# A new perspective on globular clusters, their initial mass function and their contribution to the stellar halo and the cosmic reionization 

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#### Abstract

We examine various implications from a dynamical and chemical model of globular clusters (GCs), which successfully reproduces the observed abundance patterns and the multiple populations of stars in these systems assuming chemical enrichment from fast-rotating massive stars. Using the model of Decressin et al., we determine the ratio between the observed, present-day mass of GCs and their initial stellar mass as a function of the stellar initial mass function (IMF). We also compute the mass of low-mass stars ejected and the amount of hydrogen ionizing photons emitted by the proto-GCs. Typically, we find that the initial masses of GCs must be $\sim 8-10$ times (or up to 25 times, if second-generation stars also escape from GCs) larger than the present-day stellar mass. The present-day Galactic GC population must then have contributed to approximately $5-8$ per cent ( $10-20$ per cent) of the low-mass stars in the Galactic halo. We also show that the detection of second-generation stars in the Galactic halo, recently announced by different groups, provides a new constraint on the GC IMF (GCIMF). These observations appear to rule out a power-law GCIMF, whereas they are compatible with a lognormal one. Finally, the high initial masses also imply that GCs must have emitted a large amount of ionizing photons in the early Universe. Our results reopen the question on the IMF of GCs and reinforce earlier conclusions that old GCs could have represented a significant contribution to reionize the intergalactic medium at high redshift.


Key words: stars: Population II - globular clusters: general - galaxies: star clusters: general dark ages, reionization, first stars.

## 1 INTRODUCTION

Although for a long time thought to be among the simplest stellar systems, globular clusters (GCs) have been subject to intense studies both observationally and through theory and simulations. These include, for example, detailed work on the stellar content of GCs and on chemical abundances of GC stars, searches for viable proto-GCs, studies of dynamical effects on massive star cluster evolution, cosmological simulations of their formation, estimates of their contribution to cosmic reionization and other related Galactic and extragalactic astrophysical topics (see e.g. reviews by Gratton, Sneden \& Carretta 2004; Brodie \& Strader 2006; Piotto 2009; Boily 2010; Elmegreen 2010b).

Despite these studies, many open questions remain, concerning both GCs as individual objects and as a collective population. For example, the origin of Galactic halo stars and the contribution of GCs to this population are still unclear (see e.g. Hut \& Djorgovski 1992; Parmentier \& Gilmore 2007; Bell et al. 2008; Boley et al.

[^0]2009). Similarly, the shape of the GC initial mass function (GCIMF), the nature of the present-day GC mass function and the processes and the time-scales responsible for transforming the former into the latter are debated (see Fall \& Zhang 2001; Vesperini \& Zepf 2003; Parmentier \& Gilmore 2005, 2007; Elmegreen 2010a). Also, first steps are being made in order to understand GC formation in cosmological simulations (Bromm \& Clarke 2002; Kravtsov \& Gnedin 2005; Boley et al. 2009; Griffen et al. 2010). Finally, Ricotti (2002) has shown that GCs emit enough ionizing photons to reionize the Universe, provided their escape fraction $f_{\text {esc }}$ is of the order of unity. Examining this question is also of interest in the present context, where it appears that galaxies found so far in deep surveys are insufficient to reionize the intergalactic medium (see e.g. Ouchi et al. 2009; Bunker et al. 2010; Labbé et al. 2010; McLure et al. 2010), and where the main sources responsible for cosmic reionization, presumably faint, low-mass galaxies, below the current detection limits (cf. Choudhury \& Ferrara 2007; Choudhury, Ferrara \& Gallerani 2008), remain thus to be identified.

A major paradigm shift that sheds new light on these key questions has occurred recently in the GC community. Indeed, detailed abundance studies of their long-lived low-mass stars made possible
with $8-10 \mathrm{~m}$ class telescopes, together with high-precision photometry of Galactic GCs performed with Hubble Space Telescope (HST), have revolutionized our picture of these stellar systems. It is now clear that individual GCs host multiple stellar populations as shown by their different chemical properties and by multimodal sequences in the colour-magnitude diagrams (Bedin et al. 2004; Piotto et al. 2007; Milone et al. 2008, 2010; Villanova et al. 2010). Indeed, although nearly all GCs ${ }^{1}$ appear to be fairly homogeneous in heavy elements (i.e. Fe peak, neutron capture and $\alpha$-elements; see e.g. James et al. 2004; Sneden 2005; Carretta et al. 2009b), they all exhibit large star-to-star abundance variations for light elements from C to Al that are the signatures of hydrogen burning at high temperature implanted at birth in their long-lived low-mass stars (see e.g. Gratton et al. 2001, 2004; Sneden 2005; Prantzos, Charbonnel \& Iliadis 2007; Carretta et al. 2009a; Charbonnel 2010, and reference therein). In fact, the so-called $\mathrm{O}-\mathrm{Na}$ anticorrelation is ubiquitous in Galactic GCs and is now accepted as the decisive observational criterion distinguishing bona fide GCs from other clusters (Carretta et al. 2010b).
The current explanation for these chemical patterns is the socalled 'self-enrichment' scenario that calls for the formation of at least two stellar generations in all GCs during their infancy. The first-generation stars were born with the protocluster original composition, which is that of contemporary field halo stars, while the second-generation stars formed from original gas polluted to various degrees by hydrogen-burning processed material ejected by more massive, short-lived, first-generation GC stars. Details and references can be found e.g. in Prantzos \& Charbonnel (2006) who discuss the pros and cons of two versions of this 'self-enrichment scenario', invoking either massive asymptotic giant branch (AGB) stars (e.g. Cottrell \& Da Costa 1981; Ventura et al. 2001; Ventura \& D'Antona 2009) or fast-rotating massive stars (e.g. Decressin, Charbonnel \& Meynet 2007b, hereafter DCM07) as polluters.
Whatever the actual polluting stars, an immediate consequence of this scenario is that, in order to reproduce the present proportion of first to second generation stars with acceptable values of the polluters IMF, the initial stellar masses of GCs must have been considerably larger than their present-day value (Prantzos \& Charbonnel 2006; DCM07; D'Ercole et al. 2008; Carretta et al. 2010b; Decressin et al. 2010). However, most of the extragalactic studies have not yet incorporated this revised picture, or have not yet explored the resulting implications. Furthermore, the recent discovery of stars with signatures characteristic of second-generation GC stars among the metal-poor halo population (Carretta et al. 2010b; Martell \& Grebel 2010) sheds new light on the amount of low-mass stars ejected from GCs and on the IMF of these clusters, as we shall show below.
In the present paper, we explore several consequences of this new paradigm, based on the model that was developed by DCM07 to describe the early chemical and dynamical evolution of GCs. In this model, fast-rotating massive ( $M \gtrsim 25 \mathrm{M}_{\odot}$ ) stars are responsible for the GC pollution. The model successfully explains the observed abundance patterns of present-day GC stars and has also been tested with $N$-body and hydrodynamical simulations (see Decressin, Baumgardt \& Kroupa 2008; Decressin et al. 2010). Its main assumptions are briefly described and summarized in Section 2. Within this framework, we constrain the relation between

