Persistent Currents versus Phase Breaking in Mesoscopic Metallic Samples

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Persistent currents in mesoscopic normal metal rings represent, even a decade after their first experimental observation, a challenge to both, theorists and experimentalists. After giving a brief review of the existing – experimental and theoretical – results, we concentrate on the (propsed) relationship of the size of the persistent current to the phase breaking rate. In particular, we consider effects induced by noise, scattering at two-level systems, and magnetic impurities.

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The question of the magnetic properties of ring molecules and metallic rings has a long history. In analogy to the Aharonov-Bohm effect, i.e. the tuning of the interference pattern in a double slit experiment through the enclosed flux, it can be expected that the magnetization, or the corresponding "persistent current", has a flux periodic contribution. In this sense, and for sufficiently low temperatures, metallic rings are very similar to ring molecules. But how large is the effect for a mesoscopic, disordered ring? Is it experimentally observable with today's technology?

The first successful experimental investigations which showed that persistent currents exist in mesoscopic rings have been performed at the beginning of the 1990s and were most important for the advancement of the field[1–8]. We will first present selected aspects of the theoretical concepts developed at the time; see, for example, Ref. 9 for more details. We then compare the theoretical results with the available experimental data. Since no clear picture emerges there is still room for improvement of the theory. In particular, we report on recent attempts to relate the size of the persistent current to the phase braking rate.

1. GENERAL ASPECTS OF THE THEORY

In the following we consider, for the sake of simplicity, an idealized situation where the width of the metal ring is so small compared to the circumference L that magnetic field penetration into the metal can be neglected. Then the energy and the thermodynamic potential, $K(\phi)$, of a given sample depend on the magnetic flux, ϕ, and not explicitly on the magnetic field. As the persistent current $I(\phi)$ is an equilibrium property, it can be calculated by taking the derivative according to

$$I(\phi) = -\frac{\partial K(\phi)}{\partial \phi}. \tag{1}$$

Depending on the variables to be kept constant in the experiment, $K(\phi)$ is given by $F(N, \phi)$ or $\Omega(\mu, \phi)$ for the canonical and the grand canonical ensemble, respectively. The periodicity of $K(\phi)$ with period $\phi_0 = h/e$ allows the Fourier decomposition of the current,

$$I(\phi) = I_{h/e} \sin(2\pi\phi/\phi_0) + I_{h/2e} \sin(4\pi\phi/\phi_0) + \cdots. \tag{2}$$

For simplicity we do not further discuss the subtle questions concerning differences [5–7] between $F(N, \phi)$ and $\Omega(\mu, \phi)$ and we concentrate on the grand canonical ensemble. Furthermore we will focus on diffusive rings, i.e. we include disorder with an elastic mean free path l such that $\lambda_F \ll l \ll L$. We also assume that the circumference is much smaller than the localization length.

2. NON-INTERACTING ELECTRONS

For non-interacting electrons the complete thermodynamics is determined from the single particle density of states. In order to compute the persistent current, consider first the grand canonical potential

$$\Omega(\mu, \phi) = -2\mathcal{V} k_B T \int \mathrm{d}E N(E, \mu) \ln\{1 + \exp[-(E - \mu)/k_B T]\}\,, \tag{3}$$

where the factor two is due to the spin, $\mathcal{V}$ is the volume, and $N(E)$ is the density of states. The energy levels and hence the density of states depend on the magnetic flux, which is to be calculated.

In an ensemble of weakly disordered rings the disorder configuration will change from ring to ring. Accordingly the density of states and the persistent current will be statistically distributed. It is well known that the average density of states, $N_0 = \langle N(E) \rangle$, is a flux independent quantity except for

