On the Determination of the Position of Strata in Stratified Rocks. By L. A. Necker, Honorary Professor of Mineralogy and Geology in the Academy of Geneva, \&c.
(Read 6th February 1832.)
$\mathbf{I}_{\mathrm{T}}$ has always appeared to me, that the study of the stratification of rocks and of mountain masses, ought to be one of the principal objects of a geological observer. Many of the most important facts in geology have been ascertained by the consideration of the position of strata. Among these facts, the relation existing between the direction and the inclination of the strata and the unstratified rocks, to whose presence the change in the position of the beds from an horizontal to an inclined, and sometimes even to a vertical situation, is now generally attributed, is one of the most conspicuous. It is only by an accurate determination of the position of the strata in any mountain-chain that the real direction of the line of elevation of that chain or its mineralogical axis may be determined.

However, such a determination is not always so easily accomplished as it is in mines, in quarries, or in such places where the upper or under surface of a stratum being exposed to the view of the observer, admits of the immediate application of one of the different kinds of clinometers well known to geologists. In high mountain-chains, the real direction and inclination of the strata can only be concluded from the examination of the positions of those lines which the seams of the strata form upon the face of abrupt and precipitous rocks, often inaccessible to the most hardy mountaineer. In such cases, the inexperienced observer, who would be tempted to consider the dip of those lines
or seams, as corresponding to the true inclination of the plane of the strata, would be entirely misled; and if, from this supposed inclination of the plane, he wished to infer the bearing of the strata, he would meet only with the most inextricable confusion, instead of perceiving that remarkable regularity in the direction of the strata which is so conspicuous in mountain-chains, whatever may be their extent.

Saussure, in his Agenda, has already cautioned the geologist against too hasty an inference from the horizontality of the lines of stratification on the surface of a precipitous crag: he has shown that, before pronouncing a mountain to be formed of horizontal strata, it is necessary to view it from the extremities of two lines crossing each other at an angle.

The following considerations will show that Saussure's observation is not applicable only to the case on which the seams of the strata, when observed from only one side of a mountain, appear horizontal, but that it applies equally well to inclined lines of stratification, which make with the horizon an angle smaller than the true angle of dip of the planes.

Let us suppose a system of parallel planes or strata more or less inclined, into which a section should be made by a vertical plane, whose direction or line of common intersection with the horizontal plane would be parallel to the direction of the strata themselves. In such a case, the seams of the strata on the surface laid bare by the section would appear horizontal. If, on the contrary, the direction of the vertical section was supposed to be perpendicular to the direction of the strata, then the seams of the strata would exhibit a dip identical with the true dip of the planes themselves. But between these two limits an indefinite number of vertical sections may be supposed, in which the angle of dip of the seams will vary according to the direction of the section. The more this direction approaches to a parallelism with the direction of the strata, the lesser will be the angles made by
the seams of the strata with the horizon; the more, on the contrary, the direction of the section approaches to be perpendicular to that of the strata, the greater will be the inclination of the seams.

Now, although the position of the strata in a mountain-chain often remain for a great extent of country invariably the same, the varying direction of the accidental sections will make the lines of the stratification appear changing at every step. Hence arises often the mistaken ideas that seems to prevail, that it is of little use to notice, or mark down upon geological maps, the apparently too variable position of the strata. While such information would be of the greatest interest, whenever questions relating to the upheaving of the strata, and of the chain itself, should happen to be examined.

Of some erroneous opinions to which a neglect of the preceding considerations may have given rise, I shall only notice the following instance.

