

Modelling supermassive black hole growth: towards an improved sub-grid prescription

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ABSTRACT

Accretion on to supermassive black holes (SMBHs) in galaxy formation simulations is frequently modelled by the Bondi–Hoyle formalism. Here we examine the validity of this approach analytically and numerically. We argue that the character of the flow where one evaluates the gas properties is unlikely to satisfy the simple Bondi–Hoyle model. Only in the specific case of hot virialized gas with zero angular momentum and negligible radiative cooling is the Bondi–Hoyle solution relevant. In the opposite extreme, where the gas is in a state of free-fall at the evaluation radius due to efficient cooling and the dominant gravity of the surrounding halo, the Bondi–Hoyle formalism can be erroneous by orders of magnitude in either direction. This may impose artificial trends with halo mass in cosmological simulations by being wrong by different factors for different halo masses. We propose an expression for the sub-grid accretion rate which interpolates between the free-fall regime and the Bondi–Hoyle regime, therefore taking account of the contribution of the halo to the gas dynamics.

Key words: accretion, accretion discs – methods: numerical – galaxies: evolution – galaxies: formation – galaxies: haloes – galaxies: nuclei.

1 INTRODUCTION

Over the last decade, compelling observational evidence has revealed that many galaxies in the local Universe harbour supermassive black holes (SMBHs) with masses $10^6 \lesssim M_{\text{bh}}/M_{\odot} \lesssim 10^9$ in their centres. During the same period, surveys of the distant Universe uncovered the existence of quasars at $z \sim 7$, when the Universe $\lesssim 1/10$ th of its current age; this implies that many SMBHs had already assembled their mass by this time (Mortlock et al. 2011).

Our understanding of the physics that dictates the growth of SMBHs is incomplete. Black holes grow by accreting low angular momentum material from their surroundings, yet the character of the accretion flow on to an SMBH is governed by physical processes as diverse as galaxy mergers (e.g. Hopkins & Quataert 2010), turbulence induced by stellar feedback (e.g. Hobbs et al. 2011) and black hole accretion-driven outflows (e.g. Nayakshin & Power 2010).

Black hole growth is now routinely modelled in galaxy formation simulations (for a fiducial work see Springel, Di Matteo & Hernquist 2005) and the importance of SMBHs in shaping the properties of galaxies is now well established (e.g. Bower et al. 2006; Croton et al. 2006). The majority of galaxy formation simulations published in the literature incorporate what we shall term the ‘Bondi–Hoyle model’ for black hole growth (see e.g. Springel et al. 2005; Sijacki et al. 2007; Pelupessy, Di Matteo & Ciardi 2007; Di Matteo et al.

2008; Johansson, Naab & Burkert 2009; Kim et al. 2011), which derives from the work of Bondi & Hoyle (1944) and Bondi (1952) – hereafter B&H. This model assumes the simplest possible accretion flow, where the gas is at rest at infinity and accretes steadily on to a black hole, subject only to the (Newtonian) gravity of the latter, which is modelled as a point mass.

Simulations that model the idealized physical problem as it is set out in B&H, or in idealized generalizations, produce results that are in good agreement with the analytical solution (Ruffert 1994; Barai, Proga & Nagamine 2011). In galaxy formation simulations, unfortunately, this idealized picture is far from satisfied, as the gas inflow is complicated considerably by the properties of the flow at larger scales. The most notable example of this is the presence of non-zero angular momentum that provides a natural barrier to eventual accretion by the SMBH. Gas settles into a disc whose dimensions are set by the angular momentum of the accretion flow, with only the very lowest angular momentum material able to accrete. A true estimate of the accretion rate on to the SMBH must therefore take account of this angular momentum, and indeed attempts to include it in an accretion sub-grid model have been made (Levine, Gnedin & Hamilton 2010; DeBuhr, Quataert & Ma 2011; Power, Nayakshin & King 2011) along with the presence of turbulence and/or vorticity in the gas (Krumholz, McKee & Klein 2005, 2006) finding large departures from the standard Bondi–Hoyle rate.

In this short paper we wish to make a simple and more fundamental point that in galaxy formation simulations even spherically symmetric accretion cannot be correctly modelled by the

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Bondi–Hoyle formalism, except in the most specific of cases. To make this point we suspend, for the moment, our disbelief that gaseous infall can proceed entirely radially from large scales and consider zero angular momentum accretion flows on to an SMBH embedded in the potential of a massive dark matter halo. Indeed, this is an example of a situation where one might expect the B&H formula to provide a reasonable estimate of the accretion rate.