[^1]the initial and the present stellar mass of GCs (Section 3), as well as the contribution to the stellar halo (Section 4), taking the recent observational identification of second-generation stars in the Galactic halo (Carretta et al. 2010b; Martell \& Grebel 2010) into account. Implications on the GCIMF are derived in Section 5. Finally, we derive in Section 6 a well-defined ionizing photon production rate for proto-GCs, taking all the detailed observational constraints from nearby GCs into account, and estimate their contribution to cosmic reionization. Our main conclusions are summarized in Section 7.

## 2 THE ADOPTED CHEMICAL AND DYNAMICAL EVOLUTION MODEL

DCM07 and Decressin et al. (2007a) have shown that the O-Na anticorrelation observed in GC stars can be explained if a second generation of low-mass stars form from the ejecta of first stellar generation fast-rotating massive stars mixed with some original interstellar material. In their model, the first generation forms the full mass spectrum of stars described by a power-law IMF at the high end and a lognormal below $0.8 \mathrm{M}_{\odot}$. The second generation of 'polluted' stars is assumed to form only low-mass stars, following the same lognormal IMF (see below). The model allows for dynamical cluster evolution and, more specifically, for the evaporation of stars due to primordial gas expulsion driven by supernovae as well as for longterm dynamical processes as described by Decressin et al. (2008, 2010).

The main free parameter of the model is the IMF slope $x$ above $0.8 \mathrm{M}_{\odot}$ of the first stellar generation, the low-mass IMF being set for both the first and second stellar populations to the presentday mass function observed in GCs (Paresce \& De Marchi 2000). Second-generation low-mass stars are then formed from the mass of slow wind ejecta $f_{\text {Sw }}$ predicted by the stellar evolution models of Decressin et al. (2007a) and after dilution of this material with interstellar gas. The parameter $d$ describing this dilution is inferred from the observed $\mathrm{Li}-\mathrm{Na}$ anticorrelation (see DCM07; Charbonnel $\&$ Decressin, in preparation). We adopt $d=1.15$ from DCM07 as our standard value and comment on the (relatively weak) dependence of our results on this parameter.
Allowing for the escape of a fraction of first-generation stars, the model then predicts the relative number of first- and secondgeneration stars, as well as detailed abundance ratios of these stars, which successfully reproduce observed abundance patterns and anticorrelations (see DCM07).

The fraction of 'unpolluted', pristine first-generation long-lived stars $f_{\mathrm{p}}$ still present today in GCs can be determined observationally from the distribution of stars along the $\mathrm{O}-\mathrm{Na}$ anticorrelation (see e.g. Prantzos \& Charbonnel 2006). In the nomenclature of DCM07, one has $f_{\mathrm{p}}=n_{\mathrm{LL}}^{1 \mathrm{G}} /\left(n_{\mathrm{LL}}^{1 \mathrm{G}}+n_{\mathrm{LL}}^{2 \mathrm{G}}\right)$, where $n_{\mathrm{LL}}^{1 \mathrm{G}}$ and $n_{\mathrm{LL}}^{2 \mathrm{G}}$ are the number of first- and second-generation low-mass, long-lived stars, respectively. Observations of the $\mathrm{O}-\mathrm{Na}$ anticorrelation in a large GC sample by Carretta et al. (2010b) provide a median value $( \pm 68$ per cent CL$)$ of $f_{\mathrm{p}}=0.33_{-0.08}^{+0.07}$ both for the total sample and for the lowest metallicity $([\mathrm{Fe} / \mathrm{H}]<-1)$, hence oldest subsample. The semi-analytical model of DCM07 predicts $f_{\mathrm{p}}$ as a function of the IMF slope, the dilution parameter $d$ and the fraction $e_{\mathrm{LL}}^{1 \mathrm{G}}$ of low-mass, first-generation stars being lost from the cluster due to dynamical processes (see equations 20, 23 of DCM07). We can therefore invert this problem to determine, for each value of the IMF slope, the lost stellar mass fraction $e_{\mathrm{LL}}^{1 \mathrm{G}}$ from the observed value of $f_{\mathrm{p}}$. With this at hand, all the properties of the two stellar generations mixed within the GC can be determined (see DCM07).