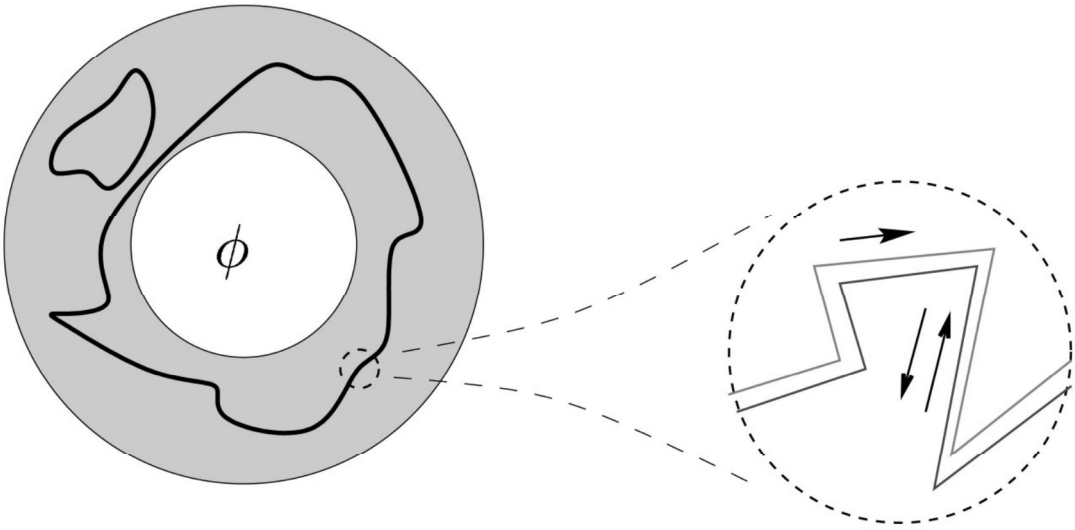

Fig. 1. Paths with winding number zero and one. The scaled-up part represents the microscopic processes with amplitudes A_i and $(A_j)^*$. A_i and A_j correspond to identical or time reversed paths.

corrections which are exponentially small and proportional to $\exp(-L/2l)$. Thus the average persistent current, as computed in the grand canonical ensemble, is negligibly small. Fluctuations of the density of states, on the other hand, are not exponentially suppressed and have to be considered. Manipulating the expression given in Ref. 10 one arrives at

$$\langle \delta N(E,\phi)\delta N(E+\hbar\omega,\phi')\rangle = \frac{L}{\pi^2\hbar^2\mathcal{V}^2}\int_0^\infty \mathrm{d}t\cos(\omega t)t \times\left[P(0,t)+\sum_{m,\pm}P(mL,t)\cos(2\pi m(\phi\pm\phi')/\phi_0)\right], \tag{4}$$

where

$$P(x,t)=\frac{1}{\sqrt{4\pi Dt}}\exp\left(-\frac{x^2}{4Dt}\right) \tag{5}$$

is the solution of the diffusion equation. This means that the density of states fluctuations are proportional to the integrated probability that a diffusing particle returns after time t. For the ring structure there are topologically different possibilities to return to the origin, see Fig. 1. Paths with zero winding number lead to the contribution $P(0,t)$ and do not depend on magnetic flux, while paths which traverse the ring m times lead to the term $P(mL,t)$ and are flux dependent. The reason is the following: First recall that a probability is in quantum mechanics always the product of amplitudes, $P\sim(\sum A_i)(\sum A_j)^*$. From all possible combinations $A_iA_j^*$, Eq. (4) then selects pairs of equal paths $j=i$ and pairs of time reversed paths $j=\bar{i}$ as illustrated in the figure. In the presence of a magnetic flux

the phase of a closed path is shifted according to $A \to A\exp(2\pi \mathrm{i}\phi/\phi_0)$, where ϕ is the enclosed magnetic flux. For a pair of two equal paths, but in the presence of two different magnetic fluxes ϕ and ϕ', one obtains $A_i A_i^* \to A_i A_i^* \exp[2\pi \mathrm{i}(\phi - \phi')]$. For a pair of time reversed paths, on the other hand, the result is $A_i A_{\bar{i}}^* \to A_i A_i^* \exp[2\pi \mathrm{i}(\phi + \phi')]$. These processes then lead to the magnetic flux dependencies given in the density of states fluctuations, Eq.(4).

From the density of states fluctuations the persistent current fluctuations are found to be given by

$$\langle I(\phi) I(\phi') \rangle = \sum_m \mathcal{C}_m \sin\left(\frac{2\pi m\phi}{\phi_0}\right) \sin\left(\frac{2\pi m\phi'}{\phi_0}\right) \tag{6}$$

$$\mathcal{C}_m = \frac{8e^2 m^2 L}{\pi^2 \hbar^2} \int_0^\infty \frac{\mathrm{d}t}{t} \left(\frac{\pi k_B T}{\sinh(\pi k_B T t/\hbar)}\right)^2 P(mL, t) \,. \tag{7}$$

The temperature dependent factor originates from the Fourier transform of the Fermi function,

$$\int \mathrm{d}\epsilon f(\epsilon) \mathrm{e}^{\mathrm{i}\epsilon t/\hbar} = \mathrm{i} \frac{\pi k_B T}{\sinh(\pi k_B T t/\hbar)} \,. \tag{8}$$

