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The Rayleigh–Taylor problem with a vertical magnetic field, including the effects of Hall current and resistivity

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The influence of resistivity and Hall current on the Rayleigh–Taylor problem involving two superposed fluids of finite density in the presence of gravitational and magnetic fields normal to the fluid interface is examined. Unlike the related problem in which the magnetic field is parallel to the interface, it appears that the dispersion relation does not exhibit singular behaviour in the zero resistivity limit. The 'potentially stable' situation is considered throughout. The results are compared with earlier ideal and resistive theories, and an apparent anomaly regarding the existence of normal modes in such systems is resolved.

1. Introduction

In Kalra *et al.* (1970), the effect of Hall current and resistivity on the stability of a gas liquid system in the presence of a horizontal magnetic field was discussed. It was found that the dispersion relation derived (cf. Singh & Tandon 1969) does not reduce exactly to the dispersion relation derived from non-resistive Hall current theory (Hosking 1965, 1968; Talwar & Kalra 1967) in the limit as resistivity tends to zero. This singular behaviour is associated with the existence of a 'dipole layer' at the gas–liquid interface in the zero resistivity limit (cf. Woods 1962, 1964). Hosking (1971) gives a detailed discussion of the boundary conditions used in studying the stability of superposed fluid systems, but to date no account has been given in which the non-resistive problem is solved using such a dipole layer.

In the problem discussed in the present paper, however, a vertical seed field replaces the horizontal magnetic field considered by both Singh & Tandon (1969) and Kalra *et al.* (1970), and one may note that the omission of resistivity does not reduce the order of the differential equation for the system (see below). Thus, in the presence of Hall current, one might expect the result of ignoring resistivity from the outset to be the same as that of including resistivity in the basic equations, but allowing it to tend to zero in the final analysis.

The Rayleigh–Taylor problem in the presence of a vertical magnetic field, and in which both fluids are perfectly conducting, was considered by Chandrasekhar (1961). He concluded that, for the case where the system is stable in the absence of a magnetic field, no normal mode solutions to the dispersion relation exist. This conclusion has also been supported by Roberts & Boardman (1962), who have exhaustively studied the effect of resistivity and viscosity on the propagation of gravity waves at a fluid vacuum interface. They conclude that, even in the presence of resistivity and viscosity, there is still a small band of wavenumbers k, for which no normal mode solutions exist. The assumption on which both papers base their conclusions is the apparently self-evident fact that perturbations, whose spatial dependence is an increasing function of the distance from the interface, are not permitted. Schatzman (1964), in a paper which discusses the problem of the oscillations of a slab of gas confined by two semi-infinite media of different temperature, contradicts this assumption. He claims that, in the absence of reflexion of the perturbations at infinity, perturbations with infinite amplitude at infinite time are perfectly acceptable normal mode solutions. The present problem is developed from the viewpoint of the Schatzman paper, and a fuller discussion of the divergence between this approach and that of Chandrasekhar and Roberts & Boardman is given in § 6.

In this paper, the potentially stable analogue of the Rayleigh–Taylor problem at the interface between a non-conducting fluid and one which supports finite resistivity and Hall current in the presence of a vertical magnetic field is discussed. Both fluids are assumed to be incompressible and initially homogeneous. A dispersion relation is obtained, which contains both the effects of Hall current and resistivity, and this is shown to be equivalent in the limit of zero resistivity to the dispersion relation obtained when resistivity is neglected from the outset. If resistivity and Hall current are both allowed to tend to zero we find a set of normal mode solutions exists for the dispersion relation in the potentially stable configuration, one of which corresponds to the 'gravity waves' studied by Rayleigh (1900) in the absence of a magnetic field. The effect of resistivity and Hall current on the modes obtained for the ideal fluid is then discussed in both the long and short wavelength limits.

