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Explosion energies, nickel masses and distances of Type II plateau supernovae

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ABSTRACT

The hydrodynamical modelling of Type II plateau supernova (SNIIP) light curves predicts a correlation between three observable parameters (plateau duration, absolute magnitude and photospheric velocity at the middle of the plateau) on the one hand, and three physical parameters (explosion energy E, mass of the envelope expelled $\mathcal{M}$ and pre-supernova radius R) on the other. The correlation is used, together with *adopted* distances from the expanding photosphere method, to estimate E, $\mathcal{M}$ and R for a dozen well-observed SNIIP. For this set of supernovae, the resulting value of E varies within a factor of 6 ($0.5 \lesssim E/10^{51}$ erg $\lesssim 3$), whereas the envelope mass remains within the limits $10 \lesssim \mathcal{M}/\mathcal{M}_{\odot} \lesssim 30$. The pre-supernova radius is typically 200–600 $R_{\odot}$, but can reach $\gtrsim$1000 $R_{\odot}$ for the brightest supernovae (e.g. SN 1992am).

A new method of determining the distance of SNIIP is proposed. It is based on the assumption of a correlation between the explosion energy E and the ^{56}Ni mass required to power the post-plateau light curve tail through ^{56}Co decay. The method is useful for SNIIP with well-observed bolometric light curves during both the plateau and radioactive tail phases. The resulting distances and future improvements are discussed.

Key words: supernovae: general – galaxies: distances and redshifts.

1 INTRODUCTION

Type II plateau supernovae (SNIIP) are believed to come from the explosion of massive supergiant stars whose envelopes are rich in hydrogen. Their light curves are easy to identify by a long plateau (sometimes up to 120–150 d), which is the result of the propagation of a cooling and recombination wave (CRW) through the supernova (SN) envelope that is in a state of free inertial expansion ($u = r/t$). The CRW physics is discussed in detail by Imshennik & Nadyozhin (1964), Grassberg, Imshennik & Nadyozhin (1971) and Grassberg & Nadyozhin (1976). The CRW propagates supersonically downwards through the expanding supernova envelope and separates almost recombined outer layers from still strongly ionized inner ones. During the plateau phase, the photosphere sits on the upper edge of the CRW front. Since the CRW downward speed turns out to be close to the velocity of the outward expansion, the photospheric radius changes only slowly during the plateau phase. If one takes into account that also the effective temperature does not change appreciably (it approximately equals the recombination temperature, 5000–7000 K), the approximate constancy of the luminosity becomes obvious.

The supernova outburst properties are determined mainly by three physical parameters: explosion energy E, mass $\mathcal{M}$ of the envelope expelled and initial radius R of the star just before the explosion (pre-supernova). Litvinova & Nadyozhin (1983, 1985) have undertaken an attempt to derive these parameters from a comparison of the hydrodynamical supernova models with observations. They constructed simple approximation formulae that allow one to estimate E, $\mathcal{M}$ and R from observations of individual SNIIP. Their results were confirmed by an independent semi-analytical study (Popov 1993). At that time, only one or two supernovae were sufficiently well observed to apply these formulae. At present, there exist detailed observational data for 14 such supernovae, including in 12 cases distances from the expanding photosphere method (EPM), which we use in Section 2 to estimate E, $\mathcal{M}$ and R by means of these formulae.

In Section 3, we propose a new method of distance determination and apply it to nine individual SNIIP that are well observed at both the plateau and radioactive tail phases. The method is based on the assumption of a correlation between the explosion energy E and the mass of ^{56}Ni in the supernova envelope. In Section 4 we compare physical parameters and distances of SNIIP as derived from the new method with those obtained previously from the EPM and discuss also other aspects of our results. Concluding remarks are given in Section 5.

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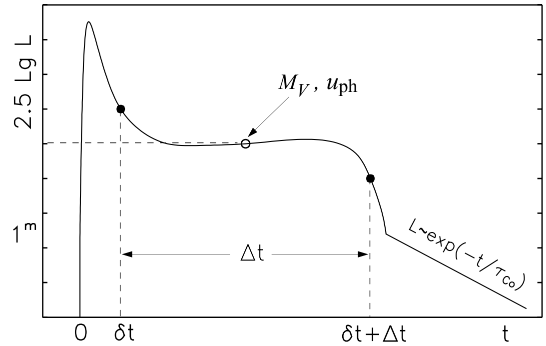

Figure 1. A schematic SNIIP light curve. The open circle marks the middle of the plateau and the two full circles show the plateau boundaries. The light curve tail powered by ^{56}Co decay is also shown ($\tau_{Co} = 111.3$ d).

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2 A COMPARISON OF HYDRODYNAMIC MODELS WITH OBSERVATIONS

Fig. 1 shows a schematic SNIIP light curve. The plateau is defined as part of the light curve on which the supernova brightness remains within 1 mag of the mean value. For some supernovae, the plateau begins almost immediately after the onset of the explosion ($t = 0$), whereas for others a short luminosity peak can precede the plateau. The peak either appears as a result of a shock wave breakout in the case of pre-supernovae of not very large initial radii ($R \lesssim 1000\,R_\odot$) or, according to Grassberg et al. (1971), originates from the emergence of a thermal wave precursor for pre-supernovae of very large radii ($R \approx 2000$–$5000\,R_\odot$) and of moderate explosion energies ($E \lesssim 1 \times 10^{51}$erg), or, finally, it may occur as a result of interaction between the supernova envelope and a dense stellar wind (Grassberg & Nadyozhin 1987). For some SNIIP the peak duration δt lasts only a few days and is difficult to observe (shock wave breakout); for others it could be as large as 10–20 d (thermal wave or dense wind). Examples of the latter may be supernovae such as SNe 1988A, 1991al and 1992af (see below).

