Consumer Choice and Data Envelopment Analysis: The Case of Discrimination on Housing Markets

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Abstract

The objective of this paper is twofold. First we discuss, whether data envelopment analysis can be applied to households' consumption activities on the basis of Kelvin Lancaster's approach to consumer choice. Then, we apply data envelopment analysis to identify inefficiencies in a subsector of the Augsburg (Germany) housing market. These inefficiencies can be attributed to differential treatment by landlords. With these inefficiencies occuring systematically for certain householdtypes we conclude that discriminatory behavior exists.

1 Introduction

Inspired by Becker's (1957 updated in 1971) neoclassical theory of discrimination various studies have been published testing for discrimination on housing-markets.

Basically, two types of analysis can be distinguished. First, Yinger (1986), Galster and Constantine (1991), Yinger (1991), Roychoudhury and Goodman (1992) and Roychoudhury and Goodman (1996) employ the direct housing audit technique. They perform controlled housing search experiments to directly test for discriminatory behaviour of landlords. Second, two indirect statistical approaches utilizing existing housing data are conducted. Kain and Quigley (1972), McDonald (1974), Roistacher and Goodman (1976), Weinberg (1978), Silberman, Yochum and Ihlanfeldt (1982), Haurin and Kamara (1992), Krishnan and Krotki (1993) and Bourassa (1994) used discrete choice models to test for significant disparities in homeownership between certain houshold groups whereas Kain and Quigley (1970 and 1975), Lapham (1971), King and Mieszkowsky (1973), Follain and Malpezzi (1981) and Chambers (1992) apply hedonic price estimation. They try to assess discrimination by observing differential prices while defining equivalent housing through Lancaster's (1966) charakteristics approach. In these studies the characteristics approach has been

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employed to evaluate price differentials on identical collections of housing attributes or characteristics. Our analysis of discrimination on housing markets follows Lancaster's approach of consumer choice. On the basis of the Lancasterian approach we set up a rather simple model of the households' choice of dwelling units where landlords discriminate against a certain group of potential tenants by systematically restricting the set of dwellings they can choose from. We show that inefficiencies in consumption can be induced by landlords. In contrast to the above mentioned studies, only testing for price differentials those inefficiencies are not restricted to be price differentials for equivalent housing, they can also mean equal price for less housing. We further show that those inefficiencies can be identified by data envelopment analysis. After measuring the inefficiency for a data set of 418 households, we regress the efficiency scores against variables indicating the household type. As we find that those variables can explain for a large proportion of the identified inefficiencies we conclude that there is systematic differential treatment based on the household type constituting discrimination.

So, the paper is structured as follows. In the second section we discuss the landlords' options to discriminate. Applying Lancaster's characteristics approach to model the houshold's choice in section three allows us to see, how discriminatory action by landlords affects the choice. In the section four we assert that a discriminatory landlord may induce inefficient decisions by the households. After presenting the data in the following section, we identify the inefficiencies by data envelopment analysis and explain them by regression analysis in section six. A conclusion is provided in section seven.

2 Discriminatory action

Becker defines discrimination in terms of a price differential for equivalent housing, that is, if "some people pay more than others for a dwelling of a given quality" (Becker 1971, p. 78) discrimination occurs.

For our analysis we want to adopt a more general view on discrimination. An action taken by one party that negatively affects a second party is seen as being discriminatory, if it is based on personal characteristics of the second party (Black et. al. 1978). Additionally, Turner (1992) objects that not any differential treatment is discriminatory, rather is it systematic differential treatment that constitutes discrimination.

Although housholds are often comprised of several members, we assume that dwelling decisions are made by the household head. So the household is generally seen as one decision making unit, that is characterized by the type of the income earning member. The other party is denoted landlord, not distinguishing whether he or she is the owner of the property or a realtor acting on behalf of the owner.[1]

Stahl (1985) with reference to Stigler (1961) points out that conditions in housing marktes are such, that landlords possess a certain degree of monopoly

[1]Without further mentioning it, we restrict our discussion on rented dwelling units only.

power, allowing them to discriminate households. The majority of the traded dwelling units are advertised in newspapers. As rents and expenses for utilities are revealed in this ads, landlords basically cannot earn a higher rent from tenants of discriminated groups. Hence a price differential due to exploiting a lower price elasticity of demand for housing of a certain group is not created. [2]

Rather, landlords discriminate via exclusionary tactics. This exclusionary treatment by landlords may include denying the availability of vacant dwellings to a potential tenant on the basis of personal characteristics as well as delaying further information on the appartment in question. Empirical results of Galster and Constantine (1991), Yinger (1991), and Turner (1992) show that, for the United States, exclusionary treatment is the major discriminatory tactic employed by agents.[3]

So, discrimination amounts to systematically not offering available dwelling units to members of a certain group.