At zero temperature only the diffusion time $\tau_d = L^2/D$ exists as a time scale in the above integral, and we obtain

$$\langle I_{h/me}^2 \rangle = \mathcal{C}_m = \frac{96}{\pi^2 m^3} \left(\frac{e}{\tau_d}\right)^2 . \tag{9}$$

At finite temperature there is an exponential cut-off for long paths, proportional to $\exp(-2\pi k_B T t/\hbar)$, due to the hyperbolic sine in the denominator. This leads to a temperature dependence of the current on the diffusive scale $k_B T \sim \hbar/\tau_d$ for the h/e component, and $k_B T \sim \hbar/m^2 \tau_d$ for the mth Fourier component of the current where the electrons have to diffuse m times around the ring.

3. INTERACTING ELECTRONS

The Coulomb interaction enhances the average current considerably above the value for non-interacting electrons, as pointed out by Ambegaokar and Eckern[8]. Again we consider the grand canonical potential and work out the flux sensitive correction in a simple approximation. Consider the Hartree expression for the Coulomb interaction contribution to the thermodynamic potential,

$$\delta\Omega_C = \frac{1}{2} \int \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}' v(\mathbf{r} - \mathbf{r}') \delta n(\mathbf{r}) \delta n(\mathbf{r}'), \tag{10}$$

where $v(\mathbf{r}-\mathbf{r}')$ is the screened interaction and $\delta n(\mathbf{r})$ is the spatially varying electron density. In full analogy to what is found for the density of states, the average electron density is flux independent but the fluctuations do depend on magnetic flux. Averaging the above equation with respect to disorder and including also the exchange (Fock) term, we find

$$\begin{aligned}\langle \delta\Omega_C(\phi)\rangle &= \mu_0 \frac{2L}{\pi\hbar}\int_0^\infty \mathrm{d}t \left(\frac{\pi k_B T}{\sinh(\pi k_B T t/\hbar)}\right)^2 \\ &\quad \times \sum_m P(mL,t)\cos(4\pi m\phi/\phi_0)\,. \end{aligned} \tag{11}$$

The temperature dependent factor is related to the Fourier transform of two Fermi functions, similar to the noninteracting theory. As above, the diffusion probability $P(mL,t)$ appears. Since only time reversed paths contribute to the average current, the primitive period is here given by $h/2e$, not h/e. The dimensionless number μ_0 characterizes the strength of the Coulomb interaction. It is given by the Fermi surface average of the screened interaction, multiplied by the density of states. For copper, for example, μ_0 has been estimated to be about 0.3. The final result for the zero temperature persistent current is $I \sim \mu_0 e/\tau_d$. For a precise estimate of the prefactor the theory has to be refined: It is well known that higher order terms renormalize the coupling constant μ_0 logarithmically, $\mu_0 \to \mu^* \approx \mu_0/\{1+\mu_0 \ln[\epsilon_F/(\hbar/\tau_d)]\}$. In addition, the attractive electron-phonon interaction reduces it even further. This could, in principle, also lead to a different sign, implying that the system undergoes a transition to a superconducting state at extremely low temperature.

Taking μ^* as a parameter which has to be put into the theory "by hand", we replace μ_0 in Eq. (11) by μ^*, and find the the zero temperature average persistent current to be given by $I(\phi) = I_{h/2e}\sin(4\pi\phi/\phi_0)+\ldots$, with $I_{h/2e} = 8\mu^*/\pi(e/\tau_d)$.

4. EXPERIMENTAL RESULTS

Experimental results obtained by different groups are summarized in Table 1. The mean free path l and the phase coherence length L_ϕ, given in the table, have been determined by transport measurements in equally prepared wires. In two of the experiments the number of rings was very large, 10^7 and 10^5 respectively. The current per ring should then correspond to the average current calculated in theory. In both experiments the h/e component of the current was not observed, in agreement with the theoretical prediction. Also the amplitude of the $h/2e$ component is reasonable, both theory and experiment find a current of the order of e/τ_d. In the first