The layout of this paper is as follows. In Section 2 we show analytically that in large-scale simulations of cosmological volumes the Bondi–Hoyle approach is invalid, and in Section 3 we present some numerical tests of this hypothesis. Finally in Sections 4 and 5 we discuss our conclusions.

2 ANALYTICAL ARGUMENTS

2.1 Classical Bondi–Hoyle accretion

We first recap the main assumptions underpinning the classical B&H papers. These are nicely summarized in the first sentence of the abstract of Bondi (1952): ‘the special accretion problem is investigated in which the motion is steady and spherically symmetrical, the gas *being at rest at infinity*’. We have italicised the part of the sentence that bears the most importance for us here.

Physically, gas can be at rest at infinity only when it is not subject to any forces. The only external force acting on the gas in the restricted B&H problem, i.e. the gravitational force, is due to the black hole. Self-gravity of the gas is neglected. The ‘infinity’ in question is a region at a distance large enough from the SMBH that the gravitational force exerted by the latter is negligible when compared to the pressure forces within the gas. This is quantified by defining the Bondi (or the ‘capture’) radius,

$$r_B = 2GM_{\text{BH}}/c_\infty^2, \quad (1)$$

where M_{BH} is the mass of the central object and c_∞ is the sound speed of the gas far from the hole. The Bondi radius divides the flow into two distinct regions (Frank, King & Raine 2002). Far from r_B , gas is hardly aware of the existence of the black hole, and the flow is very subsonic. The pressure and density of a subsonic flow are approximately constant, therefore we can set $\rho(r) \approx \rho_\infty$ at $r \gg r_B$.

Inside the capture radius, on the other hand, $\rho(r)$ begins to increase above the initial value, and the flow eventually reaches a sonic point where $|v_r| = c_\infty$, within which it plunges essentially at free-fall. The sonic point is found from $r_s = GM_{\text{BH}}/2c_s^2(r_s)$, where $c_s(r_s)$ is the sound speed at r_s . This local quantity¹ is related to the sound speed at infinity (Frank et al. 2002) via $c_s(r_s) = c_\infty(2/(5 - 3\Gamma))^{1/2}$ where Γ is the polytropic index of the gas that relates the gas pressure and density by $P = K\rho^\Gamma$, with K a positive constant.

Applying the Bondi–Hoyle formalism to black hole growth assumes that the accretion rate on to the SMBH is commensurate with the accretion rate through the Bondi radius (i.e. the flow is steady-state) and therefore is given by

$$\dot{M}_{\text{BH}} = \pi\lambda(\Gamma)r_B^2\rho_\infty c_\infty = \frac{4\pi\lambda(\Gamma)G^2M_{\text{BH}}^2\rho_\infty}{c_\infty^3} \quad (2)$$

where $\lambda(\Gamma)$ contains all the corrections arising due to the finite pressure gradient force in the problem. This function varies relatively weakly, i.e. between 1.12 for $\Gamma = 1$ and 0.25 for $\Gamma = 5/3$ (Bondi 1952). For the remainder of this paper our fiducial assumption is a soft equation of state, i.e. $\Gamma \approx 1$.

¹ Note that r_s here is the sonic radius, not the scale radius as it is commonly used in descriptions of dark matter halo profiles.

2.2 When is Bondi–Hoyle accretion applicable?

The Bondi–Hoyle formalism has been widely adopted as a ‘sub-grid’ prescription for the accretion rate on to the SMBH in large-scale cosmological simulations. The standard argument is that while one cannot usually resolve the scales of the Bondi radius, one can at least use the smallest resolved scales to approximately determine the value of the gas density and the sound speed ‘at infinity’ for use in the Bondi formula (see e.g. Booth & Schaye 2009). Indeed, the smallest resolvable scales are usually about a fraction of a kpc, whereas the Bondi radius is of the order of a few to a few tens of pc.

For the Bondi (1952) solution to be applicable even to spherical flow, we need to make sure that the ‘gas *being at rest at infinity*’ assumption is satisfied where the relevant gas properties (density and sound speed) are evaluated. In cosmological simulations we expect SMBHs to be immersed in stellar bulges and dark matter haloes that are typically $\sim 10^3$ to 10^4 times more massive than the SMBH (see e.g. Häring & Rix 2004; Guo et al. 2010). If gas in the halo (or bulge) is as hot as the halo virial temperature, it will be in hydrostatic balance (see e.g. Komatsu & Seljak 2001; Suto, Sasaki & Makino 1998). It seems that in this situation, which is common for low-luminosity SMBHs in giant elliptical galaxies where the gas is rather tenuous and hot since the cooling time is long (Churazov et al. 2005), the Bondi–Hoyle solution is potentially useful.