3 Consumer choice

Modeling the consumer choice with reference to Lancaster (1966, 1971 and 1991) is based on the assumption that households do not directly derive utility from the consumption of goods, but from bundles of objective properties associated with goods, here dwelling units. Lancaster uses the term characteristics for these properties. Dwelling unit j ($j = 1, \ldots, n$) possesses b_{ij} ($i = 1, \ldots, r$) units of characteristic i. The quantities of the i-th characteristic and the j-th dwelling X_j is denoted with z_i and x_j respectively. The amount of the r characteristics for a given bundle of n dwellings is $z_i = \sum_{j=1}^{n} b_{ij} x_j$ for any $i \in \{1 \ldots r\}$ [4]; in matrix notation $z = Bx$, where z is the r-dimensional vector of characteristics and x is the n-dimensional vector of dwellings. The $r \times n$ matrix B is the consumption technology matrix that maps from goods space into characteristics space.

As x_i is the fraction of the i-th dwelling that is consumed by a household, we want to impose an additional restriction on the total quantity of dwellings consumed: $\sum_{j=1}^{n} x_j = 1$.

So preferences refer to characteristics, whereas the budget constraint is on dwellings. $G = \{x | px \leq k, x \geq 0, \vec{1}x = 1\}$ is the budget set in goods (dwellings) space, where p denotes the n-dimensional vector of dwelling prices [5] and k is the total expenditure a household decided to spend on housing. The budget set's image in characteristics space is $K = \{z | z = Bx, px \leq k, x \geq 0, \vec{1}x = 1\}$.

Goodman (1989) and Goodman (1988) find a strong dependence of the

[2] Courant (1978) points out, that this lower elasticity could be caused simply by a household's fear to be discriminated against.

[3] No empirical research on discrimination in housing markets is available for Germany. Narrative evidence can be found in Gude (1990), though.

[4] Linearity $z_i = b_{ij} x_j$ and additivity $z_i = b_{ij} x_j + b_{ik} x_k$ being assumed.

[5] The term price is used throughout the discussion, although rent would be more appropriate in the context of rented property.

expenditure on housing and the permanent income of a household. So it can be reasonably assumed that k is independent of the dwelling decision itself, and it is therefore exogenous to the model.

The choice is modeled in two distinct steps, reflecting the separation of the two decision spaces.

3.1 First Step

In the first step of the consumption decision the household chooses, subject to the budget constraint, its utility maximizing characteristics bundle. The optimization step is performed in characteristics space.

$$\begin{aligned} &\max u(z) \\ &\text{s.t. } z \in K \end{aligned} \tag{1}$$

Lancaster's assumptions on preferences (see Lancaster 1971, pp. 20) secure that a solution z^* to problem (1) exists.

3.2 Second Step

The first step determines which bundle of characteristics is a utility maximizing one for the household. In the second step of the decision the household chooses that combination of dwellings, which supplies z^* at minimum cost. This optimization step is performed in dwellings (goods-) space.

The household consumes the dwelling bundle x^*, that corresponds to the solution of

$$\begin{aligned} &\min J(z^*) \\ &\text{where } J(z^*) = \{\bar{p} | \bar{p} = px, Bx = z^*, px \leq k, x \geq 0, \vec{1}x = 1\} \end{aligned} \tag{2}$$

J contains all dwelling expenses $\bar{p} = px$, that make z^* attainable.

We define the expenditure correspondence $L(z^*)$ to be the set of all expenses making at least z^* attainable:

$$L(z^*) = \{\bar{p} | \bar{p} = px, z^* \leq Bx, x \geq 0, \vec{1}x = 1\} \tag{3}$$

