

# Semi-analytical study on the generic degeneracy for galaxy clustering measurements

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**Abstract.** From the galaxy power spectrum in redshift space, we derive semi-analytical results on the generic degeneracy of galaxy clustering measurements. Defining the observables  $\bar{A} = Gb\sigma_8$  and  $\bar{R} = Gf\sigma_8$ , (being  $G$  the growth function,  $b$  the bias,  $f$  the growth rate, and  $\sigma_8$  the amplitude of the power spectrum), we perform a Fisher matrix formalism to forecast the expected precision of these quantities for a Euclid-like survey. Among the results we found that galaxy surveys have generically a slightly negative correlation between  $\bar{A}$  and  $\bar{R}$ , and they can always measure  $\bar{R}$  about 3.7 to 4.7 times better than  $\bar{A}$ .

**Keywords.** cosmology: observations, cosmology: theory, methods: statistical, surveys

## 1. Introduction

Future galaxy surveys will provide new opportunities to verify the current standard cosmological model, and also to constrain modified gravity theories, invoked to explain the present accelerated expansion of the universe. Before studying general parametrizations of dark energy, its however important to understand first which quantities can be really observed. From this direction recently Amendola *et al.* (2013) shown that cosmological measurements can determine, in addition to the expansion rate  $H(z)$ , only three additional variables  $R$ ,  $A$  and  $L$ , given by

$$A = Gb\delta_{m,0}, \quad R = Gf\delta_{m,0}, \quad L = \Omega_{m0}GY(1 + \eta)\delta_{m,0}. \quad (1.1)$$

with  $G$  is the growth function,  $b$  is the galaxy bias with respect to the dark matter density contrast, and  $\delta_{m,0}$  is the dark matter density contrast today. The functions  $\eta$  (the anisotropic stress  $\eta = -\frac{\Phi}{\Psi}$ ) and  $Y$  (The clustering of dark energy  $Y \equiv -\frac{2k^2\Psi}{3\Omega_m\delta_m}$ ), describe the impact of the dark energy on cosmological perturbations. In Amendola *et al.* (2014), a Fisher analysis was made using galaxy clustering, weak lensing and supernovae probes, in order to find the expected accuracy with which an Euclid like survey can measure the anisotropic stress  $\eta$ , in a model-independent way.

In this work we want to obtain some results on the intrinsic degeneracy on galaxy clustering measurements, using the quantities  $A$  and  $R$ . We use a flat  $\Lambda$ CDM fiducial model, with  $\Omega_{m,0}h^2 = 0.134$ ,  $\Omega_{b,0}h^2 = 0.022$ ,  $n_s = 0.96$ ,  $\tau = 0.085$ ,  $h = 0.694$ ,  $\Omega_k = 0$ , Euclid like survey specifications are used Amendola *et al.* (2013): we divided the redshift range  $[0.5, 2.0]$  in 5 bins of width  $\Delta z = 0.2$  and one of width  $\Delta z = 0.4$ ; an spectroscopic error  $\delta z = 0.001(1 + z)$ , a fraction of sky  $f_{\text{sky}} = 0.375$  and area of  $15\,000 \text{ deg}^2$ . Later on, the bias  $b$  is assumed to be unity.

## 2. Fisher matrix for Galaxy Clustering

Observations of the growth rate  $f$  from large scale structures using Redshift Space Distortions (RSD), give a direct way to test different dark energy models, Song & Percival (2009), Percival & White (2009), Racanelli *et al.* (2013). Lets consider now the galaxy power spectrum in redshift space

$$P(k, \mu) = (A + R\mu^2)^2 = (\bar{A} + \bar{R}\mu^2)^2 \delta_{t,0}^2(k), \tag{2.1}$$

whit  $\bar{A} = Gb\sigma_8, \bar{R} = Gf\sigma_8$ , and we explicitly use  $\delta_{m,0} = \sigma_8\delta_{t,0}$ . The Fisher matrix is in general

$$F_{\alpha\beta} = \frac{1}{8\pi^2} \int_{-1}^1 d\mu \int_{k_{\min}}^{k_{\max}} k^2 V_{\text{eff}} D_\alpha D_\beta dk, \tag{2.2}$$

where  $D_\alpha \equiv \frac{d \log P}{dp_\alpha}$ , and  $V_{\text{eff}}$  is the effective volume of the survey

$$V_{\text{eff}} = \left( \frac{\bar{n}P(k, \mu)}{\bar{n}P(k, \mu) + 1} \right)^2 V_{\text{survey}}, \tag{2.3}$$

being  $\bar{n}$  the galaxy number density in each bin. We want to study the dependence on the angular integration in the Fisher matrix for the set of parameters  $p_\alpha = \{\bar{A}(z_\alpha), \bar{R}(z_\alpha)\}$ . The derivatives of the power spectrum are

$$D_\alpha = \frac{2}{\bar{A} + \bar{R}\mu^2} \{1, \mu^2\}. \tag{2.4}$$

we consider two cases depending on the behavior of  $V_{\text{eff}}$ , equation (2.3):