However, in the epoch when SMBHs grow rapidly, their hosts are very gas rich, and the inflow of gas from large scales cannot be easily captured by gradual cooling from a tenuous hot halo (see e.g. Birnboim & Dekel 2003; Kereš et al. 2005, 2009; Dekel et al. 2009; Kimm et al. 2011). Higher density gas is likely to cool much faster and hence is likely to be much cooler than the virial temperature. In this case, the gas is not able to support its own weight, and must collapse to the centre, where it feeds the SMBH and forms stars. Therefore, we expect a radial inflow of gas to the centre rather than a hydrostatic balance ‘far’ from the SMBH. We note that Ricotti (2007) has demonstrated how the Bondi–Hoyle formalism must be modified in the presence of an external potential for the specific case of the growth of primordial black holes in a dark matter halo with a power-law density profile.

In actuality, even the meaning of the Bondi radius becomes unclear in this situation, as gas is not virialized near the SMBH, and the halo potential plays an important role. In the ‘naked SMBH’ problem, r_B delineates the region inside which the potential energy of the hole starts to become greater than the internal energy of the gas. For an SMBH plus host halo system, one should introduce a modified Bondi radius,

$$\tilde{r}_B = 2GM_h/c_\infty^2, \quad (3)$$

that takes into account the total mass of the halo. In order for the gas to accrete efficiently on to a dark matter halo, its temperature must be, at most, comparable with the virial temperature at the outer edge of the halo (White & Frenk 1991). Thus, we set $c_\infty \lesssim GM_h/r_h$, where r_h is the halo virial radius, giving us a modified Bondi radius of $\tilde{r}_B \gtrsim 2r_h$. The gravitational potential energy starts to dominate the internal energy of the infalling gas before the latter has even reached the edge of the halo, and so the standard r_B is meaningless in this case.

Fig. 1 illustrates this point graphically by comparing the potential energy of gas as a function of radius for a variety of halo profiles with $c_{\text{vir}}^2/2$, assuming that the gas temperature is virial at r_h . These profiles are described in the Appendix. The cosmology we have assumed for the haloes is λ cold dark matter with $\Omega_m = 0.27$, $\Omega_\lambda =$

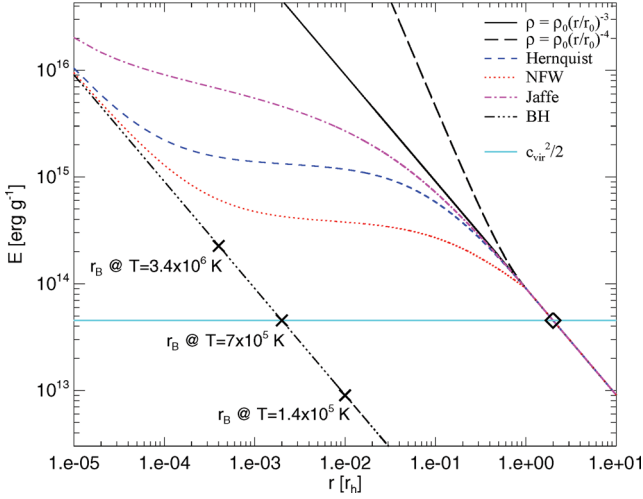


Figure 1. Plot of potential energies for a selection of dark matter halo profiles (some realistic, some instructive) modified by a central SMBH, compared to the thermal energy of the gas at the virial temperature at the halo virial radius. The halo profiles are normalized to contain a mass $M_h = 10^{11} M_\odot$ within the halo radius r_h , outside of which the potential is Keplerian by construction. The potential due to the central SMBH by itself is also plotted. The classical Bondi radius for three different values of temperature are indicated as crosses – the greater the separation between the cross denoting r_B and the curves of potential energy for a given halo the more inaccurate the standard Bondi–Hoyle approach. It is clear then that the latter becomes more accurate at higher T . The modified Bondi radius for the fiducial T_{vir} is denoted by a rhombus, demonstrating that at this temperature the potential energies are dominant for the entirety of the halo.

0.73 and a virial overdensity parameter of $\Delta = 200$ at a redshift of $z = 2$ (although we note that our conclusions are unchanged for a wide range in z , from the early Universe to the present day).