	L	l	L_ϕ	$I_{h/e}$/ring	$I_{h/2e}$/ring
10^7 rings; Cu (Ref. 2)	2.2μm	30nm	2μm	0	$\pm 0.77e/\tau_d$
single rings; Au (Ref. 3)	7.5μm 12.5μm 8.0μm	70nm 70nm 70nm	12μm 12μm 12μm	$+111e/\tau_d$ $\pm 33e/\tau_d$ $+24e/\tau_d$	
30 rings; Au (Refs. 11 and 12)	8μm	87nm	16μm	$0.42e/\tau_d$	$-0.44e/\tau_d$
10^5; GaAs/GaAlAs (Ref. 13)	8μm	3μm	7μm	0	$-0.3e/\tau_d$
16; GaAs/GaAlAs (Ref. 14)	12μm	8μm	20μm	$0.25e/\tau_d$	

Table 1. Summary of experimental results for the persistent current in diffusive rings

experiment the sign of the current was not clear. The amplitude of the experimental zero temperature current corresponds to an interaction parameter $|\mu^*| = 0.3$, which is at least a factor five larger than what we expect from the Coulomb interaction in copper. In the second experiment, Ref. 13, the sign of the current was negative (diamagnetic), which is clearly at odds with a theory taking into account the Coulomb interaction only. We emphasize that the computed temperature dependence is in perfect agreement with the experimental findings of Ref. 2, and in reasonable agreement with Ref. 13: Theory predicts in the relevant temperature range an exponential suppression, $I_{h/2e}(T) \approx I_{h/2e}(0)\exp(-T/T^*)$, with the typical temperature given by $k_B T^* \approx 3\hbar/\tau_d$. An exponential decay was seen in both experiments. In the copper experiment[2] the typical temperature was $T^* = 80$ mK with $\hbar/\tau_d \approx 25$ mK and in the semiconductor experiment $T^* = 190$ mK and $\hbar/\tau_d \approx 31$ mK.

The experiments on single or a few rings are more difficult to interpret. Theory predicts that in single rings the typical current is of the order e/τ_d and should vary from sample to sample. The measured current in single gold rings[3] was up to two orders of magnitude larger than this esimate. More recent measurements of the current in a few rings, on the other hand, are much closer to what is expected: for N rings the typical current per ring is expected to be of the order $I \sim \langle I \rangle + \sqrt{\langle \delta I^2 \rangle / N}$. Measurements for 30 gold rings and 16 semiconductor rings are consistent with this expectation, see Table 1. The measured h/e current per ring, $I_{h/e} \approx 0.42e/\tau_d$, is close to the expected current, which is of the order $\sqrt{96/(30\pi^2)} \approx 0.57e/\tau_d$. Concerning the $h/2e$ current it is not clear whether for 30 rings the average or the fluc-

tuations dominate, since the expected fluctuations are of the order $0.2e/\tau_d$, which is only a factor two smaller than the experimentally observed current. The temperature suppression of the h/e and the $h/2e$ current reported in Refs. 11 and 12 is consistent with an exponential decay, with the characteristic temperature $T^*_{h/e} = 166$ mK, and $T^*_{h/2e} = 89$ mK. The theoretical prediction in the relevant temperature range is[4] $\langle \delta I^2_{h/me} \rangle \approx \exp(-2T/T^*_m)$, with $k_B T^*_m = \pi^2 \hbar/(m^2 \tau_d)$, i.e. $T^*_{h/e} \approx 72$ mK and $T^*_{h/2e} \approx 18$ mK, when inserting $\hbar/\tau_d = 7.3$ mK. The experimentally observed temperature suppression is thus somewhat slower than expected.

In conclusion we have seen that in some aspects theory and experiments agree with each other: The periodicity of the current is predicted correctly for both the experiments with few and many rings. The amplitude of the current is of the order $\hbar/\tau_d$, and the temperature scale is reasonable, too. Nevertheless the agreement is not satisfactory: Although the $h/2e$ current is of the right order of magnitude, it is still considerably larger than predicted. Moreover the negative sign still requires an explanation.

Due to this discrepancies it is important to consider alternative mechanisms that could influence the persistent current. In the following we discuss recent suggestions which connect persistent currents and phase breaking.

5. NOISE

Recently it has been suggested[11,15] that the large persistent currents might be related to another problem in mesoscopic physics, namely the unexpectedly large electron dephasing rate. Whereas it is expected that the dephasing rate decreases to zero in the zero temperature limit[16], many experiments show a saturation in this limit. Usually this saturation is attributed to the presence of magnetic impurities or to heating. However, a saturation of the dephasing time is observable, even after excluding these possibilities[17].