2. The basic equations and the equilibrium configuration

The Eulerian forms of the equations for an electron-ion system are (in mks)

$$\nabla . \mathbf{v} = 0, \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} . \nabla \mathbf{v}\right) + \nabla p = \mu \mathbf{j} \times \mathbf{H} + \rho \mathbf{g}, \tag{2}$$

$$\mathbf{E} + \mu \mathbf{v} \times \mathbf{H} - \frac{\mu}{N_e e} \mathbf{j} \times \mathbf{H} + \frac{1}{N_e e} \nabla p_e = \eta \mathbf{j}, \tag{3}$$

$$\nabla \times \mathbf{H} = \mathbf{j}, \tag{4}$$

$$\nabla . \mathbf{H} = 0, \tag{5}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}, \tag{6}$$

where $\mathbf{v}$ is the fluid velocity, ρ is the mass density, p is the fluid pressure, μ is the magnetic permeability, $\mathbf{g}$ is the gravitational field, $\mathbf{j}$ is the current density,

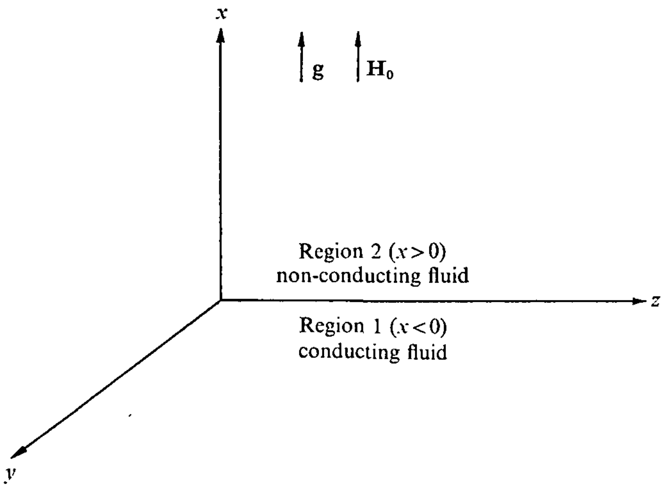

FIGURE 1. The equilibrium configuration for the Rayleigh–Taylor problem with vertical magnetic field.

H is the magnetic field intensity, **E** is the electric field, N_e is the electron number density, e is the electronic charge, p_e is the electron pressure, and η the resistivity. Implicit in (1)–(6) are the assumptions that the plasma is isotropic, incompressible and quasi-neutral.

The equilibrium configuration for the system under consideration is shown in figure 1. A finitely conducting fluid with non-zero Hall current occupies region 1 ($x < 0$), whilst a non-conducting fluid occupies region 2 ($x > 0$). Both fluids are homogeneous, and they are separated by the horizontal boundary $x = 0$. The equilibrium magnetic field, which is constant throughout, is given by

$$\mathbf{H}_0 = (H_0, 0, 0),$$

and the gravitational field by $\mathbf{g} = (g, 0, 0).$

The equilibrium fluid velocity is zero and the equilibrium hydrostatic pressure satisfies

$$Dp_0 = \rho_0 g$$

in each of the fluids, where $D \equiv d/dx$.

Throughout the following discussion, the subscripts 1, 2 on the field quantities refer to the respective regions below and above $x = 0$.

3. The perturbation equations

In the present problem, the axes may always be oriented such that the wave vector **k** is parallel to the z axis (say). Thus we may take a Fourier–Laplace transform of the field quantities of the form

$$f(x,z,t) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} dk \int_L ds\,\hat{f}(x,k,s)\exp\{ikz+st\},$$

where L indicates a path of integration in the right half plane from $-i\infty$ to $+i\infty$. The perturbation equations in each fluid are discussed separately.