It is well known that, at some distance to the south-west of Geneva, the River Rhone makes its way through the Jura chain by a narrow and rocky defile. This pass between the two mountains Credo and Varache, is named after the Fort de l'Ecluse, a small but strong fortress, builtin this place, to guard this part of the French frontiers. The direction of the strata of Jura limestone, which have been cut through to make way for the river, is nearly north and south, and the direction of the channel in which the river flows nearly east and west; the dip of the strata is to the east ; and if the two high rocks which form the sides of the channel had been exactly parallel, the direction of this accidental section would have shown, on both sides, a similar inclination of the seams of the strata corresponding with the dip of the strata themselves to the east. But as the canal is narrower at its eastern extremity than at its western, its two sides are not parallel but converge at the entrance towards the east. Hence
it follows, that in the cliff on the northern bank of the Rhone, the seams present an inclination a little to the south of east, while the seams on the cliff of the southern bank shew a dip a little to the north of east.

From such an appearance some observers have been led to conclude, that this opening in the chain had been produced by a depression of the strata. But the remarks before stated show that such appearances are merely superficial and external, and that a similar inclination in the lines of stratification does not indicate a corresponding inclination of the strata, but is owing simply to the mere accidental direction of the section through the stratified mass. In fact, the strata on both sides of the Rhone, at the Fort de l'Ecluse, although they have been traversed by a wide and deep fissure, entirely occupied by the waters of the river, have not experienced any change in their original direction and dip; so that this narrow transversal valley cannot by any means be called a valley of depression. I am convinced that the same will be found to be the case with many much larger and much more important transversal valleys of the Alps, which have been supposed to be formed by the sinking of the strata on both sides.

Now, it has occurred to me, that such external appearances as are the position of the seams of strata on the surface of cliffs, could enable us, when combined together, to determine the true position of the planes to which they belong, inasmuch as the position of a plane is determined by that of two lines in the same place.

I am indebted to my learned friend M. Gautier, Professor of Astronomy in the Academy of Geneva, for the following exact and complete solution of the given problem by means of algebraical formulæ.

The angles $\alpha$ and $\alpha^{\prime}$, which two straight lines form with their horizontal projection, being given, as well as the angles $\beta$ and $\beta^{\prime}$,
which these projections form with a fixed horizontal axis, there will be determined, 1 st, the angle $\gamma$ comprised between the plane passing through the two straight lines and the horizontal plane by the formula.

$$
\tan \gamma=\frac{\sin \left(\alpha^{\prime}+\alpha\right) \cos \phi}{\cos \alpha^{\prime} \cos \alpha \sin \left(\beta^{\prime}-\beta\right)}
$$

$\phi$ being an auxiliary angle, such that

$$
\sin \phi=\frac{\cos \frac{1}{2}\left(\beta^{\prime}-\beta\right) \sqrt{\sin 2 \alpha^{\prime} \sin 2 \alpha}}{\sin \left(\alpha^{\prime}+\alpha\right)}
$$

2 d , the angle $\omega$ comprised between the line of intersection of the two planes and the fixed axis by the formula

$$
\sin (\beta-\rho)=\frac{\operatorname{tang} \alpha}{\operatorname{tang} \gamma}
$$

in which every thing is known except $\omega$, when $\gamma$ has already been determined.

It will be readily perceived that this solution, however satisfactory it may be in a theoretical point of view, will never be advantageously employed in practice, on account of its presupposing that the data given by actual observation should be of the nicest accuracy. The measurement of the angle of dip of the lines of stratification, and of the angle which the horizontal projection of these lines make with the magnetic meridian, especially when taken at a distance, can never reach the degree of accuracy required by the nature of the formulæ employed.

In such a case, practical geometry, or that system of combined projections which is known in French under the name of Geometrie descriptive, requiring less perfect data, will be in general better adapted to the imperfect means which the geologist possesses for ascertaining the data required, and at the same time will give results sufficiently accurate for the present purposes of the science.
M. Dufour of Geneva, well known by his writings as a military engineer, has, at my request, pointed out to me the simple graphical process which I am now to explain, and which may be employed in all cases whatever, where the position of two lines in the same plane is given.

The data in this case are the same as in the preceding, viz. the angles which two seams of the strata taken in two different sections of the rock, make with their horizontal projection, and the angle which each of these projections make with the magnetic or with the true meridian.

