$, and we have therefore recovered, from our path integral approach, the distribution function of excursion set theory, equation (2.6).

When we consider a different filter, equation (2.15) is replaced by

$$\langle \delta(S_i) \delta(S_j) \rangle = \min(S_i, S_j) + \Delta(S_i, S_j), \quad (2.17)$$

where $\Delta(S_i, S_j)$ describes the deviations from a Markovian dynamics. For instance, for a sharp filter in coordinate space, which is the most interesting case, the function $\Delta(S_i, S_j)$ is very well approximated by

$$\Delta(S_i, S_j) \simeq \kappa \frac{S_i(S_j - S_i)}{S_j} \quad (2.18)$$

(for $S_i \leq S_j$, and is symmetric under exchange of S_i and S_j), with $\kappa \simeq 0.45$. The non-Markovian corrections can then be computed expanding perturbatively in κ . The computation, which is quite non-trivial from a technical point of view, has been discussed in great detail in Maggiore & Riotto (2010). Let us summarize here the crucial points. First of all, expanding to first order in Δ_{ij} and using $\lambda_i e^{i \sum_k \lambda_k \delta_k} = -i \partial_i e^{i \sum_k \lambda_k \delta_k}$, where $\partial_i = \partial / \partial \delta_i$, the first-order correction to Π_ϵ is

$$\begin{aligned} \Pi_\epsilon^{\Delta 1}(\delta_0; \delta_n; S_n) &\equiv \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \frac{1}{2} \sum_{i,j=1}^n \Delta_{ij} \partial_i \partial_j \\ &\times \int \mathcal{D}\lambda \exp \left\{ i \sum_{i=1}^n \lambda_i \delta_i - \frac{1}{2} \sum_{i,j=1}^n \min(S_i, S_j) \lambda_i \lambda_j \right\} \\ &= \frac{1}{2} \sum_{i,j=1}^n \Delta_{ij} \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \partial_i \partial_j W^{\text{gm}}(\delta_0; \delta_1, \dots, \delta_n; S_n), \end{aligned} \quad (2.19)$$

where we use the notation $\Delta_{ij} = \Delta(S_i, S_j)$. One then observes that the derivatives ∂_i run over $i = 1, \dots, n$, while we integrate only over $d\delta_1 \cdots d\delta_{n-1}$. Therefore, derivatives ∂_i with $i = n$ can be simply carried outside the integrals. Derivatives ∂_i with $i = 1, \dots, n-1$ are dealt as follows. Consider for instance the terms with $i < n$ and $j = n$ (together with $j < n$ and $i = n$, which gives a factor of 2). These are given by

$$\sum_{i=1}^{n-1} \Delta_{in} \partial_n \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \partial_i W^{\text{gm}}(\delta_0; \delta_1, \dots, \delta_n; S_n). \quad (2.20)$$

To compute this expression we integrate ∂_i by parts:

$$\begin{aligned} &\int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \partial_i W^{\text{gm}}(\delta_0; \delta_1, \dots, \delta_n; S_n) \\ &= \int_{-\infty}^{\delta_c} d\delta_1 \cdots \widehat{d\delta_i} \cdots d\delta_{n-1} W(\delta_0; \delta_1, \dots, \delta_i = \delta_c, \dots, \delta_{n-1}, \delta_n; S_n), \end{aligned} \quad (2.21)$$

where the notation $\widehat{d\delta_i}$ means that we must omit $d\delta_i$ from the list of integration variables. We next observe that W^{gm} satisfies

$$\begin{aligned} &W^{\text{gm}}(\delta_0; \delta_1, \dots, \delta_i = \delta_c, \dots, \delta_n; S_n) \\ &= W^{\text{gm}}(\delta_0; \delta_1, \dots, \delta_{i-1}, \delta_c; S_i) W^{\text{gm}}(\delta_c; \delta_{i+1}, \dots, \delta_n; S_n - S_i), \end{aligned} \quad (2.22)$$