The $\propto 1/r$ potential due to the SMBH of mass $M_{\text{BH}} = 10^8 M_\odot$ is shown with the dash–triple-dot power law. The traditional Bondi (or capture) radius is at $r = 0.002$, where the SMBH potential crosses the $c_{\text{vir}}^2/2$ line. For all the haloes considered, the potential energy of the host makes a significant contribution at all radii except those within the very inner parts of the halo, exceeding the gas internal energy by a large factor everywhere outside the SMBH radius of influence.

Therefore, if we assume that the gas has a relatively soft equation of state and accretes spherically on to the halo at or below $T_{\text{virial}}(r_h)$, the character of the inflow becomes that of supersonic inside the halo *instead of being stationary at infinity*. In the Appendix we quantify this by calculating the sonic point for isothermal gas flows at or below the virial temperature and for all of the halo mass profiles considered. We show that for typical dark matter haloes the sonic point is reached while still at very large distances from the central black hole.

From Fig. 1 it can also be seen that for hotter gas, the classical r_B estimate becomes more accurate, as the potential energy of the (halo + SMBH) system starts to asymptote to the SMBH solution and no longer dominates over the value of $c_{\text{vir}}^2/2$. If the gas thermal energy is comparable to that of the potential energy we naturally expect near hydrostatic equilibrium to be maintained and in this case the Bondi–Hoyle formalism is applicable to spherical flow.

Of course, this is a simplified picture. In reality gas that has accreted on to a halo from outside the virial radius may shock at smaller radii, heating up to the local T_{vir} at that radius. In the

‘cold mode’ where the galaxy is assembled via cold streams that penetrate far inside the halo, such shock heating is likely to occur when the infalling gas reaches the radius of the galactic disc, at a small fraction of r_h (Kereš et al. 2005). However, due to the high densities reached for the shocked gas the cooling time is likely to be short (Birnboim & Dekel 2003; Kereš et al. 2005), and in particular in the presence of Compton cooling from a quasar radiation field it will be significantly less than the free-fall time at that radius (Nulsen & Fabian 2000). Gas may therefore be stationary temporarily at the shock radius but as it cools and begins to infall it will quickly tend to the free-fall velocity, and certainly by the time a radial inflow reaches the classical Bondi radius – values for which can be seen in Fig. 1 for different temperatures – the assumption of being at rest will no longer be satisfied.

Put simply, this is an energy argument. Regardless of where the gas might begin to infall from, the halo imposes a far stronger gravitational potential energy than the SMBH everywhere outside of the SMBH radius of influence (by definition). The potential energy due to a typical halo also increases to smaller radii from any point at which the gas is likely to have reached the virial temperature (see Fig. 1). As a result, it is clear that gravity will often dominate over thermal energy at scales significantly larger than the classical Bondi radius for the SMBH.

In the interest of completeness we now demonstrate numerically the form of the radial inflow in a realistic background potential within a galaxy. We choose a dynamic range that lies inside the sonic point (as would always be the case for efficiently cooled gas at these scales, as we have shown in the Appendix) in order to highlight why the standard Bondi estimate is inaccurate here.

3 NUMERICAL TESTS

To perform the simulations we employ the three-dimensional smoothed particle hydrodynamics (SPH)/ N -body code GADGET-3, an updated version of the code presented in Springel (2005). The gas is evolved in a static external potential that includes a point mass black hole at the centre. The computational domain extends from a kiloparsec down to an ‘accretion radius’ around the black hole at $r_{\text{acc}} = 1$ pc, and we remove the particles that come within this distance of the SMBH.

For the external potential in our model we use a Jaffe cusp as per equation (A3) but with a core at the centre of our computational domain to prevent divergence in the gravitational force. The radius of the core, r_c , corresponds to approximately the dynamical influence radius of the SMBH. With this (modified) potential the mass enclosed within radius r is given by

$$M(r) = M_{\text{bh}} + \begin{cases} M_c \left(\frac{r}{r_c}\right)^3, & r < r_c \\ M_c + aM \left(\frac{1}{r_c+a} - \frac{1}{r+a}\right), & r \geq r_c, \end{cases} \quad (4)$$

where $M_c = 2 \times 10^8 M_\odot$, $M = 10^{11} M_\odot$, $r_c = 20$ pc and $a = 10$ kpc. The mass of the SMBH is set to $M_{\text{bh}} = 10^8 M_\odot$.

For simplicity, the gas is kept isothermal throughout the entirety of the simulation and self-gravity is turned off.

