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A multi-sector growth model with technology diffusion and networks*

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Abstract

This paper adds the production network into a multi-sector endogenous growth model to analyze the respective effects on technology adoption and thus on economic growth. In particular, we show that the higher the network degree, the higher is TFP and the stronger is the impact on *extensive* and *intensive margins*; hence, more likely is technology adoption and economic growth. Therefore, distinct cross-country network structures explain inter-country income differences. We then estimate the model and confirm theoretical findings: e.g., the network degree and technology invention year can together explain 81% of the technology adoption lag variation (*extensive margin*), the network degree can explain 9% of the *intensive margin* variation, and the inverse-network degree and the relative *intensive* and *extensive margins* together account for approximately 31% of the inter-country income differences.

JEL classification: O33, O41, O47.

Keywords: Income Differences, Production Network, Technology Diffusion.

1 Introduction

The core of the endogenous growth theory should be not only the production but also the diffusion of knowledge, which is responsible for Total Factor Productivity (TFP) and explains the majority of inter-country differences in per capita output (e.g., Hall and Jones (1999); Jerzmanowski (2007)). However, since the initial seminal endogenous growth models (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) until the semi-endogenous growth models (Jones, 1995), the focus of the literature was only on the production of knowledge. Given that over time the empirical literature has presented evidence more supportive

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of the endogenous growth models than of semi-endogenous growth (e.g., Dinopoulos and Thompson (2000); Laincz and Peretto (2006); Madsen (2008); Ang and Madsen (2015)), a new wave of theoretical models recovered the endogeneous growth result (e.g., Peretto (1998); Howitt (1999); Acemoglu (1998, 2002)) and increasingly highlight the knowledge diffusion (e.g., Parente and Prescott (1994a); Basu and Weil (1998); Comin and Hobijn (2010)).

Our contribution inserts itself on this new wave of models and emphasises the inter-sector production network effects on technology adoption. That is, we take advantage of the network concept to examine how the production network shape affects technology adoption decisions, which, in turn, help to explain inter-country differences in TFP.

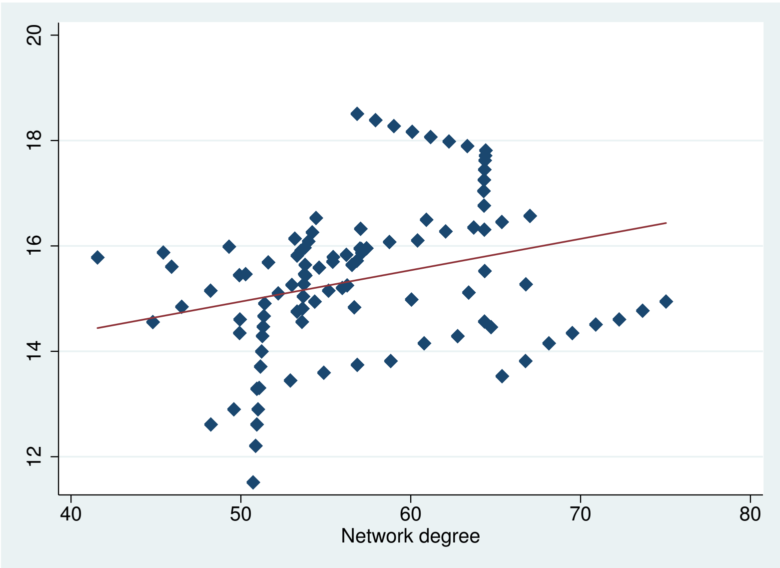
Our research is motivated by four findings. Firstly, we intend to build an endogenous growth model that accommodates the core of endogenous growth models; i.e., the production and the diffusion of knowledge, using for this purpose the concept of network. Secondly, we observe an intensification of inter-sector and inter-firm relationships in the recent years, and this intensification of relationships has been neglected by the theoretical growth models. Thirdly, recent studies have shown that social networks affect the diffusion of communication technologies (e.g., (Jackson, 2011)). Thus, we can expect inter-sector networks to show a similar result in technology diffusion and, consequently, in economic growth. Finally, some studies have shown that the network structure significantly affects the propagation of idiosyncratic shocks (e.g., Acemoglu et al. (2012)).

We therefore examine how the production network structure can affect technology adoption and, accordingly, economic growth and inter-country income differences. Our research question is then the following: can the production network structure explain the inter-country differences in adoption lags (*extensive margin*, decomposed in the *embodiment effect*, related to the time of adopting a new technology, and in the *variety effect*, connected with the number of used varieties dependent on the path of adoption), adoption intensity (*intensive margin*, associated with the number of adopted technologies), and income per capita? Concerning the extensive margin it should be emphasised that because new technologies are more productive, the adoption of new technologies raises the productivity level of the technologies used. Thus, our argument is that inter-sector buyer-supply relationships act as an important channel throughout the technological-knowledge flow and, therefore, the inter-sector network structure may play an important role on the adoption of new technologies and, consequently, on the productivity level of technologies in use (*embodiment effect*) and the range of technologies in use (*variety effect*).

To answer to the above question, we introduce an extended version of the intermediate goods sector proposed by Ngai and Samaniego (2009) in the technology diffusion model of Comin and Hobijn (2010). Each intermediate good can be consumed or used by other sectors as a production input. Thus, we account for the inter-connections between the sectors in the input-use matrix and consider input-use relationships to map the inter-sector production network. In particular, we argue that a production network structure, through its inter-sector linkages, stimulates technology adoption by other sectors. The network structure establishes links between sectors at the micro level, which, according to empirical evidence, seems to have effects at the macro level.

A simple example is illustrated in Figure 1. This Figure plots the relationship between network degree and computer diffusion for each country. The network degree indicates the average number of sectors from or to which a sector buys or sells intermediate goods; that is, the average number of trade partners (neighbors) of each sector. Thus, the higher the number of trade partners, the higher is the probability of an individual sector's decisions to be affected by its neighbors. This is called the peer effect in social network theory (Jackson, 2011).

Figure 1: Computer adoption versus network degree



This Figure plots the per capita computer adoption logarithm for the UK, Germany, France, Denmark, Canada, Netherlands, Australia, and Japan from 1968 *to* 1998 *against the inter-sector network degree of each country. The estimated relation is* $\log pc = 11.43 + 0.07$ *degree, and the network degree is statistically significant at the* 1% *level.*

As stated above, the papers of Ngai and Samaniego (2009) and Comin and Hobijn (2010) are particularly useful in our analysis. We follow the Ngai and Samaniego (2009) strategy to solve the model. When we add intermediate goods into the multi-sector model, we define the model in terms of gross output. Thus, to aggregate this into a one-sector model, we find an equivalent one-sector value-added model different from the one obtained in the traditional multi-sector models. Therefore, we argue that the inter-sector network can affect technology adoption and growth from the effects of the composition of intermediate goods in TFP. As our model will demonstrate, these effects can be captured and described by the production network properties. In relation to technology diffusion model of Comin and Hobijn (2010), we then correct the *extensive margin* by the composition of the inter-sector linkages that can be characterized through the weighted network degree, we also add and endogeneize the *intensive margin*, and we further include and treat the income differences.[1]

[1]The intensive margin is omitted in Comin and Hobijn (2010). It is however proprosed in Marti Mestieri et al. (2013), but the mechanism is quite weak since depends on an exogenously introduced parameter in the production function.

The literature has recently investigated the effects of the production network structure on aggregate fluctuations (Carvalho, 2014; Acemoglu et al., 2012; Kelly et al., 2013), aggregate productivity (Oberfield, 2013), and international trade (Chaney, 2014). Carvalho (2014) and Acemoglu et al. (2012), using a network theory perspective, analyze the role of sectoral and idiosyncratic shocks in generating aggregate fluctuations. Oberfield (2013) demonstrates that when the ratio of intermediate goods relative to labor in production is high, star suppliers appear[2] endogenously and aggregate productivity increases. Finally, Hausmann and Hidalgo (2011) relate the structure (complexity) of exported goods to the capabilities required to produce them and show that diversification increases with the number of capabilities of a country. On the other hand, Ngai and Samaniego (2009) shows that the composition of intermediate goods may affect the productivity of the value-added model. However, most researchers mapping the multi-sector model with intermediate goods into the one-sector model have neglected the distinction between productivity indices of the valued-added and gross output models.

We observe that indeed the production network structure explain the cross-country differences in the *extensive margin* (adoption lags), the *intensive margin* (adoption intensity), and income per capita. In particular, the solution shows that endogenous adoption decisions as well as the weighted degree of inter-sector networks determine the growth rate of productivity embodied in technology through the *embodiment effect* and the *variety effect* behind the *extensive margin*. That is, the average number of trade partners in an economy and the strength of their trade flows work as an externality at the aggregate level, increasing productivity of the technologies in use (*embodiment effect*); moreover, the adoption of new technologies increases the range of technologies in use, improving the productivity level of the technologies in use, and this effect, which is also proportional to the weighted network degree (*variety effect*).

From a comparison of the results of this study and Comin and Hobijn (2010), the *embodiment* and *variety effects* are then corrected by the weighted network degree, that is, by a statistical measure of the composition of the intermediate goods in the economy. In economies where the average number of inter-sector trade partners and the weight of inter-sector trade flows are high, new technologies can raise the aggregate productivity higher than in economies where the number of trade partners and trade flows are small. Thus, the growth rate of productivity obviously depends on the time of technology adoption, on whether the new technologies are more productive, and also on the inter-sector network degree due to peer and linkage effects.

To test the model, we use data from two databases, the OECD harmonized input-output (I-O) dataset (1995 and 2002 editions), and the CHAT database. We use I-O matrices to characterize the production network. Each sector of the I-O tables is an input supplier to every other sector and represents a network node, and between any two pairs of nodes, there is a direct arc. Thus, we can map the inter-sector production relations as a complete and weighted directed network. This methodology allows us to study the main features of the production network and its evolution over time. From the CHAT dataset, we collect data about technol-

[2] A star supplier is an entrepreneur who sells intermediate goods for many other entrepreneurs.

ogy adoption. We select data for 10 technologies: cell phones, computers, Internet, ATMs, magnetic resonance imaging (MRI), credit and debit transactions, electronic funds transfers (EFT), cable TV, points of services for debit and credit cards (POS), and computer tomography scanners (CATs). These technologies cover different sectors of the economy such as finance, health, telecommunications, and general sectors. For example, computers and the Internet are technologies used by all sectors of the economy. The technology selection criterion is its invention year. As the I-O tables computation started around the 1960s, we can research only the effects of the production network on technology adoption for technologies whose invention year occurs after the 1960s.

We obtain additional results on the intensity of adoption and cross-country income differences. The number of technology units depends on the intensity of the intermediate goods as well. Finally, the cross-country differences in income per capita can be decomposed into three factors: (i) differences in *intensive margin*, (ii) differences in weighted network degree, and (iii) differences in time of technology adoption. Hence, the network degree is also a determinant of the intensity and time of adoption.

Our main finding is that all our theoretical results are confirmed by our empirical test. Our model provides precise estimates of the adoption lags for 84% of our technology-country pairs. Additionally, 81% of the adoption lag variability is explained by the technology invention year and inverse-network degree. Secondly, we estimate that the median level of the adoption intensity is 83% of the US level and that the production network structure accounts for 9% of its variability. Finally, the model accounts for 48% of the income per capita variability and 31% of the cross-country income differences.

To sum up, our paper takes advantage of a parsimonious analytical structure to contributes to the growth literature by identifying the inter-sector network structure effect in TFP. We provide a new growth model that focuses on the input-use relationship between firms or sectors. We use these trade flows between sectors to set up the production network. The interpretation of sectors/firms as nodes and the trade flows between them as linkages seems normal. This network interpretation allows us to use analytical methods from the network literature and centrality network measures as proxies for the synergies between sectors and the aggregate network production level. Moreover, by using available data, we show that the theoretical network "operator" approaches well the estimated series for the technological diffusion.

The rest of the paper is organized as follows. Section 2 summarizes the inter-sector network properties and introduces the case of computer diffusion. Section 3 describes the data used in this paper. Section 4 defines a multi-sector growth model with I-O relationships between sectors. This section also derives the equivalent value-added and one-sector aggregate model. Section 5 derives technology diffusion measures. Section 6 presents our estimated results, and section 8 concludes the paper.

2 Network properties and computer diffusion

The network concept, consisting of two or more entities sharing resources and information, has been adapted and extended by many fields of study, including Economics. The importance of some network properties,[3] such as network density, shortest distance, network size, and strong or weak ties have been researched by industrial economics and social network theory as factors responsible for the propagation of network effects (Suarez, 2005; Jackson, 2011; Bertolotti et al., 2015; Guan et al., 2016). In aggregate terms, we follow Ciccone and Matsuyama (1996) and consider that economies at earlier stages of development are characterized by relatively low diversity of economic activities and of network effects, but when the economy develops and increases the diversity of economic activities, the network effects become increasingly more relevant and a roughly proportional diffusion of knowledge is expected.

In this Section, we start by briefly describing the inter-sector network properties and their temporal evolution by emphasizing three countries: Germany, Japan, and the US (Subsection 2.1). We then focus on the network degree, because, as the model shows, it is the determinant of productivity path. We examine the empirical correlation between network degree and technology diffusion, by considering computer diffusion, as the example for technology diffusion process, namely for Australia, Denmark, Germany, The UK, and the US (Subsection 2.2).

2.1 Network properties

A network has certain attributes that can be calculated to analyze the respective properties, which, in turn, define how certain models/economies contrast to each other. In a context of an economy, the input supply relation between different sectors can be represented as a weighted and directed graph, which can be used to characterize the formation and development of a network, as well as its properties – see, e.g., Jackson (2008) as pioneering study on this subject. Thus, we conjecture that the input supply network of the economy $E_n = (\mathcal{J}_n, \mathcal{W}_n)$, where $\mathcal{J}_n$ is the set of sectors and $\mathcal{W}_n$ is the input-output matrix, can be represented by the graph $\mathcal{G}_n = (\mathcal{V}_n, E_n, \mathcal{W}_n)$ with vertex set $\mathcal{V}_n$, edge set E_n, and edge weighted matrix $\mathcal{W}_n$. Each vertex in $\mathcal{V}_n$ corresponds to an economic sector; i.e., $\mathcal{V}_n = \mathcal{J}_n$. Moreover, $(i, j) \in E_n$ takes the value of 1 if sector i is an input supplier to sector j and 0 otherwise, and the weight of edge $(i, j) \in E_n$ is equal to w_{ij}; i.e., the input intensity of sector i in sector j's production. The directed edges are called arcs in graph theory. Thus, by definition, the I-O matrix is a weighted directed graph or network. Self-loops $(i, i) \in E_n$ with $w_{ii} > 0$ are allowed in our definition of supply network as observed in I-O matrices.