3.1. *Perturbation equations in the conducting fluid*

For the homogeneous incompressible fluid, $\delta\hat{\rho}_1 = 0$ and the Fourier–Laplace transform of (2) is to first order

$$\rho_{0_1} s\,\delta\hat{\mathbf{v}}_1 + \nabla\,\delta\hat{p}_1 = \mu\,\delta\hat{\mathbf{j}}_1 \times \mathbf{H}_0, \tag{7}$$

while, from (3), together with (6) one obtains

$$s\hat{\mathbf{h}}_1 = \mathbf{H}_0 . \nabla\left(\delta\hat{\mathbf{v}}_1 - \frac{1}{N_e e}\,\delta\hat{\mathbf{j}}_1\right) - \eta\nabla\times\delta\hat{\mathbf{j}}_1, \tag{8}$$

where $$\mathbf{H}_0 . \nabla = H_0 D,$$

and $\mathbf{h}$ represents the perturbed magnetic field. Taking the curl of (7) and (8) and eliminating $\nabla\times\delta\hat{\mathbf{v}}_1$ one obtains

$$\left(s - \frac{v_A^2}{s}D^2 - \eta(D^2-k^2)\right)\nabla\times\hat{\mathbf{h}}_1 = \frac{H_0}{N_e e}D(D^2-k^2)\,\hat{\mathbf{h}}_1, \tag{9}$$

where $$v_A^2 \equiv \mu H_0/\rho_{0_1}.$$

If the solutions for $\hat{\mathbf{h}}_1$ are assumed to be of the form $\mathbf{A}e^{\lambda x}$, (9) can be solved noting

$$\nabla\times\hat{\mathbf{h}}_1 = \alpha\hat{\mathbf{h}}_1, \tag{10}$$

where $$\alpha^2 = k^2 - \lambda^2.$$

One has $\alpha = 0$ or

$$(v_A^2 + \eta s)\,\lambda^2 - \frac{s}{\omega_i}v_A^2\alpha\lambda - (s^2 + \eta k^2 s) = 0, \tag{11}$$

where $$\omega_i = e\mu H_0/m_i.$$

There are either two or four distinct pairs of solutions (λ, α) to (11), depending on whether Hall current is absent or present, respectively. (In the limit of zero Hall current, two of these 'pairs' coalesce with the other two.) We note, however, that, if (λ_j, α_j) is a solution to (11), so is $(-\lambda_j, -\alpha_j)$ and clearly we reject values of λ with negative real part in the region $x < 0$. Thus, in general there is either one (Hall current absent) or there are two (Hall current present) values of λ, which may be obtained from (11).

There is one solution $\lambda = k$ corresponding to $\alpha = 0$, so that the set (λ_j, k) defines the base solutions for the perturbed field and velocity. The general solution for the perturbed field in the finitely conducting fluid is thus

$$\hat{\mathbf{h}}_1 = \mathbf{A}_1 e^{\lambda_1 x} + \mathbf{A}_2 e^{\lambda_2 x} + \mathbf{A}_3 e^{kx}, \tag{12}$$

where λ_j are the solutions to (11) with positive real part and $\mathbf{A}_j$ denote constant vectors. The corresponding solution for the perturbed velocity, obtained from (7) and (8), is

$$\delta\hat{\mathbf{v}}_1 = \frac{\mu H_0}{\rho_{0_1} s}(\lambda_i \mathbf{A}_1 e^{\lambda_1 x} + \lambda_2 \mathbf{A}_2 e^{\lambda_2 x}) + \frac{s}{kH_0}\mathbf{A}_3 e^{kx}. \tag{13}$$

Finally, from the x component of (7) one obtains

$$\delta\hat{p}_1 = -\mu H_0\,\mathbf{e}_x . \left[\mathbf{A}_1 e^{\lambda_1 x} + \mathbf{A}_2 e^{\lambda_2 x} + \frac{s^2}{k^2 v_A^2}\mathbf{A}_3 e^{kx}\right]. \tag{14}$$

In passing, one may note that the solution form (12) in the limit $\eta \to 0$ is identical with that obtained from non-resistive theory ($\eta = 0$). This non-singular behaviour is expected, since the governing differential equation (obtained by taking the curl of (9)) is

$$\left[\left\{s - \frac{v_A^2}{s} D^2 - \eta(D^2 - k^2)\right\}^2 + \left(\frac{H_0}{N_e e}\right)^2 D^2(D^2-k^2)\right](D^2-k^2)\,\hat{\mathbf{h}}_1 = 0,$$

which remains sixth-order even as $\eta \to 0$.