as can be verified directly from its explicit expression (2.16). Then

$$\begin{aligned} &\int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{i-1} \int_{-\infty}^{\delta_c} d\delta_{i+1} \cdots d\delta_{n-1} \\ &\times W^{\text{gm}}(\delta_0; \delta_1, \dots, \delta_{i-1}, \delta_c; S_i) W^{\text{gm}}(\delta_c; \delta_{i+1}, \dots, \delta_n; S_n - S_i) \\ &= \Pi_\epsilon^{\text{gm}}(\delta_0; \delta_c; S_i) \Pi_\epsilon^{\text{gm}}(\delta_c; \delta_n; S_n - S_i), \end{aligned} \quad (2.23)$$

and to compute the expression given in equation (2.20) we must compute

$$\sum_{i=1}^{n-1} \Delta_{in} \Pi_\epsilon^{\text{gm}}(\delta_0; \delta_c; S_i) \Pi_\epsilon^{\text{gm}}(\delta_c; \delta_n; S_n - S_i). \quad (2.24)$$

To proceed further, we need to know $\Pi_\epsilon^{\text{gm}}(\delta_0; \delta_c; S_i)$. By definition, for $\epsilon = 0$ this quantity vanishes, since its second argument is equal to the threshold value δ_c , compare with equation (2.5). However, in the continuum limit the sum over i becomes $1/\epsilon$ times an integral over an intermediate time variable S_i ,

$$\sum_{i=1}^{n-1} \rightarrow \frac{1}{\epsilon} \int_o^{S_n} dS_i, \quad (2.25)$$

so we need to know how $\Pi_\epsilon^{\text{gm}}(\delta_0; \delta_c; S_i)$ approaches zero when $\epsilon \rightarrow 0$. In Maggiore & Riotto (2010) we proved that it vanishes as $\sqrt{\epsilon}$, and that

$$\Pi_\epsilon^{\text{gm}}(\delta_0; \delta_c; S) = \sqrt{\epsilon} \frac{1}{\sqrt{\pi}} \frac{\delta_c - \delta_0}{S^{3/2}} e^{-(\delta_c - \delta_0)^2 / (2S)} + \mathcal{O}(\epsilon). \quad (2.26)$$

Similarly, for $\delta_n < \delta_c$,

$$\Pi_\epsilon^{\text{gm}}(\delta_c; \delta_n; S) = \sqrt{\epsilon} \frac{1}{\sqrt{\pi}} \frac{\delta_c - \delta_n}{S^{3/2}} e^{-(\delta_c - \delta_n)^2 / (2S)} + \mathcal{O}(\epsilon). \quad (2.27)$$

Therefore the two-factor $\sqrt{\epsilon}$ from equations (2.26) and (2.27) produce just an overall factor of ϵ that compensate the factor $1/\epsilon$ in equation (2.25), and we are left with a finite integral over dS_i . Terms with two or more derivative, e.g. $\partial_i \partial_j$, or $\partial_i, \partial_j \partial_k$ acting on W , with all indices i, j, k smaller than n , can be computed similarly, and have been discussed in detail in Maggiore & Riotto (2010).

3 CONTRIBUTION OF THE TRISPECTRUM TO THE HALO MASS FUNCTION

The effect of NGs can be computed similarly, expanding perturbatively equation (2.14) in terms of the higher order correlators. In Maggiore & Riotto (2009b) we have examined the three-point correlator, i.e. the bispectrum. Here we compute the effect of the trispectrum.