Figure 1. Left-hand panel: ratio between the initial and present-day mass of GCs as a function of the massive star IMF slope ( 1.35 is the Salpeter value) and for $f_{\mathrm{p}}=0.33_{-0.08}^{+0.07}$ (boundaries shown by dotted lines). The blue lines correspond to the case where no second-generation stars escape from the cluster $\left(e_{\mathrm{LL}}^{2 \mathrm{G}}=\right.$ 0 ), while green and magenta lines show the values predicted when allowing also for the evaporation of second-generation stars with fractions $e_{\mathrm{LL}}^{2 \mathrm{G}}=0.43$ and 0.65 , respectively. Right-hand panel: mass ratio between the ejected low-mass stars (total including first- and second-generation stars) and the present-day mass as a function of the massive star IMF slope. As for the left figure, blue, green and magenta lines show the predictions for $e_{\mathrm{LL}}^{2 \mathrm{G}}=0,0.43$ and 0.65 , respectively, and for the observed range of $f_{\mathrm{p}}$. The black lines show the contribution of second-generation stars for the latter two cases. The shaded area indicates the region where the present-day GCs would overpredict the amount of halo stars (assuming 2 per cent of the stellar halo mass in GCs).

Here, we are in particular interested in the relation between the present-day, observed stellar mass and its total, initial value, as well as in the mass of stars ejected from the cluster. These are derived below.

To do so, we generalize the dynamical evolution scenario discussed in depth by DCM07 that allows for mass-loss from the cluster (as described by $e_{\mathrm{LL}}^{1 \mathrm{G}}>0$ in their 'Scenario II) due to mass segregation and evaporation of stars. We consider the IMF slope $x$ above $0.8 \mathrm{M}_{\odot}$ as a free parameter, and we determine the allowed values of $x$ from the observed value of $f_{\mathrm{p}}$ given above. However, it is understood that most observations for proto-GCs indicate an IMF slope close to Salpeter ( $x=1.35$ ) in this mass range (see e.g. Chabrier 2003; Bastian, Covey \& Meyer 2010; De Marchi, Paresce \& Portegies Zwart 2010). While DCM07 assume that all secondgeneration stars are retained within the cluster $\left(e_{\mathrm{LL}}^{2 \mathrm{G}}=0\right.$ in their notation), we will subsequently relax this assumption, motivated by recent findings of some chemically polluted, second-generation stars in the Galactic halo (Carretta et al. 2010b; Martell \& Grebel 2010).

We now follow the semi-analytical model of DCM07 and their notation. The current mass of first-generation long-lived (i.e. lowmass) stars, $M_{\mathrm{LL}}^{1 G}$, in a GC is $M_{\mathrm{LL}}^{1 G}=f_{\mathrm{p}} \times M_{\mathrm{obs}}$, where $f_{\mathrm{p}}$ is the fraction of first-generation stars determined from observations (cf. above) and $M_{\text {obs }}$ is the 'observed', current stellar mass of the GC, excluding stellar remnants. ${ }^{2}$ Allowing for dynamical mass-loss of stars from the cluster, the total (initial) stellar mass of the cluster

[^2]can be written as
$M_{\mathrm{ini}}=\frac{f_{\mathrm{p}}}{\left(1-e_{\mathrm{LL}}^{1 \mathrm{G}}\right) f_{\mathrm{LL}}^{1 \mathrm{G}}} \times M_{\mathrm{obs}}$,
where $f_{\mathrm{LL}}^{1 \mathrm{G}}$ stands for the fraction of stellar mass forming low-mass stars (in the first generation), i.e. the mass fraction of the IMF found at $\lesssim 0.8 \mathrm{M}_{\odot}$, and where $e_{\mathrm{LL}}^{1 \mathrm{G}}$ is the fraction of low-mass stars from the first generation having escaped from the cluster during its history. The total mass of ejected low-mass stars is then
$M_{\mathrm{eje}}^{\mathrm{LL}}=\left(\frac{f_{\mathrm{p}} e_{\mathrm{LL}}^{1 \mathrm{G}}}{1-e_{\mathrm{LL}}^{1 \mathrm{G}}}+\frac{\left(1-f_{\mathrm{p}}\right) e_{\mathrm{LL}}^{2 \mathrm{G}}}{1-e_{\mathrm{LL}}^{2 \mathrm{G}}}\right) \times M_{\mathrm{obs}}$,
where we also allow for a fraction $e_{\mathrm{LL}}^{2 \mathrm{G}}$ of low-mass stars of the second generation to escape (see Section 4).

In the dynamical GC scenario discussed by $\mathrm{DCM} 07, e_{\mathrm{LL}}^{1 \mathrm{G}}$ can be determined from the observed fraction $f_{\mathrm{p}}$ of first-generation stars for a given slope $x$ of the massive stars IMF, assuming a value for the global dilution factor $d$ (see their fig. 4). ${ }^{3}$ Since $f_{\mathrm{LL}}^{1 \mathrm{G}}$ depends only on the IMF, it is straightforward to compute the relation between the observed and the total initial mass of GCs (equation 1) and the amount of stellar mass ejected (equation 2).