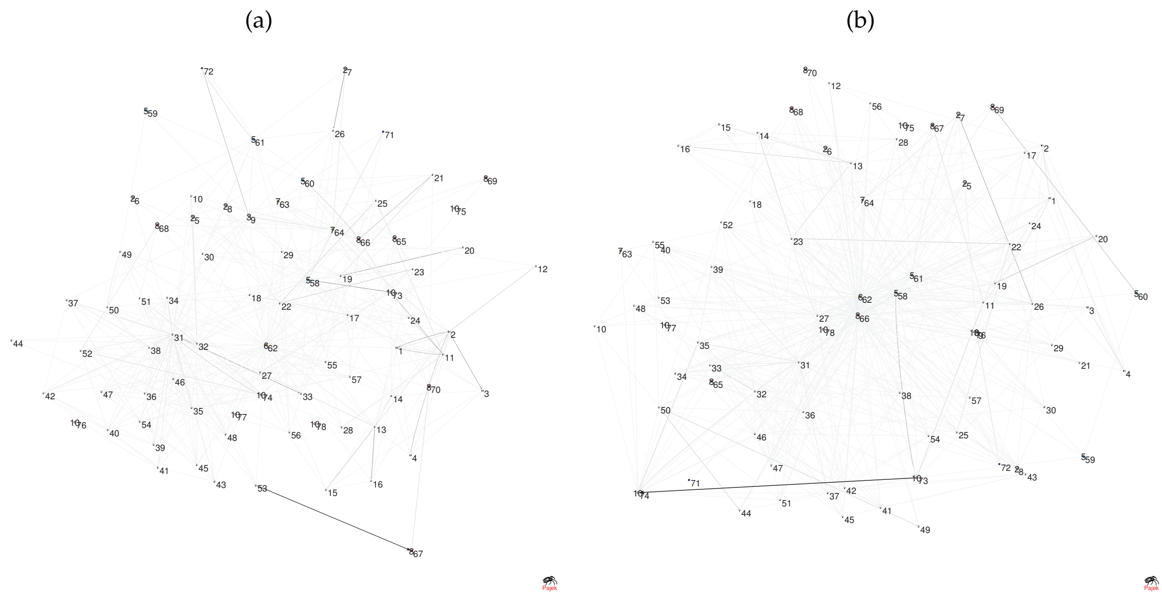
Figure 2 depicts the US inter-sector network as a graph. Each vertex (dot) corresponds to an economic sector defined at the North American Industry Classification System (NAICS) two-digit-level desegregation in the Bureau of Economic Analysis (BEA) Use Tables,[4] for a total of

[3] See, e.g., Jackson (2008) and Magalhães et al. (2016) for an extensive analysis of network properties.

[4] The I-O data for 1945 are extrapolated from the 1947 benchmark I-O tables and are based on the 1957 Standard Industrial Classification (SIC) system, whereas the data for 1995 were prepared to fit the 1957 SIC system.

78 sectors. Each link is shaded by the correspondent weight, with darker shades indicating higher values. The links represent an in-flow or out-flow between sectors i and j that is greater than 2%. The left-hand side and right-hand side show the intermediate I-O flows for the US in 1945 and 1995, respectively. Obviously, a large number of links (arcs) exist between these 78 sectors. The Figure suggests the existence of a small group of sectors connected by a large number of links and an increase in network connectivity in 1995. These observations can be confirmed by some network statistics. For example, the network density, network links, and the average degree for I-O transactions above 2% are 0.063, 382, and 9.79 in 1945 and 0.069, 424, and 10.87 in 1995, respectively. This increase in intermediate input intensity, trade partners, and flows are a result of higher production fragmentation and sector integration.

Figure 2: Inter-sectoral network for the US.



The I-O network corresponds to the US I-O matrix in 1945*(left) and* 1995 *(right) for every I-O transition* 2% *above the total input purchase. Each vertex (dot) corresponds to a sector.*

To complement this analysis, we provide the network summary statistics for the US in Table1, which compares the US production networks at the beginning of the 1970s and end of the 1990s. The data were collected by the OECD and are based on the International Standard Industrial Classification (ISIC) Rev. 2 at the two-digit level. These statistics can be summarized in two groups, network dimension and sector importance. The number of nodes, number of crossings, number of lines, and network degree describe the network size; all these statistics show an increase from 1972 to 1997. The production network is bigger and the integration between sectors higher in the 1990s. For example, in 1997, on average, each sector has connections with 10 more sectors than it had in 1972. In sum, the statistics' closest vertices and the shortest and longest lines describe the relative importance of each sector within the network. However, the Table shows some differences here. For instance, in 1972, other manufacturing and governing services (sectors 24 and 34) were the closest sectors. Yet, the closest sectors in 1997 were public administration (equivalent to government services) and the renting of machinery and equipment. This indicates that the public sector is critical to the economy, but its spending and finance system remained chained in the last 30 years.

Table 1: The US inter-sector network properties

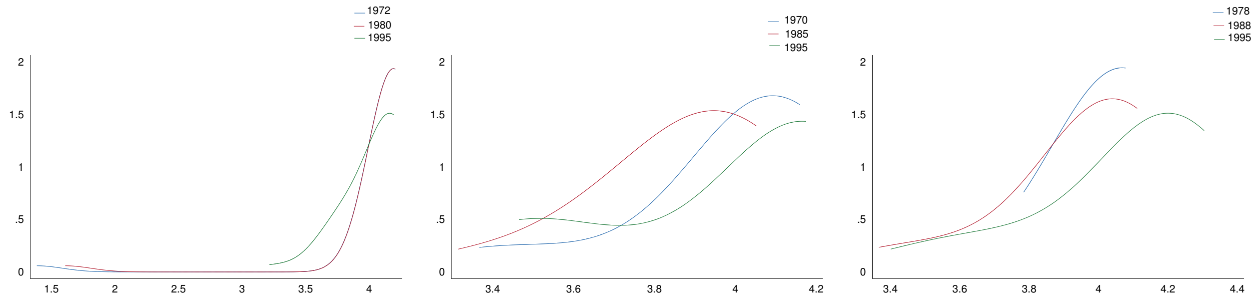
	1972				1997		
Closest vertices	24; 34	Distance	0.03978	Closest vertices	33; 37	Distance	0.04448
Smallest angle	2-1-2	Angle	0.00000	Smallest angle	2-1-2	Angle	0.00000
Shortest line	5-33	Length	0.10830	Shortest line	25-34	Length	0.10910
Longest line	16-19	Length	0.76241	Longest line	1-35	Length	0.83152
N. of crossings	134050	N. of lines	1089	N. of crossings	252616	N. of lines	1483
Closest vertex to line	11 to 22;26	Distance	0.00005	Closest vertex to line	38 to 25;17	Distance	0.00002
Density (no loops)	0.91512	Degree	62.22858	Density (no loops)	0.90427	Degree	72.34146
N. of nodes	35			N. of nodes	41		

Statistics obtained with Pajek for the I-O matrix at the two-digit level and for transitions 1% above the total input purchase of a sector.

The network density distribution gives us an idea of the relative importance of all sectors in an economy. Figure 3 plots the empirical density of intermediate input shares for the US, Germany, and Japan in the 1970s, 1980s, and 1990s. Our first observation is that the density distributions are quite different. The US has a smaller number of sectors with a reduced number of connections than does Japan or Germany. On average, the US sectors directly connect with a higher number of sectors than do the other two countries' sectors because of the highly fragmented production system. However, over the recent years, Germany and Japan have followed the US trend. The German and Japanese densities shifted to the right during the recent decades. Furthermore, the US sectors are more homogeneous than the Japanese and German sectors. The density distribution in these two countries is flatter, with higher standard deviation. On the contrary, almost all sectors in the US have a number of connections close to the average.

Figure 3: Empirical density of intermediate I-O shares

(a) the United States (b) Japan (c) Germany



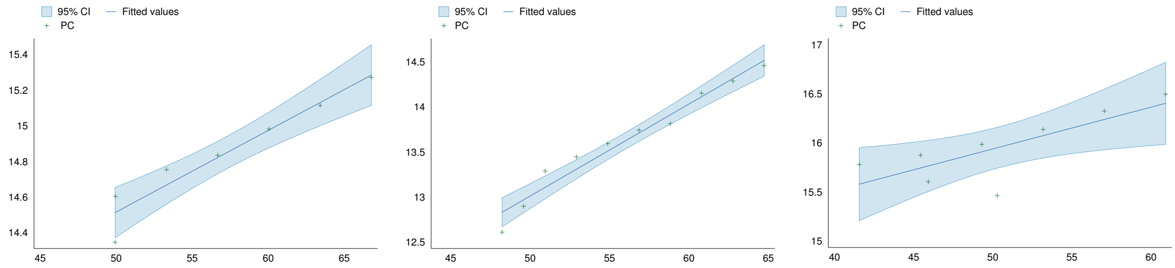
The I-O network 1% above the total input purchase. The y-axis plots the network density and x-axis plots the log of the network degree. We use Gaussian distribution as the kernel smoothing function with a bandwidth of 0.2.

2.2 Computer adoption case

Let us now examine the relationship between the inter-sector network structure and the technology diffusion process, taking computers as example namely for Australia, Denmark, Germany, the UK and the US.

Figure 4: Correlation between network degree and computer adoption.

(a) Australia (b) Denmark (c) Germany



The estimated relations are: (i) $\log pc = 12.23 + 0.05\, degree$, $Adj.R^2 = 0.91$ *for Australia; (ii)* $\log pc = 7.90 + 0.10\, degree$, $Adj.R^2 = 0.95$ *for Denmark; and* $\log pc = 13.81 + 0.04\, degree$, $Adj.R^2 = 0.55$ *for Germany. The network degree is statistically significant at the* 1% *level for Australia and Denmark and at the* 5% *level for Germany.*

Figure 4 shows the logarithm of computer adoption and network degree with the linear fit and mean confidence interval. As expected, a strong and positive correlation exists between units of the adopted computer and network degree. The linear regression slope varies between 0.04 (Australia) and 0.10 (Denmark). Thus, for each additional trade partner in Denmark, the purchase of computers increases by 10.5%. Even when we control for industry-fixed effects, the relationship remains unaltered, implying that it is not an industry composition effect. At a higher level of desegregation,[5] the relationship is also positive and statistically significant. Hence, we find a positive correlation between technology diffusion and degree of production network in computers.

However, our purpose is to go further than simply investigating the technology diffusion process. Comin and Mestieri (2014) identified two differences in the technology diffusion path between countries: adoption lags (*extensive margin*) and penetration rate (*intensive margin*). These margins are illustrated in Figure 5. Adoption lags are related to the adoption time, whereas penetration rate is related to the number of units adopted. Thus, countries adopt various technologies at different periods of time and different number of units per capita. Thus, we analyze how the production networks explain these two differences. The adoption lags and intensity of adoption will be reflected on the aggregate productivity of each country. When a country adopts new (and more productive) technologies, its productivity growth rate goes up. Similarly, when a country buys more technology units, its productivity increases.

3 Data

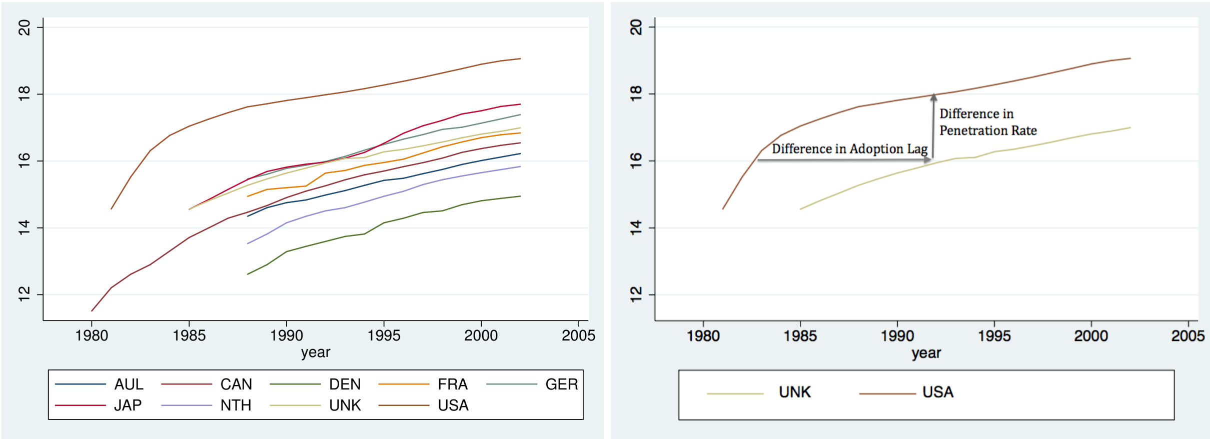
The data used in for study come from two databases:[6] (i) Technology, income, and population data come from the CHAT database,[7] collected by Comin and Hobijn (2009), which covers the diffusion of 104 technologies from 161 countries over the last 200 years; (ii) the production

[5] We test this correlation for the US at the three-digit level and we also find a positive relation.

[6] See Appendix I for further details.

[7] The income and population data included in the CHAT database were previously collected by Maddison (2004).

Figure 5: Personal computers



This Figure, adapted from Marti Mestieri et al. (2013), on the left-hand side shows the log of personal computer adoption, and that on the right-hand side shows the same data for the US and the UK, evidencing adoption lag and penetration rate differences between the two countries.

network data come from the OECD I-O edition 1995 (ISIC Rev. 2) and edition 2002 (ISIC Rev. 3).

Since the I-O tables started to be computed only around the 1950s, our analysis focuses on the technologies invented after that period until 1998. We select 10 technologies (see Table 9 in the Appendix) from several economic sectors such as communication and information technologies, services, and medical and general sectors. The technology measure in the CHAT database can be either the amount of output produced with the technology (e.g., number of transactions using payment cards at points of service) or the number of units of capital embodying the technology (e.g., number of computers).