3.1 Initial conditions

The starting condition for our simulations is that of a uniform density, spherically symmetric thick gaseous shell with mass $M_{\text{shell}} = 10^8 M_\odot$, which ranges from $r_{\text{in}} = 0.1$ kpc to $r_{\text{out}} = 1$ kpc and is centred on the black hole. The temperature of the gas is varied

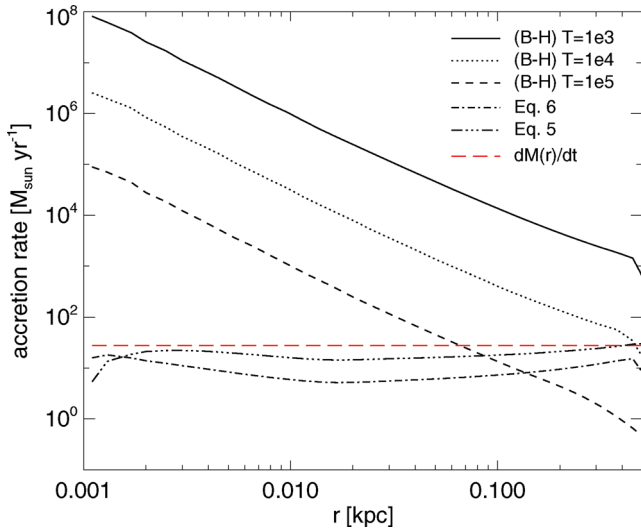


Figure 2. The mass flux (steady-state value) from the simulations through each radius (red), i.e. $\dot{M}(r)$, together with the Bondi–Hoyle estimate as per equation (2) and the free-fall estimate as per equations (5) and (6).

between tests, ranging from 10^3 to 10^5 K. To minimize initial inhomogeneities we cut the shell from a relaxed, glass-like configuration.

The gas infalls from rest within our static potential. After a time of the order of the dynamical time at the outer edge of the shell, a steady-state radial mass flux is reached for the majority of the gas and it is at this time that we make our comparisons between the various accretion rates in the next section.

3.2 Results

We define a radius-dependent ‘measured’ accretion rate as $\dot{M}(r) = 4\pi r^2 \rho v_r$, where v_r is the radial velocity of the gas. Since the system has been allowed to settle into an approximate steady-state, this function (shown as an average by the long dashed red line) is almost constant with radius, and is the same as the time-averaged accretion rate measured at the black hole.

The solid, the dotted and the dashed curves in Fig. 2 show the standard Bondi–Hoyle estimate for the accretion rate (equation 2) as a function of radius for three different values of gas temperature. Each of these curves uses a ‘local’ ρ , the density at each radius, in order to represent where the ρ_∞ might be evaluated in a large-scale simulation.

Clearly, as Fig. 2 shows, the Bondi–Hoyle estimate is very inaccurate for these isothermal simulations. At intermediate temperatures, e.g. 10^5 K, the formula in the inner parts results in a significant overestimate of the accretion rate and an underestimate at large radii.

3.2.1 Free-fall rate

A simple but physically well motivated alternative to the Bondi–Hoyle formula for efficiently cooled spherically symmetric flows is a free-fall rate estimate,

$$\dot{M}_{\text{ff}}(r) = \frac{M_{\text{gas,enc}}(r)}{t_{\text{ff}}(r)}, \quad (5)$$

where $M_{\text{gas,enc}}(r)$ is the enclosed gas mass within radius r , and $t_{\text{ff}} = (r^3/2GM(r))^{1/2}$ is the free-fall time. Alternatively, one may wish

to approximate the enclosed gas mass in the above equation as $(4\pi/3)r^3\rho_{\text{gas}}(r)$, so that

$$\dot{M}_{\text{ff}}(r) \sim \frac{4\pi r^3 \rho_{\text{gas}}(r)}{3t_{\text{ff}}(r)}. \quad (6)$$

Both of the above estimates provide a much better match to the accretion rate in our numerical tests than the Bondi–Hoyle approach, as shown in Fig. 2. It should be noted too that since the free-fall estimates have no dependence on c_s , the profiles are converged regardless of the temperature.

4 DISCUSSION

Based on our analytical arguments and numerical tests, we conclude that the Bondi (1952) formula, designed for accretion on to ‘naked black holes’, can only be applied to accretion on to astrophysical SMBHs, i.e. those embedded in massive dark matter haloes, if the gas in the latter is at or near hydrostatic equilibrium. It is only in this case that one of the key assumptions of Bondi (1952) – the ‘gas being at rest at infinity’ – is satisfied (infinity meaning outside the Bondi radius). If the gas cooling time is long then it is possible that this state may be reached, and indeed this is probably the case in giant gas-poor elliptical galaxies, where the gas is tenuous and hot (see e.g. Churazov et al. 2005).

