mean-independent. However, if we take a non-compact group and consider the space of all continuous functions with the topology of convergence uniform on every compact set, a novel situation arises in so far as the set of translates $f(xa)$ is generally unbounded. I hope to return to this question in a later paper. Another problem of some interest would be to apply Theorem A to the study of the mean-invariant envelope of a specified set of translates of a given function, a question which appears not to have been discussed at all amidst the vast literature on linear envelopes of translates.

References.

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BOUNDS FOR THE GREATEST LATENT ROOT OF A POSITIVE MATRIX

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1. Let $A = (a_{\mu\nu})$ be an $n \times n$-matrix with arbitrary non-zero $a_{\mu\nu}$.

Put
$$R_\mu = \sum_{\nu=1}^{n} |a_{\mu\nu}| \quad (\mu = 1, \ldots, n), \tag{1}$$

$$R = \max_\mu R_\mu, \tag{2}$$

$$r = \min_\mu R_\mu, \tag{3}$$

$$\kappa = \min_{\mu\nu} |a_{\mu\nu}|, \tag{4}$$

$$\sigma = \sqrt{\left(\frac{r-\kappa}{R-\kappa}\right)}. \tag{5}$$

* Received 10 October, 1951; read 15 November, 1951.

† This paper was prepared under contract of the National Bureau of Standards with the American University, Washington, D.C.

Then the root ω with the greatest modulus of the equation

$$|\lambda E - A| = 0 \tag{6}$$

satisfies the inequality

$$|\omega| \leqslant R-(1-\sigma)\kappa \tag{7}$$

and, if all $a_{\mu\nu}$ are positive, the further inequality

$$\omega \geqslant r+\left(\frac{1}{\sigma}-1\right)\kappa. \tag{8}$$

2. The results (7) and (8) are an improvement of the corresponding inequalities given a year ago in an interesting note* by W. Ledermann.

Put

$$\delta = \max_{R_\mu < R_\nu} \frac{R_\mu}{R_\nu}; \tag{9}$$

then the inequalities of Ledermann are obtained from (7) and (8) on replacing σ by $\sqrt{\delta}$ and $\leqslant$ by $<$.

3. Since the modulus of ω is majored by the greatest fundamental root of the matrix $(|a_{\mu\nu}|)$, it is sufficient to consider the case in which all $a_{\mu\nu}$ are *positive*. Then by a theorem of Perron and Frobenius, ω is positive and there exists a fundamental vector $(x_1, \ldots, x_n)$ of A corresponding to ω, with positive x_ν:

$$\omega x_\mu = \sum_{\nu=1}^{n} a_{\mu\nu} x_\nu \quad (\mu = 1, \ldots, n). \tag{10}$$

We shall prove a little more than the result stated, namely, assuming $r < R$,

$$\frac{\kappa}{R-r+\kappa} < \frac{\min_\nu x_\nu}{\max_\nu x_\nu} \leqslant \sigma. \tag{11}$$

We can assume, by permuting the rows and columns of A in a cogredient manner† and multiplying all x_ν by a convenient constant, that

$$1 = x_1 \geqslant x_2 \geqslant \ldots \geqslant x_n. \tag{12}$$

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* W. Ledermann, " Bounds for the greatest latent root of a positive matrix ", *Journal London Math. Soc.*, 25 (1950), 265–268.

† A cogredient transformation is the application of the *same* permutation to the rows and the columns.

Then we have from (10), (12) and (4), for any μ,

$$x_\mu \omega \geqslant a_{\mu 1} + \left(\sum_{\nu=2}^{n} a_{\mu\nu}\right) x_n = a_{\mu 1}(1-x_n) + R_\mu x_n,$$

$$\omega \geqslant \frac{1}{x_\mu}\left[x_n R_\mu + (1-x_n)\kappa\right]. \tag{13}$$

In a similar manner it follows that, for any index λ,

$$x_\lambda \omega \leqslant \sum_{\nu=1}^{n-1} a_{\lambda\nu} + a_{\lambda n} x_n = R_\lambda - (1-x_n) a_{\lambda n},$$

$$\omega \leqslant \frac{1}{x_\lambda}\left[R_\lambda - (1-x_n)\kappa\right]. \tag{14}$$

4. We now specialize (13) and (14) by taking μ and λ such that

$$R_\mu = R, \quad R_\lambda = r. \tag{15}$$

Then it follows from (13) and (14), since $x_\mu \leqslant 1$, $x_\lambda \geqslant x_n$, that

$$x_n(R-\kappa)+\kappa \leqslant \omega \leqslant \frac{r-\kappa}{x_n}+\kappa,$$

i.e.

$$x_n(R-\kappa) \leqslant \omega-\kappa \leqslant \frac{r-\kappa}{x_n}, \tag{16}$$

and therefore

$$x_n \leqslant \sqrt{\left(\frac{r-\kappa}{R-\kappa}\right)} = \sigma. \tag{17}$$

We write now (13) and (14) for $\mu = n$, $\lambda = 1$, and obtain, since

$$R_n \geqslant r, \quad R_1 \leqslant R, \quad x_n \leqslant \sigma:$$

$$R_n + \left(\frac{1}{x_n}-1\right)\kappa \leqslant \omega \leqslant R_1 - \kappa + x_n \kappa, \tag{18}$$

$$r + \left(\frac{1}{\sigma}-1\right)\kappa \leqslant \omega \leqslant R-\kappa+\sigma\kappa, \tag{19}$$

that is (7) and (8).

5. On the other hand we have from (18), since the bound on the right-hand side in (18) is less than R and $R_n \geqslant r$,

$$r + \left(\frac{1}{x_n}-1\right)\kappa < R;$$

and solving this with respect to x_n, we obtain

$$x_n > \frac{\kappa}{R-r+\kappa}, \tag{20}$$

that is (11).

6. It may be remarked finally that the inequalities (7), (8) and (11) can be still further improved, by introducing the expressions

$$\kappa_1 = \min_{\mu} a_{\mu\mu}, \quad \kappa_2 = \min_{\mu \neq \nu} a_{\mu\nu}. \tag{21}$$

Then in these inequalities we can replace σ by

$$\sigma_1 = \sqrt{\left(\frac{r-\kappa_1}{R-\kappa_1}\right)} \tag{22}$$

and κ by κ_2.

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CORRIGENDA

ON A THEOREM DUE TO M. RIESZ

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P. 289. In the conclusion of Theorem E, $o(e^{-\sigma\omega})$ should be replaced by $o(\omega^k e^{-\sigma\omega})$.

THE ASYMPTOTIC EXPANSION OF THE GENERALISED HYPERGEOMETRIC FUNCTION

E. M. WRIGHT†.

P. 287, LEMMA, line 3, for A_m read κA_m;

line 5, for $\kappa^{\frac{1}{2}-\vartheta}$ read $\kappa^{-\frac{1}{2}-\vartheta}$.

I am indebted to Dr. E. C. Bullard for drawing my attention to this error.

* This *Journal*, 26 (1951), 285–290.

† This *Journal*, 10 (1935), 286–293.