It is quite clear that the middle of the plateau is to be used as the main reference point to compare the theoretical models with observations. Litvinova & Nadyozhin (1983, 1985, hereafter LN83, LN85) calculated a grid of supernova models for E, $\mathcal{M}$ and R within limits of 0.18–2.91 × 10^{51} erg, 1–16 $\mathcal{M}_\odot$ and 300–5000 $R_\odot$. They found E, $\mathcal{M}$ and R to be strongly correlated with the plateau duration Δt, the mid-plateau value of the absolute V magnitude M_V, and the expansion velocity u_{ph} at the level of the photosphere (Fig. 1). According to LN85, the following approximate relations can be used to derive E, $\mathcal{M}$ and R from observations:

$$\lg E = -0.135M_V + 2.34 \lg \Delta t + 3.13 \lg u_{ph} - 4.205, \tag{1}$$

$$\lg \mathcal{M} = -0.234M_V + 2.91 \lg \Delta t + 1.96 \lg u_{ph} - 1.829, \tag{2}$$

$$\lg R = -0.572M_V - 1.07 \lg \Delta t - 2.74 \lg u_{ph} - 3.350, \tag{3}$$

where E is expressed in units of 10^{51} erg, $\mathcal{M}$ and R are in solar units, Δt in days, and u_{ph} in 1000 km s^{-1}. Here M_V can be expressed through the apparent V magnitude by the relation

$$M_V = V - A_V - 5 \lg(D/1\,\mathrm{Mpc}) - 25, \tag{4}$$

where D is the distance to a supernova and A_V is the total absorption on the way to the supernova. One can find from equations (1)–(3) that E, $\mathcal{M}$ and R scale with the distance as

$$E \sim D^{-0.675}, \qquad \mathcal{M} \sim D^{-1.17}, \qquad R \sim D^{2.86}. \tag{5}$$

Thus, it is very important to know D with as high accuracy as possible. We have selected 14 SNe, whose observational data are collected in Table 1. The entries are: the heliocentric recession velocity v_0 (from the NASA/IPAC Extragalactic Database, NED) in column 3; the total absorption A_V in column 4; the apparent V magnitude of the mid-point of the plateau in column 5; the duration Δt of the plateau in column 6; and the photosphere expansion velocity u_{ph} in column 7. References are given in column 8.

Table 1. Observational data for 14 SNIIP.

SN (1)	Host galaxy (2)	v_0 (km s^{-1}) (3)	A_V (mag) (4)	V (mag) (5)	Δt (d) (6)	u_{ph} (km s^{-1}) (7)	Ref.[a] (8)
1968L	NGC 5236	516	0.219	12.0	80	4100	1,2,3
1969L	NGC 1058	518	0.203	13.4	100	4000	1,2
1986L	NGC 1559	1292	0.099	14.7	110	4000	4
1988A	NGC 4579	1519	0.136	15.0	110	3000	1,2,4,5,6
1989L	NGC 7339	1313	1.00	16.5	140	3000	7,19
1990E	NGC 1035	1241	1.083	16.0	120	4000	2,4,8,9
1991al	LEDA 140858	4572	0.318	17.0	90	6000	4
1992af	ESO 340-G038	6000	0.171	17.3	90	6000	4,7
1992am	anon 0122-04	14600	0.464	19.0	110	4800	4,10
1992ba	NGC 2082	1104	0.193	15.43	100	2900	4,7
1999cr	ESO 576-G034	6069	0.324	18.6	100	3600	4
1999em	NGC 1637	717	0.314	14.0	110	3000	4,11,12,13,17
1999gi	NGC 3184	592	0.65	15.0	110	2900	14,15,16,18
1987A	LMC	278	0.465	3.3	110	2900	4

[a]References: (1) Patat et al. (1993); (2) Schmidt, Kirshner & Eastman (1992); (3) Wood & Andrews (1974); (4) Hamuy (2001); (5) Ruiz-Lapuente et al. (1990); (6) Turatto et al. (1993); (7) Schmidt et al. (1994a); (8) Schmidt et al. (1993); (9) Benetti et al. (1994); (10) Schmidt et al. (1994b); (11) Hamuy et al. (2001); (12) Haynes et al. (1998); (13) Baron et al. (2000); (14) Schlegel (2001); (15) Smartt et al. (2001); (16) Li et al. (2002); (17) Elmhamdi et al. (2003); (18) Leonard et al. (2002b); (19) Pennypacker & Perlmutter (1989).

Table 2. The supernova physical properties.

SN	v_{220} (km s^{-1})	$D_{\rm H}$ (Mpc)	$D_{\rm EPM}$[a] (Mpc)	M_V (mag)	E (10^{51} erg)	$\mathcal{M}$ ($M_\odot$)	R (R$_\odot$)	$\mathcal{M}_{\rm Ni0}$ ($M_\odot$)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1968L	291	4.85	4.5[(1)]	−16.65	0.83	10.3	286	
1969L	766	12.77	10.6[(1)]	−17.33	1.05	13.0	595	
1986L	1121	18.68	16.0[(1)]	−16.76	1.56	23.5	251	0.026
1988A	1179	19.65	20.0[(1)]	−16.60	0.67	14.5	452	0.082
1989L	1556	25.93	17.0[(1)]	−16.57	1.18	29.8	334	
1990E	1238	20.63	18.0[(1)]	−16.66	1.98	31.9	200	0.052
1991al	4476	74.60	70.0[(5)]	−17.68	2.61	17.6	347	0.12
1992af	6000	100.00	83.70[(5)]	−17.87	2.46	15.9	445	0.24
1992am	14600	243.33	180.0[(1)]	−18.40	1.66	13.9	1321	0.36
1992ba	1096	18.27	22.0[(2)]	−16.07	0.57	13.7	272	0.029
1999cr	6069	101.15	86.0[(2)]	−16.75	0.90	14.5	368	0.085
1999em	743	12.38	8.20[(3)]	−16.78	0.63	13.2	569	0.058
1999gi	707	11.78	11.10[(4)]	−16.01	0.72	18.7	226	0.025[b]
1987A	–	0.05	0.05	−15.66	0.80	22.6	143	0.065

[a]References: [(1)]Eastman et al. (1996); [(2)]Hamuy (2001); [(3)]Leonard et al. (2002a); [(4)]Leonard et al. (2002b); [(5)]Hamuy (2001), based on an adopted value of $H_0 = 65$ (see text).
[b]Derived from $V = 17.86$ mag at $t = 174.3$ d (Leonard et al. 2002b), and Hamuy's recipe (Section 5.3 of his thesis) to convert V into luminosity L.