(a) “Enough data”  $\bar{n}P(k, \mu) \gg 1$ , then we have  $V_{\text{eff}} \approx V_{\text{survey}}$  and the Fisher matrix could be written as

$$F_{\alpha\beta} \approx \frac{1}{2\pi^2} \int_{k_{\min}}^{k_{\max}} k^2 V_{\text{survey}} M_{\alpha\beta} dk, \tag{2.5}$$

where

$$M_{\alpha\beta} = 4 \begin{pmatrix} \frac{\sqrt{\bar{S}} + (\bar{A} + \bar{R}) \tan^{-1} \sqrt{\bar{P}_1}}{\bar{A}^{3/2} \sqrt{\bar{R}}(\bar{A} + \bar{R})} & \frac{-\sqrt{\bar{S}} + (\bar{A} + \bar{R}) \tan^{-1} \sqrt{\bar{P}_1}}{\bar{R}^{3/2} \sqrt{\bar{A}}(\bar{A} + \bar{R})} \\ \frac{-\sqrt{\bar{S}} + (\bar{A} + \bar{R}) \tan^{-1} \sqrt{\bar{P}_1}}{\bar{R}^{3/2} \sqrt{\bar{A}}(\bar{A} + \bar{R})} & \frac{\bar{R}(3\bar{A} + 2\bar{R}) - 3(\bar{A} + \bar{R})\sqrt{\bar{S}} \tan^{-1} \sqrt{\bar{P}_1}}{\bar{R}^3(\bar{A} + \bar{R})} \end{pmatrix} \tag{2.6}$$

being  $\bar{S} = \bar{A}\bar{R}$ .

