

# Corotation resonances for gravity waves and their impact on angular momentum transport in stellar interiors

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**Abstract.** Gravity waves, which propagate in radiation zones, can extract or deposit angular momentum by radiative and viscous damping. Another process, poorly explored in stellar physics, concerns their direct interaction with the differential rotation and the related turbulence. In this work, we thus study their corotation resonances, also called critical layers, that occur where the Doppler-shifted frequency of the wave approaches zero. First, we study the adiabatic and non-adiabatic propagation of gravity waves near critical layers. Next, we derive the induced transport of angular momentum. Finally, we use the dynamical stellar evolution code STAREVOL to apply the results to the case of a solar-like star. The results depend on the value of the Richardson number at the critical layer. In the first stable case, the wave is damped. In the other unstable and turbulent case, the wave can be reflected and transmitted by the critical layer with a coefficient larger than one: the critical layer acts as a secondary source of excitation for gravity waves. These new results can have a strong impact on our understanding of angular momentum transport processes in stellar interiors along stellar evolution where strong gradients of angular velocity can develop.

**Keywords.** hydrodynamics, waves, turbulence, stars: rotation, stars: evolution

## 1. Damping or over-reflection

Critical layers (e.g. Booker & Bretherton 1967) occur in stellar radiation zones where the frequency of the wave is resonant with the mean-flow rotation rate. Mathematically, this means  $\sigma(r) = \sigma_w + m\Delta\Omega(r) = 0$ , where  $\sigma_w$  is the excitation frequency of the wave,  $m$  corresponds to a Fourier expansion along the longitudinal direction, and  $\Delta\Omega(r) = \bar{\Omega}(r) - \Omega_{CZ}$  is the difference between the angular velocity at the radius  $r$  and at the border with the convective zone, where the waves are excited. In the adiabatic case, the equation of propagation of IGWs in spherical coordinates is

$$\frac{d^2\Psi_{l,m}}{dr^2} + \left[ \frac{l(l+1)}{m^2} \frac{\text{Ri}_c}{(r-r_c)^2} - k_{Hc}^2 \right] \Psi_{l,m} = 0, \quad (1.1)$$

where  $\Psi_{l,m}(r) = \bar{\rho}^{\frac{1}{2}} r^2 \hat{\xi}_{r;l,m}$ ,  $\bar{\rho}$  is the average density and  $\hat{\xi}_{r;l,m}$  the vertical displacement expanded in the spherical harmonics basis. The local Richardson number  $\text{Ri}_c = [N^2 / (r^2 (d\bar{\Omega}/dr)^2)]_{r=r_c}$  depends on the Brunt-Väisälä frequency  $N$ . Its value determines the behaviour of the wave passing through a critical layer as shown in Fig. 1.

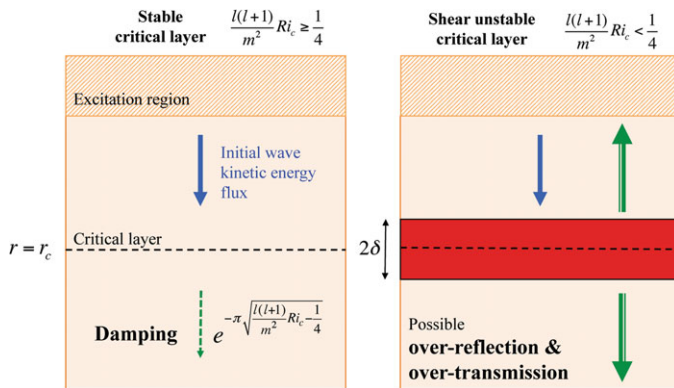


Figure 1. Two possible behaviours for a wave passing through a critical layer.  $2\delta$  is the thickness of the turbulent region.

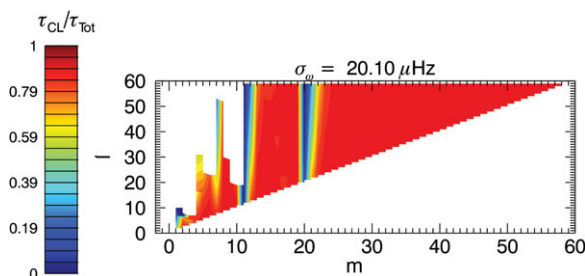


Figure 2. Comparison between  $\tau_{CL}$  and  $\tau_{tot} = \tau_{CL} + \tau_{rad}$ . Red/dark tones correspond to regions where the action of the critical layers is dominant in comparison with the radiative damping.

### 2. Application to a solar-type star

We applied our theoretical prescriptions to a solar-type star calculated with the evolution code STAREVOL. It appears that only the first stable case can occur in the studied case. We show in Fig. 2 that the damping  $\tau_{CL}$  produced by the critical layers competes with the radiative damping  $\tau_{rad}$  described by Zahn *et al.* (1997). Both phenomena are complementary to explain the transport of angular momentum by IGWs (e.g. Charbonnel & Talon 2005, Alvan *et al.* 2013).

### 3. Conclusion

Further details about this study are available in Alvan *et al.* (2013). These results indicate that critical layers can lead to major modification of the transport of angular momentum in stellar interiors. A systematic exploration of different types of stars for different evolutionary stages will be undertaken and we expect to find stars where the unstable regime and possible tunneling or over-reflection/transmission take place.

### References

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