## 3 THE INITIAL MASS OF GLOBULAR CLUSTERS

The ratio between the initial and present-day stellar mass computed in this manner is shown in the left-hand panel of Fig. 1. Blue lines
${ }^{3}$ One obtains $e_{\mathrm{LL}}^{1 \mathrm{G}}=1-\frac{1}{f_{\mathrm{LL}}^{1 \mathrm{G}}}\left(f_{\mathrm{SW}}(1+d)\left(1-e_{\mathrm{LL}}^{2 \mathrm{G}}\right)\left[\frac{1}{1-f_{\mathrm{p}}}-1\right]\right)$ from their equations (3) and (23), for the assumptions of scenario II, but relaxing the hypothesis of $e_{\mathrm{LL}}^{2 \mathrm{G}}=0$, i.e. allowing also for loss of second-generation stars.
show the case of $e_{\mathrm{LL}}^{2 \mathrm{G}}=0$ as in DCM07, the green and magenta curves when accounting for $e_{\mathrm{LL}}^{2 \mathrm{G}}=0.43$ and 0.65 , respectively (see below). As can be seen, the steeper the IMF, the higher the ratio $m$ between the initial and the current mass. This is the case since in the present framework massive stars are responsible for the chemical pollution of the GC that leads to the formation of second-generation low-mass stars. Hence, for a steeper IMF, a larger total mass of stars is required to compensate the relative decrease of massive stellar polluters in order to reproduce $f_{\mathrm{p}}$. In this way, the same total mass of massive star ejecta incorporated into the second-generation stars can be produced. As mentioned above, all results shown here are computed adopting $d=1.15$ for the dilution parameter. A stronger dilution of the ejecta from the rapidly rotating massive stars (i.e. increasing $d$ ) would imply lower values of $M_{\mathrm{ini}} / M_{\mathrm{obs}}$ and $M_{\mathrm{eje}}^{\mathrm{LL}} / M_{\mathrm{obs}}$, since more material is then available to form the second-generation stars. In practice, changing $d$ by a factor of 2 (3) around our 'standard' value implies changes of $\sim 40-50(100)$ per cent in the initial and ejected masses, comparable to the differences between the different cases illustrated in Fig. 1.
For a Salpeter IMF as assumed in DCM07, the initial cluster mass is found to be $\sim 8-10$ times larger than the current (observed) mass when no second-generation stars are lost, as seen in Fig. 1. Considering massive AGBs as potential GC polluters (instead of massive stars), and assuming that none of the second-generation stars is lost, Prantzos \& Charbonnel (2006) and Carretta et al. (2010b) also found that the original cluster population should have been larger than the current one by approximately 1 order of magnitude for a Salpeter IMF. This agreement is dictated by the amount of initial mass of polluters needed to provide enough material for the second stellar generation. If we attribute the recently observed second-generation stars in the halo to the present population of GCs (cf. Section 4.), we must allow for a loss of second-generation stars (i.e. $e_{\mathrm{LL}}^{2 \mathrm{G}}>0$ ), which implies even higher initial masses, as shown by the green and magenta curves in Fig. 1. In this case, we typically find initial cluster masses (in stars) 15-25 times the present-day mass, for a Salpeter slope. Of course, the masses of proto-GC clouds must be even higher, depending on their star-formation efficiency.

## 4 CONTRIBUTION TO THE GALACTIC STELLAR HALO

The ratio between the ejected stellar mass and the present-day mass and its dependence on the IMF slope is illustrated in the right-hand panel of Fig. 1. The total amount of low-mass ( $<0.8 \mathrm{M}_{\odot}$ ), longlived stars (both the first and second generation) ejected from the cluster shown by the blue lines corresponds to $\sim(2.5-3.2)$ times the observed, present-day mass of GCs, for a Salpeter IMF in the case where no second-generation stars are lost as assumed by DCM07. This amount increases for a steeper IMF. These stars must contribute to the population of the Galactic halo.

Recent observations have found indications for chemically polluted, second-generation stars in the Galactic halo, with a frequency of $f_{\mathrm{s}} \sim 1.4-2.5$ per cent (Carretta et al. 2010b; Martell \& Grebel 2010). While Vesperini et al. (2010) have examined the ejection of these stars with hydrodynamic cluster models, we follow here a different approach. If these stars originate from the population of present-day GCs, we can easily infer the escape fraction of secondgeneration stars $e_{\mathrm{LL}}^{2 \mathrm{G}}=\left[1+\left(1-f_{\mathrm{p}}\right) /\left(f_{\mathrm{s}} \times M_{\text {halo }} / M_{\text {now }}^{\mathrm{GC}}\right)\right]^{-1}$, where $M_{\mathrm{GC}}^{\text {now }} / M_{\text {halo }} \approx 2$ per cent is the fraction of present-day GC of the total stellar halo mass (Freeman \& Bland-Hawthorn 2002). For $f_{\mathrm{s}}=1$ (2.5) per cent, we obtain $e_{\mathrm{LL}}^{2 \mathrm{G}}=0.43$ (0.65), i.e. a loss of approximately half of the second-generation low-mass stars. With
such a loss, the initial cluster masses and the total amount of ejected stars must be even higher than discussed above. The corresponding values are shown in Fig. 1 with green and magenta lines. Typically, both initial and ejected mass are increased by a factor of 1.7-3.5 compared to the case of $e_{\mathrm{LL}}^{2 \mathrm{G}}=0$. For a Salpeter slope, the mass of ejected low-mass stars is then $\sim 5-10$ times the present-day GC mass.