If in equation (2.14) we only retain the four-point correlator, and we use the top-hat filter in coordinate space, we have

$$\begin{aligned} \Pi_\epsilon(\delta_0; \delta_n; S_n) &= \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \int \mathcal{D}\lambda \\ &\times \exp \left\{ i\lambda_i \delta_i - \frac{1}{2} [\min(S_i, S_j) + \Delta_{ij}] \lambda_i \lambda_j + \frac{(-i)^4}{24} \langle \delta_i \delta_j \delta_k \delta_l \rangle \lambda_i \lambda_j \lambda_k \lambda_l \right\}. \end{aligned} \quad (3.1)$$

Expanding to first order, Δ_{ij} and $\langle \delta_i \delta_j \delta_k \delta_l \rangle$ do not mix, so we must compute

$$\Pi_\epsilon^{(4)}(\delta_0; \delta_n; S_n) \equiv \frac{1}{24} \sum_{i,j,k,l=1}^n \langle \delta_i \delta_j \delta_k \delta_l \rangle \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \partial_i \partial_j \partial_k \partial_l W^{\text{gm}}, \quad (3.2)$$

where the superscript (4) in $\Pi_\epsilon^{(4)}$ refers to the fact that this is the contribution linear in the four-point correlator.

In principle this expression can be computed with the same technique discussed above, separating the various contributions to the sum according to whether an index is equal or smaller than n . In this way, however, the computations faces some technical difficulties.² Fortunately, the problem simplifies considerably in the limit of large halo masses, which is just the physically interesting limit. Large masses mean small values of S_n . In equation (3.2) the arguments S_i, S_j, S_k and S_l in the correlator $\langle \delta_i \delta_j \delta_k \delta_l \rangle \equiv \langle \delta(S_i) \delta(S_j) \delta(S_k) \delta(S_l) \rangle$ range over the interval $[0, S_n]$ and, if S_n goes to zero, we can expand the correlator in a multiple Taylor series around the point $S_i = S_j = S_k = S_l = S_n$. We introduce the notation

$$G_4^{(p,q,r,s)}(S_n) \equiv \left[\frac{d^p}{dS_i^p} \frac{d^q}{dS_j^q} \frac{d^r}{dS_k^r} \frac{d^s}{dS_l^s} \langle \delta(S_i) \delta(S_j) \delta(S_k) \delta(S_l) \rangle \right]_{S_i=S_j=S_k=S_l=S_n}. \quad (3.3)$$

Then

$$\begin{aligned} \langle \delta(S_i) \delta(S_j) \delta(S_k) \delta(S_l) \rangle &= \sum_{p,q,r,s=0}^{\infty} \frac{(-1)^{p+q+r+s}}{p!q!r!s!} \\ &\times (S_n - S_i)^p (S_n - S_j)^q (S_n - S_k)^r (S_n - S_l)^s G_4^{(p,q,r,s)}(S_n). \end{aligned} \quad (3.4)$$

Terms with more and more derivatives give contributions to the function $f(\sigma)$, defined in equation (1.2), that are subleading in the limit of small σ , i.e. for $\sigma/\delta_c \ll 1$, and the leading contribution to the halo mass function is given by the term in equation (3.4) with $p = q = r = s = 0$. At next-to-leading order we must also include the contribution of the terms in equation (3.4) with $p + q + r + s = 1$, i.e. the four terms ($p = 1, q = 0, r = 0, s = 0$), ($p = 0, q = 1, r = 0, s = 0$), ($p = 0, q = 0, r = 1, s = 0$) and ($p = 0, q = 0, r = 0, s = 1$); at next-to-next-to-leading order we must include the contribution of the terms in equation (3.4) with $p + q + r = 2$, and so on. For the purpose of organizing the expansion in leading term, subleading terms, etc., we can reasonably expect that, for small S_n

$$G_4^{(p,q,r,s)}(S_n) \sim S_n^{-(p+q+r+s)} \langle \delta^4(S_n) \rangle, \quad (3.5)$$

i.e. each derivative $\partial/\partial S_i$, when evaluated in $S_i = S_n$, gives a factor of order $1/S_n$. This ordering will be assumed when we present our final result for the halo mass function below. However, our formalism allows us to compute each contribution separately, so our results below can be easily generalized in order to cope with a different hierarchy between the various $G_4^{(p,q,r,s)}(S_n)$.