In order to check the extrapolative capability of equations (1)–(3), we have included SN 1987A in our analysis. It is well known that the pre-supernova radius of SN 1987A was as small as ≈50 R$_\odot$ – i.e. outside the interval of 300–5000 R$_\odot$ encompassed by the above equations. Moreover, the major part of the SN 1987A plateau (about 70 of 110 d) was powered by ^{56}Co decay (see the review of Imshennik & Nadyozhin 1989, and references therein).

Derived properties of the 14 SNIIP are given in Table 2. Column 2 lists the recession velocity v_{220} of the supernova corrected for a self-consistent Virgocentric infall model with a local infall vector of 220 km s^{-1} as described by Kraan-Korteweg (1986). Column 3 gives the distance $D_{\rm H} = v_{220}/H_0$ assuming arbitrarily a value of $H_0 = 60$ km s^{-1}Mpc^{-1}. For comparison, column 4 gives the distance $D_{\rm EPM}$ obtained with the use of the expanding photosphere method (EPM) in the references listed at the bottom of the table. The SNe 1991al and 1992af are the exception. Owing to the incompleteness of the observational data, it is hard to determine the EPM distance to SN 1991al (Hamuy 2001). For the same reason, the EPM distance of 55 Mpc for SN 1992af obtained by Schmidt et al. (1994a) seems to be quite uncertain as pointed out by Hamuy (2001). For these two SNe, we present in column 4 the distances calculated by Hamuy (2001) from the cosmic microwave background (CMB) redshifts and the Hubble constant $H_0 = 65$ km s^{-1} Mpc^{-1}. Columns 5–8 are the absolute magnitude M_V of the mid-point of the plateau, the explosion energy E, the mass expelled $\mathcal{M}$ and the pre-supernova radius R – all derived from equations (1)–(4) for the $D_{\rm H}$ distances listed in column 3. Column 9 gives the mass of ^{56}Ni, ejected by some supernovae, which was estimated by reducing the radioactive tail luminosities, measured by Hamuy (2001), to the distances $D_{\rm H}$ given in column 3.

For SN 1987A, the resulting values of E and $\mathcal{M}$ (Table 2) differ by no more than a factor of 1.5 from current estimates based on a detailed study. However, the pre-supernova radius turned out to be too large. This happened because the LN85 approximations do not take into account the radioactive heating. An advanced study (Grassberg & Nadyozhin 1986) demonstrates that radioactive heating only weakly influences E and $\mathcal{M}$ furnished by equations (1)–(3), whereas the R values can be overestimated by a factor of 3. In this connection, one should have in mind that for some supernovae the R values from Table 2 can be larger than actual pre-supernova radii.

According to Table 2, the resulting values of E, $\mathcal{M}$ and R seem to be reasonable enough: the expelled mass, explosion energy and pre-supernova radius remain approximately in limits 10–30 $\mathcal{M}_\odot$, 0.6–2.6 × 10^{51} erg and 200–1300 R$_\odot$, respectively. Hamuy (2001) assumed that SNe 1991al and 1992af were discovered several weeks after the explosion. Their plateaux, therefore, could have lasted for $\Delta t \approx 110$ d. It is quite probable, however, that their peak duration was $\delta t \approx 20$ d for the reasons mentioned above. Having this in mind, in Table 1 we have chosen $\Delta t = 90$ d, which results in $u_{\rm ph} = 6000$ km s^{-1}. In the case of $\Delta t = 110$ d, we would have to assume $u_{\rm ph} = 7000$ km s^{-1} and would obtain very large values of E and $\mathcal{M}$ for both supernovae: $E \approx 7 \times 10^{51}$ erg and $\mathcal{M} \approx 40\ \mathcal{M}_\odot$. No other special adjustments of the observational data given in Table 1 were made.

3 PLATEAU–TAIL DISTANCE DETERMINATION

The SNIIP light curve tails are believed to be powered by ^{56}Co decay. The temporal behaviour of the bolometric luminosity is given by (see e.g. Nadyozhin 1994)

$$L = 1.45 \times 10^{43} \exp\left(-\frac{t}{\tau_{\rm Co}}\right) \frac{\mathcal{M}_{\rm Ni0}}{\mathcal{M}_\odot} \, {\rm erg\,s^{-1}}, \qquad (6)$$

where t is measured from the moment of explosion ($t = 0$), $\mathcal{M}_{\rm Ni0}$ is the total mass of ^{56}Ni at $t = 0$ which decays with a half-life of 6.10 d into ^{56}Co, and $\tau_{\rm Co} = 111.3$ d.

Equation (6) can be written in the form

$$\mathcal{M}_{\rm Ni0} = \frac{D^2}{145} Q, \qquad Q \equiv F_{41}(t) \exp\left(\frac{t}{111.3}\right), \qquad (7)$$

where $\mathcal{M}_{\rm Ni0}$ is in $\mathcal{M}_\odot$, t in days and D in Mpc. The quantity $F_{41}(t)$ is the bolometric tail luminosity measured at time t in units of 10^{41} erg s^{-1} under the assumption that the supernova is at distance $D = 1$ Mpc. Equation (7) contains a single observational parameter

Q, which is independent of t and also makes no assumption on D as long as F_{41} is fixed by observations.

Thus, it is irrelevant at which t the luminosity is actually measured – one has only to be sure that the supernova has really entered its tail phase. Columns 8–10 of Table 3 give t and corresponding values of $F_{41}(t)$ and Q derived from Hamuy et al.'s (2001) figs 5.7 and 5.8, except SN 1999gi, for which the values were calculated from the data of Leonard et al. (2002b).

If the value of $\mathcal{M}_{\rm Ni0}$ was known, one could easily find the distance D from equation (7). So, we have to look for a way to estimate $\mathcal{M}_{\rm Ni0}$ independently. It seems reasonable to assume that the supernova explosion energy E should correlate with $\mathcal{M}_{\rm Ni0}$ produced during the explosion. This means that

$$E = f(\mathcal{M}_{\rm Ni0}) = f\left(\frac{D^2}{145}Q\right), \qquad (8)$$

where f represents a *statistically admissible correlation function* rather than a strict mathematical relation. Inserting this expression for E into equation (1) and using equation (4) for M_V, we obtain an equation that can be solved for D when $V - A_V$, $u_{\rm ph}$, Δt and Q are known from observations. Then for given D, we can find E, $\mathcal{M}$, R and $\mathcal{M}_{\rm Ni0}$ from equations (1)–(4) and (7), respectively.