Historical I-O tables are available for nine countries: Australia, Canada, Denmark, France, Germany, Japan, Netherlands, the UK, and the US at the two-digit level. The 1995 edition considers 35 industries and the 2002 edition considers 41 industries. To homogenize these editions, we aggregate some sectors, to end up with 35 sectors. Next, as the I-O tables are collected every five years, we compute the yearly I-O tables. We used the linear interpolation procedure following the BEA, and OECD methodology. We discard very small transactions between the sectors and consider only input transactions at least 1% above the total input purchase.

To define the production network structure, we follow Acemoglu et al. (2012) and Carvalho (2007). We set the I-O table as a network of input-flows, where each sector is a node and each input-supply relationship is a weighted directed arc linking two nodes. Thus, the I-O matrix is, by definition, a weighted directed network.

4 Model

Next, we present a multi-sector endogenous growth model extended to include production network in order to analyze the effects on technology adoption and on economic growth. Thus, in addition to the importance of innovation in growth, empirically emphasized by for instance Hasan and Tucci (2010) we intent to take into account the production network to analyze the respective effects on technology adoption and on economic growth. It is an extension of the Comin and Hobijn (2010) model by adding an intermediate goods sector similar to the one considered by Ngai and Samaniego (2009). Goods produced by each industry are used as intermediate goods by different industries of the economy, and the intermediate goods composition differs across sectors, as evidenced by the I-O data. Our goal is to prove that, firstly, I-O linkages affect the productivity indices. Secondly, using the network theory perspective, we demonstrate that the impact on TFP depends on the production networks' properties, in particular the network degree.

4.1 Preferences

A representative household exists in the economy, which consumes and supplies inelastically one unit of labor at the wage rate W. The household utility is given by

$$U = \int_{t_0}^{\infty} e^{-\rho t} \log(C_t) \mathrm{d}t, \tag{1}$$

where ρ denotes the discount rate and C_t denotes the consumption per capita at time t. Capital markets are perfectly competitive and consumers can lend and borrow at the real rate $\tilde{r}$.[8] The household maximizes the inter temporal utility (1) subject to the budget constraint and the transversality condition. From the first order conditions (FOC) the consumption Euler Equation is $\frac{\dot{C}}{C} = \tilde{r} - \rho$.

4.2 Production

4.2.1 Technology

At each instant t, a new, more productive production method or technology vintage v appears exogenously. Technology vintages are capital embodied, and vintages are indexed by the time they appear. The set of vintages available at period t is given by $V = (-\infty, t)$. The productivity of new vintages is sector-specific and grows constantly over time at the rate of γ_i across vintages,

$$Z_{v_i} = Z_{0_i} e^{\gamma_i v}, \tag{2}$$

[8] Henceforth, we omit subscript t to simplify the notation.

where: subscript i denotes a sector, $i = 1, ..., M$; and Z_{0_i} is a normalization parameter that gives the productivity level of vintage $\underline{v}$, such that $Z_{0_i} = Z_{\underline{v}_i} e^{-\gamma_i \underline{v}}$.

Not all technology vintages available in the world are used since, as shown below, making capital vintages available for production is costly. The set of technology vintages actually used is given by $V = (-\infty, t - D]$, where $D \geq 0$ denotes the adoption lag, that is, the amount of time taken by a country to adopt the best technology from the time such technology was invented. Following Comin and Hobijn (2010), we assume that a technology, denoted by τ, is a set of production methods used to produce closely related intermediate goods. In particular, assume only two technologies: an old technology denoted by o and a new one denoted by n. The former consists of a set V_o of production methods introduced until time $\underline{v}$, such that $V_o = (-\infty, \underline{v}]$, while the latter includes a set V_n of production methods invented after $\underline{v}$, such that $V_n = [\underline{v}, t - D]$.

4.2.2 Final output

The aggregate final output, Y, is competitively produced by a constant elasticity of substitution (CES) production function

$$Y = \left(\int_{-\infty}^{t-D} Y_v^{\frac{1}{\mu}} \, \mathrm{d}v \right)^{\mu} \equiv \left(\sum_{\tau \in \{o,n\}} Y_\tau^{\frac{1}{\mu}} \right)^{\mu}, \tag{3}$$

where: $\mu > 1$ denotes the elasticity of substitution between technologies, and Y_v denotes the final output produced with technology vintage v. The final output producers' optimization problem subject to the production function (3) is $max \; PY - \int_v P_v Y_v \, \mathrm{d}v$ and from the FOCs result

$$Y_v = \left(\frac{P_v}{P} \right)^{-\frac{\mu}{\mu-1}} Y, \tag{4}$$

and $P = \left[\int_{\underline{v}}^{\overline{v}} (P_v)^{-\frac{1}{\mu-1}} \, \mathrm{d}v \right]^{-(\mu-1)}$ for the price index.

4.2.3 Intermediate sectors

The economy consists of M productive units or sectors, each producing a different good i, $i = 1, ..., M$. Let Q_{vi} be the gross output produced by technology vintage v in sector i, which can be used either as a final good, Y_{vi}, or as an intermediate good, H_{vi}. The market clearing for each sector requires $Q_{vi} = Y_{vi} + H_{vi}$.

The perfect competive gross output Q_{vi} is produced under a Cobb-Douglas technology:

$$Q_{vi} = Z_{vi} M_{vi}^{\alpha_m} \left(K_{vi}^{\alpha} L_{vi}^{1-\alpha} \right)^{1-\alpha_m}, \tag{5}$$

where: Z_{vi} is (remember) the productivity level of the vintage v in sector i, which is likely to change over time; M_{vi} is the composition of the intermediate goods used by sector i;[9] K_{vi} and L_{vi} represent the amount of capital and labor used as input in the production of Q_{vi}; α_m is the shares of M_{vi} in production, $\alpha(1-\alpha_m)$ is the share of capital in production, and $(1-\alpha)(1-\alpha_m)$ is the share of labor in production. That is, Q_{vi} combines labor L_{vi}, capital K_{vi}, and intermediate goods M_{vi} of a particular vintage v.

The composition of the intermediate goods differs across sectors and is modeled as in Ngai and Samaniego (2009). The intermediate goods used by sector i are produced according the following technology:

$$M_{vi} = \prod_j \left(\frac{H_{vji}}{\psi_{ji}}\right)^{\psi_{ji}}, \tag{6}$$

where: H_{vji} is the intermediate good of sector j used in the production of good i, and the matrix Ψ with the elements $\psi_{ji} \geq 0$, $\sum_j \psi_{ji} = 1$, can be mapped into the I-O table linking the flow of intermediate goods across sectors. The market clearing condition for each sector i is $\sum_j H_{vij} = H_{vi}$.

4.2.4 Capital goods production and technology adoption

Each capital good producer must invest an up-front fixed cost in order to be monopolist, protected by a patent, which represents the adoption cost of either the production method or technology vintage v. Following Comin and Hobijn (2010), the adopting cost of vintage v at time t is given by

$$\Gamma_{vt} = \overline{\psi}\,(1+b)\left(\frac{Z_v}{Z_t}\right)^{\frac{1+\nu}{\mu-1}}\left(\frac{Z_t}{A_t}\right)^{\frac{1}{\mu-1}} Y_t, \tag{7}$$

where: $\nu > 0$; $\overline{\psi}$ is the steady-state stock market capitalization-to-GDP ratio, which is included for normalization purposes; b indicates the barriers to adoption; A_t denotes the TFP associated with the final aggregate output Y_t set out below; and Z_t denotes the productivity of technology vintage t. The term $\left(\frac{Z_v}{Z_t}\right)^{\frac{1+\nu}{\mu-1}}$ illustrates that it is costly to adopt technologies closer to the technology frontier. Finally, The last two terms, $\left(\frac{Z_t}{A_t}\right)^{\frac{1}{\mu-1}}$ and Y_t, show that the adoption cost is increasing with the market size. As shown by Comin and Hobijn (2010) and Parente and Prescott (1994b), this cost formulation ensures the existence of an aggregate balanced growth path.

To produce one unit of any capital good, only one unit of the final output Y is required and the production process is fully reversible. Since the final good is chosen as *numeraire*, the marginal production cost of capital goods is equal to unity. For simplicity, assume no physical capital

[9]Since the data of technology adoption are not sector specific, we assume, by simplification, that the intermediates used as inputs in the production of each intermediate good are of the same vintage of the output.

depreciation. Capital good supplier of vintage v rents its good at the rate R_v.

4.3 Factor demand, prices, and aggregation of all sectors

We have technology vintages, sectors, and the consumption of intermediate goods. To compute a balanced growth equilibrium, we need to carry out several aggregations in order to end up with the equivalent one-sector growth model. The procedure is as follows. Firstly, we find the equivalent multi-sector model in terms of value added; that is, without intermediate goods. Otherwise, the effects of the intermediate goods composition on the aggregate productivity index are neglected (Ngai and Samaniego, 2009; Greenwood and Yorukoglu, 1997). Secondly, we aggregate the value-added multi-sector model in the one-sector model; that is, we aggregate the output produced by each sector. At this point, we have the aggregate output for each vintage v. Finally, we aggregate the technology vintages.

In order to find the equivalent multi-sector model in terms of value added, we need to derive the factor demand, prices, and productivity for each sector i in terms of gross output – Section 4.3.1. By using them, we can find the equivalent value-added model and derive the demand, prices, and productivity indices for each sector i in terms of value-added output – Section 4.3.2.

4.3.1 Factor demands, prices, and productivity indices for each sector in terms of gross output

In each sector i, firms produce gross output Q_{vi} in (5) by maximizing their profits; i.e., $\max_{K_{vi},L_{vi},M_{vi}} P_{vi}Q_{vi} - R_{vi}K_{vi}W_{vi}L_{vi}p_{mvi}M_{vi}$, where: P_{vi} is the price of the gross output produced by sector i with technology vintage v; R_{vi} is the rental rate of capital used by sector i with technology v; W_{vi} is the wage of labor used by sector i with technology v; and p_{mvi} is the price of the composition of intermediate goods used by sector i and technology v. From the FOCs the optimal usage of intermediate goods, labor and capital in the sector i are:

$$p_{mvi}M_{vi} = \alpha_m P_{vi}Q_{vi}, \tag{8}$$

$$W_{vi}L_{vi} = (1-\alpha_m)(1-\alpha)P_{vi}Q_{vi}, \tag{9}$$

$$R_{vi}K_{vi} = (1-\alpha_m)\alpha P_{vi}Q_{vi}, \tag{10}$$

i.e., firms spend a constant expenditure share on all inputs M_{vi}, L_{vi}, and K_{vi}.

As Y_{vi} is produced competitively, its price is equal to the marginal cost. The revenue share of labor, revenue share of intermediate goods, and rental cost of capital exhaust the remaining revenue. Labor is homogeneous, competitively supplied, and perfectly mobile across sectors, meaning that $W_{vi} = W$. Further, the rental rate of capital is equal across all sectors for each technology v, meaning that $R_{vi} = R_v$. Therefore, $P_{vi} = \frac{1}{1-\alpha_m}(1-\alpha_m)^{\alpha_m}\frac{1}{Z_{vi}}\left(\frac{W}{1-\alpha}\right)^{(1-\alpha)(1-\alpha_m)}\left(\frac{R_v}{\alpha}\right)^{\alpha(1-\alpha_m)}$ $\left(\frac{p_{mvi}}{\alpha}\right)^{\alpha_m}$.

From the free mobility of inputs across sectors, the capital-labor ratio is equalized across sectors, $\frac{K_{vi}}{L_{vi}} = \frac{K_{vj}}{L_{vj}}$, and the intermediate-labor ratio may differ across sectors due to differences in p_{mvi}. The optimal usage of intermediate and labor inputs implies that for any sectors i and j, $\frac{p_{mvi}M_{vi}}{L_{vi}} = \frac{p_{mvj}M_{vj}}{L_{vj}}$. By equating the marginal product of labor across sectors, we obtain the relative prices

$$\frac{P_{vi}}{P_{vj}} = \frac{Z_{vj}}{Z_{vi}}\left(\frac{p_{mvi}}{p_{mvj}}\right)^{\alpha_m}, \tag{11}$$

which depend on not only the relative productivity term $\frac{Z_{vj}}{Z_{vi}}$, but also on the productivity of other sectors if the composition of the intermediate goods used by sectors i and j differs. The optimal composition for the intermediate goods producers is obtained by solving the intermediate good producers' profit maximization problem $\max_{H_{vi}} p_{mvi}M_{mvi} - \sum_j P_{vj}H_{vji}$, bearing in mind ((6)), resulting

$$P_{vj}H_{vji} = \psi_{vji}p_{mvi}M_{vi}, \ \forall j. \tag{12}$$

By using the profit zero condition for intermediate good producers and substituting H_{vji} by the optimal demand condition derived in (12), we obtain the index price of the intermediate goods composition for sector i,

$$p_{mvi} = \prod_j P_{vj}^{\psi_{vji}}. \tag{13}$$

Next, by using equation (13), we derive the relation between relative prices,

$$\frac{p_{mvi}}{P_{vi}} = \prod_j \left(\frac{P_{vj}}{P_{vi}}\right)^{\psi_{vji}}, \tag{14}$$

and by substituting (14) into (11), we also derive the relationship between relative prices and relative productivity

$$q_{vi} = \frac{P_{vk}}{P_{vi}} = \frac{Z_{vi}}{Z_{vk}}\left[\prod_j \left(\frac{P_{vk}}{P_{vj}}\right)^{(\psi_{vji}-\psi_{vjk})}\right]^{\alpha_m} = \tilde{z}_{vi}\left(\prod_j q_{vj}^{\phi_{vji}}\right)^{\alpha_m}, \tag{15}$$

where: $q_{vi} = \frac{P_{vk}}{P_{vi}}$; $\tilde{z}_{vi} = \frac{Z_{vi}}{Z_{vk}}$; $q_{vj} = \frac{P_{vk}}{P_{vj}}$; and $\phi_{vji} = \psi_{vji} - \psi_{vjk}$, which measures the intensity of good j in the production of sector i relative to sector k. By taking the logarithms and rewriting (15) in matrix form, we have

$$\begin{bmatrix}\log q_{v_i=1}\\ \vdots \\ \log q_{v_i=m}\end{bmatrix} = (\boldsymbol{I} - \alpha_m\boldsymbol{\Phi})^{-1}\begin{bmatrix}\log \tilde{z}_{v_i=1}\\ \vdots \\ \log \tilde{z}_{v_i=m}\end{bmatrix}.$$

Thus, the equilibrium growth rate of productivity and prices are related by

$$\begin{bmatrix} \log \gamma_{q_{v_i=1}} \\ \vdots \\ \log \gamma_{q_{v_i=m}} \end{bmatrix} = \left(\boldsymbol{I} - \alpha_m \boldsymbol{\Phi}\right)^{-1} \begin{bmatrix} \log \gamma_{\tilde{z}_{v_i=1}} \\ \vdots \\ \log \gamma_{\tilde{z}_{v_i=m}}, \end{bmatrix}.$$

where $\boldsymbol{\Phi} = \left(\boldsymbol{1} - \boldsymbol{\alpha_m}\right) \boldsymbol{\alpha_m^{\frac{\alpha_m}{1-\alpha_m}}} \boldsymbol{Z_{\underline{v}_k}^{\frac{1}{1-\alpha_m}}} \prod_{j=1}^{M} \boldsymbol{z_{\tau_{jo}}^{\psi_{vjk}\frac{\alpha_m}{1-\alpha_m}}}$ is a matrix $(m \times m)$, and when the intermediate goods usage is the same across sectors $\boldsymbol{\Phi} = \boldsymbol{0}$ and $q_{vi} = \tilde{z}_{vi}, \forall i$.