3.2. *Perturbation equations in the non-conducting fluid*

In the non-conducting fluid, the perturbed quantities are given by

$$\hat{\mathbf{h}}_2 = \mathbf{A}_4 e^{-kx}, \quad \delta\hat{\mathbf{v}}_2 = \mathbf{A}_5 e^{-kx}, \quad \delta\hat{p}_2 = \frac{\rho_{0_2} s}{k}\,\delta\hat{\mathbf{v}}_{2_x}. \quad (15), (16), (17)$$

It may be remarked in passing that the form of the perturbations in the non-conducting fluid is quite different from that for an infinitely conducting fluid. Thus, it is not possible to compare the equations derived by Chandrasekhar (1961) with those derived in the present paper in the limit of zero resistivity and Hall current, except in the special case of a fluid-vacuum interface where the forms of the perturbation are identical.

4. The boundary conditions

We now show that the boundary conditions for the present problem may be derived without reference to Ohm's law, so that they are independent of the presence (or absence) of either resistivity or Hall current or both.

Assuming (1), (2), (4) and (5) are valid in a narrow transition region near the surface of discontinuity, integration in the usual way (cf. Stix 1962) yields

$$\mathbf{n}.[\mathbf{v}] = 0, \tag{18}$$

$$\mathbf{n}[p] - \mu\mathbf{j}^* \times \overline{\mathbf{H}} = 0, \tag{19}$$

$$\mathbf{n} \times [\mathbf{H}] = \mathbf{j}^*, \tag{20}$$

$$\mathbf{n}.[\mathbf{H}] = 0, \tag{21}$$

where $\mathbf{n}$ is the unit normal to the interface, $\overline{\mathbf{H}}$ is the average of the magnetic field strengths at the two sides of the interface, the brackets denote the change in the quantity across the interface, and

$$\mathbf{j}^* = \lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} \mathbf{j}\, dx.$$

To first order (18) is

$$\mathbf{e}_x.[\delta\mathbf{v}]. \tag{22}$$

By definition the zeroth-order magnetic field satisfies $[\mathbf{H}_0] = 0$. Hence, from (20), $j_0^* = 0$; thus, from (19), $[p_0] = 0$. Thus, to first order, (19) may be written

$$[\delta p + \boldsymbol{\xi}.\nabla p_0]\,\mathbf{e}_x = \mu H_0 \delta j^* \times \mathbf{e}_x,$$

where $\boldsymbol{\xi} = \frac{1}{s}\delta\mathbf{v}$. Therefore

$$[\delta p + \boldsymbol{\xi}.\nabla p_0] = 0 \tag{23}$$

and $\delta\mathbf{j}^* \times \mathbf{e}_x = 0$. Equation (21) is to first order

$$\mathbf{e}_x.[\mathbf{h}] = 0 \tag{24}$$

and hence, from (20),

$$[\mathbf{h}] = 0. \tag{25}$$

Conditions (22), (23) and (25) form a complete set.

In deriving the dispersion relation, it is convenient to replace (25) by condition (24), together with the conditions

$$\mathbf{e}_x.[D\mathbf{h}] = 0 \quad \text{and} \quad \mathbf{e}_x.[\delta\mathbf{j}] = 0, \qquad (26), (27)$$

which follow from (25) and the solenoidal character of $\mathbf{h}$.