What can be said about the function $f(\mathcal{M}_{\rm Ni0})$ at present, when the details of the SNII mechanism still remain ambiguous? First of all, it is reasonable to assume that a good fraction of E comes from the recombination of free neutrons and protons into ^{56}Ni just at the bottom of the envelope to be finally expelled (Nadyozhin 1978; Bethe 1996). The hydrodynamical modelling of the collapse (Nadyozhin 1978) has indicated that, under favourable conditions, a neutron–proton shell could be accumulated just under the steady accreting shock wave. When the mass of such a shell reaches some critical value (presumably of the order of $\approx 0.1\ \mathcal{M}_\odot$), the shell can become unstable with respect to recombination into the 'iron group' elements (specifically into ^{56}Ni) to supply the stalled shock wave with the energy of $\approx 10^{51}$ erg necessary to trigger the supernova. Here, there is a physical analogy with the origin of planetary nebulae from red giants, where the energy from the recombination of hydrogen and helium causes the expulsion of a red giant rarefied envelope. The recent study (Imshennik 2002, and references therein), of the 'neutrino crown' – the region enclosed within the neutrinosphere and accreting shock – turns out to be in line with such a picture of the supernova mechanism. However, some Ni can be produced through the explosive carbon–oxygen burning induced by the outgoing shock wave. In this case the energy release per unit Ni mass is lower by an order of magnitude than for the neutron–proton recombination.

The energy released by the neutron–proton recombination, producing a ^{56}Ni mass of $\mathcal{M}_{\rm Ni0}$, is given by

$$E(\text{np}\rightarrow\text{Ni}) = 1.66 \times 10^{52}\frac{\mathcal{M}_{\rm Ni0}}{\mathcal{M}_\odot}\ \text{erg}. \qquad (9)$$

Thus, the production of only $\sim 0.06\ \mathcal{M}_\odot$ of ^{56}Ni is sufficient to provide the standard explosion energy of 10^{51} erg. The current hydrodynamic models of the SNII explosions (Woosley & Weaver 1995; Rauscher et al. 2002) do not show a correlation between E and $\mathcal{M}_{\rm Ni0}$ because in these models ^{56}Ni comes from explosive silicon and carbon–oxygen burning near to the envelope bottom, and its yield is sensitive to the mass cut point. The photometric and spectroscopic properties of the SN models are virtually independent of the mass cut. On the contrary, the nucleosynthesis yields are very sensitive to the mass cut. In the current SN models the explosion is usually simulated by locating a piston at the internal boundary $m = \mathcal{M}_{\rm cut}$. The piston moves with time according to a prescribed law, $R_{\rm pis}(t)$, with velocity ($\dot{R}$) amplitude being chosen to ensure that the final kinetic energy of the expelled envelope is of the order of 10^{51} erg. There are two major uncertainties at this point. First, for a given velocity amplitude the resulting nuclear yields are still sensitive to the form of the function $R_{\rm pis}(t)$. Secondly, the pre-supernova structure (especially chemical composition) in the vicinity of $m = \mathcal{M}_{\rm cut}$ will always remain ambiguous until the detailed mechanism of the SN disintegration on to the collapsed core and thrown envelope is established. The point is that such 2D effects as rotation and large-scale mixing can result in a pre-supernova structure different from that predicted by spherically symmetric models. Under such circumstances, it is difficult to find a serious argument against the possibility to expel a noticeable amount of ^{56}Ni from the recombination of the neutron–proton shell. Thus, we propose a neutron–proton layer that is located somewhat deeper than the value of $\mathcal{M}_{\rm cut}$ assumed in the current SN models. This layer recombines into ^{56}Ni, providing energy sufficient to convert a steady-state accretion shock into an outgoing blast wave. In this case a good correlation between E and $\mathcal{M}_{\rm Ni0}$ is to be expected.

The proposed correlation can have a complex nature. It is quite probable that the function f in equation (8) depends also on $\mathcal{M}$ since the supernova mechanism is expected to be sensitive to the pre-supernova mass. For us only the existence of some correlation is important, which in combination with equations (1)–(3) allows us to determine the distance independently.

To demonstrate how such a method can work, we make the simplest assumption that E is proportional to $E(\text{np} \rightarrow \text{Ni})$. Then one can write

$$E = \xi E(\text{np}\rightarrow\text{Ni}) = 16.6\xi\mathcal{M}_{\rm Ni0} = 0.1145\xi D^2 Q, \qquad (10)$$

where, as usual, E is in 10^{51} erg, $\mathcal{M}_{\rm Ni0}$ in $\mathcal{M}_\odot$ and D in Mpc. This equation implies that the function f, introduced in equation (8), reads as $f(x) = 16.6\xi x$, where ξ is an adjustable parameter that can be either less than or greater than unity. If there is a noticeable contribution to $\mathcal{M}_{\rm Ni0}$ from the explosive carbon–oxygen burning then $\xi < 1$; if a noticeable contribution to the explosion energy comes from other sources rather than the neutron–proton recombination then $\xi > 1$.

Inserting E from equation (10) and M_V from equation (4) into equation (1) and solving for D, we obtain

$$\lg D = -0.374\lg(\xi Q) + 0.0504(V - A_V) + 0.875\lg\Delta t + 1.17\lg u_{\rm ph} - 2.482, \qquad (11)$$

where D is in Mpc, Δt in days, and $u_{\rm ph}$ in 1000 km s^{-1}. We will refer to distances derived from equation (11) as 'plateau–tail distances', $D_{\rm P\text{–}T}$, hereafter. The results are given in Table 3 for nine supernovae selected from Table 2. We did not include SNe 1992am and 1999cr in our analysis because their last available observations may not yet reflect the radioactive tail phase. Specifically, there are only two observations of SN 1992am at the post-plateau phase of the light curve. Since the observations are separated by a short time interval of 3 d, it is difficult to derive the inclination of the bolometric light curve with a required accuracy to be sure that SN 1992am is already in the radioactive tail phase. Moreover, one has to remember that, in addition to Co decay, the tail luminosity can also be contributed by the ejecta–wind interaction (see Chugai 1991, and references therein). SN 1992am is suspicious in this respect because its pre-supernova radius seems to be larger than 1000 R$_\odot$ (Table 2). Hence, the $\mathcal{M}_{\rm Ni0}$ values for these SNe in Table 2 could actually be upper limits.