Several attempts have been made to explain this low temperature saturation[18–21]. In particular it has been argued by Altshuler *et al.*[19] that non-equilibrium electromagnetic noise can contribute to decoherence without heating the electrons. Extending earlier work[22] on the effect of a high frequency electromagnetic field in mesoscopic rings, Kravtsov and Altshuler[15] have shown that non-equilibrium noise induces a directed non-equilibrium current. This leads to the suggestion that both the "large" currents observed[2,11,13] and the strong dephasing are related, and are non-equilibrium phenomena.

The starting point of the theory is the weak localization correction to the current in the presence of magnetic flux and a time dependent electric

field,

$$I_{wl}(t) = \frac{\mathcal{C}_\beta e^2 D}{2\hbar L} \int_0^\infty \mathrm{d}\tau C_{t-\tau/2}\left(\frac{\tau}{2}, -\frac{\tau}{2}\right) E(t-\tau) \ , \tag{12}$$

where $C_t(\tau,\tau') = \sum_q C_t(q,\tau,\tau')$ is the cooperon at coinciding space points, and $C_t(q,\tau,\tau')$ is determined by

$$\frac{\partial C_t}{\partial \tau} + D\left(q - \frac{e}{\hbar}(A_{t+\tau} + A_{t-\tau})\right)^2 C_t = \delta(\tau - \tau') \ . \tag{13}$$

The momentum q is for a ring structure given by $q = (2\pi/L)(n - 2\phi/\phi_0)$. The constant $\mathcal{C}_\beta$ depends on the Dyson symmetry class: For pure potential disorder ($\beta = 1$) $\mathcal{C}_\beta = -4/\pi$, while $\mathcal{C}_\beta = 2/\pi$ in the presence of strong spin-orbit scattering ($\beta = 4$). Altshuler and Kravtsov calculated the DC-component of the current, i.e. the time average $\overline{I_{wl}(t)}$, in presence of a random electric field with zero time average, $\bar{A}_t = 0$, and correlations $\overline{A_t A_{t'}}$ that decrease at $|t - t'| > t_c$, where $t_c < \tau_\phi, \tau_d$. The final result for the average persistent current is

$$I_{h/2me} = \mathcal{C}_\beta \left(\frac{e}{\tau_\phi}\right) \exp\left(-m\frac{L}{L_\phi}\right), \tag{14}$$

where

$$\frac{1}{\tau_\phi} = 2D(e^2/\hbar^2)\overline{A_t^2} \tag{15}$$

is the noise-induced dephasing rate. If in one of the experiments dephasing is due to such a non-equilibrium electric noise, then the noise-induced current is also relevant: Since in all experiments the phase coherence length is of the order of the phase breaking length, the noise induced current is of the order e/τ_d. The noise induced current is paramagnetic for rings with strong spin-orbit coupling (like in gold or copper), and is diamagnetic in the absence of spin-orbit coupling, and could thus be the explanation of the semiconductor experiments in Ref. 13.

6. IMPURITY MEDIATED INTERACTIONS

In the presence of impurities, in particular, whenever the defects have an internal structure, the persistent current may have a sizeable contribution through what can be called an "effective interaction"; this mechanism was dsicussed, for example in Refs. 23 and 24. We first consider the interaction of conduction electrons with nonmagnetic impurities, which we assume to couple to the electron density. The Hamiltonian is of the form

$$\hat{H}_{\mathrm{int}} = \int \mathrm{d}x \hat{n}(\mathbf{x}) \hat{V}(\mathbf{x}) \ . \tag{16}$$

The operator $\hat{V}(\mathbf{x})$ that is due to the impurities, will be specified more explicitly below. To second order in this interaction one finds a correction to the free energy which is the sum of a Hartree and a Fock like term, $\delta\Omega = \delta\Omega_{\rm H} + \delta\Omega_{\rm F}$. We concentrate on the Hartree term given by ($\beta = 1/kT$)

$$\begin{aligned}\delta\Omega_{\rm H} &= -\frac{1}{2}\int_0^\beta {\rm d}\tau \int {\rm d}\mathbf{x} \int {\rm d}\mathbf{x}' \langle\hat{n}(\mathbf{x})\rangle_{\rm th}\langle\hat{n}(\mathbf{x}')\rangle_{\rm th} \\ &\quad \times \left[\langle\hat{V}(\mathbf{x},\tau)\hat{V}(\mathbf{x}',0)\rangle_{\rm th} - \langle\hat{V}(\mathbf{x})\rangle_{\rm th}\langle\hat{V}(\mathbf{x}')\rangle_{\rm th}\right], \qquad (17)\end{aligned}$$