To determine with the aid of these data the true dip and direction of the plane, it is necessary, first, to bring the angles into the same horizontal plane, then through the intersection of two vertical planes passing through the horizontal projections of the seams, to draw a vertical plane. This plane will cut the plane containing the two seams, and the horizontal plane, at an angle, which is the angle required.

Fig. 1.


In Fig. 1. the vertical plane $a b \mathrm{G}$ is placed parallel to the horizontal projection AB of one of the seams. The point A is
first to be projected vertically as well as the angle $\mathrm{G} a b$, which the seam having for its horizontal projection the line AB makes with the horizon.

The known angle ECF, which is seen drawn on the horizontal plane, is to be projected vertically; for which operation it is necessary that it should be made to turn upon DC till it should be returned in its true position, then the point E will be projected horizontally in F and vertically in $e$. The line $e f$ will be the vertical projection of EF , and the angle $e c f$ will be the vertical projection of the angle ECF.

Two vertical planes are then drawn through the horizontal projections of the seams, and the intersection of these two planes, which is a line perpendicular to the horizontal plane, is vertically projected according to the line $m n$. The point where the line $m n$ meets with the line $a b$ will be the vertical projection of the point where the two horizontal projections of the seams meet.

The point $s$ being common to the two lines, the angle e $c f$ is made to slide till the line $b c$ meets the point $s$. The angles will be then in the same plane, without their size being altered.

The point $G$ being projected in $g$, the point where the seam, having for its horizontal projection the line DC, cuts the horizontal plane, will be found, and Ag will be a horizontal line drawn upon the plane passing through the two seams, or, in other words, will be the true direction of the strata.

Nothing remains now but to find the angle of dip, which will be easily obtained in drawing a line perpendicular to the direction or line Ag, and passing through the line of intersection. Its horizontal projection will be found in drawing from the point N the perpendicular NP , so that a triangle will be formed, of which two sides are known, viz. SN and PN , which will be brought to $n p$, and the angle $\mathrm{S} p n$ will be the angle made with the horizon by the plane passing through the two seams. PN will be the horizontal projection of the dip.

Fig. 2.


Fig. 3.


The Figures 2. and 3. are intended to show, that whatsoever may be the direction of the horizontal projection of the seams, and whatever may be the position of the vertical plane of the drawing in relation to these projections, the same process will lead to the wished-for result.

Fig. 4.


Fig. 4. shows the operation which takes place, in case of the true direction of the strata being one of the given lines of stratification, or seams, in such a case where the angle made by this line with its horizontal projection $=0$, the process is much simplified, although always carried on according to the same principle. The same letters being used in all the four figures, to denote the analogous points, the explanation above given will apply to them all.

Although this last way of resolving the problem did not require any more precise data than those which the less skilled observer can easily obtain in confining himself to the simple use of the compass and the plumb-line; and although this proceeding requires but a little attention, and the use of the most elementary processes of geometry, it is nevertheless to be feared that the practical geologist, to whose care the solutions of such often important problems are trusted, would often think it too long and too complicated a method to be prevailed upon to use it.

Such an idea, and at the same time the wish that numerous and accurate observations of the position of the strata should be vol. xil. part if. в b
given by all those who have the opportunity of making geological observations in any part of the world, has induced me to contrive a very simple mechanical apparatus, by which all such problems could be easily, and in a very short time, solved. It is this instrument which I call the Clinometrical Compass, of which I am now to give the description, at the same time that I have the honour to lay it before the Royal Society.