4.3.2 Factor demands, prices, and productivity indices for each sector in terms of value added

Now, we map the multi-sector model with intermediate goods into a multi-sector value-added model. That is, we re-write the equivalent firms' problem in terms of value added. Through the optimal use of the intermediate goods in (8), the firms' maximizing problem in terms of gross output is equivalent to maximizing the profits in terms of the value-added $Y_{vi} = Z_{vi}^{va} F^{va}\left(K_{vi}, L_{vi}\right)$, where Z^{va} is the value-added productivity level for sector i and vintage v.

By definition, the value-added revenue is

$$P_{vi}^{va} Y_{vi} = P_{vi} Q_{vi} - p_{mvi} M_{vi} = \left(1 - \alpha_m\right) P_{vi} Q_{vi}, \tag{16}$$

where P_{vi}^{va} is the value-added price index. We obtain the last equality by substituting M_{vi} with the optimal intermediate goods usage (8). By substituting again equation (8) into the production function of gross intermediate goods (5), we obtain

$$Q_{vi} = Z_{vi}^{\frac{1}{1-\alpha_m}} K_{vi}^{\alpha} L_{vi}^{1-\alpha} \left(\frac{\alpha_m P_{vi}}{p_{mvi}}\right)^{\frac{\alpha_m}{1-\alpha_m}}. \tag{17}$$

Finally, by re-writing the firm's problem in terms of value added, $\max\limits_{L_{vi}, K_{vi}, M_{vi}} P_{vi}^{va} Y_{vi} - R_{vi} K_{vi} - W L_{vi}$, and substituting $P_{vi}^{va} Y_{vi}$ with (16) and Q_{vi} with (17), we obtain the value-added price index

$$P_{vi}^{va} = \left(\frac{P_{vi}}{p_{mvi}^{\alpha_m}}\right)^{\frac{1}{1-\alpha_m}}, \tag{18}$$

the productivity index for the value-added model

$$Z_{vi}^{va} = \left(1 - \alpha_m\right) \alpha_m^{\frac{\alpha_m}{1-\alpha_m}} Z_{vi}^{\frac{1}{1-\alpha_m}}, \tag{19}$$

and the production for the value-added final good Y_{vi}

$$Y_{vi} = Z_{vi}^{va} K_{vi}^{\alpha} L_{vi}^{1-\alpha}. \tag{20}$$

To obtain the value-added price index in terms of the cost of all factors, we follow the procedure described in Section 4.3.1. We find the ratio between the optimal demand factor for K_{vi} and L_{vi} by sector i for the value-added model. We then re-write the value-added production function in terms of capital, to obtain the optimal demand for capital by each unity of Y_{vi}. Finally, we substitute this optimal demand into the total cost function, to obtain the total cost by each unity of Y_{vi}.

Since prices equalize the marginal cost, we can write the value-added price index as $P_{vi}^{va} = \frac{1}{Z_{vi}^{va}}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_v}{\alpha}\right)^{\alpha}$. By substituting 11 into 18, we obtain the relative price index for the value-added model,

$$\frac{P_{vi}^{va}}{P_{vj}^{va}} = \frac{Z_{vj}^{va}}{Z_{vi}^{va}} = \left(\frac{Z_{vj}}{Z_{vi}}\right)^{\frac{1}{1-\alpha_m}}, \tag{21}$$

which is the inverse of relative productivity indices.

4.3.3 Aggregate the value-added model for sectors

Now, we aggregate the M-sector, $i = 1, ..., M$, value-added model into one-sector value-added model for each vintage v. We aggregate the valued added in terms of good k as

$$Y_v = \sum_i \frac{P_{vi}^{va} Y_{vi}}{P_{vk}}. \tag{22}$$

By dividing and multiplying the previous equation with P_{vk}^{va} and using (21) and (20), we obtain

$$P_{vk} Y_v = Z_{vk}^{va} P_{vk}^{va} K_v^{\alpha} L_v^{1-\alpha}, \tag{23}$$

where: $K_v = \sum_i K_{vi}$ and $L_v = \sum_i L_{vi}$.

By substituting (18) into (23), we find the aggregate value added,

$$Y_v = Z_{vk}^{va}\left(\frac{P_{vk}}{p_{vmk}}\right)^{\frac{\alpha_m}{1-\alpha_m}} K_v^{\alpha} L_v^{1-\alpha}. \tag{24}$$

Finally, by substituting (14) and (19) into (24), we obtain the aggregate value added for each technology v in terms of good k: $Y_v = Z_v K_v^{\alpha} L_v^{1-\alpha}$, where

$$Z_v = (1-\alpha_m)\, \alpha_m^{\frac{\alpha_m}{1-\alpha_m}}\, Z_{vk}^{\frac{1}{1-\alpha_m}} \left[\prod_j \left(\frac{P_{vk}}{P_{vj}}\right)^{\psi_{vjk}}\right]^{\frac{\alpha_m}{1-\alpha_m}}. \tag{25}$$

Since Y_v is produced under perfect competition and by a Cobb-Douglas production function, its price is

$$P_v = \frac{1}{Z_v}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_v}{\alpha}\right)^{\alpha}. \tag{26}$$

Let $\mathcal{F}$ be a matrix $(m \times m)$ with elements $\mathfrak{f}_{ji}$ such that $\mathcal{F} = (\boldsymbol{I} - \alpha_m \boldsymbol{\Phi})^{-1}$ and $\mathcal{F}_j$ is the row vector with the elements $(\mathfrak{f}_{ji}), j = 1, ..., m \; \forall i = 1, ..., m$. Thus, from (15), it results the relative price ratio $q_{v_j} = \tilde{z}_{v_{jo}} e^{\digamma_j \gamma_{\tilde{z}_{v_j}} v}$, where $\gamma_{\tilde{z}_{v_j}}$ is a vector column with elements $(\gamma_j - \gamma_k)$. $\digamma$

By rewriting the product of the relative prices, we obtain

$$\Pi_j \left(\frac{P_{vk}}{P_{vj}} \right)^{\psi_{vjk}} = \Pi_j \left(\tilde{z}_{\tau_{jo}}^{\psi_{vjk}} \right) e^{\sum_j \tilde{\digamma}_j \gamma_{\tilde{z}_{\tau_j}} v}, \tag{27}$$

where $\tilde{\digamma}_j$ is a row vector $(1 \times m)$ and $\tilde{\digamma}_j = \psi_{\tau_{jk}} \digamma_j$.

The growth rate of relative prices is a combination of the sectoral technology growth rates and the influence of each sector on the inter-sector network through the Leontief inverse, which can be expressed in terms of the I-O matrix and has a straightforward relation to the sector's centrality or degree. As shown by Acemoglu et al. (2012), the Leontief inverse and the vector of multipliers can be written as a convergent power series, $\digamma \mathbf{1} = (\boldsymbol{I} - \alpha_m \boldsymbol{\Phi})^{-1} \mathbf{1} = \left(\sum_{k=0}^{+\infty} (\alpha_m \Phi)^k \right) \mathbf{1} \approx \mathbf{1} + \alpha_m \boldsymbol{\Phi} \mathbf{1}$, which is well approximated by the sum of the first terms, since the elements of $\boldsymbol{\Phi}$ are sufficiently small. In turn, $\boldsymbol{\Phi}\mathbf{1} = \mathbf{d^{out}}$, where $\mathbf{d^{out}}$ is a column vector with elements $\mathbf{d_j^{out}}$; i.e., $\mathbf{d_j^{out}}$ is the weighted out-degree of sector j and $\sum_j \mathbf{d_j^{out}}$ is the average weighted out-degree of the inter-sector network, which, for simplicity, we call weighted degree. The weighted degree is the share of its output in the input supply of each sector of the economy and is equal to the sum of the corresponding row elements of matrix $\boldsymbol{\Phi}$, such that $\mathbf{d_j^{out}} = \sum_i^m \boldsymbol{\Phi}_{\mathbf{ij}}$; thus, $\digamma_j \approx 1 + \alpha_m \mathbf{d_j^{out}}$.

4.4 Capital goods demand and vintage prices

Once we aggregate the model for each sector i, we can obtain the capital demand and prices for each vintage v. The supplier of each capital vintage charges the rental price R_v, and the revenue share of capital is α; that is, the demand for capital of a vintage v by all sectors $i = 1, ..., M$ is given by $R_v K_v = \alpha P_v Y_v$, where $K_v = \sum_{i=1}^{M} K_{vi}$. The revenue generated from the output produced with the vintage v is determined by the demand function and we can rewrite it as being $R_v K_v = \alpha P_v \left(\frac{P_v}{P} \right)^{-\frac{\mu}{\mu-1}} Y = \alpha P_v^{-\frac{1}{\mu-1}} P^{\frac{\mu}{\mu-1}} Y$. Then, substituting P_v by the equation (26) and solving with respect to K_v the demand curve faced by the capital good supplier is

$$K_v = \left(\frac{1}{Z_v} \right)^{-\frac{1}{\mu-1}} \left(\frac{1-\alpha}{W} \right)^{\frac{\beta}{\mu-1}} \left(\frac{\alpha}{R_v} \right)^{\epsilon} Y, \tag{28}$$

where: $\epsilon = 1 + \frac{\alpha}{\mu-1}$ is the price elasticity of demand, which is constant. Capital suppliers maximize their profit at time t equals $\mathcal{L} = \int_0^{+\infty} \mathcal{H}_{vs} \, e^{-\int_t^s r_s \, ds'} \, ds$, where $\mathcal{H}_{vs}$ is the current Hamiltonian.[10] As a result the flow profit is $\pi_v = \frac{1}{\epsilon-1} \tilde{r} K_v = \frac{1}{\epsilon} R_v K_v$. Thus, the optimal rental price equals

[10]For simplicity reasons, we drop the time subscript s from now on and thus

$$\mathcal{H}_v = R_v K_v - Q I_v + \lambda_v \left(R_v K_v - \alpha Y \left(\frac{1}{Z_v} \right)^{-\frac{1}{\mu-1}} \left(\frac{1-\alpha}{W} \right)^{\frac{\beta}{\mu-1}} \left(\frac{\alpha}{R_v} \right)^{\epsilon-1} \prod_{j=1}^{M} \left(\frac{\psi_{jv}}{P_{vj}} \right)^{\frac{\psi_{vj}}{\mu-1}} \right) + v_v \left(I_v - \delta K_v \right),$$

the mark-up times the marginal cost of a unit of capital. Given the durability and reversibility of capital, the marginal production cost is the user-cost of capital. Hence, the optimal rental price is

$$R_v = R = \frac{\epsilon}{\epsilon - 1}\tilde{r}. \tag{29}$$

4.5 Optimal adoption

The time of adoption is endogenously decided by the monopolist. From equations (26), (4), and (44), we derive the flow profits earned by the capital good producer of vintage v, $\pi_v = \frac{1}{\epsilon} R_v K_v = \frac{\alpha}{\epsilon} P_v Y_v$. Since the price of intermediate good i produced with vintage v and the aggregate price of final good i are

$$P_v = \frac{1}{Z_{vi}} \left(\frac{1-\alpha}{W}\right)^{-(1-\alpha)} \left(\frac{\alpha}{R_v}\right)^{-\alpha} \prod_{j=1}^{M} \left(\frac{\psi_j}{P_{vj}}\right)^{-\psi_j}, \tag{30}$$

$$P = \frac{1}{A_i} \left(\frac{1-\alpha}{W}\right)^{-(1-\alpha)} \left(\frac{\alpha}{R_v}\right)^{-\alpha} \prod_{j=1}^{M} \left(\frac{\psi_j}{P_j}\right)^{-\psi_j}, \tag{31}$$

then

$$P_v Y_v = P_v \left(\frac{P_v}{P}\right)^{-\frac{\mu}{\mu-1}} Y \equiv \alpha \left(\frac{P}{P_v}\right)^{\frac{1}{\mu-1}} Y = \left(\frac{Z_v}{A}\right)^{\frac{1}{\mu-1}} \prod_{j=1}^{M} \left(\frac{P_{vj}}{P_j}\right)^{\frac{1}{\mu-1}} Y \tag{32}$$

and thus $\pi_v = \frac{\alpha}{\epsilon}\left(\frac{Z_v}{A}\right)^{\frac{1}{\mu-1}} Y$. Therefore, the present discounted value of flow profits is

$$M_{v,t} = \int_t^{+\infty} \pi_{vs}\, e^{-\int_t^s r_s\, ds'}\, ds = \left(\frac{Z_v}{Z_t}\right)^{\frac{1}{\mu-1}} \left(\frac{Z_t}{A_t}\right)^{\frac{1}{\mu-1}} \Psi Y_t, \tag{33}$$

where

$$\Psi = \frac{\alpha}{\epsilon} \int_t^{+\infty} \left(\frac{A_t}{A_s}\right)^{\frac{1}{\mu-1}} \left(\frac{Y_s}{Y_t}\right) e^{-\int_t^s r_s\, ds'}\, ds \tag{34}$$

is the stock market capitalization-to-GDP ratio.