where the brackets $\langle\ldots\rangle_{\rm th}$ are the thermal average. Comparing Eqs. (10) and (17) one realizes that the Coulomb interaction is replaced by an effective interaction,

$$v(\mathbf{x}-\mathbf{x}') \to -\int_0^\beta {\rm d}\tau \left[\langle\hat{V}(\mathbf{x},\tau)\hat{V}(\mathbf{x}',0)\rangle_{\rm th} - \langle\hat{V}(\mathbf{x})\rangle_{\rm th}\langle\hat{V}(\mathbf{x}')\rangle_{\rm th}\right] \qquad (18)$$

mediated by the defects. If $\hat{V}(\mathbf{x})$ describes pure potential scattering, then $\hat{V}(\mathbf{x})$ is a c-number with the result that this effective interaction vanishes. The situation is more interesting whenever the impurity has an internal degree of freedom. For a two-level system, for example, which may be realized by an impurity which sits in a double well potential with nearly degenerate minima at $\mathbf{r}$ and $\mathbf{r}+\mathbf{d}$, we write the scattering potential as $\hat{V}(\mathbf{x}) = V[\hat{n}_A\delta(\mathbf{x}-\mathbf{r}) + \hat{n}_B\delta(\mathbf{x}-\mathbf{r}-\mathbf{d})]$. Here $\hat{n}_A$ and $\hat{n}_B$ are the number operators for the impurity in the respective potential minimum. Since the impurity is in either of these minima, $\hat{n}_A + \hat{n}_B = 1$.

Next we average the interaction over the Fermi surface. The dimensionless interaction "constant", i.e. the analog to μ_0 defined earlier for the Coulomb interaction, reads

$$\mu_{\rm TLS} = -\frac{N_0 V^2}{\mathcal{V}}\frac{1}{2}F\int_0^\beta {\rm d}\tau\left[\langle\hat{n}_A(\tau)\hat{n}_A(0)\rangle_{\rm th} - \langle\hat{n}_A\rangle_{\rm th}\langle\hat{n}_A\rangle_{\rm th}\right], \qquad (19)$$

with $F = \left[1-\sin^2(k_F d)/(k_F d)^2\right]$, and $\mathcal{V}$ the volume. In absence of spin-orbit scattering the zero temperature persistent current thus is obtained as

$$I(\phi) = \frac{16\mu_{\rm TLS}}{\pi}\frac{e}{\tau_d}\sin\left(\frac{4\pi\phi}{\phi_0}\right) + \cdots. \qquad (20)$$

In the presence of strong spin-orbit scattering this result has to be divided by four. In order to calculate $\mu_{\rm TLS}$ explicitly, we need details of the impurity Hamiltonian. We characterize the impurity by an asymmetry ϵ, and a tunneling amplitude Δ. The relevant correlation function is given by

$$\int_0^\beta {\rm d}\tau\left[\langle\hat{n}_A(\tau)\hat{n}_A(0)\rangle_{\rm th} - \langle\hat{n}_A\rangle_{\rm th}\langle\hat{n}_A\rangle_{\rm th}\right] = \begin{cases} \frac{1}{4}\frac{1}{kT} \\ \frac{1}{4}\frac{\Delta^2}{\epsilon^2+\Delta^2}\frac{1}{\sqrt{\epsilon^2+\Delta^2}} \end{cases}, \qquad (21)$$

in the two limits $\epsilon^2 + \Delta^2 < (kT)^2$, and $\epsilon^2 + \Delta^2 > (kT)^2$, respectively. For a given concentration c of two-level systems, we find under the standard assumption[25] of a flat distribution of ϵ between zero and $\epsilon_{\max}$ ($> T$), and a distribution of Δ that is proportional to $1/\Delta$ between $\Delta_{\min}$ and $\Delta_{\max}$, the result $\mu_{\rm TLS} \sim -FcN_0V^2/\epsilon_{\max}$. This number should be of the order one, for this mechanism to be relevant for the persistent current experiments considered above. Estimates, however, are difficult due to the large number of parameters. Although the required density of TLS is not unreasonable[20,26], it has been objected[26,27] – in another context – that the required concentration is larger than typical values in metallic glasses.