5. The complete dispersion relation

If both resistivity and Hall current are included, the x components of the perturbed magnetic field and velocity can be written in the form

$$\hat{h}_{1_x} = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + C_3 e^{kx},$$

$$\delta\hat{v}_{1_x} = \frac{\mu H_0}{\rho_{0_1} s}(\lambda_1 C_1 e^{\lambda_1 x} + \lambda_2 C_2 e^{\lambda_2 x}) + \frac{s}{kH_0} C_3 e^{kx},$$

where C_j are constants, while from (14) the perturbed pressure is given by

$$\delta\hat{p}_1 = -\mu H_0 (C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}) - \frac{s^2}{k^2 v_A^2} C_3 e^{kx}.$$

For the non-conducting, incompressible fluid

$$\hat{h}_{2_x} = C_4 e^{-kx}, \quad \delta\hat{v}_{2_x} = C_5 e^{-kx}, \quad \delta\hat{p}_2 = (\rho_{0_2} s/k) C_5 e^{-kx},$$

where C_4, C_5 are constants. From the Fourier–Laplace transforms of the boundary conditions (22)–(24), (26) and (27), one obtains

$$\frac{v_A^2}{s}(\lambda_1 C_1 + \lambda_2 C_2) + \frac{s}{k} C_3 = H_0 C_5, \tag{28}$$

$$v_A^2 (C_1 + C_2) + \frac{s^2}{k^2} C_3 + \left(\frac{P_s}{k} - \frac{(1-P)g}{s}\right) H_0 C_5 = 0, \tag{29}$$

$$C_1 + C_2 + C_3 = C_4, \tag{30}$$

$$\lambda_1 C_1 + \lambda_2 C_2 + kC_3 = -kC_4, \tag{31}$$

$$\alpha_1 C_1 + \alpha_2 C_2 = 0, \tag{32}$$

where

$$P \equiv \rho_{0_2}/\rho_{0_1}.$$

A non-trivial solution of (28)–(32) exists, provided

$$\frac{\lambda_1\alpha_2 - \lambda_2\alpha_1}{k(\alpha_2 - \alpha_1)} = \frac{s^2(2k^2 v_A^2 - [(1+P)s^2 - (1-P)gk])}{(s^2 - 2k^2 v_A^2)[(1+P)s^2 - (1-P)gk] + 2k^2 v_A^2 s^2}. \tag{33}$$

One may note that dispersion relation (33) limits to the non-resistive form when $\eta \to 0$, as is expected from the earlier comment that the form of the perturbation solutions and the boundary conditions are independent of whether or not resistivity is included. From (33), one finds that reversing the direction of the gravitational field is equivalent to interchanging the fluid densities. In the present paper, discussion is limited to the potentially stable case; one may consider (without loss of generality) that $\rho_{0_2} < \rho_{0_1}$ $(P < 1)$ and $g < 0$.

In § 6 the limit in which the conducting fluid is ideal (resistivity and Hall current negligible) is discussed as a basis for the theory in which resistivity and Hall current are considered finite, which is given in § 7.

6. The ideal limit

In the limit of zero resistivity and Hall current, one has, from (11),

$$\lambda_1^2, \lambda_2^2 \rightarrow \lambda^2 = s^2/v_A^2, \tag{34}$$

together with the condition that $\operatorname{Re}\lambda > 0$. From the definition of the Laplace transform $(\operatorname{Re} s > 0)$, the value $\lambda = -s/v_A$ must be rejected, and we are left with the value $\lambda = s/v_A$. We may now use this value of λ in (33) to obtain a set of modes for the system. Replacing s by ω in (33), where ω represents the analytic continuation of s in the entire complex plane, we obtain

$$(\omega - kv_A)\,[(1+P)\,\omega^3 + 2kv_A(1+P)\,\omega^2 + \{2kv_A^2 - (1-P)\,g\}\,k\omega - 2(1-P)\,gk^2v_A] = 0. \tag{35}$$

After removal of the spurious root $\omega = kv_A$ (i.e. $\lambda = k$) from (35), one may note that the remaining equation,

$$(1+P)\,\omega^3 + 2kv_A(1+P)\,\omega^2 + \{2kv_A^2 - (1-P)\,g\}\,k\omega - 2(1-P)\,gk^2v_A = 0, \tag{36}$$

agrees with the dispersion relation obtained by Chandrasekhar (1961), provided $P \equiv 0$. The modes described by (36) are not, in the strict mathematical sense of the term, 'normal modes'. Nevertheless, they are perfectly valid modes for the system, and correspond to what one normally means by 'normal modes' in a physical context.