To determine the optimal adoption time, the market value of the firm producing capital good v is, at every instant, at least as large as the adoption cost,

$$\Gamma_v \leq M_v. \tag{35}$$

since $\dot{K}_v = I_v - \delta K_v$ and $R_{vi} \equiv R_v$, implying that $R_{vi} K_{vi} = R_v K_{iv}$ and $R_v K_v = R_v \sum_{i=1}^{M} K_{iv}$. The co-state variables λ_v and v_v are associated with the demand function for capital good v faced by the supplier and the capital accumulation equation. From the optimality conditions for the Hamiltonian result $\lambda_v = -\frac{1}{\epsilon}$ and $R_v = \frac{\epsilon}{\epsilon-1} Q\,(\tilde{r} + \delta + q)$, where $-q = \frac{\dot{Q}}{Q}$.

This holds with equality when a positive adoption lag exists. Consequently, the optimal adoption lag satisfying equation (35) solves $\left(\frac{Z_v}{Z_t}\right)^{\frac{\nu}{\mu-1}} = \min\left\{1, \frac{1}{1+b}\frac{\Psi}{\overline{\Psi}}\right\}$, such that $\ln Z_v - \ln Z_t = \min\left\{0, -\frac{\mu-1}{\nu}\left(\ln(1+b) - \ln\Psi + \ln\overline{\Psi}\right)\right\}$ and

$$D_v = \max\left\{0, \frac{(\mu-1)(1-\alpha_m)}{\left(\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{\tilde{z}_{\tau_i}}\right)\nu}\left[\log(1+b) + \left(\log\overline{\Psi} - ln\Psi\right)\right]\right\} \equiv D. \tag{36}$$

Equation (36) shows that adoption lags are constant across vintages v, increase with the adoption costs b and with the deviation of the stock market-to-output ratio from its steady state level, and decrease with the weighted network degree. On the other hand, because of specifications of the production function and adoption cost, the market size symmetrically affects the benefits and costs. Therefore, adoption lags are affected not by variations in the market size or technology productivity at time zero, Z_0, but rather by the production network structure. Additionally, as on the balanced growth path $\overline{\Psi} = \Psi$, steady-state adoption lags are independent of the stock market capitalization-to-GDP ratio.

The aggregate TFP level is given by

$$A_t = A_0 e^{\gamma_a (t - D_t)}, \tag{37}$$

where $A_0 = (1-\alpha_m)\,\alpha_m^{\frac{\alpha_m}{1-\alpha_m}} Z_{\underline{v}_k}^{\frac{1}{1-\alpha_m}} \prod_i \tilde{z}_{\tau_{io}}^{\psi_{vik}\frac{\alpha_m}{1-\alpha_m}} \left(\frac{(1-\alpha_m)(\mu-1)}{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{\tilde{z}_{\tau_i}}}\right)^{\mu-1} > 0$ and $\gamma_a = \frac{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{\tilde{z}_{\tau_i}}}{1-\alpha_m}$ are constants and depend on model parameters. In turn, the aggregate adoption cost is given by

$$\Gamma = \overline{\psi}(1+b)\left(\frac{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{\tilde{z}_{\tau_i}}}{(\mu-1)(1-\alpha_m)}\right) e^{-\frac{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{\tilde{z}_{\tau_i}}}{(\mu-1)(1-\alpha_m)}\nu D} Y\left(1 - \dot{D}\right), \tag{38}$$

which affects the TFP because it determines the adoption lag and, as a consequence, the range of technologies available for production.

4.6 Technology aggregation: factor demands and prices

Now, we aggregate the model for all technology vintages v existing in the economy in order to reach the aggregate final output Y. To aggregate the model, we use the optimal factor demand for K_v and L_v as well as the optimal demand for Y_v. The CES production function (3) implies that the demand for the output produced by technology v is given by (4), where $P = \left(\int_v P_v^{-\frac{1}{\mu-1}} \mathrm{d}v\right)^{\mu-1}$, and that the marginal revenue of labor equates the real wage rate:

$$W_v L_v = (1-\alpha) P_v Y_v. \tag{39}$$

Using (32) to get the optimal price P_v in function of Y_v and Y and substituting it into (39), we get

$$W_v L_v = (1-\alpha) Y_v^{\frac{1}{\mu}} Y^{\frac{\mu-1}{\mu}} P. \tag{40}$$

Similarly, the marginal revenue of capital also equates its user cost

$$R_v L_v = \alpha Y_v^{\frac{1}{\mu}} Y^{\frac{\mu-1}{\mu}} P. \tag{41}$$

Integrating over technologies both of these optimality conditions and recalling that there is no restriction on movements of labor and capital; i.e., $W_v = W$ and $R_v = R$, we obtain $WL = (1-\alpha)\left(\int_v Y_v^{\frac{1}{\mu}} \mathrm{d}v\right) Y^{\frac{\mu-1}{\mu}} P$ and $RK = \alpha \left(\int_v Y_v^{\frac{1}{\mu}} \mathrm{d}v\right) Y^{\frac{\mu-1}{\mu}} P$. Finding the ratio between the optimal conditions for the value-added of the intermediate good produced with technology v and the value-added of whole technologies, we obtain $\frac{K_v}{K} = \frac{L_v}{L} = \left(\frac{Y_v}{Y}\right)^{\frac{1}{\mu}}$. Then, substituting into the value-added production function of intermediate goods, we have

$$Y_v = Z_v \left(\frac{Y_v}{Y}\right)^{\frac{1}{\mu}} L^{1-\alpha} K^{\alpha}. \tag{42}$$

Now, integrating (42) over v, $Y^{\frac{1}{\mu}} = \int_v Y_v^{\frac{1}{\mu}} \mathrm{d}v = \int_v Z_v^{\frac{1}{\mu-1}} \left(\frac{L^{1-\alpha}K^{\alpha}}{Y^{\frac{1}{\mu}}}\right)^{\frac{1}{\mu-1}} \mathrm{d}v$, which can be written as $Y = ZL^{1-\alpha}K^{\alpha}$ or as

$$Y = AL^{1-\alpha}K^{\alpha}, \tag{43}$$

where: $Z = \left[\int_v Z_v^{\frac{1}{\mu-1}} \mathrm{d}v\right]^{\frac{1}{\mu-1}}$, $A = \left(\int_v Z_v^{\frac{1}{\mu-1}} \mathrm{d}v\right)^{\mu-1}$, $L = \int_v L_v \,\mathrm{d}v$ and $K = \int_v K_v \,\mathrm{d}v$.

Since the aggregate output production function is also Cobb-Douglas and the output is produced under perfect competition, the output price is

$$P = \frac{1}{A}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}. \tag{44}$$

5 Diffusion of new technology

We now describe and characterize the technology diffusion process,[11] in order to analyze the network effects on technology diffusion. The competitive equilibrium in this aggregate economy can be expressed in terms of eight equilibrium variables $\{C, L, Y, K, \Gamma, A, D, \Psi\}$, being the capital stock, K, the only state variable as in the standard neoclassical growth model; i.e., the equilibrium dynamics is given by: (i) the consumption Euler equation, $\frac{\dot{C}_t}{C_t} = r_t - \rho$; (ii) the aggregate resources constraint,[12] $Y = C + I + \Gamma$; (iii) the capital accumulation equation, $\dot{K} = \delta K + I$;

[11] We focus on the balanced growth path of the economy in which adoption lags D are constant and the economy grows at a constant rate equal to $\gamma_a/(1-\alpha)$.

[12] It is assumed that the adoption cost are measured as part of the final output, such that Y can be interpreted as the GDP.

(iv) the production function, (43), taking into account that in equilibrium $L = 1$; (v) the adoption cost function, (38); (vi) the technology adoption equation, (36); (vii) the stock market to GDP ratio, (34); (viii) the aggregate TFP level, (37).

Given the technology measures in our dataset, we are interested in the total demand for capital goods and the output produced with a production method using new technology, $\tau = n$. The model provides expressions for both aggregate output and capital at the vintage level. From (3) and (43), we can express the output produced with technology τ as $Y = A_\tau K_\tau^\alpha L_\tau^{1-\alpha}$, where $K = \int_{v \in V_\tau} K_v \, dv$, $L_\tau = \int_{v \in V_\tau} L_v \, dv$, and $A = \left(\int_{V_\tau} Z_v^{\frac{1}{\mu-1}} \, dv \right)^{\mu-1}$, which by substituting Z_v with equation (25) we obtain the following equation:

$$A_\tau = \left(\int_{V_\tau} \left[(1-\alpha_m)\, \alpha_m^{\frac{\alpha_m}{1-\alpha_m}} Z_{vk}^{\frac{1}{1-\alpha_m}} \left(\prod_j \left(\frac{P_{vk}}{P_{vj}} \right)^{\psi_{vjk}} \right)^{\frac{\alpha_m}{1-\alpha_m}} \right]^{\frac{1}{\mu-1}} dv \right)^{\mu-1}. \tag{45}$$

Then, by replacing (27) into (45), we obtain

$$A_\tau = \left(\frac{(1-\alpha_m)\,(\mu-1)}{\gamma_{\tau_k} + \alpha_m \sum_j \tilde{F}_j \gamma_{\tilde{z}_{\tau_j}}} \right)^{\mu-1} \underbrace{Z_{A_\tau}}_{\text{intensive margin}} \underbrace{e^{\frac{\gamma_{\tau_k} + \alpha_m \sum_j \tilde{F}_j \gamma_{\tilde{z}_{\tau_i}}}{1-\alpha_m}(t - D_t - \underline{v})}}_{\text{embodiment effect}} \underbrace{\left[1 - e^{-\frac{\gamma_{\tau_k} + \alpha_m \sum_j \tilde{F}_j \gamma_{\tilde{z}_{\tau_j}}}{(1-\alpha_m)(\mu-1)}(t - D_t - \underline{v})} \right]^{\mu-1}}_{\text{variety effect}}, \tag{46}$$

where:

$$Z_{A_\tau} = (1-\alpha_m)\, \alpha_m^{\frac{\alpha_m}{1-\alpha_m}} Z_{\underline{v}_{k_0}}^{\frac{1}{1-\alpha_m}} \prod_j \tilde{z}_{\tau_{jo}}^{\psi_{vjk} \frac{\alpha_m}{1-\alpha_m}}. \tag{47}$$

Equation (46) defines the TFP for new technologies and its path is affected by both *extensive* and *intensive margins*.

As regards the *intensive margin* effect, the inter-sector network structure leads to efficiency gains. The higher the intensity of intermediate goods, ψ_{vjk}, number of trade partners, $j : \psi_{vjk} > 0$, and total weight of intermediate goods in production, α_m, the higher are the efficiency gains. Comin and Mestieri (2014) explain the cross-country differences in intensity of adoption by introducing a Hicks-neutral exogenous parameter b_v in the production function; i.e., $Y_v = b_v Z_v F(L_v, K_v)$, which captures the effect of the country-specific factors that would explain country differences in the number of users of a technology and in the efficiency level with which a technology is used. In our model, these differences are endogenously explained by the production network structure of each country. As the consumption of different intermediate goods composition by each sector is country specific, the aggregate productivity level of each technology vintage Z_v is also country specific. Thus, we can claim that the country-specific effect is nothing but inter-sector country linkages, that is, the inter-sector country network.

As regards the *extensive* effect, equation (36) shows the determinant factors of adoption lags. Now, equation (46) shows how the adoption lags affect the TFP level through two mechanisms, the *embodiment effect* and the *variety effect*. The former defines the growth trend of the TFP, which depends on the growth rate of the traditionally embodied productivity γ_τ, and on the inter-sector network production through the term $\frac{1+\alpha_m \sum_j \tilde{F}_j}{1-\alpha_m}$ and adoption lags. Thus, the higher the adoption lags, the higher is the TFP growth rate due to the catching-up effect. Furthermore, the higher the network degree $\tilde{F}_j$ and the weight of intermediate goods in the economy α_m, the higher is the TFP growth due to inter-sector linkages. The *variety effect* is the increase in productivity due to the range of technology varieties in use $t - D_t - \underline{v}$, which, in equation (46), shows that the productivity gains from an additional technology are decreasing. Moreover, the higher are $\tilde{F}_j$ and α_m, the bigger the diminishing returns of an additional technology variety.

In sum, the inter-sector network structure has a multiplicative effect on both the *embodiment effect* and the *variety effect*; this has not been captured by the previous growth models using technology diffusion. This network effect is due to the imperfect substitutability between the products of each sector. If instead of aggregator (22) we had a CES aggregator, because of the neutral technology and perfect substitutability between sectors, there would be no linkage effect on technology adoption. The only effect would be due to the weight of the intermediate inputs in the economy, α_m, and adoption would be independent of the I-O matrix, that is, ψ_{ij} and relative prices $\frac{P_{vk}}{P_{vj}}$.

The price index of output from technology τ is

$$P_\tau = \frac{1}{A_\tau} \left(\frac{w}{1-\alpha} \right)^{1-\alpha} \left(\frac{R}{\alpha} \right)^{\alpha}, \tag{48}$$

and the demand equals

$$Y_\tau = Y \left(P_\tau \right)^{-\frac{\mu}{\mu-1}}. \tag{49}$$

6 Technology dynamics

We now characterize the diffusion path, margin effects, and inter-sector network effects and then test our model. We follow the Comin and Hobijn (2010) identification and estimation procedure. We estimate how the inter-sector linkages affect the adoption margins and compare our results to theirs. From equation (46), we expect a positive inter-sector network effect on both margins. We correct the margin effects by a multiplicative term, which characterizes the inter-sector network or linkages.