The leading term in $\Pi^{(4)}$ is

$$\Pi_\epsilon^{(4,L)}(\delta_0; \delta_n; S_n) = \frac{\langle \delta_n^4 \rangle}{24} \sum_{i,j,k,l=1}^n \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \partial_i \partial_j \partial_k \partial_l W^{\text{gm}}, \quad (3.6)$$

where the superscript ‘L’ stands for ‘leading’. Since in the end we are interested in the integral over $d\delta_n$ of $\Pi_\epsilon(\delta_0; \delta_n; S_n)$, see equation (2.7), we can write directly

$$\int_{-\infty}^{\delta_c} d\delta_n \Pi_{\epsilon=0}^{(4,L)}(0; \delta_n; S_n) = \frac{\langle \delta^4(S_n) \rangle}{24} \sum_{i,j,k,l=1}^n \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} \partial_i \partial_j \partial_k \partial_l W^{\text{gm}}. \quad (3.7)$$

²In particular, when we have terms with three or more derivatives, we need to generalize equation (2.27), including terms up to $O(\epsilon^{3/2})$, which is quite non-trivial.

This expression can be computed very easily by making use of identities that we proved in Maggiore & Riotto (2009b, 2010). Namely, we consider the derivative of Π_ϵ^{gm} with respect to the threshold δ_c [which, when we use the notation $\Pi_\epsilon^{\text{gm}}(\delta_0; \delta_n; S_n)$, is not written explicitly in the list of variable on which Π_ϵ^{gm} depends, but of course enters as upper integration limit in equation 2.11]. Then one can show that

$$\sum_{i=1}^n \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_n \partial_i W^{\text{gm}} = \frac{\partial}{\partial \delta_c} \int_{-\infty}^{\delta_c} d\delta_n \Pi_\epsilon^{\text{gm}}, \quad (3.8)$$

$$\sum_{i,j=1}^n \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_n \partial_i \partial_j W^{\text{gm}} = \frac{\partial^2}{\partial \delta_c^2} \int_{-\infty}^{\delta_c} d\delta_n \Pi_\epsilon^{\text{gm}} \quad (3.9)$$

and similarly for all higher order derivatives, so in particular

$$\sum_{i,j,k,l=1}^n \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_n \partial_i \partial_j \partial_k \partial_l W^{\text{gm}} = \frac{\partial^4}{\partial \delta_c^4} \int_{-\infty}^{\delta_c} d\delta_n \Pi_\epsilon^{\text{gm}}. \quad (3.10)$$

Therefore, in the continuum limit, the right-hand side of equation (3.7) is computed very simply, just by inserting in equation (3.10) the value of Π_ϵ^{gm} for $\epsilon \rightarrow 0$,

$$\Pi_{\epsilon=0}^{\text{gm}}(\delta_0 = 0; \delta_n; S_n) = \frac{1}{\sqrt{2\pi S_n}} \left[e^{-\delta_n^2/(2S_n)} - e^{-(2\delta_c - \delta_n)^2/(2S_n)} \right], \quad (3.11)$$

and therefore, in the continuum limit,

$$\int_{-\infty}^{\delta_c} d\delta_n \Pi_{\epsilon=0}^{(4,L)}(0; \delta_n; S_n) = \frac{\langle \delta_n^4 \rangle}{12\sqrt{2\pi} S_n^{3/2}} \delta_c \left(3 - \frac{\delta_c^2}{S_n} \right) e^{-\delta_c^2/(2S_n)}. \quad (3.12)$$

We now insert this result into equations (2.7) and (2.8) and we express the result in terms of the normalized kurtosis:

$$S_4(\sigma) \equiv \frac{1}{S^3} \langle \delta^4(S) \rangle. \quad (3.13)$$

Putting the contribution of $\Pi^{(4,L)}$ together with the Gaussian contribution, and writing $S = \sigma^2$, we find

$$f(\sigma) = \left(\frac{2}{\pi} \right)^{1/2} \frac{\delta_c}{\sigma} e^{-\delta_c^2/(2\sigma^2)} \left[1 + \frac{\sigma^2 S_4(\sigma)}{24} \left(\frac{\delta_c^4}{\sigma^4} - \frac{4\delta_c^2}{\sigma^2} - 3 \right) + \frac{1}{24} \sigma^2 \frac{dS_4}{d \ln \sigma} \left(\frac{\delta_c^2}{\sigma^2} - 3 \right) \right]. \quad (3.14)$$