When $g < 0$, the three roots of (36) have a negative real part. Thus, whilst the amplitude of the waves at a fixed point decreases with time, at a given time the amplitude grows with the distance from the interface. This apparent anomaly is similar to one considered by Schatzman (1964). Its explanation lies in the fact that Alfvén waves take energy from the disturbed interface, and hence the amplitude of the waves on it decreases with time. As a result of this, we would expect that those waves which have progressed the furthest from the interface at a certain time would be the ones with the greatest amplitudes. Both Chandrasekhar (1961) and Roberts & Boardman (1962) assumed the non-existence of normal modes in the above situation; but, at the end of their paper, Roberts & Boardman (1962) solved an initial-value problem in which behaviour similar to the above is observed.

Thus we conclude that (36) provides a valid set of modes for the system under consideration. In the limit where $k \rightarrow 0$, one finds

$$\omega = \frac{2k^2v_A^3}{(1-P)\,g} \pm i\left(\frac{1-P}{1+P}\right)^{\frac{1}{2}} (|g|\,k)^{\frac{1}{2}}\left[1 - \frac{kv_A^2}{(1-P)\,g}\right] \tag{37}$$

and

$$\omega = -2kv_A - \frac{4k^2v_A^3}{(1-P)\,g}, \tag{38}$$

while, in the limit $k \to \infty$, one obtains

$$\omega = kv_A[-1 \pm i(1-P^2)^{\frac{1}{2}}] \tag{39}$$

and

$$\omega = \frac{(1-P)g}{v_A}\left[1 - \frac{(1+2P)(1+P)g}{2kv_A^2}\right]. \tag{40}$$

The mode described by (37) shows the effect of a long wavelength perturbation, or a weak magnetic field, on the 'gravity' waves found by Rayleigh (1900) in the absence of a magnetic field. In the presence of a large magnetic field, or for short wavelength perturbations, (39) and (40) show that these gravity waves are completely swamped by the presence of the magnetic field.

7. The effect of resistivity and Hall current

We now proceed to examine the effects of resistivity and Hall current on the 'ideal' modes described in § 6. The algebra involved in obtaining the dispersion relation in the form of a polynomial in ω is extremely tedious, and in § 7 we introduce a simplification by considering a simple plasma-vacuum interface (i.e. $P \equiv 0$). In this case, (33) becomes

$$\frac{\lambda_1\alpha_2 - \lambda_2\alpha_1}{k(\alpha_2-\alpha_1)} = \frac{\omega^2(2k^2v_A^2 - \omega^2 + gk)}{(\omega^2 - 2k^2v_A^2)(\omega^2 - gk) + 2k^2v_A^2\omega^2}, \tag{41}$$

where, following the argument of § 6, we have replaced s by ω, requiring $\operatorname{Re}\lambda_1(s)$, $\operatorname{Re}\lambda_2(s) > 0$. One may note that, in the absence of Hall current, (41) is the equation found by Roberts & Boardman (1962) for a resistive fluid.