6.1 Identification and estimation strategy

Identification. We next estimate the *extensive* and *intensive margins* for different technology-country pairs. To estimate the technology diffusion measure equations, we use the demand for output produced with τ, equation (49), the price deflator, equation (48), the equilibrium wage rate,[13] and A_τ, equation (46). By rearranging and taking logs, we obtain the following output demand:

$$y_\tau = y + \frac{\mu}{\mu - 1}\left[a_\tau - (1-\alpha)(y-l) - \alpha r + \alpha \log \alpha\right], \tag{50}$$

where the small letters denote logarithms. Similarly, using the demand for capital $K_\tau R = P_\tau Y_\tau$, price deflator index, equation (48), and taking logs, we obtain the reduced form for capital,

$$k_\tau = (1-\alpha)\log\alpha - a_\tau + (1-\alpha)(y-l) - (1-\alpha) r + y_\tau. \tag{51}$$

These two equations depend on D_τ, through the technology lags effect on a_τ. They also depend on r, which we assume to be constant in steady state and a part of the constant term. For the first-order approximation of equation (46), γ_τ, affects only y_τ and k_τ through a linear trend. Thus, by approximating $\log A_\tau$ around $\gamma_\tau \to 0$,[14] we obtain

$$a_\tau \approx \phi + (\mu - 1)\log(t - T_\tau) + \frac{1 + \alpha_m \sum_j \tilde{F}_j}{2(1-\alpha_m)}(t - T)\left(\gamma_\tau + \gamma_{z_{\tau_j}}\right), \tag{52}$$

where

$$\phi = \frac{\alpha_m}{1-\alpha_m}\log(\alpha_m(1-\alpha_m)) + \frac{1}{1-\alpha_m}\log Z_{\underline{v}k} + \frac{\alpha_m}{1-\alpha_m}\sum_j \psi_{vjk}\log \tilde{z}_{\tau_{jo}}, \tag{53}$$

and $T_\tau = \underline{v}_\tau + D_\tau$ is the time of technology adoption. By substituting (52) into equations (50)-(51), we obtain the reduced form of the estimated equation for output,[15]

$$y_\tau = y + \beta_1 + \beta_2 \frac{1 + \alpha_m \sum_j \tilde{F}_j}{2(1-\alpha_m)} t + \beta_3 \log(t - T_\tau) + \beta_4 (1-\alpha)(y-l) + \epsilon_\tau, \tag{54}$$

and capital,

$$k_\tau = y + \beta_1 + \beta_2 \frac{1 + \alpha_m \sum_j \tilde{F}_j}{2(1-\alpha_m)} t + \log(t - T_\tau) + \beta_4 (1-\alpha)(y-l) + \epsilon_\tau, \tag{55}$$

where ϵ_τ is the error term and the βs are the reduced-form coefficients. With these two reduced equations, we come to the two technologies measured in our data. Some technologies are measured as the number of units embodying the technology (e.g., number of ATMs or computers), whereas others are measured as the output produced with a specific technology (e.g., the volume of debit and credit transactions). Equations (54)-(55) describe the log of the output produced with technology τ, y_τ, and the log of the number of units embodying technology τ, k_τ, as the summation of a fixed effect; β_1, a log-linear term in t weighted by the

[13] At the equilibrium, $W = (1-\alpha)\frac{Y}{L}$.

[14] See Appendix B for details.

[15] See Appendix C for details.

intermediate goods weight in production α_m and the weighted network degree $\tilde{F}_j$; two log-linear terms of income y and the income per capital $y - l$; and a log-log term of the adoption lag $\ln(t - T)$.

Theoretically, fixed effect β_1 captures the differences in units of technology measure, the TFP levels across countries, adoption lags, and the relative prices of investment goods. These vary across countries and, therefore, we assume that coefficient β_1 is country-specific. In turn, parameters β_2 and β_4 depend on the elasticity of substitution between technologies, μ, and the elasticity of substitution of capital, α. From the evidence, both elasticities are approximately constants across countries. Thus, we assume that β_2 and β_4 are common across countries. Similarly, parameter β_3 depends on technology parameters and it is also assumed to be constant across countries. In particular, for k_τ, parameter $\beta_3 = 1$.

Besides estimating the coefficients βs, we estimate the adoption lags (*extensive margin*) throughout the estimative for T_τ in equations (54) and (55). As regards the coefficient β_1, again, we can write β_1 as[16]

$$\beta_1^y = \beta_3 \left[\log \left((1 - \alpha_m)\, \alpha_m^{\frac{\alpha_m}{1-\alpha_m}}\, Z_{\underline{\tau}_k}^{\frac{1}{1-\alpha_m}} \prod_j \tilde{z}_{\tau_{jo}}^{\psi_{vjk}\frac{\alpha_m}{1-\alpha_m}} \right) - \alpha \log \frac{R}{\alpha} - \frac{\left(1 + \alpha_m \sum_j \tilde{F}_j\right)\left(\gamma_\tau + \sum_j \gamma_{z_{\tau_j}}\right)}{2\,(1 - \alpha_m)} T_\tau \right], \tag{56}$$

and

$$\beta_1^k = \beta_3 \left[\log \underbrace{\left((1 - \alpha_m)\, \alpha_m^{\frac{\alpha_m}{1-\alpha_m}}\, Z_{\underline{\tau}_k}^{\frac{1}{1-\alpha_m}} \prod_j \tilde{z}_{\tau_{jo}}^{\psi_{vjk}\frac{\alpha_m}{1-\alpha_m}} \right)}_{Z_{v_\tau}} - \log \frac{R}{\alpha} - \frac{\left(1 + \alpha_m \sum_j \tilde{F}_j\right)\left(\gamma_\tau + \sum_j \gamma_{z_{\tau_j}}\right)}{2\,(1 - \alpha_m)} T_\tau \right]. \tag{57}$$

Thus, our model can explain the cross-country differences in the intercept term. The explicative factors are the weight of intermediate inputs in production, α_m, the input-use flows of trade between sectors, ψ_{vjk}, and the time of adoption T_τ. The first two features explain the cross-country differences in the *intensive margin*, Z_{v_τ}. Basically, the number of units adopted depends on the technology and network factors Z_{v_τ}, which we call *intensive margin*, and on the novelty of technology. Note that coefficient β_3 and parameters α and R are assumed to be equal for all countries. We have obtained the estimates for the fixed effect β_1 and for β_3. In order to obtain the estimate for the cross-country differences in the *intensive margin* Z_{v_τ}, we rewrite (56) and (57) as follows:

$$\Delta \log Z_{v_\tau} = \frac{\beta_1 - \beta_1^{USA}}{\beta_3} + \frac{\gamma_\tau + \sum_i \gamma_{z_{\tau_i}}}{2\,(1 - \alpha_m)} \left[\left(1 + \alpha_m \sum_i \tilde{F}_i\right) T - \left(1 + \alpha_m \sum_i \tilde{F}_i^{USA}\right) T^{USA} \right], \tag{58}$$

where $\Delta \log Z_{v_\tau}$ defines the logarithm of the *intensive margin* relative to the US.

[16] See Appendix C for additional details.

Estimation strategy. Our estimation strategy follows a two-step approach. For each technology, we estimate equations (54)-(55) for the US. This provides us with values for β_1-β_4 and T. Through the estimate of T, we obtain the estimate for adoption lags, D_τ. In a second step, we use the US estimate values for β_2-β_4, and again estimate equations (54)-(55) for each technology-country pair, to obtain the conditional estimates of β_1 and D_τ. All equations are estimated by nonlinear least squares.

Preference ρ and technology parameters γ, μ, α, $z_{v_{\tau_0}}$, are assumed to be the same across countries, implying that the interest rate, r, is also the same across countries. The adoption cost, b, and the relative intensity of the intermediate goods in production, ψ_{ji}, differ across countries. Thus, D_τ and β_1 can also differ across countries. All countries in our sample are developed countries with identical weight for intermediate goods. Therefore, we do not estimate α_m and α, but calibrate $\alpha = 0.3$ and $\alpha_m = 0.5$, based on estimates of the economic literature (Ngai and Samaniego, 2009; Comin and Hobijn, 2010).

6.2 Estimation results

Table 2 reports our summary statistics. We follow Comin and Hobijn (2010) and classify the estimates in three groups: (i) implausible, if it implies that the technology was adopted more than 10 years before it was invented; (ii) plausible but imprecise, if the estimate standard errors are high; (iii) plausible and precise, in all remaining cases. From the plausibility and precision criteria, we find both criteria for the majority of the technology-country pairs (84%). For these technology-country pairs, the estimated equations (54)–(55) and the imposed the US parameters produce a good fit for the data with an average detrended R^2 of 0.92 across countries and technologies. All the remaining results are for plausible and precise estimates.

Table 2: Summary statistics of the estimated adoption lags

	Invention	Number of		Plausible			R^2	
Technology	year	countries	Implausible	Imprecise	Precise	R^2	mean	sd
Cellphones	1973	9	1	0	8	7	.97	.02
Computer	1973	9	0	0	9	9	.97	.03
Internet	1983	9	0	1	8	8	.95	.06
MRI	1977	9	1	0	8	8	.95	.03
ATM's	1967	7	2	0	5	1	.91	.14
CableTv	1949	8	1	0	7	7	.93	.07
Credit Debit	1950	8	1	0	7	6	.93	.06
EFT	1979	9	3	0	6	4	.92	.05
Cat's	1972	9	0	0	9	8	.75	.19
Pos	1950	9	3	1	5	6	.88	.07
Total		86	12	2	72	69	.92	.11

Table 3 summarizes the estimates of adoption lags for all technology-country pairs. The average adoption lag across all technologies and countries is 15 years, with a median lag of 12 years. Across technologies, we observe a significant variation in average adoption lags, ranging from 2 years (electric fund transfers) to 35 years (points of service for debit/credit cards). As regards the cross-country variation in adoption lags, the range is from 1 year for Internet to

9 years for debit/credit card transactions. Our results also show that, on average, the adoption lags are decreasing over time. Technologies are adopted faster than ever before.

Our technology case example: computers; the estimate adoption years, T_τ, for Australia, Canada, Denmark, France, Germany, Japan, Netherlands, the UK, and the US are 1983, 1980, 1986, 1984, 1983, 1983, 1985, 1983 and 1981, respectively. These results are consistent with the computer patent in 1973. Thus, Canada adopted computers with an average lag of 7 years, while the estimated computer adoption lag for Netherlands was 12 years.

Table 3: Estimate adoption lags

Technology	Invention year	Number of countries	Adoption lags Mean	sd	1%	10%	50%	90%	99%
Cellphones	1973	8	9	5	-3	-3	11	14	14
Computer	1973	9	10	2	7	7	10	13	13
Internet	1983	8	12	1	11	11	12	13	13
MRI	1977	8	7	2	4	4	7	9	9
ATM's	1967	5	13	8	0	0	17	18	18
CableTv	1949	7	31	6	23	23	33	38	38
Credit Debit	1950	7	28	9	17	17	30	37	37
EFT	1979	6	2	5	-5	-5	1	8	8
Cat's	1972	9	9	2	8	8	8	12	12
Pos	1950	5	35	2	33	36	36	38	38
Total		72	15	11	-5	4	12	36	38

These results are consistent with those in Comin and Mestieri (2010). We estimate a different equation and use a smaller technology sample because I-O tables are not available for all countries. In addition, I-O data are available only after the 1950s. The novelty in our model is the inclusion of inter-sector linkages throughout the inter-sector network properties, which have been ignored by the literature. When we compare the estimates for the technologies common in these two papers, we find that our results are more consistent, with a smaller standard deviation for adoption lags, smaller adoption lags, and higher R^2. We also carried out an additional control test that comprised estimating Comin and Mestieri (2010) equation with our data. This test also resulted in the same conclusions.

Tables 2 and 3 demonstrate that a model incorporating the inter-sector network provides better estimates for adoption lags, that is, *extensive margin*, but do not directly provide any estimate for the value of the network effect in the adoption lags. For a direct impact of networks in adoption lags and to confirm our model validation, in equations (54)-(55) we estimate the time of adoption of $T_\tau = \underline{v}_\tau + D_\tau$, where $\underline{v}_\tau$ is the invention year. Once we estimate T and thus D, the model provides an inverse relationship between the inter-sector network degree and the adoption lags – see equation (36). This relationship is valid for each technology vintage v. Thus, by using equation (36) and $T_\tau = \underline{v}_\tau + D_\tau$ and aggregating them for all technology vintages such as $v \in V_\tau$, we can obtain the direct impact of networks into diffusion lags,

$$D = \alpha_1 + \alpha_2 \underline{v} + \alpha_3 D_e^{-1} + \epsilon, \tag{59}$$

where $D_e^{-1} = \frac{(1-\alpha_m)}{\left(\gamma_{\tau_k} + \alpha_m \sum_i \tilde{\Gamma}_i \gamma_{\tilde{z}_{\tau_i}}\right)}$. The network degree is common for all technologies and thus we can estimate (59) for all technologies and countries. We assume that technology growth is the same across countries and sectors and equal to $\gamma = \gamma_i = 1\%, \forall i$.[17]

Table 4: Estimated inter-sector network effect on adoption lags

	α	std. err.	$P > \|t\|$
D_e^{-1}	2.47	0.28	0.00
$\underline{v}$	-0.69	0.03	0.00
const	1260.62	13.02	0.00
R^2	0.81		
Pr $F > 0$	0.00		
N. obs	9510		

Notes: Estimates are obtained with OLS and robust standard errors.

Table 4 illustrates that adoption lags are inversely proportional to the network degree. The higher the network degree, the quicker does a country adopt a technology. This network effect is statistically significant. Moreover, the technology invention year has a negative effect on technology adoption lags, as expected, which is based on the empirical evidence that newer technologies are adopted faster than ever before. Finally, the network degree and the invention year explain 81% of the adoption lag variance for the period and countries considered in this study.