Let us emphasize that the variance σ^2 is to be computed applying the linear transfer function to the primordial gravitational potential (1.1) containing the extra g_{NL} piece. The result given in equation (3.14) agrees with the one obtained in LoVerde et al. (2008) by performing the Edgeworth expansion of a non-Gaussian generalization of PS theory. However, just as we have discussed in Maggiore & Riotto (2009b) for the case of the bispectrum, equation (3.14) cannot be taken as the full result beyond leading order. If we want to compute consistently to NL order, we need to include the terms with $p + q + r + s = 1$ in equation (3.4), which is given by

$$\begin{aligned} \int_{-\infty}^{\delta_c} d\delta_n \Pi_\epsilon^{(4,\text{NL})}(\delta_0; \delta_n; S_n) &= -\frac{4}{24} G_4^{(1,0,0,0)}(S_n) \sum_{i=1}^n (S_n - S_i) \\ &\times \sum_{j,k,l=1}^n \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} d\delta_n \partial_j \partial_k \partial_l W^{\text{gm}}, \end{aligned} \quad (3.15)$$

where the superscript ‘NL’ in $\Pi_\epsilon^{(4,\text{NL})}$ stands for next-to-leading, and we used the fact that the four terms ($p = 1, q = 0, r = 0, s = 0$), \dots , ($p = 0, q = 0, r = 0, s = 1$) give the same contribution. We now use the same trick as before to eliminate $\sum_{j,k,l=1}^n \partial_j \partial_k \partial_l$ in favour of $\partial^3/\partial \delta_c^3$:

$$\int_{-\infty}^{\delta_c} d\delta_n \Pi_\epsilon^{(4,\text{NL})}(\delta_0; \delta_n; S_n) = -\frac{4}{24} G_4^{(1,0,0,0)}(S_n) \sum_{i=1}^n (S_n - S_i) \frac{\partial^3}{\partial \delta_c^3} \int_{-\infty}^{\delta_c} d\delta_1 \cdots d\delta_{n-1} d\delta_n \partial_i W^{\text{gm}}. \quad (3.16)$$

The remaining path integral can be computed using the technique discussed in equations (2.21) and (2.27), and we get

$$\begin{aligned} \int_{-\infty}^{\delta_c} d\delta_n \Pi_\epsilon^{(4,\text{NL})}(\delta_0; \delta_n; S_n) &= -\frac{4}{24\pi} G_4^{(1,0,0,0)}(S_n) \int_0^{S_n} dS_i \frac{1}{S_i^{3/2} (S_n - S_i)^{1/2}} \\ &\times \frac{\partial^3}{\partial \delta_c^3} \left[\delta_c e^{-\delta_c^2/(2S_i)} \int_{-\infty}^{\delta_c} d\delta_n (\delta_c - \delta_n) \exp \left\{ -\frac{(\delta_c - \delta_n)^2}{2(S_n - S_i)} \right\} \right]. \end{aligned} \quad (3.17)$$

Observe that this expression involves an integral over all values of an intermediate ‘time’ variable S_i which ranges over $0 \leq S_i \leq S_n$. As we have discussed in detail in Maggiore & Riotto (2009b, 2010) these terms are ‘memory’ terms that depend on the whole past history of the trajectory, and reflect the non-Markovian nature of the stochastic process.