After much manipulation, including the squaring of (41) twice, we arrive at the fifteenth-order polynomial in ω,

$$\begin{aligned}
&\omega^{15} - 4gk\omega^{13} + 4\eta v_A^2 k^4\omega^{12} + [5g^2 + (g+2kv_A^2)^2 + 4gkv_A^2]\,k^2\omega^{11} - 8g\eta v_A^2 k^5\omega^{10} \\
&\quad - 2[8g^2kv_A^2 + (g+2kv_A^2)^2 g + g^3 + 2v_A^8 k^5\omega_i^{-2}]\,k^3\omega^9 + 16\eta g v_A^2 k^7\omega^8 \\
&\quad + [16gv_A^2 k + (g+2kv_A^2)(g+4kv_A^2) + 4k^2v_A^4]\,g^2k^4\omega^7 + 8(g-2kv_A^2)\,\eta g^2 v_A^2 k^7\omega^6 \\
&\quad - 4[g^2 + 6gkv_A^2 + (g+2kv_A^2)^2 - 2v_A^6 k^4\omega_i^{-2}]\,g^2v_A^2k^6\omega^5 \\
&\quad - 4[g^2 + 4gkv_A^2 - 4k^2v_A^4]\,\eta g^2 v_A^2 k^8\omega^4 + 4[4g^2 + (g+2kv_A^2)^2]\,g^2v_A^4k^8\omega^3 \\
&\quad + 16\eta g^4 v_A^4 k^{10}\omega^2 - 4(4 + k^2v_A^2\omega_i^{-2})\,g^4v_A^6k^{10}\omega - 16\eta v_A^6 g^4 k^{12} = 0.
\end{aligned} \tag{42}$$

In the limit $k \to 0$ one finds, as expected, that the effect of Hall current becomes negligible for all modes. The effect of resistivity on the gravity waves in (37) is found to be of order $\eta^2 k^{\frac{5}{2}}$ in this limit, and thus may also be neglected, while a modification is introduced to the mode described by (38) such that

$$\omega = -2kv_A + \tfrac{1}{2}k^2(\eta - 8v_A^3/g). \tag{43}$$

In the limit $k \to \infty$ with either finite resistivity or Hall current or both, one is able to reproduce the gravity waves of (37). When both Hall current and resistivity are included, one finds

$$\omega = \frac{\eta - (\eta^2 + v_A^4\omega_i^{-2})^{\frac{1}{2}}}{2v_A^4\omega_i^{-2}} \pm i(|g|\,k)^{\frac{1}{2}}, \tag{44}$$

while for zero Hall current (i.e. $\omega_i \to \infty$) one finds the modes described by Roberts & Boardman (1962, § 6, (6.14*a*)):

$$\omega = -v_A^2/4\eta \pm i(|g|\,k)^{\frac{1}{2}}. \tag{45}$$

Thus, in the short wavelength limit, one finds a set of gravity waves which are heavily damped in the presence of small resistive and/or Hall current terms. As $\eta \to 0$ *and* $\omega_i \to \infty$, the above approximation breaks down and we are forced to consider the modes given by (39) and (40). All other modes found from (42) in the limit $k \to \infty$ are spurious roots introduced by the double squaring process.

8. Conclusions

It is apparent from the discussion of the dispersion relation in this paper that, in contrast to the problem discussed by Kalra *et al.* (1970), singular behaviour does not occur in the limit of zero resistivity. The effect of allowing resistivity to tend to zero in the final analysis is identical to that obtained when resistivity is zero from the outset. We have noted that, in the present problem, inclusion of finite resistivity does not alter the order of the differential equations, and that the boundary conditions are independent of the presence, or absence, of either resistivity or Hall current or both.

In contrast to Chandrasekhar (1961) and Roberts & Boardman (1962), we conclude that under all conditions we are able to find a set of modes for the system, to describe the effect of a small perturbation at the interface between a plasma and a non-conducting fluid when a vertical magnetic field is present. We have attributed the explanation of this discrepancy to Schatzman (1964), but note that, in discussing an initial-value problem near the end of their paper, Roberts & Boardman (1962) do in fact find similar behaviour to that described in the present paper for the case of an ideal fluid.

Finally, we find the introduction of finite resistivity and Hall current is of little consequence in the long wavelength ($k \to 0$) limit, whereas in the short wavelength limit these terms permit the existence of 'gravity waves', which were not found in the ideal theory.

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