From the current total mass of halo Galactic GCs of $M_{\text {now }}^{\mathrm{GC}} \sim$ $2 \times 10^{7} \mathrm{M}_{\odot}$ and the total halo mass $M_{\text {halo }} \approx 10^{9} \mathrm{M}_{\odot}$ (Freeman \& Bland-Hawthorn 2002), i.e. the above 2 per cent, we therefore find that low-mass stars ejected from the present-day population of GCs make up $\sim 5-8$ per cent of the mass of halo stars if $e_{\mathrm{LL}}^{2 \mathrm{G}}=$ 0 , or $10-20$ per cent (for the above values $f_{\mathrm{s}}$ ) if all halo secondgeneration stars also come from these clusters. These numbers could be a factor of 0.75 lower if a lower mass-to-light ratio of $M / L_{V}=$ 1.5 (cf. Dubath \& Grillmair 1997; Larsen et al. 2002) instead of 2 adopted by Freeman \& Bland-Hawthorn (2002, and others) was more appropriate.
For comparison, Carretta et al. (2010b) estimate a minimum contribution of GC stars to the stellar halo of 2.8 per cent, but up to a factor of $\sim 10$ more, while from Martell \& Grebel (2010), one obtains a contribution of $\sim 17.5$ per cent. ${ }^{4}$ These estimates, based on the observed fraction of second-generation stars in the halo and on models accounting for multiple stellar generations, agree with ours. Other authors, using calculations of the GC survival fraction or their destruction rate (cf. Gnedin \& Ostriker 1997), estimate contributions of $\lesssim 10$ per cent (Parmentier \& Gilmore 2007) or less (38 per cent; Boley et al. 2009) to the Galactic halo from the presentday GC population. Other estimates, based on a variety of different IMFs of stellar clusters, range e.g. from 4 to 40 per cent (Fall \& Zhang 2001), $\sim 40$ to 50 per cent (Baumgardt, Kroupa \& Parmentier 2008), or from 30 to 80 per cent (Boley et al. 2009). However, these studies neglect the observed multiple stellar generations in GC and their implications.

## 5 IMPLICATIONS FOR THE INITIAL GLOBULAR CLUSTER MASS FUNCTION

The fact that GCs show - now basically by definition (cf. Carretta et al. 2010b) - stellar generations, distinct by their chemical abundances, allows us to take a new step in constraining their initial cluster mass function. Indeed, since stars showing the abundances characteristic of second-generation stars have recently been found in the Galactic halo (Carretta et al. 2010b; Martell \& Grebel 2010), their frequency among normal halo stars provides interesting, new constraints within the framework of the model examined here.

Consider two limiting cases. First, let us assume that all secondgeneration stars found in the halo originate from the present-day population of GCs. As discussed above, one then finds for the initial mass of GCs typically $M_{\mathrm{ini}} \approx(15-25) \times M_{\text {obs }}$. The GCIMF must then be equal to the observed one - commonly described by a lognormal with a characteristic mass $M_{c} \sim 1.4 \times 10^{5} \mathrm{M}_{\odot}$ (cf. Harris 1991) - but with $M_{c}$ increased by a factor of $15-20$, i.e. $M_{\mathrm{c}} \sim(2.1-3.5) \times 10^{6} \mathrm{M}_{\odot}$. Besides this, there is no room left for other proto-GCs, since these would otherwise contribute - by definition - additional second-generation stars to the halo.
Now assume the other extreme, namely none of the observed second-generation halo stars is from the observed GCs. This

[^3]corresponds to $e_{\mathrm{LL}}^{2 \mathrm{G}}=0$, and we know then that the present-day GC population had an IMF with a characteristic mass $\sim(8-10)$ times the present value ( $M_{\mathrm{c}} \sim(1.1-1.4) \times 10^{6} \mathrm{M}_{\odot}$ ) and a total mass of $M_{\mathrm{ini}}^{\mathrm{GC}} \sim(1.6-2) \times 10^{8} \mathrm{M}_{\odot}$. In addition, however, other, dissolved GCs must be invoked to explain the presence of these peculiar stars in the halo. Let us assume that the total GCIMF is given by a power law with a slope $\beta=-2$, as often studied (e.g. Fall \& Zhang 2001; Boley et al. 2009). The lowest normalization we can chose is the one tangential to, i.e. osculating the lognormal IMF of the present-day GCs, as discussed e.g. in Boley et al. (2009). From this, we compute the total initial mass of GCs to be dissolved, $M_{\mathrm{ini}}^{\mathrm{GC}--d i s s}$, by subtracting $M_{\mathrm{ini}}^{\mathrm{GC}}$ from $M_{\mathrm{ini}}^{\text {tot }}$ given by the integral of the GCIMF over the same range considered by Boley et al. (2009). ${ }^{5}$ Since our osculating mass is ( $8-10$ ) times higher than that of Boley et al. (2009), we obtain $M_{\text {ini }}^{\text {tot }}=(1.6-2) \times 10^{9} \mathrm{M}_{\odot}$, from which 90 per cent (or more) is in GCs which must be dissolved. Since the fraction of low-mass ( $\lesssim 0.8 \mathrm{M}_{\odot}$ ) stars is $f_{\mathrm{LL}}^{1 \mathrm{G}} \sim 0.35$ for a Salpeter slope, all the globulars then contribute approximately $50-70$ per cent of the present-day stellar halo mass. ${ }^{6}$