The integral over $d\delta_n$ is easily performed writing

$$(\delta_c - \delta_n) \exp \left\{ -\frac{(\delta_c - \delta_n)^2}{2(S_n - S_i)} \right\} = (S_n - S_i) \partial_n \exp \left\{ -\frac{(\delta_c - \delta_n)^2}{2(S_n - S_i)} \right\}, \quad (3.18)$$

so it just gives $(S_n - S_i)$. Carrying out the third derivative with respect to δ_c and the remaining elementary integral over dS_i we get

$$\int_{-\infty}^{\delta_c} d\delta_n \Pi_\epsilon^{(4,\text{NL})}(\delta_0; \delta_n; S_n) = \frac{\delta_c}{3\sqrt{2\pi}} \frac{G_4^{(1,0,0,0)}(S_n)}{S_n^{3/2}} e^{-\delta_c^2/(2S_n)}. \quad (3.19)$$

We now define

$$\mathcal{U}_4(\sigma) \equiv \frac{4G_4^{(1,0,0,0)}(S)}{S^2}, \quad (3.20)$$

where as usual $S = \sigma^2$. When the ordering given in equation (3.5) holds, $\mathcal{U}_4(\sigma)$ is of the same order as the normalized kurtosis $\mathcal{S}_4(\sigma)$. Computing the contribution to $f(\sigma)$ from equation (3.19) and we finally find

$$f(\sigma) = \left(\frac{2}{\pi}\right)^{1/2} \frac{\delta_c}{\sigma} e^{-\delta_c^2/(2\sigma^2)} \left[1 + \frac{\sigma^2 \mathcal{S}_4(\sigma)}{24} \left(\frac{\delta_c^4}{\sigma^4} - \frac{4\delta_c^2}{\sigma^2} - 3 \right) + \frac{1}{24} \sigma^2 \frac{d\mathcal{S}_4}{d \ln \sigma} \left(\frac{\delta_c^2}{\sigma^2} - 3 \right) - \frac{\sigma^2 \mathcal{U}_4(\sigma)}{24} \left(\frac{\delta_c^2}{\sigma^2} + 1 \right) - \frac{1}{24} \sigma^2 \frac{d\mathcal{U}_4}{d \ln \sigma} \right]. \quad (3.21)$$

The above result only holds up to NL order. If one wants to use it up to NNL order, the terms in square bracket where $\sigma^2 \mathcal{S}_4(\sigma)$, $\sigma^2 d\mathcal{S}_4/d \ln \sigma$, $\sigma^2 \mathcal{U}_4(\sigma)$ and $\sigma^2 d\mathcal{U}_4/d \ln \sigma$ are multiplied by factors of $\mathcal{O}(1)$ must be supplemented by the computation of the terms with $p + q + r + s = 2$ in equation (3.4), which will give a contribution analogous to the term $\mathcal{V}_3(\sigma)$ computed in Maggiore & Riotto (2009b). However, with present numerical accuracy, NNL terms are not yet relevant for the comparison with N -body simulations with non-Gaussian initial conditions.

Our result may be refined in several ways. Until now we have worked with a barrier with a fixed height δ_c and we neglected the corrections due to the filter. We can now include the modifications due to the fact that the height of the barrier may be thought to diffuse stochastically, as discussed in Maggiore & Riotto (2009a), and also the corrections due to the filter. To compute the non-Gaussian term proportional to the four-point correlator with the diffusing barrier we recall, from Maggiore & Riotto (2009a), that the first-passage time problem of a particle obeying a diffusion equation with diffusion coefficient $D = 1$, in the presence of a barrier that moves stochastically with diffusion coefficient D_B , can be mapped into the first-passage time problem of a particle with effective diffusion coefficient $(1 + D_B)$, and fixed barrier. This can be reabsorbed into a rescaling of the ‘time’ variable $S \rightarrow (1 + D_B)S = S/a$, and therefore $\sigma \rightarrow \sigma/\sqrt{a}$. At the same time the four-point correlator must be rescaled according to $\langle \delta_n^4 \rangle \rightarrow a^{-2} \langle \delta_n^4 \rangle$ since, dimensionally, $\langle \delta_n^4 \rangle$ is the same as S^2 , which means that $\mathcal{S}_4 \rightarrow a\mathcal{S}_4$. As a final ingredient, we must add the effect of the top-hat filter function in coordinate space. For a top-hat filter in coordinate space, we have found (Maggiore & Riotto 2010) that the two-point correlator is given by equations (2.17) and (2.18). Including the non-Markovianity induced by the top-hat smoothing function in real space, using the computations already performed in Maggiore & Riotto (2009b, 2010), we end up with