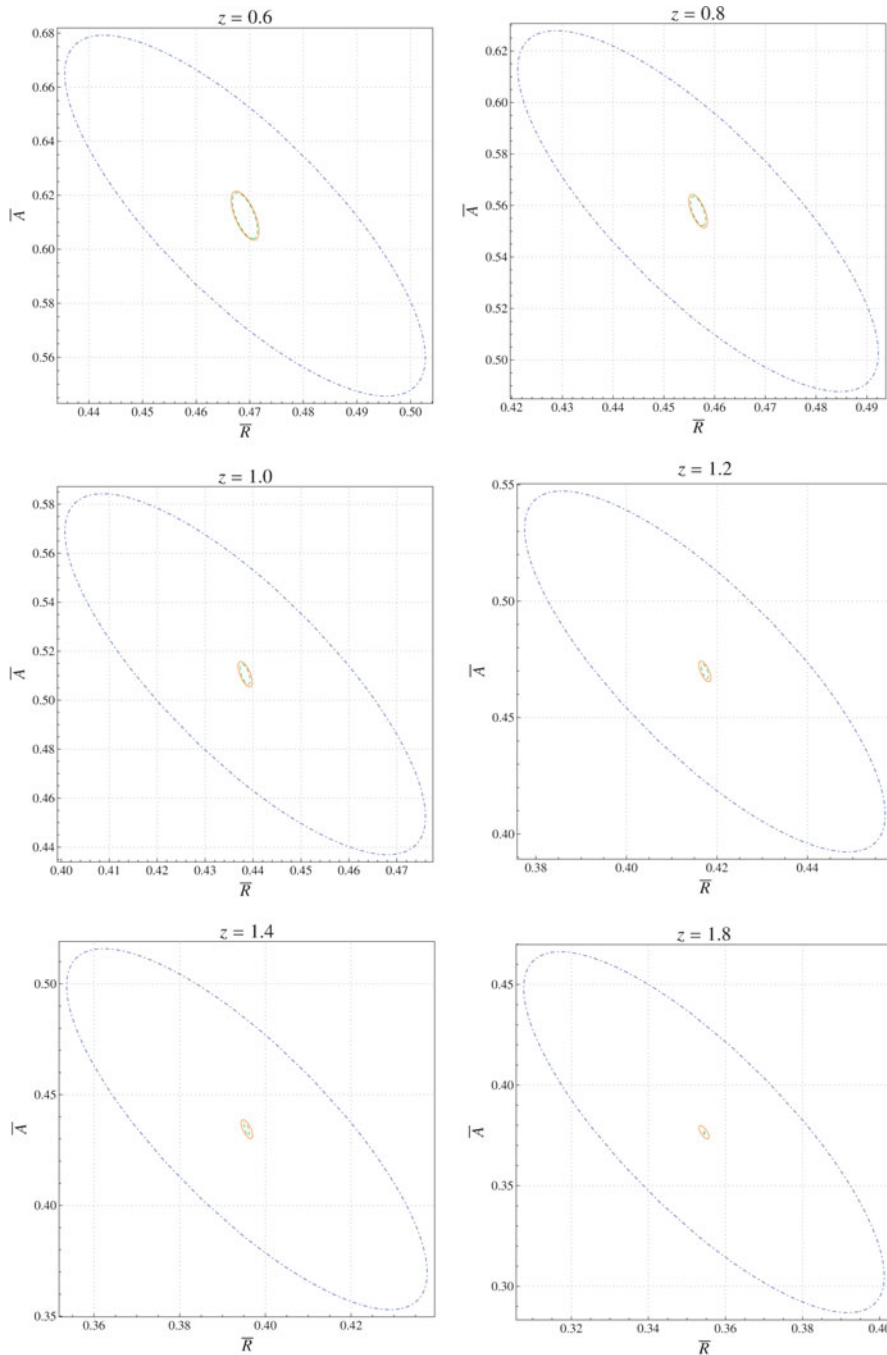
(b) Shot-noise dominated  $\bar{n}P(k, \mu) \ll 1$ , and then  $V_{\text{eff}} \approx (\bar{n}P(k, \mu))^2 V_{\text{survey}}$  and since we are interesting only in the  $\mu$  dependence we can write  $V_{\text{eff}} \approx P(k, \mu)^2$ . Then the Fisher matrix becomes

$$F_{\alpha\beta} \approx \frac{1}{2\pi^2} \int_{k_{\min}}^{k_{\max}} k^2 \delta_{t,0}^4(k) N_{\alpha\beta} dk, \tag{2.7}$$

with

$$N_{\alpha\beta} = 8 \begin{pmatrix} \bar{A}^2 + \frac{2\bar{A}\bar{R}}{3} + \frac{\bar{R}^2}{5} & \frac{\bar{A}^2}{3} + \frac{2\bar{A}\bar{R}}{5} + \frac{\bar{R}^2}{7} \\ \frac{\bar{A}^2}{3} + \frac{2\bar{A}\bar{R}}{5} + \frac{\bar{R}^2}{7} & \frac{\bar{A}^2}{5} + \frac{2\bar{A}\bar{R}}{7} + \frac{\bar{R}^2}{9} \end{pmatrix}. \tag{2.8}$$

We notice that in the two limiting cases above, we can move the matrices  $M_{\alpha\beta}$  and  $N_{\alpha\beta}$  outside of the integral, as for the fiducial model  $\bar{A}$  and  $\bar{R}$  do not depend on  $k$ . This



**Figure 1.** Confidence contours for  $\bar{A}$  and  $\bar{R}$  in the three cases: Solid line  $V_{\text{eff}}$ , dot dashed line  $V_{\text{eff}} \approx V_{\text{survey}}$ , and dashed line  $V_{\text{eff}} \approx P(k, \mu)^2$ .

means that, although the absolute size of the error ellipse depends on the integral, the relative size and orientation do not. In other words, we can obtain ‘generic expectations’ for the shape of the degeneracy between  $\bar{A}$  and  $\bar{R}$  from galaxy clustering surveys. These results are quite representative for the full range of  $\bar{A}$  and  $\bar{R}$ , i.e. galaxy surveys have

generically a slightly negative correlation between  $\bar{A}$  and  $\bar{R}$ , and they can always measure  $\bar{R}$  about 3.7 to 4.7 times better than  $\bar{A}$ , see Figure 1. In comparison to the results of Song & Kayo (2010), we remove the dependence on  $\delta_{t,0}$ , eq. (1.1), which is a quantity that depends on inflation or other primordial effects.

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