Table 3. The tail-calibrated supernova physical properties ($\xi = 1$).

SN	$D_{\rm P-T}$ (Mpc)	M_V (mag)	E (10^{51} erg)	$\mathcal{M}$ ($M_\odot$)	R ($R_\odot$)	$\mathcal{M}_{\rm Ni0}$ ($M_\odot$)	$F_{41}(t)$ (10^{41} erg (s Mpc^2)$^{-1}$)	t (d)	Q
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1986L	29.67	−17.76	1.14	13.7	944	0.067	2.25×10^{-3}	180	0.0113
1988A	15.21	−16.05	0.79	19.6	217	0.048	4.96×10^{-3}	200	0.0299
1990E	29.16	−17.41	1.57	21.3	539	0.094	2.67×10^{-3}	200	0.0161
1991al	85.31	−17.97	2.38	15.0	509	0.14	8.13×10^{-4}	140	0.00286
1992af	86.45	−17.55	2.71	18.8	293	0.16	9.02×10^{-4}	140	0.00317
1992ba	19.85	−16.25	0.53	12.4	346	0.032	1.97×10^{-3}	200	0.0119
1999em	11.08	−16.54	0.68	15.0	414	0.041	1.26×10^{-2}	150	0.0485
1999gi	14.53	−16.46	0.63	14.5	411	0.038	5.41×10^{-3}	174	0.0259
1987A	0.045	−15.42	0.87	25.6	104	0.053	8.16×10^{2}	170	3762

The different columns of Table 3 give the following quantities: column 2 lists the distance $D_{\rm P-T}$ from equation (11) setting $\xi = 1$; the corresponding absolute V magnitude of the mid-point of the plateau M_V is in column 3; columns 4–7 give the quantities E, $\mathcal{M}$, R and $\mathcal{M}_{\rm Ni0}$ as in Table 2, but now using the distance $D_{\rm P-T}$ as in column 1; and columns 8–10 are explained above.

The values of E, $\mathcal{M}$, R and $\mathcal{M}_{\rm Ni0}$ for ξ values different from unity can be found using the following scaling relations, which result from equations (5), (7) and (11):

$$E \sim \xi^{0.252}, \quad \mathcal{M} \sim \xi^{0.438}, \quad R \sim \xi^{-1.07}, \quad \mathcal{M}_{\rm Ni0} \sim \xi^{-0.748}. \quad (12)$$

For a fixed Q, the dependence of the distance $D_{\rm P-T}$, defined by equation (11), on extinction A_V proves to be very weak: an error in A_V of ±1 mag changes $D_{\rm P-T}$ by only ±12 per cent. However, if the tail luminosity F_{41} is derived from the V measurements (just the case of Hamuy's F_{41} values we use here), then lg F_{41}, and consequently lg$Q$, scales as $0.4A_V$ and lg$D_{\rm P-T}$, derived from equation (11), actually varies with A_V in a standard way, as $-0.2A_V$. If the tail luminosity were derived from infrared measurements, then the resulting $D_{\rm P-T}$ distances would be largely independent of extinction. Note also the rather weak dependence on ξQ: $D_{\rm P-T} \sim (\xi Q)^{-0.374}$. For instance, the decrease in ξQ by a factor of 2 results in an *increase* of $D_{\rm P-T}$ by 30 per cent only.

The random errors typically of ±10 per cent for the δt and $u_{\rm ph}$ values assumed in Table 1 result in an uncertainty factor of ≈1.2 for $D_{\rm P-T}$ and ≈1.5 for $\mathcal{M}_{\rm Ni0}$ ($\sim D^2$) given in Table 3. However, one has to keep in mind two main sources of systematic errors: (i) probable deviation of the theoretical models (on which equations 1–3 are based) from real SNe, and (ii) the presentation of the E–$\mathcal{M}_{\rm Ni0}$ correlation in the form of the straight proportionality (equation 10). Both types of systematic errors are difficult to estimate at present. Although the SN models calculated in LN83 and LN85 rest upon a very simplified pre-supernova structure, they consistently take into account the ionization and recombination of hydrogen and helium thereby still remaining useful. When a new grid of the SN models, based on modern evolutionary pre-supernova structure, is created, the systematic error (i) certainly will be reduced. The reduction of the systematic error (ii) requires a more profound knowledge of the SN mechanism. Empirically, this problem can be solved by adjusting the factor ξ for each individual SN. It is necessary, however, to collect much richer statistics (at least by a factor of 3) than currently available (only nine SNe in Table 3).

4 DISCUSSION

The plateau–tail distances derived in Section 3 and listed in column 2 of Table 3 are plotted in a Hubble diagram in Fig. 2 (except for SN 1987A, which is not in the Hubble flow). The eight SNIIP define a Hubble line with $H_0 = 55 \pm 5$ km s^{-1} Mpc^{-1}. Also shown in Fig. 2 are the 11 SNIIP for which EPM distances have been published (column 3 of Table 2). They define a Hubble line of $H_0 = 70 \pm 4$ km s^{-1} Mpc^{-1}, i.e. the EPM distances are smaller than the plateau–tail distances by 25 per cent on average.