$$f(\sigma) = (1 - \tilde{\kappa}) \left(\frac{2}{\pi}\right)^{1/2} \frac{a^{1/2} \delta_c}{\sigma} e^{-a\delta_c^2/(2\sigma^2)} \times \left[1 + \frac{\sigma^2 \mathcal{S}_4(\sigma)}{24} \left(\frac{a^2 \delta_c^4}{\sigma^4} - \frac{4a\delta_c^2}{\sigma^2} - 3 \right) + \frac{1}{24} \sigma^2 \frac{d\mathcal{S}_4}{d \ln \sigma} \left(\frac{a\delta_c^2}{\sigma^2} - 3 \right) - \frac{\sigma^2 \mathcal{U}_4(\sigma)}{24} \left(\frac{a\delta_c^2}{\sigma^2} + 1 \right) - \frac{1}{24} \sigma^2 \frac{d\mathcal{U}_4}{d \ln \sigma} \right] + \frac{\tilde{\kappa}}{\sqrt{2\pi}} \frac{a^{1/2} \delta_c}{\sigma} \Gamma \left(0, \frac{a\delta_c^2}{2\sigma^2} \right), \quad (3.22)$$

where $\tilde{\kappa} = \kappa/(1 + D_B)$. This is our final result. More generally, also the term proportional to the incomplete Gamma function could get non-Gaussian corrections, which in principle can be computed evaluating perturbatively a ‘mixed’ term proportional to $\Delta_{ij} \langle \delta_\kappa \delta_i \delta_m \delta_n \rangle \partial_i \partial_j \partial_k \partial_l \partial_m \partial_n$. However, we saw in Maggiore & Riotto (2010) that in the large mass limit, where the NGs are important, the term proportional to the incomplete Gamma function is subleading, so we will neglect the NG corrections to this subleading term.

4 CONCLUSIONS

In this paper we have computed the DM halo mass function as predicted within the excursion set theory when a NG is present under the form of a trispectrum. We thus have extended our previous results presented in Maggiore & Riotto (2009b), where a similar computation was performed in the presence of a NG bispectrum. Our computation accounts for the non-Markovianity of the random walk of the smoothed density contrast, which inevitably arise when deviations from Gaussianity are present. While our result coincides at the leading order $\mathcal{O}(\delta_c^4/\sigma^4)$ with that obtained in LoVerde et al. (2008) through PS theory, it is different at the order $\mathcal{O}(\delta_c^2/\sigma^2)$. This is due to the memory effects induced by the non-Markovian excursion set which are not present in the PS approach. Our final expression (3.22) takes into account as well the non-Markovian effects due to the choice of the top-hat filter in real space and the proper exponential decay at large DM masses.

ACKNOWLEDGMENTS

The work of MM is supported by the Fond National Suisse. The work of AR is supported by the European Community’s Research Training Networks under contract MRTN-CT-2006-035505.

REFERENCES

- Acquaviva V., Bartolo N., Matarrese S., Riotto A., 2003, Nuclear Phys. B, 667, 119
 Afshordi N., Tolley A., 2008, Phys. Rev. D, 78, 123507
 Arroya F., Koyama K., 2008, Phys. Rev. D, 77, 083517
 Arroya F., Mizuno S., Koyama K., Tanaka T., 2009, Phys. Rev. D, 80, 043527