However, in the most interesting phase of SMBH and galaxy buildup, when the halo is likely to be awash with gas to feed both the SMBH and star formation, densities are high and the cooling time is expected to be short, with gas cooling far below the virial temperature. Hydrostatic balance is then extremely unlikely for gas in the halo, and if it is not supported by either shock heating or angular momentum it will tend to the free-fall velocity. We have run a series of simple numerical tests to explore this limit, allowing a thick, spherical shell of gas to accrete on to an SMBH at the centre of a background halo potential. These tests showed that a free-fall accretion rate estimate is indeed much more accurate than the Bondi–Hoyle formalism in this case. What is most concerning is that the error of the latter strongly depends on gas temperature (and thus the cooling function) and cannot be ‘predicted’. The Bondi–Hoyle accretion formalism may thus be wrong by significant (and unknown) factors in either direction. The error may well depend on the galaxy mass and type systematically. Predictions based on the Bondi–Hoyle formalism alone are therefore unlikely to be robust.

We note too that this problem persists even for simulations that are able to resolve down to the classical Bondi radius of the SMBH. While the properties of the gas (density, sound speed) evaluated at this radius may be correct in the sense that they do not suffer from resolution problems, it is unlikely that this radius will be a true ‘infinity’ where the gas is in hydrostatic equilibrium – again, the reason for this is the influence of the more massive background halo potential on the dynamics of the gas.

For realistic flows of course the situation is more complex. Here we find that the best way to present our argument is in terms of ‘supporting’ mechanisms for the gas at the radius where one evaluates the gas properties in a simulation. To start with, we have the two extremes: (i) Bondi–Hoyle, where the gas is completely supported and stationary at the evaluation radius; and (ii) free-fall, where the gas is entirely radial and influenced only by gravity, i.e. has zero support. It is important to recognize that the reality will lie somewhere in-between. Specific cases that lie between these extremes are: (iii) the gas is unsupported at the evaluation radius but is shocked at smaller radii, after which it may begin to tend to free-fall once again if the cooling time is short, and (iv) the gas possesses

angular momentum, and is thus supported by rotation but inflows through viscous processes.

What we therefore desire is a formula for the sub-grid accretion rate that interpolates between the two extremes based on the relevant supporting mechanism. As we mentioned in the Introduction, angular momentum concerns are extremely important in determining the correct accretion rate; and indeed the picture can be further complicated by the effect of star formation, where it is not clear whether forming stars would deprive the SMBH of fuel (as may have been the case with Sgr A* – see e.g. Nayakshin & Cuadra 2005), or if feedback from star formation would actually enhance accretion through a broadening of the angular momentum distribution (Hobbs et al. 2011). Feedback from the AGN itself must also be a consideration, and in particular the interplay between SMBH feeding and AGN feedback, due to the fact that these processes connect the small to the large scales. Indeed, a number of authors have conducted detailed investigations into the multi-scale coupling between feeding and feedback (see e.g. Cattaneo & Teyssier 2007; Dubois et al. 2010; Kim et al. 2011), often finding that SMBH growth enters into a self-regulated state that oscillates between periods of high and low activity. We plan to investigate all of the processes relevant to SMBH growth and thereby to further develop the sub-grid model; for now, however, we focus on a direct interpolation between the Bondi–Hoyle regime and the free-fall regime as a starting point.

The formula we propose is a minor modification of the full Bondi–Hoyle–Lyttleton (Hoyle & Lyttleton 1939; Bondi 1952) expression for an accretor that is moving relative to the gas, namely

$$\dot{M}_{\text{BHL}} = \frac{4\pi\lambda(\Gamma)G^2M_{\text{BH}}^2\rho_{\infty}}{(c_{\infty}^2 + v_{\text{rel}}^2)^{3/2}} \quad (7)$$

where v_{rel} is the relative velocity between the SMBH and the gas, which in purely radial flow is zero. In order to interpolate between this and the free-fall rate (equation 5) we make two changes: (1) replace the relative velocity with the velocity dispersion for the external potential, $\sigma \sim (GM_{\text{enc}}(r)/r)^{1/2}$, and (2) replace the black hole mass with the enclosed mass of the external potential, $M_{\text{enc}}(r)$. The resulting expression,