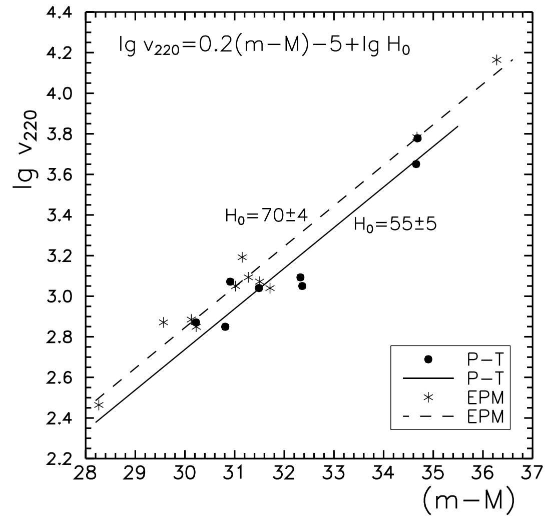

Figure 2. The Hubble diagram of eight SNIIP with $D_{\rm P-T}$ distances from plateau and tail observations (full circles). Also shown are the 11 SNIIP with known EPM distances (asterisks). The respective Hubble lines are fitted to the data. The abscissa is the distance modulus $(m - M) = 5\,{\rm lg}D + 25$.

At this point it is not possible to decide which of the two results is more nearly correct. Both methods, the plateau–tail distances and the EPM distances, depend on assumptions that are difficult to verify. The EPM faces the problem of the dilution factor in an expanding atmosphere and the definition of the photospheric radius, which depends on the uncertainties connected with the opacity of an expanding medium. However, it may be noted that the EPM distance of SN 1987A agrees well with the generally adopted distance to the Large Magellanic Cloud (LMC) of 50 kpc (Eastman, Schmidt & Kirshner 1996) and the EPM distance of SN 1968L is indistinguishable from the Cepheid distance of NGC 5236 (M83) (Thim et al. 2003).

The main assumption that affects the plateau–tail distances concerns the nature of the proposed E–$\mathcal{M}_{\rm Ni0}$ correlation. For our simplified example of such a correlation, all the uncertainties turn out to be accumulated in the proportionality factor ξ between the explosion energy E and the nickel mass $\mathcal{M}_{\rm Ni0}$. In Table 3 we have adopted a

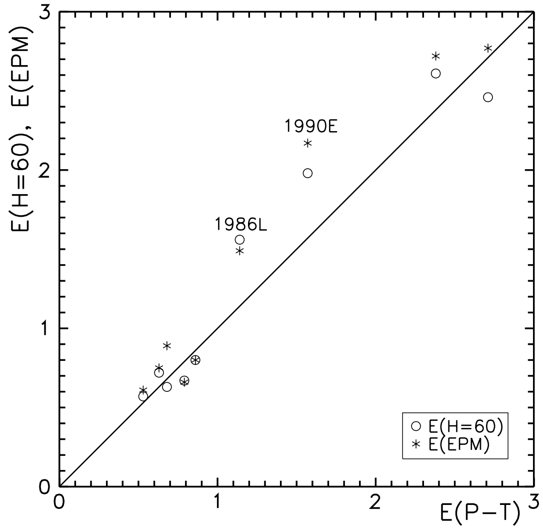

Figure 3. The explosion energies $E(H_0 = 60)$ and E(EPM) versus E(P–T) (see the text).

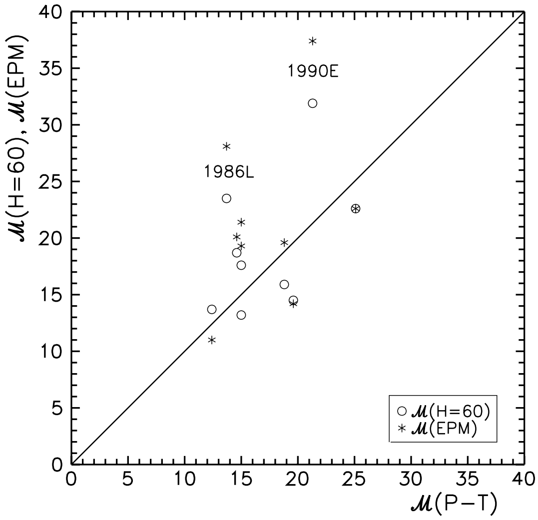

Figure 4. The expelled masses $\mathcal{M}(H_0{=}60)$ and $\mathcal{M}$(EPM) versus $\mathcal{M}$(P–T) (see the text).

plausible value of $\xi = 1$, but it cannot be excluded that ξ is as low as 0.5 or as high as 2. Since the Hubble constant scales as $H_0 \sim \xi^{0.374}$, an average value as high as $\xi = 1.9$ would be needed to bring the plateau–tail distances into general accord with the EPM distances. Such a high average value of ξ is, however, not supported by SNe 1987A and 1999gi. If the $D_{\rm P-T}$ distance of SN 1987A from Table 3 is scaled to the canonical LMC distance of 50 kpc, ξ becomes 0.75. If the host galaxy NGC 3184 of SN 1999gi with a $D_{\rm P-T}$ distance of 14.53 Mpc is a member of the same group as NGC 3198 and 3319, for which Freedman et al. (2001) give a mean Cepheid distance of 13.5 Mpc, ξ becomes 1.2. Eventually additional SNIIP with large distances, where the influence of peculiar motions are negligible, will better determine the scatter of the Hubble diagram and allow a meaningful determination of the actual range of ξ.

We have considered three sets of the physical supernova parameters E, $\mathcal{M}$ and R: (i) for the Hubble distances $D_{\rm H}$ with $H_0 = 60$ km s^{-1} Mpc^{-1} (Table 2, column 3); (ii) for the EPM distances $D_{\rm EPM}$ (Table 2, column 4); and (iii) for the plateau–tail calibrated distances $D_{\rm P-T}$ (Table 3, column 2).

Although the above parameters derived from the EPM distances are not presented in Table 2, the corresponding E and $\mathcal{M}$ values can be read from Figs 3 and 4, which compare E and $\mathcal{M}$ for sets (i) and (ii) with those for set (iii). For seven SNe E and $\mathcal{M}$ are rather insensitive to the adopted distances. However, for SNe 1986L and 1990E, labelled in Figs 3 and 4, the deviations from the P–T values are rather large, especially in the case of the envelope mass $\mathcal{M}$. These SNe differ from others by having a long plateau of 110–120 d in combination with still a substantial expansion velocity of 4000 km s^{-1}. As a result, their envelope masses $\mathcal{M}$, derived from the distances defined by the $D_{\rm H}$ and $D_{\rm EPM}$ values, exceed those for other SNe. Such a discrepancy for these two SNe is considerably weakened if $\xi \approx 2$. Such a high value of ξ implies that half of the explosion energy is supplied by a source different from the neutron–proton recombination. This may indicate that for massive SNe the envelope mass $\mathcal{M}$ (in addition to $\mathcal{M}_{\rm Ni0}$) is involved in the correlation given by equation (8).