This instrument is composed of a circular plate of brass A B, divided in thirty-two parts, corresponding to as many points of the compass. A semicircular plate of brass IC, concentric to the above mentioned circle, is made to turn upon it around the common centre. This semicircle is also divided by lines in sixteen divisions, exactly corresponding to those of the under circle. A semicircular portion of a ring of brass $E$, concentric with both plates, is connected by a hinge $F$, with the diameter of the semicircular plate, in such a way that the diameter of this plate and that of the half ring are made to coincide in the hinge. In this manner the ring may be moved upwards or downwards, so as to take any inclination whatever upon the horizontal plane, or upon the plane of the lower circular plate, by which the horizontal
plane is represented, and at the same time it may be made to revolve round the centre of the instrument, so as to be placed in any required position relatively to any of the points of the compass.

In such a state of things, the half ring will represent the plane of a stratum, its hinge the direction of the stratum, or its common intersection with the horizontal plane. And all the imaginary lines which may be drawn from the centre of the instrument to the circumference of the ring represent all the possible sections which may be made by vertical planes in the plane of the stratum, and so correspond exactly to the seams or lines of stratification often mentioned above. The divisions on the semicircular plate under the half ring will represent the horizontal projection of those lines which coincide with the given divisions of the compass.

Now, by means of a graduated arc, applied perpendicularly to the plane of the semicircle, and touched in one point by the sharpened interior edge of the half ring, we will be able to measure the angle made by any line in the plane of the half ring with its horizontal projection.

I am indebted to Mr Adie, member of this Society, for a material improvement, that of substituting a simple half ring, such as I have described, for a semicircular plate traversed by fissures, by which I had at first thought of representing the plane of the stratum. To him also is due the contrivance by which the centre of the divided arc is made to coincide with that of the instrument, and the arc itself to stand vertically, which is absolutely necessary in all cases. For these purposes, a small projecting pin occupies the centre of the instrument, and is received into a small hole corresponding to the centre of the protractor, while a small support fixed at a point of its diameter contributes also in keeping it in a vertical position.

An instance will now show the mode of using the instrument,
which, though it may be thought at first rather complicated, will soon be attained by a little practice, and found very simple.

Let the given lines or seams be,
1st, A line dipping $45^{\circ} \mathbf{S}$. b. E., or rising $45^{\circ}$ N. b. W., which is the same thing.
2 d , A line dipping $56^{\circ}$ E. b. N., or rising the same number of degrees W. b. S.

We have, first, to observe, that, as has been said before, the hinge or diameter of the brass semicircle correspond to the direction of the stratum, in which direction all the seams appear horizontal. Secondly, that the line marked on the semicircle perpendicularly to the hinge or diameter, is the horizontal projection of the true line of dip or inclination of the stratum, and that this true line of dip is, of all the lines which may be drawn in the plane, that which makes the greatest angle with its horizontal projection. The horizontal projection of this line perpendicular to the hinge I shall name the Index Line.

Now, we must begin by supposing, that, of the two lines given, the one that dips with the greater angle may be the true line of dip, and accordingly we will direct the index line to the W.b.S. of the brass circle ; then placing the protractor vertically on the index line, we will move upwards the half ring till its sharpened edge comes in contact with the 56th degree of the vertical arc. Looking then to the direction of the other given line, or to the N. b. W. of the compass, and moving the protractor around the centre till it comes to be vertical upon this line, we will look whether the edge of the ring touches its 45 th degree, for, in that case, the position of the ring would correspond exactly to that of the required plane. But, in the present instance, it is not necessary even to move the protractor to see that the N. b. W. line is that which corresponds to the dia-
meter, and in consequence to the direction of the stratum ; so that in this case the given line, instead of rising $45^{\circ}$ above the horizon, ought to be horizontal. The position of the ring does not then correspond to the position of the required plane. This first operation teaches us two things, 1st, That the angle of dip is greater than $56^{\circ}$, so that the half ring may be made to turn upwards upon its hinge. 2d, That the semicircle ought to turn upon its centre, in such a way that the index line shall move gradually towards the west, and afterwards towards the north, in order to bring the N. b. W. line nearer to that of the true dip, to get a line or seam of a greater angle of inclination.