A complete decomposition of $L(z^*)$ into disjoint subsets $L_i(z^*)$ with $i = 1 \ldots 6$ such that

$$L(z^*) = L_1(z^*) \cup L_2(z^*) \cup L_3(z^*) \cup L_4(z^*) \cup L_5(z^*) \cup L_6(z^*) \tag{4}$$

with

$$L_1(z^*) = \{\bar{p} | \bar{p} = px, z^* < Bx, px > k, x \geq 0, \vec{1}x = 1\} \cup \tag{5}$$

$$L_2(z^*) = \{\bar{p} | \bar{p} = px, z^* = Bx, px > k, x \geq 0, \vec{1}x = 1\} \cup \tag{6}$$

$$L_3(z^*) = \{\bar{p} | \bar{p} = px, z^* < Bx, px = k, x \geq 0, \vec{1}x = 1\} \cup \tag{7}$$

$$
\begin{aligned}
L_4(z^*) &= \{\bar{p}|\bar{p} = px, z^* = Bx, px = k, x \geq 0, \vec{1}x = 1\} \cup & (8)\\
L_5(z^*) &= \{\bar{p}|\bar{p} = px, z^* < Bx, px < k, x \geq 0, \vec{1}x = 1\} \cup & (9)\\
L_6(z^*) &= \{\bar{p}|\bar{p} = px, z^* = Bx, px < k, x \geq 0, \vec{1}x = 1\} & (10)
\end{aligned}
$$

reveals the following properties of the subsets:

1. $\bar{p} > k$ for all $\bar{p} \in L_1(z^*) \cup L_2(z^*)$, which means that they are not attainable.

2. $L_3(z^*) \cup L_5(z^*)$ are empty. Suppose, there exists a $\bar{p}'$ with $\bar{p}' \in L_3(z^*) \cup L_5(z^*)$, then there exists a collection of dwellings x' which maps into a characteristics bundle z', such that $z' = Bx$, $z' > z^*$, and $px' \leq k$, $x' \geq 0, \vec{1}x = 1$. So z' is an element of K in characteristics space. This leads to a contradiction with z^* still being the solution of (1).

3. $J = L_4(z^*) \cup L_6(z^*)$ and $\bar{p} \leq k$ for $\bar{p} \in J$.

Hence, the minimum of $J(z^*)$ is contained in $L_4(z^*)$ or in $L_6(z^*)$ and x^* that corresponds to the solution of (2) also corresponds to the solution of:

$$
\begin{gathered}
\min L(z^*) \\
\text{with } L(z^*) \text{ being defined in (3)} \qquad (11)
\end{gathered}
$$

From all dwelling collections supplying at least z^*, the household chooses that one, which minimizes the housing expenditure. The budget constraint is obeyed by virtue of problem (1).

4 Inefficiencies in Consumption

4.1 Sources of inefficiency

The household's decision crucially depends on the consumption technology a household regards as binding. The decision of the household is efficient with respect to this consumption technology. A bundle of characteristics is efficient, if there exists no other bundle, attainable at a less or equal price, that possesses more of some characteristic and not less of others. A collection of dwellings is efficient, if it supplies an efficient characteristic bundle at minimum cost.

If any household faced an identical consumption technology no inefficient collections are traded on the market.

But as information on available dwellings may vary across households consumption technologies that housholds face may differ. The landlords' differential behavior is the only source for differing information concerning the consumption technology. We assume that the severity of the dwelling decision induces the household to process the available information rationally.[6]

[6]For home buyers Turnbull and Sirmans (1993) could not find differences in perfomance between first-time versus repeat buyers and between out-of-town versus in-town buyers, which supports our assumption.

Although the household's decision may be efficient from the individual technology's point of view, it may be inefficient under a global perspective.

Concerning the inefficiencies we can distinguish the following four cases:

1. We may find dwelling units, that comply with the above definition of efficiency. They supply an efficient bundle of characteristics at minimum cost.

2. If we find a household consuming a dwelling unit X_m[7] at a price p_m larger than the price p_l of another dwelling unit X_l that supplies the same characteristics $z_m = z_l$, we can conclude that X_l could not have been offered to the household. Suppose both X_m and X_l are offered to the household. Both p_l and p_m are in the expenditure correspondence set $L(z_l) = L(z_m) = \{\bar{p}|\bar{p} = px, z_m \leq Bx, x \geq 0, \vec{1}x = 1\}$. As $p_l < p_m$, p_m cannot be the solution to the problem (11).

   The consequence is that compared to some other unit X_l dwelling X_m is too expensive. The source of the inefficiency is found in step two of the consumption decision.

3. Now suppose a dwelling unit X_l supplies a characteristics bundle z_l that is larger in at least one component and equal in all others than the characteristics bundle z_m of another dwelling X_m. Both dwellings are available at the same price $p_l = p_m$ and therefore $z_m \in K$ and $z_l \in K$ with $K = \{z|z = Bx, px \leq k, x \geq 0, \vec{1}x = 1\}$, where k is the total expenditure the household consuming X_m decided to spend on housing. Clearly $p_m \leq k$ and $p_l \leq k$. As z_l offers more of at least one characteristic than z_m and all partial derivatives of $u(z)$ are everywhere positive, it holds that $u(z_l) > u(z_m)$ and therefore z_m cannot be the solution to (1).