The random errors of E and $\mathcal{M}$ from our approximate equations (1)–(3) are estimated to be about ±30 per cent. Observational errors especially in the expansion velocity $u_{\rm ph}$ and the plateau duration Δt can modify E and $\mathcal{M}$ by another factor of 1.3. Thus it seems reasonable to assume a random uncertainty of a factor of ∼1.5 for the *individual* values of E and $\mathcal{M}$ in Tables 2 and 3. The pre-supernova radii R are very sensitive to distance errors (cf. equation 5) and may carry random errors of a factor of 2. The radii of SNe with large nickel masses like SN 1991al, 1992af and perhaps 1992am may carry additional systematic errors because equations (1)–(3) do not take radioactive heating into account in a consistent way.

The expelled masses $\mathcal{M}$ are plotted against the explosion energies E in Fig. 5 for two cases, i.e. based on EPM and plateau–tail distances. In the case of the $D_{\rm P-T}$ distances, the mean mass of the eight SNIIP is 16 $\mathcal{M}_\odot$ with an rms deviation of only 3 $\mathcal{M}_\odot$. This narrow mass range is contrasted by a wide range of explosion energies of 0.5–2.7 × 10^{51} erg. The conclusion that there is no correlation between the expelled mass (which is only 1.4–2 $\mathcal{M}_\odot$ smaller than the

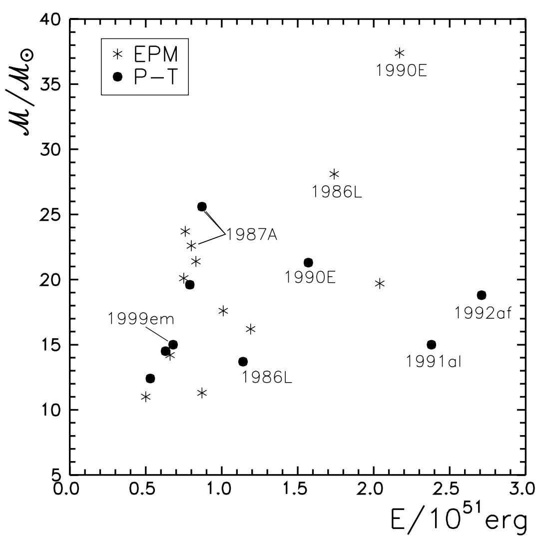

Figure 5. The explosion energy–envelope mass diagram for the case of EPM distances $D_{\rm EPM}$ from column 4 of Table 2 (asterisks; SNe 1991al and 1992af are excluded) and for the case of the plateau–tail distances $D_{\rm P-T}$ from column 2 of Table 3 (full circles). Some SNe are identified (see text).

pre-supernova mass) and the explosion energy is somewhat weakened by the values of $\mathcal{M}$ and E based on the EPM distances suggesting a marginal correlation between $\mathcal{M}$ and E, which is mainly due to only two SNe: 1986L and 1990E.

One can think of a number of parameters that may explain the wide range of explosion energies. It could be rotation and magnetic fields inherited by the collapsing stellar core. It could be also non-spherical jet-like perturbations of a random nature arising from the macroscopic neutrino-driven advection below the accretion shock. Such perturbations could launch the outgoing blast wave earlier when the recombination nuclear energy stored in a hot neutron–proton gas was not yet as large as it should be in the case of spherical symmetry. If this is correct, one may expect that the asphericity of the explosion *anticorrelates* with the explosion energy.

Recently, a promising project has been started (Van et al. 1999; Smartt et al. 2001, 2002, and references therein) with the ultimate aim to identify the supernova progenitors (pre-supernovae) or at least to impose conclusive constraints on their masses by inspecting the pre-discovery field of nearby supernovae. In particular, Smartt et al. derived upper mass limits of 12 and 9 $\mathcal{M}_{\odot}$ for the progenitors of the SNe 1999em and 1999gi, assuming distances D for the host galaxies NGC 1637 and 3184 of 7.5 and 7.9 Mpc, respectively. Note that these *upper* limits depend on D and have to be adjusted for other values of D to $12\,\mathcal{M}_{\odot}(D/7.5\,\mathrm{Mpc})^{0.6}$ for SN 1999em and $9\,\mathcal{M}_{\odot}(D/7.9\,\mathrm{Mpc})^{0.6}$ for SN 1999gi. This follows from the fact that the mass–luminosity relation can be approximated as $L \sim \mathcal{M}^{3.3}$ in the mass interval 10–15 $\mathcal{M}_{\odot}$. For SN 1999em at $D_{\mathrm{P-T}} = 11.08$ Mpc (Table 3) it follows that $15.2\,\mathcal{M}_{\odot}$ is the upper mass limit for the SN 1999em progenitor. Hence, our result $\mathcal{M} = 15.0\,\mathcal{M}_{\odot}$ (Table 3) does not contradict the observations *as long as* D(1999em) $\gtrsim 10$ Mpc. The situation for SN 1999gi is similar. The upper mass limit for $D(1999\mathrm{gi}) = 14.53$ Mpc (Table 3) is $\mathcal{M} < 9 \times (14.53/7.9)^{0.6} = 13.0\,\mathcal{M}_{\odot}$, i.e. not in significant contradiction with the $\mathcal{M}$ value of 14.5 $\mathcal{M}_{\odot}$ from Table 3. There is no contradiction either with the upper mass limit of $15^{+5}_{-3}\,\mathcal{M}_{\odot}$ for the SN 1999gi progenitor imposed recently by Leonard et al. (2002b).

Equations (1)–(3) by LN85, derived from a grid of 23 SNIIP models covering a wide parameter space, imply a correlation between the absolute magnitude M_V (and hence luminosity L – both measured at the mid-point of the plateau) and the expansion velocity u_{ph}. The correlation is shown in Figs 6 and 7, where 23 grid models are shown by full circles; the straight lines are the least-squares fits. In Fig. 7 are also shown the eight observed SNIIP from Table 3 marked by open circles, their absolute magnitudes M_V (Table 3, column 2) being calculated from equation (4), where the plateau–tail distances $D_{\mathrm{P-T}}$ were used from Table 3, column 2. These real SNe roughly follow the slope of the models, but at a fixed value of u_{ph} they are fainter by ≈0.6 mag on average.