Certainly the estimate could be improved if some of the parameters could be determined experimentally. In Ref. 20 it has been demonstrated that in the presence of a sufficient number of TLS, the electron dephasing rate becomes temperature independent in a certain range of temperature. The dephasing rate is[20]

$$\frac{1}{\tau_\phi} \sim \begin{cases} \Delta_{\max} FcN_0V^2/(\epsilon_{\max}\lambda) & \text{if } \hbar/\tau_\phi < \Delta_{\max} < kT \\ \Delta_{\max}(FcN_0V^2/\hbar\lambda\epsilon_{\max})^{1/2} & \text{if } \Delta_{\max} < \hbar/\tau_\phi < kT \end{cases} \tag{22}$$

with $\lambda = \ln(\Delta_{\max}/\Delta_{\min})$. In an experiment where this mechanism is responsible for dephasing, we can express the persistent current amplitude, $I \sim \mu_{\rm TLS}(e/\tau_D)$, in terms of the experimentally accessible dephasing rate as

$$|\mu_{\rm TLS}| \sim \begin{cases} \lambda(\hbar/\tau_\phi)/\Delta_{\max} \\ \lambda(\hbar/\tau_\phi)^2/\Delta_{\max}^2 \end{cases} \tag{23}$$

in the two limits considered. A possible candidate is the gold sample of Ref. 11: Below 500 mK the dephasing rate is T-independent with $\hbar/\tau_\phi \sim 2$ mK. For the mechanism considered here, the lowest measured temperature (~ 40 mK) is an upper limit for $\Delta_{\max}$. This leads to the estimate $|\mu_{\rm TLS}| > \lambda/20$. Note that the persistent current from TLS mediated interactions is diamagnetic, and could thus be responsible for the diamagnetic $I_{h/2e}$ in the array of gold rings of Refs. 11 and 12.

7. MAGNETIC DEFECTS

Finally, it is of course important to consider magnetic impurities. First, these are difficult to avoid in preparing the experimental samples, and thus magnetic defects often have to be taken into account for the interpretation of the results. Second, however, magnetic impurities are well suited to change the properties in an controlled way. Thus, in order to check the predictions given in this section, we would strongly encourage such experiments.

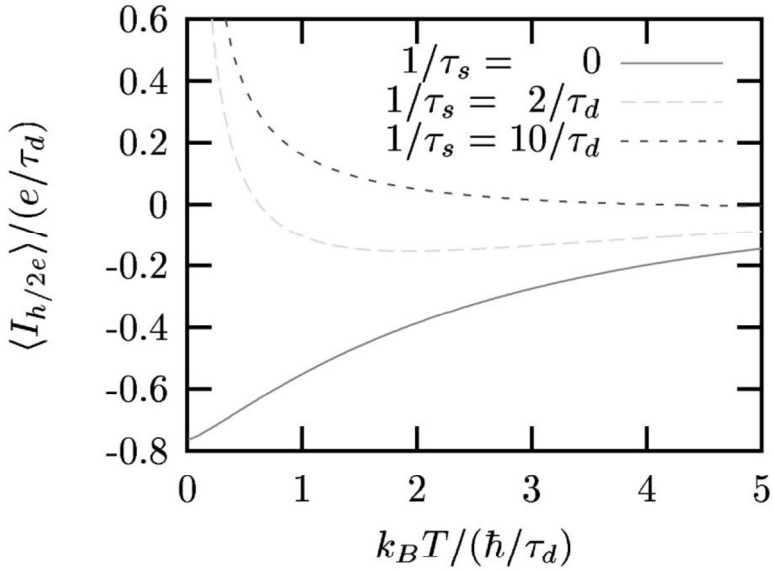

Fig. 2. Prediction for the average persistent current in the presence of magnetic impurities for rings with strong spin-orbit scattering. The interaction parameter μ^* is chosen $\mu^* = -0.3$.

The sensitivity of quantum coherence with respect to magnetic scattering is well known. Magnetic impurities suppress quantum interference from time reversed paths. Therefore one expects that the average persistent current in the presence of magnetic impurities is suppressed; explicitly

$$\begin{aligned}\langle I(\phi)\rangle &= \frac{4\mu^* emL}{\pi\hbar^2}\int_0^\infty \mathrm{d}t\left(\frac{\pi k_B T}{\sinh(\pi k_B T t/\hbar)}\right)^2 \mathrm{e}^{-t/2\tau_s} \\ &\quad\times\sum_m P(mL,t)\sin(4\pi m\phi/\phi_0)\ ,\end{aligned} \tag{24}$$

where τ_s is the spin-flip scattering time. On the other hand the coupling of the conduction electrons with a local spin,

$$\hat{H}_{\mathrm{int}} = -J\hat{\mathbf{s}}(\mathbf{x})\cdot\hat{\mathbf{S}}, \tag{25}$$