   The n-vector of the consumed quantities $x^l = (x_1, \ldots, x_m, \ldots, x_n) = (0, \ldots, 0, 1, 0, \ldots, 0)$ associated with the consumption of dwelling X_l is not in the consumption technology that the household consuming X_m regards as binding for itself. Thus X_l is not offered to the household and the source of the inefficiency is found in the first step of the consumption decision.

   Relative to the reference unit X_l the unit X_m leads to a loss of utility.

4. A dwelling X_m with $p_m > p_l$ offers less of at least one characteristic and not more of all others compared to a dwelling X_l. This dwelling X_l could not have been available for the household renting X_m. The reasoning of the above cases 2. and 3. applies simultaneously.

   Dwelling unit X_m is too expensive relative to the unit X_l. Furthermore, the household consuming X_m faces a loss of utility compared to the consumption of X_l.

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[7]Note that X_m denotes the m-th dwelling unit. The associated vector of the consumed quantities is $x^m = (x_1, \ldots, x_m, \ldots, x_n) = (0, \ldots, 1, \ldots, 0)$.

In the cases 2. to 4. we assumed implicitly that the unit of comparison denoted a single dwelling unit X_l. The comparisons hold true for a collection of dwellings which obeys the constraint $\vec{1}x = 1$. The unit of comparison can be a convex combination of real dwelling units.

One might doubt that such a compound dwelling serves as an appropriate unit for comparison. It can be argued, however, assuming equal distribution of power between a potential tenant and a landlord, a bargaining process can be induced by one of the parties to reduce the rental price p_m to a level where efficiency substitution comes into effect.[8] In this context a compound dwelling unit indicates that a point exists, where efficiency substitution comes into effect. This compound dwelling unit is a convex combination of existing dwelling units, that is not necessarily existing. So we can speak of a virtual dwelling unit that can be created by a sufficient reduction of the price of the unit Xm.

4.2 Detecting inefficiency

We assume all dwelling units, traded on a local market at time of the household decision, to be the consumption technology, a household ideally could have choosen from. So the (piecewise linear) expenditure correspondence

$$L(z) = \{\bar{p} | \bar{p} = px, z \leq Bx, x \geq 0, \vec{1}x = 1\} \tag{12}$$

where B incorporates all traded dwelling units, describes the (consumption-) technological relationship between the expenditure $\bar{p}$ and the characteristics bundle z. As noted above the expenditure correspondence contains all expenditures that allow to attain at least the characteristics bundle z. Relative inefficiencies will be measured against the reference technology represented by $L(z)$. Farell (1957) defines an input measure of technical efficiency by

$$TE(\bar{p}, z) = \min\{\theta | \theta \cdot \bar{p} \in L(z)\} \tag{13}$$

where $L(z)$ is given by equation (12). This measure of efficiency can be calculated for any dwelling decision $(\bar{p}_0, z_0)$ by the Banker, Charnes and Cooper (1984) DEA model.

$$\min \qquad \theta_0 - \varepsilon \vec{1} s_0^+ \tag{14}$$

subject to

$$\begin{aligned} Bx - s_0^+ &= z_0 \\ \theta_0 \bar{p}_0 - px &= 0 \\ \vec{1}x &= 1 \\ x, s_0^+ &\geq 0 \end{aligned}$$

The subscript '0' denotes variables associated with the analyzed dwelling unit.

Note the following points:

[8] Efficiency substitution simply means, that a sufficient reduction in price can cause efficiency for any good. Households will substitute into that commodity as soon as it is efficient.

1. In the terminology of a production problem here we have a one-input multi-output problem. The rent p can be interpreted as an one-dimensional input that generates an r-dimensional output of z.

2. As there is only one input variable, it follows that any excess input is captured by the proportional reduction of the rental price. Hence any excess input, typically denoted by s^-, is excluded from the model.

3. By virtue of the employed model, the input is measured in monetary units exclusively. One could argue that the quality of the dwelling decision strongly depends on real inputs, like e.g. time used for search. As Lancaster's simple model does not allow for real inputs we want to confine our analysis to the monetary input only. An analysis that includes the time used for search might be possible under a two stage model which, in contrast to the simple model, accounts for several activities.