Table 5: Estimated *intensive margin*

	Number of	Intensive margin						
Technology	countries	Mean	sd	1%	10%	50%	90%	99%
Cellphones	7	-0.18	0.09	-0.32	-0.32	-0.18	0.00	0.00
Internet	7	-0.14	0.09	-0.31	-0.31	-0.16	0.00	0.00
MRI's	7	-3.10	8.50	-26.8	-26.8	-0.07	0.39	0.39
Computer	8	-0.33	0.46	-1.09	-1.09	-0.30	0.34	0.34
CableTv	6	-0.42	0.66	-1.48	-1.48	-0.52	0.45	0.45
Credit Debit	6	-3.99	10.31	-30.31	-30.31	0.00	0.30	0.74
EFT	5	-0.29	0.32	-0.47	-0.47	-0.44	0.54	0.54
Cat's	8	-0.26	0.52	-1.15	-1.15	-0.30	0.90	0.90
Pos	5	-3.04	4.63	-14.33	-14.33	-0.25	0.00	0.00
Total	59	-1.28	4.87	-30.31	-30.31	-0.19	0.25	0.90

Table 5 shows the relative estimated margins.[18] Relative margins are reported as log differences relative to the US (see equation (58)). To compute the *intensive margin*, we keep the calibration of $\gamma = \gamma_i = 1\%$ and use the estimate values of β_1 and β_4. The average *intensive margin* is -1.28. Thus, the level of adoption of an average country is 28% that of the US. Note that there are three particular technologies that push down the average adoption level. They are MRI, credit-debit card transactions, and points of service for debit-credit cards. If we exclude these technologies, the average adoption level is 79% of the US adoption level. Thus, a better measure to compare the adoption levels may be the median. The median level of

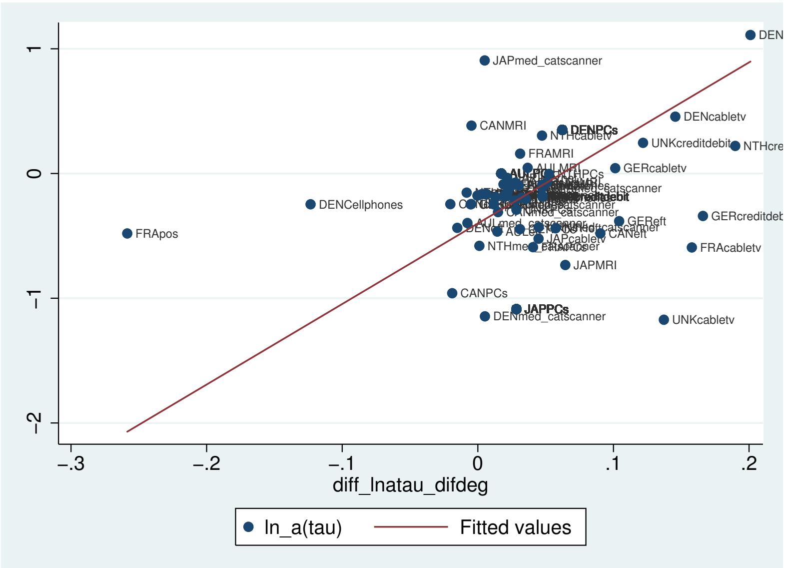
[17] Comin and Mestieri (2014) also assumes that the technology growth rate equals 1%.

[18] As the *intensive margin* is measured relative to the US and the US ATM estimations do not converge, we exclude this technology.

adoption is 83% of the US adoption level. A significant cross-country variation in the *intensive margin* is also observed. The range goes from 2% for credit-debit transactions to 87% for the Internet.

We find the *intensive margin* for each country-technology pair, and using equation (58), illustrate in Figure 6 the inter-sector network impact on the *intensive margin*. First of all, there is a multiplicative effect between the inter-sector network degree and time of adoption on the *intensive margin*. The number of units a country adopts increases with network degree, and this increase is proportional to the adoption lags. This statistically significant positive relation is found for the set of technologies analyzed in this paper as well as for individual technologies, except for Cable TV and MRI, where the production network degree is not expected to affect the number of units in a country.

Figure 6: Inter-sector network degree effect on *intensive margin*.



This Figure plots the relative intensive margin (in logs) for all country-technology pairs against the inter-relative sector network degree weighted by respective time of adoption of each country. The estimated relation is $\log Z_{v_\tau} = -0.40 + 6.45\Omega$, *where* $\Omega = \frac{\gamma_\tau + \sum_i \gamma_{z\tau_i}}{2(1-\alpha_m)}\left[(1+\alpha_m \sum_i \tilde{F}_i)\, T - \left(1+\alpha_m \sum_i \tilde{F}_i^{US}\right) T^{US}\right]$, Ω *is statistically significant at the* 1% *level and* $R^2 = 0.09$. *The equation is estimated with OLS and robust standard errors.*

7 Income differences

Finally, we study the implications of inter-sector networks in the evolution of income differences. We first derive the explicative factors of economic growth, and then research how much of the cross-country income differences can be explained by production network structures.

The aggregate TFP is given by $A = \left[\sum_{\tau\in o,n} (Z_\tau)^{\frac{1}{\mu-1}}\right]^{\mu-1} = (1-\alpha_m)\,\alpha_m^{\frac{\alpha_m}{1-\alpha_m}}\, Z_{\underline{v}_k}^{\frac{1}{1-\alpha_m}} \prod_i z_{\tau_{io}}^{\psi_{vik}\frac{\alpha_m}{1-\alpha_m}} \left(\frac{(1-\alpha_m)(\mu-1)}{\gamma_{\tau_k}+\alpha_m \sum_i \tilde{F}_i \gamma_{z\tau_i}}\right)^{\mu-1}$ $e^{\frac{\gamma_{\tau_k}+\alpha_m \sum_i \tilde{F}_i \gamma_{z\tau_i}}{1-\alpha_m}(t-D_t-\underline{v})}$, which describes the TFP growth factors – i.e., the adoption lags, D, and the production networks degree, $\sum_i \tilde{F}_i$. The smaller the adoption lags, the higher are the productivity embodied in the best technology vintage available for production and the TFP. This effect on TFP is amplified by the network degree.

The income per capita can be written as $\frac{Y}{L} = A^{\frac{1}{1-\alpha}} \left(\frac{K}{L}\right)^{\frac{\alpha}{1-\alpha}} \left((1-\alpha_m)\, \alpha_m^{\frac{\alpha_m}{1-\alpha_m}} Z_{\underline{v}_k}^{\frac{1}{1-\alpha_m}} \prod_i z_{\tau_{io}}^{\psi_{vik} \frac{\alpha_m}{1-\alpha_m}} \left(\frac{(1-\alpha_m)(\mu-1)}{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{z\tau_i}} \right)^{\mu-1} e^{\frac{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{z\tau_i}}{1-\alpha_m} (t - D_t - \underline{v})} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}}$. By taking logs, we obtain:

$$y - l = \frac{1}{1-\alpha} \left[\log Z_{v_\tau} + (\mu - 1) \log \frac{(1-\alpha_m)(\mu-1)}{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{z\tau_i}} - \alpha \log \frac{\alpha}{R} + \frac{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{z\tau_i}}{1-\alpha_m} (t - D_t - \underline{v}) \right]. \tag{60}$$

The term Z_{v_τ} is the *intensive margin* of adoption. Subtracting the same expression for the US yields $(y-l) - (y^{us} - l^{us}) = \frac{1}{1-\alpha} \left[(\log Z_{v_\tau} - \log Z_{v_\tau}^{us}) + (\mu - 1) \left(\log D_e^{-1} - \log D_e^{us^{-1}} \right) + \gamma (Dl - Dl^{us}) \right]$, i.e.,

$$\Delta(y - l) = \alpha_1 \Delta \log Z_{v_\tau} + \alpha_2 \Delta \log D_e^{-1} + \alpha_3 \Delta Dl, \tag{61}$$

where $Dl = \frac{\gamma_{\tau_k} + \alpha_m \sum_i \tilde{F}_i \gamma_{z\tau_i}}{1-\alpha_m} (t - D_t - \underline{v})$ and Δ denotes the variable X in country i relative to the US.

Under our model, income cross-country differences are due to variations in *intensive margin*, network degree, and diffusion lags. The last two terms in the equation (61) come from variations in *extensive margin*, in particular, from the embodiment and variety effect variations. As shown by the parameters of equation (61), we expect a positive effect of the relative *intensive margin* of adoption. If a country adopts more technology units than the US, its income per capita relative to the US improves, $(\hat{\alpha_1} > 0)$. The second term in equation (61) is due to *the variety effect*. The higher the network degree, the more important is the negative impact decreasing the range of technology varieties (see 46). Thus, an increase in the relative inverse-network degree also improves the relative income per capita $(\hat{\alpha_2} > 0)$. The last term of our equation shows *the embodiment effect*, which is an interaction between the network degree and adoption lags. This is expected to have a positive sign as well $(\hat{\alpha_3} > 0)$, because, while the network degree raises the technology productivity effect, a decrease in D_t means that the expression $(t - D_t - \underline{v})$ increases along with the productivity and income per capita gains.

To test our model, we estimate equations (60)-(61). We assume that our estimates for the *intensive margin* are representative of the average *intensive margins* of adoption across all the technologies used in production by each country. Table 6 presents the estimates for the difference in income per capita and relative income per capita. The expected signs and explicative variables are the same for the two estimated equations with the respective adaption. For relative income per capital, all the explicative variables are measured in relative terms. The results show that all variables are statistically significant at the 1% level with the expected signs. Moreover, the *intensive margin* and variety effect via the network degree have an important role in explaining the cross-country income per capita differences. For example, an increase in the relative margin by 1% increases the relative income by 23%. An increase in the relative inverse-network degree by 1% increases the income per capital by 36% relative to the US. In sum, our accounting exercise allows us to explain 48% and 31% of the income per capita variation and cross-income per capital differences variation, respectively, for our sample countries and period.

Table 6: Estimated cross-country income differences

	$(y-l)$	$\Delta(y-l)$
$\ln a_\tau$	0.004	0.236
	(0.000)	(0.002)
$\ln D_e^{-1}$	0.115	0.361
	(0.042)	(0.129)
Dl	0.007	0.006
	(0.000)	(0.000)
$const$	2.833	
	(0.030)	
R^2	0.481	0.312
$R^2 adj.$	0.481	0.312
$\Pr F > 0$	0.000	0.000
N. obs	44064	47906

Note: All estimates are obtained with OLS and robust standard errors. All estimates are significant at the 1% level. For relative income differences regression $\Delta(y-l)$, the explicative variables are the relative differences $(\Delta Z_{v_\tau}, \Delta D_e^{-1}, \Delta Dl)$.

8 Conclusion

In this paper, we investigate the role of the inter-sector structure in explaining the differences in technology adoption and income across countries.

We extend a multi-sector growth model with technology adoption by adding the consumption of intermediate goods as inputs. Thus, we can explicitly define the inter-sector linkages throughout the I-O relationships between sectors. Using the social network theory, we map these I-O relationships into a weighted direct network. This procedure allows us to take into account the inter-sector network structure and examine the network structure effects in TFP, the *intensive* and *extensive margins* of technology adoption, and cross-country income differences.

We find that the inter-sector network properties, in particular, the weighted out-degree, affect both the *intensive* and *extensive margins* of technology adoption as well as the income differences across countries through the impact on TFP. Indeed, our model explains the cross-country differences in intensity of adoption and corrects the adoption lags by a multiplicative term with the inter-sector network properties. The intuition is that higher the number of connections between sectors and the strength of these connections, the bigger is the increase in productivity (TFP) due to the adoption of new technologies. Thus, the inter-sector network structure plays an important role in the impacts on technology adoption lags and intensity of adoption.

We estimate the model for 10 technologies and 9 OECD countries during the second half the 20th century. Our results confirm all our theoretical results. The model provides precise estimates of adoption lags for 84% of our technology-country pairs, explaining 81% of the adoption lag variability by the difference in technology invention year and network properties. Moreover, our estimates show that the median level of the adoption intensity of our country sample is 83% of the US and the network structure accounts for 9% of its variability. Finally, our model explains 48% and 31% of the income per capital variations and cross-income per capita differences.

Before concluding, it is worth to mention some policy implications and possible future extensions. The results of the paper suggest that government should promote polices in order to increase/ incentive the inter-sector/firm relationships once knowledge diffusion and network effects increase the TFP and technology diffusion. Moreover, it should discriminate sectors with higher number of connections or strength. In turn, this paper can be extended to study intra-sector network properties in different countries: industry network, finance network, health network, social services network, agriculture network and analyze in detail their differences and contribution for growth and diffusion, seeking to explain and understand in detail how networks have affected countries' performance.

A Data Appendix

This appendix describes in detail the data used in our analysis. We use data from two data sources:

1) The OECD input-output tables, edition 1995 and edition 2002.

The edition 1995, cover 10 countries: Australia, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, the UK and the US, from 1968 to 1990. This coverage increased in the Edition 2002 from 10 to 20 countries. The countries added are Czech Republic, Finland, Greece, Hungary, Korea, Norway, Poland, Spain, Brazil and China. This edition spans from 1992 to 1997. The data is by industry type and it is assumed that each one produces only one product. Tables 7-8 describe the industry classification according to ISIC Rev.2 and ISIC Rev.3.

Table 7: ISIC industry classification before 1995.

Code	SITC 2 digit description
1	Agriculture, forestry and fishing
2	Mining and quarrying
3	Food, beverages and tobacco
4	Textiles, apparel and leather
5	Wood products and furniture
6	Paper, paper products and printing
7	Industrial chemicals
8	Drugs and medicines
9	Petroleum and coal products
10	Rubber and plastic products
11	Non-metallic mineral products
12	Iron and steel
13	Non-ferrous metals
14	Metal products
15	Non-electrical machinery
16	Office and computing machinery
17	Electrical apparatus, nec
18	Radio, TV and communication equipment
19	Shipbuilding and repairing
20	Other transport
21	Motor vehicles
22	Aircraft
23	Professional goods
24	Other manufacturing
25	Electricity, gas and water
26	Construction
27	Wholesale and retail trade
28	Restaurants and hotels
29	Transport and storage
30	Communication
31	Finance and insurance
32	Real estate and business services
33	Community, social and personal services
34	Producers of government services
35	Other producers

Table 8: ISIC industry classification after 1995

Code	SITC 2 digit description
1	Agriculture, hunting, forestry and fishing
2	Mining and quarrying
3	Food products, beverages and tobacco
4	Textiles, Textile products, leather and footwear
5	Wood and products of wood and cork
6	Pulp, paper, paper products, printing and publishing
7	Coke, refined petroleum products and nuclear fuel
8	Chemicals excluding pharmaceuticals
9	Pharmaceuticals
10	Rubber and plastics products
11	Other non-metallic mineral products
12	Iron and steel
13	Non-ferrous metals
14	Fabricated metal products, except machinery and equipment
15	Machinery and equipment, N.E.C.
16	Office, accounting and computing machinery
17	Electrical machinery and apparatus, NEC
18	Radio, television and communication equipment
19	Medical, precision and optical instruments
20	Motor vehicles, trailers and semi-trailers
21	Building and repairing of ships and boats
22	Aircraft and spacecraft
23	Railroad equipment and transport equipment N.E.C.
24	Manufacturing NEC, recycling
25	Electricity, gas and water supply
26	Construction
27	Wholesale and retail trade, repairs
28	Hotels and restaurants
29	Transport and storage
30	Post and telecommunications
31	Finance, Insurance
32	Real estate activities
33	Renting of machinery and equipment
34	Computer and related activities
35	Research and development
36	Other business activities
37	Public admin. and defence, compulsory social security
38	Education
39	Health and social work
40	Other community, social and personal services
41	Private households with employed persons (and extra territorial organizations and bodies)

2) The CHAT databases: Cross-country Historical Adoption of Technology.