From equation (1) and counting the fraction $\left(1-f_{\mathrm{p}}\right)$ low-mass stars, we finally obtain the total amount of second-generation stars produced in these clusters $M_{2 \mathrm{G}}^{\mathrm{GC}}$-diss $\approx 0.075 \times M_{\mathrm{ini}}^{\mathrm{tot}}$, which corresponds to a fraction $f_{\mathrm{s}} \sim 12-15$ per cent of the halo mass. If we adopt a lower mass-to-light ratio $\left(M / L_{V}=1.5\right)$ and assume that 30 per cent of the present GC mass consists of stellar remnants, the expected fraction of second-generation stars in the halo may be somewhat lower, $\sim 6-8$ per cent. To compute this value, we have implicitly assumed - in the absence of other information - that all GCs can be described by the same values of $f_{\mathrm{p}}$ and $e_{\mathrm{LL}}^{1 \mathrm{G}}$ as those derived from the present-day GCs.

As can be seen, our theoretical prediction for the fraction $f_{\mathrm{s}}$ of second-generation stars in the halo is considerably larger than the current observational values of $\sim 1.4-2.5$ per cent from Carretta et al. (2010b) and Martell \& Grebel (2010). The simplest conclusion from this contradiction is that the GCIMF cannot be a simple power law with $\beta=-2$, as suggested by numerous authors (cf. Fall \& Zhang 2001; Boley et al. 2009; Elmegreen 2010a), at least not over the mass range considered here. However, to reconcile our prediction with the observed value of $f_{\mathrm{s}}$, one would need to strongly reduce range of the initial cluster masses, since each decade in mass contains the same amount of total mass for this power-law distribution. In other words, reducing the predicted $f_{\mathrm{s}}$ by a factor of 4 or more would imply an initial cluster mass function over less than 1 dex, compared to our assumption of $\log (m) \in[3.3,7.3] \mathrm{M}_{\odot}$. Similarly, postulating e.g. that clusters below a certain mass (say the present-day value of $M_{\text {obs }} \lesssim 4 . \times 10^{4} \mathrm{M}_{\odot}$ suggested by Carretta et al. 2010b) will not become globulars does not solve our problem. Alternatively, in most clusters, the fraction $f_{\mathrm{p}}$ of unpolluted stars could be higher than the value observed in present-day GCs, in which case one could avoid 'overproducing' the number of second-generation stars in the halo. In this case, however, we cannot properly speak of an IMF for GCs, since these objects with much higher values of $f_{\mathrm{p}}$ cannot be the progenitors of the present-day GC population.

We are therefore naturally drawn to abandon the picture of a 'universal' initial power-law mass function for all clusters, including super star clusters, young massive clusters, etc., and for progenitors

[^4]of present-day GCs. Then, as already discussed above, the observations of the second-generation halo stars can be understood if the GCIMF is lognormal as e.g. proposed by Vesperini \& Zepf (2003) and Parmentier \& Gilmore (2005, 2007). However, other IMFs, e.g. a power law with a turnover or Schechter-type functions, cannot yet be excluded.

In any case, we have shown here that the observed fraction second-generation stars in the halo can in principle provide very useful information on the distribution of the initial masses of GCs, the GCIMF. Of course, our analysis does not constrain the IMF of other (non-globular) clusters. In fact, since the percentage of halo stars originating from GCs is typically $\lesssim 20$ per cent in our scenarii, there is room for other clusters, accreted satellites, or others to provide the rest of the present-day stellar halo. After many recent studies proposing a 'unified' picture for the formation and evolution of clusters of all kinds including GCs (cf. Meurer 1995; Fall \& Zhang 2001; Vesperini \& Zepf 2003; Brodie \& Strader 2006; Elmegreen 2010a, and references therein), it may well be that recent progress on GC stars and the finding of second-generation stars among the halo population force us again to revise this picture. One of the main questions arising now is actually what distinguishes 'normal' clusters from globulars and 'globulars-to-become', i.e. what causes a cluster to form one or two separate stellar populations. Is this e.g. related to their initial central density, to external conditions, or maybe to completely different formation scenarios, as e.g. suggested by Searle \& Zinn (1978), Freeman (1993) and Böker (2008)? Or can this be understood within the framework of current hydrodynamic and cosmological formation models (e.g Boley et al. 2009; Elmegreen 2010a)?

New detailed (hydro) dynamic models of cluster formation and evolution taking into account recent insights gained from scenarii explaining the detailed behaviour of observed abundance pattern in GC stars (e.g. Decressin et al. 2008, 2010; D'Ercole et al. 2008) are clearly likely needed to progress further on this issue. In parallel, it will be useful to firm up the first studies of second-generation stars found in the Galactic halo, as they currently suffer e.g. from poor statistics (Carretta et al. 2010b) or from uncertainties in observational criteria identifying these stars (Martell \& Grebel 2010).

## 6 THE CONTRIBUTION OF GLOBULAR CLUSTERS TO REIONIZATION

Since the initial masses of GCs may be substantially larger than their present-day values, their output of ionizing radiation and their contribution to cosmic reionization also needs to be revised.