$$\dot{M}_{\text{interp}} = \frac{4\pi\lambda(\Gamma)G^2M_{\text{enc}}^2\rho_{\infty}}{(c_{\infty}^2 + \sigma^2)^{3/2}}, \quad (8)$$

tends to the standard Bondi–Hoyle formula (equation 2) in the limit that $c_{\infty} \gg \sigma$, i.e. the gas internal energy dominates over the potential energy of the halo, and $M_{\text{enc}} \rightarrow M_{\text{BH}}$, i.e. as we approach the SMBH radius of influence. In the opposite limit, where the halo potential energy dominates, $\sigma \gg c_{\infty}$, we recover the free-fall estimate, equation (5). Finally, in order to incorporate (in a crude fashion) the effect of AGN feedback on the accretion rate should be capped at Eddington, namely

$$\dot{M}_{\text{acc}} = \max(\dot{M}_{\text{interp}}, \dot{M}_{\text{Edd}}) \quad (9)$$

as is commonly employed in the majority of the simulations of SMBH growth that we have mentioned so far.

We hope that with this interpolated expression the sub-grid SMBH accretion rate will take account of the presence of the halo and indeed any other external potential (e.g. stellar bulge, stellar halo, etc.) that contributes significantly to the dynamics of the infalling gas. We note that some simulations may already use a velocity dispersion as a cap on the v_{rel} parameter in order to avoid excessive relative velocities as a result of poor resolution (e.g. Dubois et al. 2010), in which case the change to the sub-grid expression is minimal.

This interpolation approach is particularly relevant to the picture of the ‘hot’ and ‘cold’ modes of accretion in galaxy formation, which emerged in the 1970s with the paradigm of hot halo gas that is at or near T_{vir} after being shock heated at the virial radius (see the original papers by e.g. Rees & Ostriker 1977; Silk 1977; White & Rees 1978), and has since been adjusted by results from simulations (although was suggested also in the 1970s based on analytical arguments – see Binney 1977) to include cold gas with a soft equation of state that penetrates down to small scales through streams and filaments (see e.g. Katz et al. 1994; Fardal et al. 2001; Kereš et al. 2005, 2009).

There is typically a halo mass scale which delineates the relative importance of each mode, although this varies somewhat in the literature, with Birnboim & Dekel (2003) finding $M_{\text{halo}} \sim 10^{11}$ as the result of 1D numerical calculations and Kereš et al. (2005) finding a factor of 2–3 higher from fully 3D N -body/hydrodynamical simulations. However, in a subsequent paper, Kereš et al. (2009) find that although this transition mass marks the point above which hot, virialized gas atmospheres develop in haloes, the actual contribution to the accretion rate to small scales is still dominated by the cold filamentary mode even at higher mass, with hot mode accretion only starting to become important at late times ($z < 1$). In addition, there is (indirect) observational evidence for cold accretion flows in galaxies through the H I column density distribution (van de Voort et al. 2011). This suggests that utilizing purely a ‘gas being fully supported’ Bondi–Hoyle accretion prescription is unlikely to capture enough of the relevant accretion behaviour in galaxy formation. Using the interpolation expression above (equation 8) would automatically adjust for the hot and cold mode dominance, via the relative importance of the c_{∞}^2 and σ^2 terms in the denominator. We note that semi-analytic models of galaxy formation often employ a distinction in the treatment of accretion rate between the two modes (see e.g. Hirschmann et al. 2012).

We would like, however, to emphasize again the most important caveat with the picture we have presented – the lack of angular momentum. If indeed the free-fall mode of accretion becomes more dominant as a result of employing the interpolated accretion rate expression then one must be very careful, for it is possible that the residual angular momentum of gas infalling at free-fall velocities from large scales becomes even more relevant to the accretion rate (Nulsen & Fabian 2000), due to the formation of a centrifugally supported disc at small scales. In this case we argue that any expression which does not take account of angular momentum should be viewed as more of a ‘capture’ rate, with the actual accretion on to the SMBH modelled with a viscous time-scale, the parameters of which are set by the properties of the gas at the transition point between ‘infall’ and ‘disc-mode’ provided it is sufficiently resolved (see Power et al. 2011). We note for completeness, however, that an alternative picture where the formation of a disc is not in fact a hindrance to SMBH growth was presented by Mayer et al. (2010), who found that global instabilities in the disc-like structure that formed from a merger led it to collapse and feed the SMBH on a dynamical time-scale (and see also the multi-scale simulations by Hopkins & Quataert 2010).