will induce an effective, spin-dependent electron-electron interaction,

$$v_{\alpha\beta\gamma\delta}(\mathrm{i}\omega) = -\frac{J^2}{\mathcal{V}}\sum_{a,b=x,y,z}\sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta}\chi^{ab}(\mathrm{i}\omega) \tag{26}$$

$$\chi^{ab}(\mathrm{i}\omega) = \int_0^\beta \mathrm{d}\tau \mathrm{e}^{\mathrm{i}\omega\tau}\left[\langle\hat{S}^a(\tau)\hat{S}^b(0)\rangle_{\mathrm{th}} - \langle\hat{S}^a\rangle_{\mathrm{th}}\langle\hat{S}^b\rangle_{\mathrm{th}}\right] \tag{27}$$

which may enhance the persistent current. The effective interaction is proportional to the impurity susceptibility χ^{ab}. This quantity, in weak magnetic fields, is proportional to the inverse temperature, at least when Kondo correlations and spin-spin interactions are neglected. Figures 2 and 3 show the

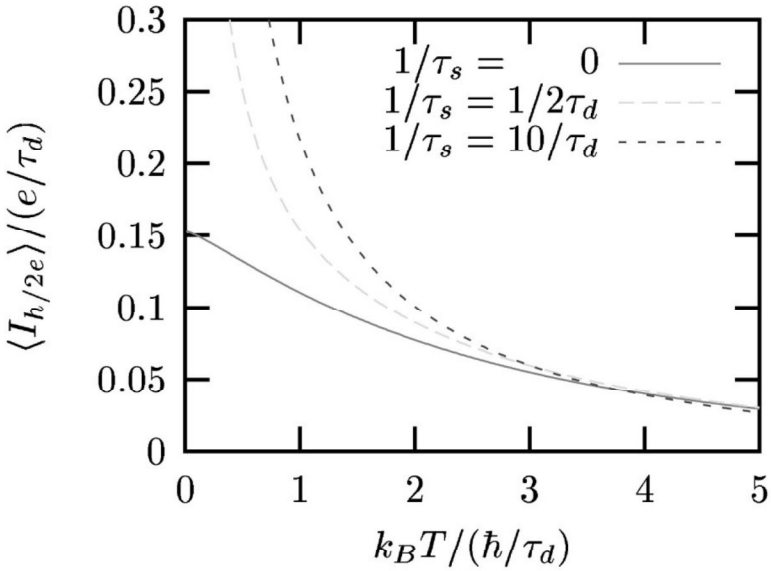

Fig. 3. Average persistent current in the presence of magnetic impurities; here the interaction parameter is $\mu^* = 0.06$.

persistent current in the presence of the magnetic impurities and for strong spin-orbit scattering, which is relevant for gold or copper. The current was obtained by adding to Eq. (24) the impurity induced current as given in Ref. 23. Notice the qualitative difference of the results for negative interaction parameter μ^* (Fig. 2), compared to positive μ^* (Fig. 3). In the first case, a low concentration of magnetic impurities strongly suppresses the current, and at very low temperature there is a change in sign. In the second case, where the persistent current in the absence of magnetic impurities is paramagnetic, the current is not reduced by a low concentration of impurities; at low temperature it is even enhanced considerably.

8. SUMMARY

One decade after the first experimental observation of persistent currents in normal conducting rings the central question is still open: Which mechanism is responsible for the amplitude of the current? Existing theories capture correctly certain aspects of the experimental observations, like the periodicity, the scale of the amplitude of the current and its temperature dependence. Nevertheless a theory of disordered electrons including the Coulomb interaction seems not to be complete.

In this article we summarized recent ideas of the relationship between persistent currents and dephasing. All mechanisms considered have one feature in common: If there is enough noise, (two-level systems, magnetic impurities) in order to explain the experimentally observed dephasing, then the noise is sufficient to explain the observed amplitude of the persistent

current.

Experimentally the relation between dephasing and persistent currents may be checked by measuring the persistent current for different materials. For silver, where no saturation of the dephasing time has been observed[28], we expect a smaller persistent current than in gold or copper where the dephasing time saturates at low temperature. We also suggest a study of the persistent current in samples doped with magnetic impurities. By varying the impurity concentration one controls the strength of the impurity-mediated electron-electron interaction. For the gold rings of Refs. 11 and 12 for example, we predict a sign change of the $h/2e$ current with increasing concentration of magnetic impurities.

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