It is an unbalanced panel databases with information on technology adoption for more than 100 technologies in more than 150 countries since 1800. Table below describes the technologies used in this study.

Table 9: Technologies and classification

Technologies	Category	Invention year	Adoption year
ATMs	Finance	1967	1988
Debit and Credit card Transactions	Finance	1950	1988
Electric fund transfers (EFT)	Finance	1979	1988
Points of service for debit/credit cards (Pos)	Finance	1950	1988
Internet users	General	1973	1990
Personal computers	General	1973	1981
MRI units	Health	1981	1983
Computed tomography scanners (Cats)	Health	1972	1981
Cable television subscribers	Telecommunications	1949	1975
Cell Phones	Telecommunications	1947	1984

See Comin et al. (2006) and Comin and Hobijn (2009) for additional information about technologies.

B Derivation of the 2nd order approximation around $\gamma = 0$.

Let's denote the adoption time by $T_\tau = D_t + \underline{v}$. Then, the TFP of technology τ is given by

$$A_\tau = \Phi \left(\frac{(1-\alpha_m)(\mu-1)}{\gamma_{\tau_k} + \alpha_m \sum_j \tilde{F}_j \gamma_{z_{\tau_j}}} \right)^{\mu-1} \left[e^{\frac{\gamma_{\tau_k}}{(1-\alpha_m)(\mu-1)}(t-T_\tau)} e^{\sum_j \tilde{F}_j \gamma_{z_{\tau_j}} \frac{\alpha_m}{(1-\alpha_m)(\mu-1)}(t-T_\tau)} - 1 \right]^{\mu-1},$$

where $\Phi = (1-\alpha_m)\, \alpha_m^{\frac{\alpha_m}{1-\alpha_m}} Z_{\underline{v}_k}^{\frac{1}{1-\alpha_m}} \prod_{j=1}^{M} z_{\tau_{jo}}^{\psi_{vjk} \frac{\alpha_m}{1-\alpha_m}}$.

We are approximating the TFP around $\gamma = 0$. In this case, there is no embodiment productivity growth, and the rise of the productivity is only due to an increase of the number of varieties over time. Note that $\gamma_{z_{\tau_j}}$ and $\tilde{F}$ are vector related to each sector $j = 1, ..., m$. Thus, we are finding the Taylor approximation for one specific j, assuming these vectors only have one element. Afterwards, we generalize the results for $j = 1, ..., m$. Thus, in order to find the first-order approximation, let's find the $\lim\{\gamma_\tau, \gamma_{z_{\tau_j}} \to 0\}$, using the l'Hopital's rule:

$$\begin{aligned}
&\lim_{\gamma_{\tau_k}, \gamma_{z_{\tau_i}} \to 0} \left\{ \Phi \left(\frac{(1-\alpha_m)(\mu-1)}{\gamma_{\tau_k} + \alpha_m \sum_j \tilde{F}_j \gamma_{z_{\tau_j}}} \right)^{\mu-1} \left[e^{\frac{\gamma_{\tau_k}}{(1-\alpha_m)(\mu-1)}(t-T_\tau)} e^{\sum_j \tilde{F}_j \gamma_{z_{\tau_j}} \frac{\alpha_m}{(1-\alpha_m)(\mu-1)}(t-T_\tau)} - 1 \right]^{\mu-1} \right\} \\
&= \Phi \lim_{\gamma_{\tau_k}, \gamma_{z_{\tau_j}} \to 0} \left\{ \frac{(1-\alpha_m)(\mu-1)}{\gamma_{\tau_k} + \alpha_m \sum_j \tilde{F}_j \gamma_{z_{\tau_j}}} e^{\frac{\gamma_{\tau_k}}{(1-\alpha_m)(\mu-1)}(t-T_\tau)} e^{\sum_j \tilde{F}_j \gamma_{z_{\tau_j}} \frac{\alpha_m}{(1-\alpha_m)(\mu-1)}(t-T_\tau)} - \frac{(1-\alpha_m)(\mu-1)}{\gamma_{\tau_k} + \alpha_m \sum_j \tilde{F}_j \gamma_{z_{\tau_j}}} \right\}^{\mu-1} \\
&\equiv \Phi \left[(t - T_\tau) \right]^{\mu-1}.
\end{aligned}$$

Taking the first order Taylor approximation around $\gamma_{\tau_k} = 0$ and $\gamma_{z_{\tau_i}} = 0$ yields that

$$\begin{aligned} A_\tau \approx & \Phi \left[(t - T_\tau)\right]^{\mu-1} + \Phi (\mu - 1) \left[(t - T_\tau)\right]^{\mu-2} \frac{1 + \alpha_m \sum_i \tilde{F}_i}{2(1-\alpha_m)(\mu-1)} (t - T_\tau)^2 \left(\gamma_\tau + \sum_i \gamma_{z_{\tau_i}}\right) \\ \approx & \Phi \left[(t - T_\tau)\right]^{\mu-1} \left[1 + \frac{1 + \alpha_m \sum_i \tilde{F}_i}{2(1-\alpha_m)} (t - T) \left(\gamma_\tau + \sum_i \gamma_{z_{\tau_i}}\right)\right]. \end{aligned}$$

Hence, for γ_{τ_k} and $\gamma_{z_{\tau_j}}$ close to zero, we get

$$a_{\tau_i} \approx \phi + (\mu - 1) \log (t - T_\tau) + \frac{1 + \alpha_m \sum_j \tilde{F}_j}{2(1-\alpha_m)} (t - T) \left(\gamma_\tau + \gamma_{z_{\tau_j}}\right), \tag{62}$$

where $\phi = \frac{\alpha_m}{1-\alpha_m} \log (\alpha_m (1 - \alpha_m)) + \frac{1}{1-\alpha_m} \log Z_{\underline{v}k} + \frac{\alpha_m}{1-\alpha_m} \sum_j \psi_{vjk} \log z_{\tau_{jo}}$.

C Estimated equations:

To derive the estimated equations (54)-(55), we combine the log-linearized equations (50), (51), (52) and (53), as follows

$$\begin{aligned} y_\tau = & \frac{\mu}{\mu-1} \left[\log\left((1-\alpha_m)\alpha_m^{\frac{\alpha_m}{1-\alpha_m}}\right) + \frac{1}{1-\alpha_m} \log Z_{\underline{v}k} + \frac{\alpha_m}{1-\alpha_m} \sum_j \psi_{vjk} \log z_{\tau_{jo}}\right] + \mu \log (t - T\tau) - \frac{\mu}{\mu-1} \alpha \log\left(\frac{R}{\alpha}\right) \\ & + \frac{\mu\left(1 + \alpha_m \sum_j \tilde{F}_j\right)\left(\gamma_\tau + \sum_j \gamma_{z_{\tau_j}}\right)}{2(\mu-1)(1-\alpha_m)} t - \frac{\mu\left(1 + \alpha_m \sum_j \tilde{F}_j\right)\left(\gamma_\tau + \sum_j \gamma_{z_{\tau_j}}\right)}{2(\mu-1)(1-\alpha_m)} T_\tau - \frac{\mu}{\mu-1} (1-\alpha)(y-l) + y \\ = & y + \beta_1 + \beta_2 \frac{1 + \alpha_m \sum_j \tilde{F}_j}{2(1-\alpha_m)} t + \beta_3 \log (t - T_\tau) + \beta_4 (1-\alpha)(y-l) \end{aligned}$$

$$\begin{aligned} k_\tau = & \frac{1}{\mu-1} \left[\log\left((1-\alpha_m)\alpha_m^{\frac{\alpha_m}{1-\alpha_m}}\right) + \frac{1}{1-\alpha_m} \log Z_{\underline{v}k} + \frac{\alpha_m}{1-\alpha_m} \sum_j \psi_{vjk} \log z_{\tau_{jo}}\right] + \log (t - T_\tau) - \frac{\alpha}{\mu-1} \log\left(\frac{R}{\alpha}\right) \\ & + \frac{\left(1 + \alpha_m \sum_j \tilde{F}_j\right)\left(\gamma_\tau + \sum_j \gamma_{z_{\tau_j}}\right)}{2(\mu-1)(1-\alpha_m)} t - \frac{\left(1 + \alpha_m \sum_j \tilde{F}_j\right)\left(\gamma_\tau + \sum_j \gamma_{z_{\tau_j}}\right)}{2(\mu-1)(1-\alpha_m)} T_\tau - \frac{1-\alpha}{\mu-1} (y-l) + y \\ = & y + \beta_1 + \beta_2 \frac{1 + \alpha_m \sum_j \tilde{F}_j}{2(1-\alpha_m)} t + \log (t - T_\tau) + \beta_3 (1-\alpha)(y-l) \end{aligned}$$

References

Acemoglu, D. (1998). Why do new technologies complement skills? directed technical change and wage inequality. *Quarterly Journal of Economics*, 113(4):1055–1089.

Acemoglu, D. (2002). Directed technical change. *Review of Economic Studies*, 69(4):781–809.

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Salehi, A. T. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016.

Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60(2).

Ang, J. B. and Madsen, J. B. (2015). What drives ideas production across the world? *Macroeconomic Dynamics*, 19(01):79–115.

Basu, S. and Weil, D. (1998). Appropriate technology and growth. *Quarterly Journal of Economics*, 113(4):1025–1054. cited By (since 1996) 108.

Bertolotti, F., Mattarelli, E., Vignoli, M., and Macrì, D. M. (2015). Exploring the relationship between multiple team membership and team performance: The role of social networks and collaborative technology. *Research Policy*, 44(4):911–924.

Carvalho, V. (2007). Aggregate fluctuations and the network structure of intersectoral trade. Economics Working Papers 1206, Department of Economics and Business, Universitat Pompeu Fabra.

Carvalho, V. M. (2014). From Micro to Macro via Production Networks. *Journal of Economic Perspectives*, 28(4):23–48.

Chaney, T. (2014). The network structure of international trade. *American Economic Review*, 104(11):3600–3634.

Ciccone, A. and Matsuyama, K. (1996). Start-up costs and pecuniary externalities as barriers to economic development. *Journal of Development Economics*, 49(1):33–59.

Comin, D. and Hobijn, B. (2010). An Exploration of Technology Diffusion. *American Economic Review*, 100(5):2031–59.

Comin, D., Hobijn, B., and Rovito, E. (2006). Five facts you need to know about technology diffusion. NBER Working Papers 11928, National Bureau of Economic Research, Inc.

Comin, D. and Mestieri, M. (2014). Technology adoption and growth dynamics. Technical report.

Comin, D. A. and Hobijn, B. (2009). The chat dataset. Working Paper 15319, National Bureau of Economic Research.

Comin, D. A. and Mestieri, M. (2010). The intensive margin of technology adoption. Harvard Business School Working Papers 11-026, Harvard Business School.

Dinopoulos, E. and Thompson, P. (2000). Endogenous growth in a cross-section of countries. *Journal of International Economics*, 51(2):335–362.

Greenwood, J. and Yorukoglu, M. (1997). 1974. *Carnegie-Rochester Conference Series on Public Policy*, 46(1):49–95.

Grossman, G. and Helpman, E. (1991). Innovation and growth in the global economy cambridge, massachusetts.

Guan, J., Zuo, K., Chen, K., and Yam, R. C. (2016). Does country-level r&d efficiency benefit from the collaboration network structure? *Research Policy*, 45(4):770–784.

Hall, R. and Jones, C. (1999). Why do some countries produce so much more output per worker than others? *Quarterly Journal of Economics*, 114(1):83–116. cited By (since 1996) 1232.

Hasan, I. and Tucci, C. L. (2010). The innovation–economic growth nexus: Global evidence. *Research Policy*, 39(10):1264–1276.

Hausmann, R. and Hidalgo, C. (2011). The network structure of economic output. *Journal of Economic Growth*, 16(4):309–342.

Howitt, P. (1999). Steady endogenous growth with population and r & d inputs growing. *Journal of Political Economy*, 107(4):715–730.

Jackson, M. O. (2008). *Social and Economic Networks*. Princeton University Press, Princeton, NJ, USA.

Jackson, M. O. (2011). Chapter 12 - an overview of social networks and economic applications*. In Jess Benhabib, A. B. and Jackson, M. O., editors, *Handbook of Social Economics*, volume 1 of *Handbook of Social Economics*, chapter 12, pages 511 – 585. North-Holland.

Jerzmanowski, M. (2007). Total factor productivity differences: Appropriate technology vs. efficiency. *European Economic Review*, 51(8):2080–2110.

Jones, C. I. (1995). R-and-d-based models of economic-growth. *Journal of Political Economy*, 103(4):759–784.

Kelly, B., Lustig, H., and Nieuwerburgh, S. V. (2013). Firm Volatility in Granular Networks. NBER Working Papers 19466, National Bureau of Economic Research, Inc.

Laincz, C. A. and Peretto, P. F. (2006). Scale effects in endogenous growth theory: An error of aggregation not specification. *Journal of Economic Growth*, 11(3):263–288.

Maddison, A. (2004). Contouts of the world eonomy and the art of macro-measurement 1500-2001. Technical report, Ruggles Lecture.

Madsen, J. B. (2008). Semi-endogenous versus schumpeterian growth models: testing the knowledge production function using international data. *Journal of Economic growth*, 13(1):1–26.

Magalhães, M., Osório, C., and António, M. (2016). The u.s. inter-sector network dynamics and its effects on technology adoption. *Available at SSRN 2787961*.

Marti Mestieri, F., Diego, C., and Martí, M. (2013). If technology has arrived everywhere, why has income diverged? *NBER Working Papers*.

Ngai, L. and Samaniego, R. (2009). Mapping prices into productivity in multisector growth models. *Journal of Economic Growth*, 14(3):183–204.

Oberfield, E. (2013). Business Networks, Production Chains, and Productivity: A Theory of Input-Output Architecture. 2013 Meeting Papers 120, Society for Economic Dynamics.

Parente, S. and Prescott, E. (1994a). Barriers to technology adoption and development. *Journal of Political Economy*, 102(2):298–321. cited By (since 1996) 242.

Parente, S. L. and Prescott, E. C. (1994b). Barriers to technology adoption and development. *Journal of Political Economy*, 102(2):298–321.

Peretto, P. F. (1998). Technological change and population growth. *Journal of Economic Growth*, 3(4):283–311.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5):S71–102.

Suarez, F. F. (2005). Network effects revisited: The role of strong ties in technology selection. *Academy of Management Journal*, 48(4):710–720.