Adv. Nonlinear Anal. 2 (2013), 147–150

Erratum
On the positive, "radial" solutions of a semilinear elliptic equation in $\mathbb{H}^N$

[Adv. Nonlinear Anal. 1 (2012), no. 1, 1–25]

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1 Singular solutions at $t = 0$

In Section 2.2 we have studied solutions of the equation

$$u'' + (N-1)\coth(t)u' + \lambda u + u^p = 0$$

near $t = 0$. Performing the Emden–Fowler transformation

$$x = (2-N)\log(t), \quad v = t^{\frac{2}{p-1}}u, \quad \sigma := \frac{2}{(p-1)(N-2)},$$

we get, by substituting $\coth(t) = t^{-1} + \frac{t}{3} + O(t^3)$ (taking the second term into account) and setting $v' := dv/dx$

$$\begin{aligned} v'' - (1-2\sigma)v' + v\Big\{\sigma^2 - \sigma + e^{\frac{-2x}{N-2}}\Big(\frac{\lambda}{(N-2)^2} - \frac{\sigma}{3}\frac{N-1}{N-2}\Big)\Big\} \\ + v^p(N-2)^{-2} + O(e^{-\frac{2x}{N-2}})v' + O(e^{-\frac{4x}{N-2}})v = 0. \end{aligned} \tag{1.1}$$

In case of $\sigma < 1$, we look for a local solution of the form

$$v = \{\sigma(1-\sigma)(N-2)^2\}^{1/(p-1)} + \eta(x).$$

It turns out that such a solution exists and that we have $\eta \sim e^{\frac{-2x}{N-2}}$ as $x \to \infty$. Lemma 2.7 (iii) has to be corrected as follows:

If $p > \frac{N}{N-2}$, there exists a singular solution of the form

$$u(t) = t^{-2/(p-1)}(c + o(t))$$

near the origin.

This correction is consistent with a result in the preprint by Wu, Chen, Chern and Kabeya entitled "Existence and uniqueness of singular solutions for elliptic equation on the unit ball".

2 Pohozaev type identity for singular solutions

In Section 3.2, we have introduced the Pohozaev type identity to

$$u''(t) + (N-1)\coth(t)u'(t) + \lambda u(t) + u^p(t) = 0 \quad \text{in } \mathbb{R}^+, \quad u > 0. \tag{2.1}$$

We argued as follows:

In a first step we transform (2.1) into an equation without first order derivatives. For this purpose set

$$u(t) = \sinh^{-\frac{N-1}{2}}(t)v(t) = \sinh^{-\lambda_0}(t)v(t).$$

Then $v(t)$ solves

$$v'' - a(t)v + b(t)v^p = 0, \tag{2.2}$$

where

$$a(t) = \lambda_0 - \lambda + \lambda_0 \frac{N-3}{2}\coth^2(t) \quad \text{and} \quad b(t) = \sinh^{-\lambda_0(p-1)}(t).$$

If we multiply (2.2) with $v'g$ and integrate, we obtain

$$\begin{aligned} \frac{1}{2}\int_0^T g'v'^2\,dt = \frac{v'^2 g}{2}\Big|_0^T - \frac{agv^2}{2}\Big|_0^T + \frac{bgv^{p+1}}{p+1}\Big|_0^T \\ + \frac{1}{2}\int_0^T (ag)'v^2\,dt - \frac{1}{p+1}\int_0^T (bg)'v^{p+1}\,dt. \end{aligned} \tag{2.3}$$

Multiplication of (2.2) with $g'v$ and integration yields

$$\begin{aligned} \frac{1}{2}\int_0^T g'v'^2\,dt = \frac{1}{2}g'vv'\Big|_0^T - \frac{1}{4}v^2 g''\Big|_0^T \\ + \int_0^T \left[\frac{g'''}{4} - \frac{ag'}{2}\right]v^2\,dt + \int_0^T \frac{g'b}{2}v^{p+1}\,dt. \end{aligned} \tag{2.4}$$

Suppose that

$$v(0) = v(T) = 0, \quad |v'(T)| < \infty \quad \text{and} \quad \lim_{t\to 0} v(t)v'(t) = 0. \tag{2.5}$$

Then (2.3) and (2.4) lead to the following *Pohozaev* type identity:

$$\frac{v'^2 g}{2}\Big|_0^T + \int_0^T \left[\frac{a'g}{2} + ag' - \frac{g'''}{4}\right]v^2\,dt = \int_0^T \left[\frac{(bg)'}{p+1} + \frac{g'b}{2}\right]v^{p+1}\,dt. \tag{2.6}$$

Now we apply this Pohozaev type identity to solutions which are singular at $t = 0$. For later use, we put $g = \sinh t$.

Since $u(t) \sim t^{-2/(p-1)}$ near $t = 0$, we have

$$v(t) = (\sinh^{(N-1)/2} t)u(t) \sim t^{\{(N-1)p-(N+3)\}/2(p-1)}$$

near $t = 0$. For our purpose, we seek conditions so that all the boundary value in (2.3) and (2.4) vanish. In (2.3), the following three conditions are necessary:

$$\lim_{t \to 0} (v')^2 g = 0, \tag{2.7}$$

$$\lim_{t \to 0} v^2 g = 0, \tag{2.8}$$

$$\lim_{t \to 0} b(t)(\sinh t)v^{p+1} = 0. \tag{2.9}$$

For (2.7), near $t = 0$, we have

$$(v')^2 g \sim t^{\{(N-1)p-(N+3)\}/(p-1)-1}$$

and thus

$$\frac{(N-1)p-(N+3)}{p-1} - 1 > 0$$

is a necessary condition. This is equivalent to

$$p > \frac{N+2}{N-2}. \tag{2.10}$$

From (2.10), (2.8) follows immediately. Concerning (2.9), we see that

$$b(t)(\sinh t)v^{p+1} \sim t^{-(N-1)(p-1)/2} t^{\{(N-1)p(p+1)-(N+3)(p+1)\}/2(p-1)} t.$$

Thus we have a necessary condition

$$\frac{2(N-3)p-2(N+1)}{2(p-1)} > 0.$$

This condition is equivalent to

$$p > \frac{N+1}{N-3}.$$

Since

$$\frac{N+1}{N-3} > \frac{N+2}{N-2},$$

it follows that

$$p > \frac{N+1}{N-3}$$

is a necessary condition.

From (2.4), we see that

$$\lim_{t \to 0} g' v v' = 0 \tag{2.11}$$

and

$$\lim_{t \to 0} v^2 g'' = 0, \tag{2.12}$$

are also necessary conditions. Since $g = \sinh t$, (2.12) is nothing but (2.8) and (2.11) is equivalent to

$$\lim_{t \to 0} v v' = 0. \tag{2.13}$$

This is equivalent to

$$\frac{(N-1)p - (N+3)}{p-1} - 1 > 0$$

and thus

$$p > \frac{N+2}{N-2}.$$

Finally, the Pohozaev type identity holds for $p > (N+1)/(N-3)$. The statement of Lemma 3.1 (ii) should be as follows:

Suppose that $N \geq 4$. *If* $p > (N+1)/(N-3)$ *and* $\lambda \leq N(N-2)/4$, *then we have* $B_s = \emptyset$.

Received October 9, 2012.

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