In Fig. 2, we plot the predicted H ionizing photon output (i.e. the total number of photons emitted above 13.6 eV ) during the life of a GC normalized to its current number of baryons, ${ }^{7} \eta^{\prime}$, as a function of the IMF slope for the dynamical scenario described previously. The Lyman continuum flux was computed using the evolutionary synthesis code of Schaerer (2003) and Raiter, Schaerer \& Fosbury (2010) for a low metallicity $Z=1 / 20 Z_{\odot}$ typical of GCs. ${ }^{8}$

Interestingly, the predicted ionizing photon output is quite independent of the IMF slope (see Fig. 2), since both the metals

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Figure 2. Ionizing photon production of GCs normalized per baryon currently locked up in stars, $\log \eta^{\prime}$, plotted as a function of the massive star IMF slope $x$. Blue, green and magenta lines show values of $\eta^{\prime}$ for IMF slopes allowing one to reproduce the observed fraction $f_{\mathrm{p}}$ (boundaries shown by dotted lines) assuming $e_{\mathrm{LL}}^{2 \mathrm{G}}=0,0.43$ and 0.65 respectively. The value of $\eta$ from Ricotti (2002) computed for metallicity $Z=0.03 Z_{\odot}$ and the Salpeter slope ( $x=1.35$ ) is shown for comparison.
explaining the observed abundance pattern and the ionizing photons are made primarily in massive stars. Should the 'pollution' of second-generation stars be related to massive AGB stars, we expect a similarly large emission of ionizing photons per baryon, since the ratio between the initial and present-day mass is similar in both scenarios (cf. above). In the AGB scenario, the dependence of $\eta^{\prime}$ on the IMF should, however, be somewhat stronger, since the masses responsible for the stellar ejecta and the ionizing photons are more distinct than in the fast-rotating, massive star scenario. Of course, one also finds an increased output of ionizing photons per unit present-day mass (or baryon number) if loss of second-generation stars is allowed (green and magenta lines) compared to the case where $e_{\mathrm{LL}}^{2 \mathrm{G}}=0$ (blue lines). This increase is simply due to higher initial mass of GCs, discussed previously.

Ricotti (2002) has estimated the output of GCs per baryon as $\eta^{\prime}=\eta \times f_{\text {di }}$, with $\eta \approx 9000^{9}$ and $f_{\mathrm{di}} \sim 2-10$ (with a maximum of $f_{\mathrm{di}}<100$ ), where $f_{\mathrm{di}}$ - the equivalent of our ratio $M_{\mathrm{ini}} / M_{\text {obs }}-$ is a factor accounting for dynamical disruption of GCs during their lifetime. In our case, the total photon output is $\log \left(\eta^{\prime}\right) \approx 4.8-$ 4.9 photon baryon ${ }^{-1}$ if all second-generation stars remain within the cluster, which is approximately a factor of 5 higher than the typical value adopted by Ricotti (2002) for $f_{\mathrm{di}} \sim 2$. The amount of emitted ionizing photons must be even higher if second-generation stars were lost from these clusters $\left(e_{\mathrm{LL}}^{2 \mathrm{G}}>0\right)$, as illustrated by the green and magenta lines.

Using simple but elegant arguments to estimate the number of ionizing photons emitted per baryon during the formation of GCs, and using a Press-Schechter model to compute the GC formation rate, Ricotti (2002) has estimated the contribution of GCs to cos-

[^6]mic reionization. He has shown that the old GCs produced enough ionizing photons to reionize the intergalactic medium at $z \approx 6$, if the escape fraction of ionizing photons, $f_{\text {esc }}$, from these objects was close to unity. Our finding of a high ionizing photon production in GCs reinforces the conclusions of Ricotti (2002) and leaves room for lower values of $f_{\text {esc }}<1$ or for other less favourable assumptions (e.g. uncertainties in the age of GCs). It appears that old GCs, formed as massive super star clusters shortly after the big bang, could provide a significant if not dominant source of ultraviolet (UV) radiation to reionize the Universe at high redshift. In any case, it seems unavoidable to seriously consider their contribution.

With a typical mass of $\sim 1.4 \times 10^{6} \mathrm{M}_{\odot}$, a young proto-GC is expected to have a peak UV (say $1500 \AA$ ) magnitude of $M_{\mathrm{AB}} \approx-16.7$ at an age $\sim 2-3 \mathrm{Myr}$, before fading by $\sim 4 \mathrm{mag}$ within $\sim 20 \mathrm{Myr}$ just due to stellar evolution (see e.g. models of Leitherer et al. 1999; Schaerer 2003) or faster when evaporation (mass-loss) sets in. At redshifts of $z \sim 7-10$, this would correspond to a typical UV restframe magnitude of $m_{\mathrm{AB}} \sim 30.3$ at the peak, just slightly fainter than the current detection limits of the deepest near-infrared (IR) images taken with the WFC3 camera onboard HST (cf. Bouwens et al. 2010). In any case, single, massive proto-GCs during their youth might be detectable in situ with current instrumentation and are certainly well within the reach of even deeper near-IR observations, which will be achieved with the James Webb Space Telescope (JWST). However, whether the two proposed scenarii (massive AGB or fast-rotating, massive stars as the origin of the bulk of material out of which second-generation stars form in GCs) can be distinguished observationally at high redshift appears a priori quite difficult, if not impossible.

## 7 SUMMARY AND CONCLUSIONS

In light of the recently recognized general existence of multiple stellar generations in GCs implying significant losses of first-generation stars from these clusters, we have re-examined the initial masses of GCs, the contribution of low-mass stars ejected from GCs to the stellar halo of our Galaxy and the contribution of GCs to the ionizing photon production necessary to reionize the intergalactic medium at high redshift.