5 CONCLUSION

In this paper we have shown that the de facto industry standard, namely the Bondi–Hoyle formalism for the accretion rate on to the SMBH, fails for more than one reason in a realistic cosmological simulation whenever gas cooling is efficient. We have shown that a free-fall estimate is more appropriate in this case (provided that

angular momentum does not impede accretion of gas even further). We suggested an approximate interpolation formula that bridges the rapidly cooling and the inefficient cooling regimes which we hope will be useful for cosmological simulations that cannot resolve gas flow all the way down to the Bondi radius (a few to few tens of parsecs).

Finally, we conclude with the answer to the question posed in the paper title, which we feel is worth re-iterating: what matters in a simulation is the *character* of the flow where one evaluates the accretion/capture rate. If it is (a) spherical, (b) at rest and (c) sufficiently far away to count as ‘infinity’, with (d) the enclosed mass dominated by the SMBH, and finally (e) the flow is uninterrupted between the evaluation radius and the black hole, then and only then is Bondi–Hoyle accretion applicable as a sub-grid model. As we have mentioned, these latter requirements may be met to a sufficient degree by massive gas-poor ellipticals if sufficiently resolved in a simulation, but are not by any situation where there is appreciable infall from large scales. We note that the interpolation formula we have proposed is applicable to both cases, although it requires further development in order to take account of the full range of accretion regimes as discussed.

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APPENDIX A: THE SONIC POINT

For an isothermal gas flow, the sonic point for an extended mass distribution satisfies $r = GM(r)/2c_s^2(r)$, where $M(r)$ is the enclosed mass. Depending on the mass profile, this may be inside or outside the halo. We now consider several profiles for dark matter haloes, solve for the sonic point and plot its location versus gas temperature in Fig. A1. In particular, for a power-law density profile with

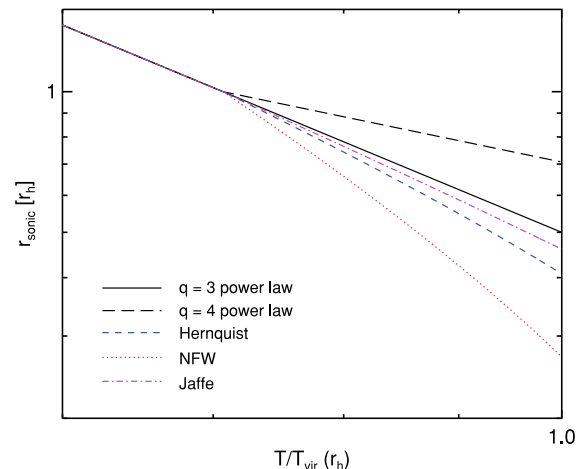


Figure A1. Sonic radius, scaled in units of r_h , for a variety of halo profiles, assuming a halo of $10^{11} M_\odot$.

index q ,

$$M(r) = M_h \left(\frac{r}{r_h} \right)^{3-q} \quad (\text{A1})$$

so that $r_s = 2^{1/(2-q)} r_h$ for gas at $T_{\text{vir}}(r_h)$. The enclosed mass in the Navarro, Frenk and White (NFW) profile (Navarro, Frenk & White 1996) is

$$M(r) = 4\pi\rho_0 a^3 \left[\ln \left(1 + \frac{r}{a} \right) - \frac{r}{a(r+a)} \right] \quad (\text{A2})$$

where a is the scale radius of the halo and ρ_0 is a characteristic density. The scale radius a depends on the concentration $c = r_h/a$ of the halo, where $5 \leq c \leq 15$ for haloes of mass $10^{14} - 10^{10} M_\odot$ (e.g. Bullock et al. 2001).

Elliptical galaxies and bulges that follow the $R^{1/4}$ law of de Vaucouleurs (1948) can usually be modelled by one of a family of

spherical density profiles characterized by an exponent γ (Dehnen 1993). Here the enclosed mass goes as

$$M(r) = 4\pi\rho_0 a^3 \left(\frac{r}{r+a} \right)^{3-\gamma} \quad (\text{A3})$$

with the Hernquist profile (Hernquist 1990) and the Jaffe cusp (Jaffe 1983) corresponding to $\gamma = 1$ and $\gamma = 2$, respectively.

Referring now to Fig. A1, we can see that the sonic point is in all these cases a significant fraction of the halo radius at $T = T_{\text{vir}}(r_h)$, and becomes larger than r_h for cooler gas. Thus, for a spherically symmetric problem, the infalling gas at or below the virial temperature (defined at r_h) *must quickly tend to a free-fall solution* once inside the halo.

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