Empirically, Hamuy & Pinto (2002) have also found, using the CMB redshift-based distances, such a correlation. The slopes of their least-squares fits are virtually the same as shown in Figs 6 and 7. Thus our models confirm their finding.

The main conclusion one can draw from Figs 6 and 7 is that our three-parameter grid of only 23 SNIIP properly chosen models is ample enough to reproduce the main features of the real SNe.

5 CONCLUSIONS

Model calculation by LN83 and LN85 of SNIIP, leading to equations (1)–(3), are combined with available EPM distances and velocity distances ($H_0 = 60$) to derive the explosion energy E, ejected mass $\mathcal{M}$ and pre-supernova radius R of 14 SNIIP. Only the apparent,

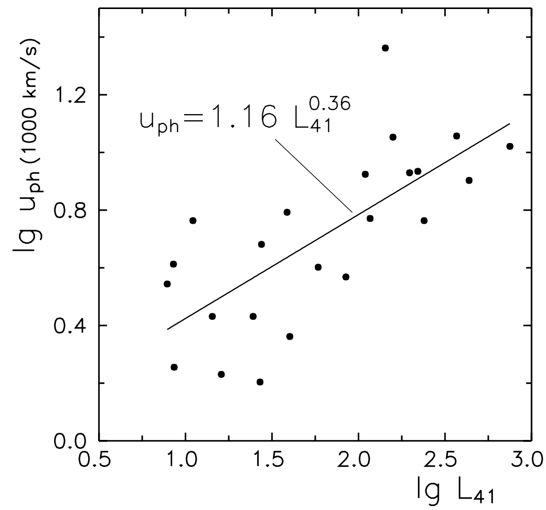

Figure 6. The correlation of the luminosity L_{41} (in units 10^{41} erg s^{-1}) of the mid-point of the plateau with the expansion velocity u_{ph} (in 1000 km s^{-1}) for 23 SN models (full circles).

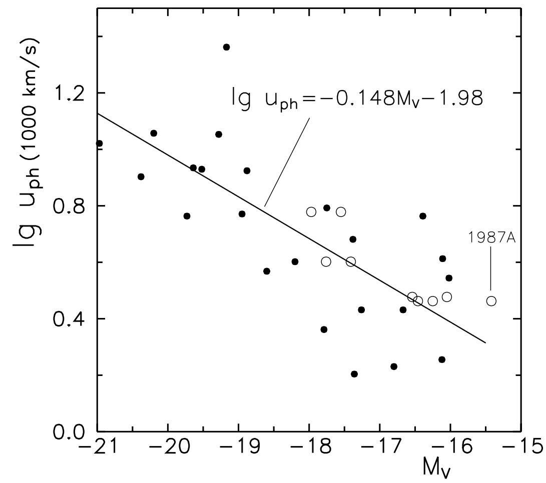

Figure 7. The correlation of the absolute magnitude M_V of the mid-point of the plateau with the expansion velocity u_{ph} for 23 SN models (full circles); open circles relate to nine real SNe including SN 1987A.

absorption-corrected magnitude V and the expansion velocity u_{ph} at the mid-point of the plateau together with its total duration Δt are needed as additional input parameters. The results are presented in Table 2.

Instead of using EPM or velocity distances it is also possible to use the bolometric fluxes observed during the SNIIP tail phase to determine the Ni mass and hence new, independent distances called here plateau–tail distances $D_{\mathrm{P-T}}$ (cf. equation 11). The $D_{\mathrm{P-T}}$ distances yield new values of E, $\mathcal{M}$ and R given in Table 3 for nine SNe which were observed during both their plateau and tail phases. The values of E and $\mathcal{M}$, based on EPM and P–T distances, agree well, with the exception of SNe 1986L and 1990E, whose masses $\mathcal{M}$ from P–T distances are lower by a factor of 2 than those from EPM distances (see Fig. 4).

The P–T distances are larger than the EPM distances by ∼25 per cent on average. The former suggests a value of $H_0 = 55 \pm 5$. The main uncertainty of this result comes from the assumption that $\xi = 1$, where ξ is the ratio between the total explosion energy and the energy liberated by the neutron–proton recombination into ^{56}Ni (cf. equation 10). To reduce the P–T distances to the level of the EPM distances, which correspond to $H_0 = 70$, an average value of $\xi = 1.9$ is required. The consequence that about half of the total energy E comes from sources other than neutron–proton recombination into ^{56}Ni seems rather extreme. In fact, it is not supported by two SNIIP (1987A and 1999gi) with independent distance information, which suggest that ξ is of the order of unity. Moreover, very recently Leonard et al. (2003) have obtained a Cepheid distance of 11.7 ± 1 Mpc to NGC 1637 (the host galaxy of SN 1999em), which is larger by a factor of 1.4 than the EPM distance (Table 2). Our $D_{\mathrm{P-T}}$ distance of 11.1 Mpc to SN 1999em (Table 3) is in good agreement with this result. If it happens that the same factor is applicable also to the EPM distances to SNe 1986L and 1990E, there will be no need to resort to large ξ values, such as $\xi \approx 2$ (Section 4), to remove the discrepancy between $D_{\mathrm{P-T}}$ and D_{EPM} for these SNe.

In conclusion, we emphasize the necessity of constructing a new grid of hydrodynamic SNIIP models based on current evolutionary pre-supernova models and taking into account ^{56}Ni as an additional parameter in a consistent way. Such a grid would allow one to create more precise analytic approximations for a number of correlations between the physical parameters of SNIIP and their observable properties.

The 'plateau–tail' method of distance determination needs, of course, further critical analysis requiring a close collaboration between astronomers observing supernovae and theorists modelling their explosions. If the proposed E–$\mathcal{M}_{\mathrm{Ni}}$ correlation is confirmed, it promises to become a tool to explore the mechanism of SNII with the aid of optical and spectroscopic observations.

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