These quantities have been estimated from the chemical and dynamical model of DCM07, which successfully reproduces the main observational constraints from first- and second-generation stars, by invoking pollution from fast-rotating massive stars. The main free parameters of this model are the slope of IMF for high masses $\left(>0.8 \mathrm{M}_{\odot}\right.$, the IMF being fixed to the observed lognormal distribution for lower masses), the relative number of first/second-generation stars, given by the fraction $f_{\mathrm{p}}=0.33_{-0.08}^{+0.07}$ of first-generation stars determined from the detailed spectroscopic observations of Carretta et al. (2010b), and a dilution parameter $d \approx 1.15$ inferred from the $\mathrm{Li}-\mathrm{Na}$ anticorrelation observed in GCs (DCM07; Charbonnel \& Decressin, in preparation).

The dynamical scenario we have explored allows for the evaporation of stars from the first generation (corresponding to an escape fraction of second-generation stars of zero, $e_{\mathrm{LL}}^{2 \mathrm{G}}=0$ ), or from both generations, as suggested by recent observations finding stars characteristic of the second generation in GCs in the Milky Way halo (Carretta et al. 2010b; Martell \& Grebel 2010). The latter case translates to $e_{\mathrm{LL}}^{2 \mathrm{G}} \sim 0.43-0.65$.

We have obtained the following main results for an IMF with a Salpeter slope above $0.8 \mathrm{M}_{\odot}$.
(i) The initial stellar masses of GCs must have been $\sim 8-10$ times larger than the current (observed) mass, when no second-generation stars are lost, in agreement with the earlier results of Prantzos \& Charbonnel (2006) and Carretta et al. (2010b). If all secondgeneration halo stars originate from the present population of GCs, the initial cluster masses must have been $\approx 25$ times larger than the current mass.
(ii) The mass in low-mass stars ejected from GCs must be $\sim 2.5-$ 3.2 times their observed stellar mass if all second-generation stars were retained, or $\sim 5-10$ times the present-day mass if $e_{\mathrm{LL}}^{2 \mathrm{G}} \sim 0.43-$ 0.65 . These numbers translate to a contribution of $5-8$ per cent or 10-20 per cent, respectively, of the ejected low-mass stars to the Galactic stellar halo mass. We have compared our estimate with earlier values obtained from various methods (cf. Section 4).
(iii) The observations of second-generation stars in the Galactic halo can constrain the IMF of the GC population (GCIMF). In particular, we have shown that a power law with a slope $\beta \approx-2$, as often assumed, is in contradiction with recent determinations of the fraction of second-generation stars in the halo, whereas a lognormal GCIMF is compatible with these observations. This finding revives the question about a common mass function and about the physical processes leading to a distinction between GCs with multiple stellar populations and other clusters.
(iv) Due to their high initial masses, the amount of Lyman continuum photons emitted by GCs during their youth must have been substantial. Indeed, we find that their output corresponds to a total number of ionizing photons emitted per baryon, $\eta^{\prime} \approx 10^{4.8-4.9}$ for $e_{\mathrm{LL}}^{2 \mathrm{G}}=0$, or $\sim 1.7-3.5$ times more if $e_{\mathrm{LL}}^{2 \mathrm{G}} \sim 0.4-0.6$. Our results reinforce the conclusion of Ricotti (2002) that GCs should contribute significantly to reionize the IGM at very high redshift $(z \gtrsim 6)$. Individual, young proto-GCs with typical masses few times $\sim 10^{6} \mathrm{M}_{\odot}$ could just be detectable at high redshift in ultradeep images with the HST and are certainly within the reach of the JWST.

The dependence of the initial and ejected masses on the IMF slope has been illustrated in Fig. 1. The ionizing photon production is found to be quite insensitive to the high-mass IMF, since both the ejecta 'polluting' the second-generation stars and the Lyman continuum flux originate from massive stars. Our main results should also be valid for the massive AGB scenario, at least qualitatively.

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[^1]:    ${ }^{1}$ With the notable exception of $\omega$ Cen, M22 and M54 (see e.g. Siegel et al. 2007; Da Costa et al. 2009; Carretta et al. 2010a; Johnson \& Pilachowski 2010, and references therein).

[^2]:    ${ }^{2}$ Following standard stellar evolution, stellar remnants constitute approximately 30 per cent of the total cluster mass after 12 Gyr . This fraction may, however, depend also on the dynamical evolution of the cluster (cf. Kruijssen \& Lamers 2008).

[^3]:    ${ }^{4}$ Their estimated 50 per cent of the halo mass corresponds to the total stellar mass including the full mass spectrum. For a Salpeter slope, one has a fraction $f_{\mathrm{LL}}^{1 \mathrm{G}} \sim 0.35$ of low-mass stars, i.e. 17.5 per cent.

[^4]:    ${ }^{5}$ Approximately over $\log (M) \in[3.27,7.27]$, corresponding to a range of magnitudes $V$ from -12 to -2 for the present-day GC mass function.
    ${ }^{6}$ Assuming $M / L_{V}=1.5$ instead of 2 (cf. above), this percentage would be lower by a factor of 0.75 .

[^5]:    ${ }^{7}$ To convert photon/baryon into photon/mass, one has e.g. $1 \mathrm{M}_{\odot}=1.19 \times$ $10^{57}$ baryon.
    ${ }^{8}$ Adopting a different metallicity leads to small changes (typically $\lesssim 0.1-$ 0.2 dex). Similarly, using the stellar evolutionary tracks of fast-rotating stars used in the study of DCM07 would lead to relatively small changes compared to the effects discussed here.

[^6]:    ${ }^{9}$ Making the same assumptions as Ricotti, we confirm this value.