Let us then raise the half ring two degrees more, and turn the semicircle two points of the compass towards the N.W., so that the index line will correspond to the W. b. N. The dip of the line corresponding to the W.b.S. will still be $56^{\circ}$; and if that of the line corresponding to the N. b. W. be $45^{\circ}$, we shall have found the true plane, directed from N. b. E. to S. b. W., and rising $58^{\circ}$ towards the $\mathrm{W} . \mathrm{b}$. N. Let then the protractor be placed vertically on the line N. b. W., the angle found being $34^{\circ}$ instead of $45^{\circ}$, we shall have still to raise the half ring, and move the semicircle in the same direction as before.

In raising the half ring to $60^{\circ}$, and pointing the index line to W.N.W., we shall find a dip of $45^{\circ}$ on the N. b. W. line, and of $56^{\circ}$ on the W. b. S. one, corresponding with our data. So that we shall have determined the direction of the stratum to be S.S.W. and N.N.E., and its inclination $60^{\circ}$ rising to the $W$. N.W. or dipping to the E.N.E.

If the direction of the plane, or that of a horizontal seam S.S.W. and N.N.E., had been one of the given lines, the operation would have been much simplified; it would have been required only to bring the hinge in that direction, remembering, however, that the other given point of the compass should be comprised in the points embraced in the semicircle; then to
place the protractor vertically on the N. b. W. line, for instance, and to raise the half ring till it should touch the 45th degree; finally, to place the protractor on the index line, indicating the rise to be towards the W.N.W., and to note the angle $=60^{\circ}$.

We hope that this simple little mechanical contrivance will be of use to the practical geologist, who may not have the means or leisure to use the more accurate modes of determination before described. The habit which he will acquire of giving his attention to observations connected with this subject of inquiry, will enable him not only to ascertain in all cases the exact position of the strata whenever they are real planes, but also to discover when two distinct sets of strata are lying in the same plane, and if not, to recognise modes, hitherto little attended to, of unconformable stratification.

Finally, although all the modes of determination above alluded to are only appropriated to the plane strata, the readiness of tact and eye-sight which such considerations will have given to the geologist, will not be without use, when he shall have to study the waving disposition of strata with curved surfaces, whether they are parts of parallel portions of cylinders with their axes horizontal, in which case the seams parallel to these axes or the direction will also appear horizontal, as is observed in that part of the Lammermuir Hills which forms the coast of Berwickshire ; or whether they are parts of oblique cylinders with their axes inclined to the horizon, in which case there is no seam which can appear horizontal, because there is a dip even in the direction of the bearing. Of this case the Alps show an amazing variety of instances, among which the more or less dismantled oblique cylindrical structure of the Mount Saléve near Geneva, and of the whole mountain group on the southern bank of the Arve, between Bonneville and Sallenches, are the most instructive and worthy of remark.

## Postscript.

As it is necessary that the protractor should stand perfectly vertical, another mode of adjusting it to the centre of the instrument must be devised, instead of that which is alluded to in the paper, and exemplified by the instrument itself. For this purpose two different methods may be adopted. The first is represented in the annexed figure ; the brass circle AB or $a b$, should

bear as its central part a solid brass cylinder $l^{\prime \prime}$ of at least two or three lines in length, and one or two in diameter, it would be received in a hole $l^{\prime}$ of the same diameter in the half circle $c d$.

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And the protractor $g h$ would have at its central part a hollow tube $l$, fitting exactly to the cylinder $l^{\prime \prime}$ of the brass circle $a b$.

The second method, which is still better, and has been sug. gested to me by Mr Robison, is that of adapting the brass cylinder at the centre $L$ or $l$ of the protractor. This cylinder should be made to enter into a brass cylindrical tube occupying the centre $l^{\prime}$ of the half circle $c d$, and projecting under the half circle to a length equal to the thickness of the brass circle AB or $a b$. This tube would be received in a hole of equal size corresponding to the central part $l^{\prime \prime}$ of the circle $a b$.