- Bartolo N., Komatsu E., Matarrese S., Riotto A., 2004, *Phys. Rep.*, 402, 103
Bartolo N., Matarrese S., Riotto A., 2005a, *J. Cosmology Astropart. Phys.*, 0508, 010
Bartolo N., Matarrese S., Riotto A., 2005b, *J. Cosmology Astropart. Phys.*, 0510, 010
Bond J. R., Cole S., Efstathiou G., Kaiser N., 1991, *ApJ*, 379, 440
Byrnes C., Sasaki M., Wands D., 2006, *Phys. Rev. D*, 74, 123519
Carbone C., Verde L., Matarrese S., 2008, *ApJ*, 684, 1
Chen X., Huang M. X., Shiu G., 2006, *Phys. Rev. D*, 74, 121301
Chen X., Hu B., Huang M. X., Shiu G., 2009, *J. Cosmology Astropart. Phys.*, 0908, 008
Dalal N., Dore' O., Huterer D., Shirokov A., 2008, *Phys. Rev. D*, 77, 123514
D'Amico A., Bartolo N., Matarrese S., Riotto A., 2008, *J. Cosmology Astropart. Phys.*, 0801, 005
Desjacques V., Seljak U., 2010, *Phys. Rev. D*, 81, 023006
Dimastrogiovanni E., Bartolo N., Matarrese S., Riotto A., 2009, *J. Cosmology Astropart. Phys.*, 10, 015
Enqvist K., Takahashi T., 2008, *J. Cosmology Astropart. Phys.*, 9, 12
Gao X., Hu B., 2009, 0908, 012
Grinstein B., Wise M. B., 1986, *ApJ*, 310, 19
Kogo N., Komatsu E., 2006, *Phys. Rev. D*, 73, 083007
Koyama K., Soda J., Taruya A., 1999, *MNRAS*, 310, 1111
LoVerde M., Miller A., Shandera S., Verde L., 2008, *J. Cosmology Astropart. Phys.*, 0804, 014
Lucchin F., Matarrese S., Vittorio N., 1988, *ApJ*, 330, L21
Maggiore M., Riotto A., 2009a, preprint (arXiv:0903.1250)
Maggiore M., Riotto A., 2009b, preprint (arXiv:0903.1251)
Maggiore M., Riotto A., 2010, *ApJ*, 711, 907
Maldacena J., 2003, *J. High Energy Phys.*, 0305, 013
Matarrese S., Verde L., 2008, *ApJ*, 677, L77
Matarrese S., Verde L., 2009, *ApJ*, 706, L91
Matarrese S., Lucchin F., Bonometto S. A., 1986, *ApJ*, 310, L21
Matarrese S., Verde L., Jimenez R., 2000, *ApJ*, 541, 10
Moscardini L., Matarrese S., Lucchin F., Messina A., 1991, *MNRAS*, 248, 424
Munshi D., Heavens A., Cooray A., Smidt J., Coles P., Serra P., 2009, preprint (arXiv:0910.3693)
Okamoto T., Hu W., 2002, *Phys. Rev. D*, 66, 063008
Peacock J. A., Heavens A. F., 1990, *MNRAS*, 243, 133
Press W. H., Schechter P., 1974, *ApJ*, 187, 425
Robinson J., Baker J. E., 2000, *MNRAS*, 311, 781
Robinson J., Gawiser E., Silk J., 2000, *ApJ*, 532, 1
Sasaki M., Valiviita J., Wands D., Wands D., 2006, *Phys. Rev. D*, 74, 103003
Seery D., Lidsey J. E., 2007, *J. Cosmology Astropart. Phys.*, 0701, 008
Seery D., Lidsey J. E., Sloth M., 2007, *J. Cosmology Astropart. Phys.*, 0701, 027
Seery D., Sloth M., Vernizzi F., 2009, *J. Cosmology Astropart. Phys.*, 0903, 018
Sheth R. K., Tormen G., 1999, *MNRAS*, 308, 119
Sheth R. K., Mo H. J., Tormen G., 2001, *MNRAS*, 323, 1
Slosar A., Hirata C., Seljak U., Ho S., Padmanabhan N. E., 2008, *J. Cosmology Astropart. Phys.*, 08, 031
Smith K. M., Senatore L., Zaldarriaga M., 2009, *J. Cosmology Astropart. Phys.*, 006
Zentner A. R., 2007, *Int. J. Modern Phys. D*, 16, 763

This paper has been typeset from a $\text{\TeX}/\text{\LaTeX}$ file prepared by the author.