In order to get a one-dimensional measure of total inefficiency under the presence of positive slacks s^+, Ali and Lerme (1990) introduce the efficiency measure ι that incorporates both proportional reduction and slacks.

Thus a dwelling unit is efficient if and only if $\theta = 1$ and $s^+ = \vec{0}$, or equivalently $\iota = 1$. Consequently, a reduction of the price p_0 to the level of θ and removal of the slacks s^+ can achieve efficiency for the analyzed dwelling.

Under the presence of positive slacks s^+ this way of achieving efficiency is quite different compared to the above mentioned reduction of the price to induce efficiency substitution. Until the price reaches the level θ relative to the initial level, both procedures are equal. Once this level is reached, efficiency substitution calls for further reduction of the price to make the unit efficient, whereas the procedure here calls for a removal of slacks. Furthermore the latter procedure accepts the current frontier as it is and moves the unit onto it, whereas any further reduction of the price generates a new frontier.[9]

4.3 Efficiency scores and consequences for the household

As discussed in section 4.1, four categories of efficiency can be distinguished. It can be seen from Table 1, how these categories relate to the efficiency scores θ and ι .

5 Data

For the analysis we use data from the housing market in Lechhausen, which is a municipal district of Augsburg, Germany. Data are taken from the dataset of

[9] Although this is to be considered rather problematic in the case of a production frontier (because we do not know these techniques), in our case a further reduction of the 'input' poses no difficulty in interpretation.

Table 1: Efficiency scores and the consequences

θ	ι	slacks	consequences compared to reference unit	case
$\theta = 1$	$\iota = 1$	no slacks	efficient	1.
$\theta = 1$	$\iota < 1$	slacks	inefficient: loss of utility	3.
$\theta < 1$	$\iota = \theta$	no slacks	inefficient: expenditure to high;	2.
$\theta < 1$	$\iota < \theta$	slacks	inefficient: expendiure to high; loss of utility	4.

the 1987 census.[10] The data record the monthly rent and the dwelling characteristics as well as personal characteristics of the household head. Also the data set indicates the year, the household moved into the dwelling. We restrict our analysis on dwelling units where moves occured in 1987. As the target date for the census was May 25th 1987, those flats must have been traded in the first half of the year. So the requirement is met, that the dwellings, which compose the reference consumption technology, must have been traded at approximately the same time.

The data set records 420 households, that moved in 1987. By close inspection of the data set two dwellings were identified containing implausible data. Those records were excluded from the data set, leaving a total of 418 dwellings for the analysis.

5.1 Variables in the analysis

The variables included in the analysis are displayed in table 2. Dwelling characteristics were available both in cardinal and ordinal scale.

`SIZE` and `ROOMS` represent the cardinal characteristics, whereas the ordinal quality of the dwelling is captured by `YEARBLT`, `BATH`, `TOILET`, `HEATING`, `FUEL` and `KITCHEN`. For the incorporation of `YEARBLT` we assume that the quality of a dwelling depends on its vintage; quality decreases with age. The composition of the intervals reflects the structure of the data recorded in the dataset. For `HEATING` we assume that the quality of a heating facility decreases with the care required by the household. A stove in every room (`HEATING`=1) requires more care by the household than central heating, that serves only the analyzed dwelling (`HEATING`=2). This, however requires more care than central heating (`HEATING`=3) . And so forth. Ranking the `FUEL`, we account for convenience of the fuel supply only if the heating consists of stoves or central heating for one appartment only. Coal (`FUEL`=0) has to be supplied manually by the household, whereas oil (`FUEL`=1) and gas (`FUEL`=2) are supplied automatically. Concerning

[10] We thank the Stadt Augsburg, especially the Amt für Statistik und Stadtentwicklung for kindly making the data available.

Table 2: Variables for DEA

Variable	value	label
RENT		Rent for the dwelling unit, including utilities, in DM
SIZE		Size of the dwelling unit in m^2
ROOMS		Number of rooms larger than $6m^2$
YEARBLT		Year the dwelling unit was built
	1=	before 1949
	2=	1949-1962
	3=	1963-1969
	4=	1970-1979
	5=	1980-1987
BATH	1=	Dwelling has a bathroom
	0=	else
TOILET	1=	Dwelling has a separate toilet
	0=	else
HEATING		Type of heating
	1=	Stove
	2=	Central heating serving one dwelling unit
	3=	Central heating
	4=	District heating
FUEL		Fuel for heating
	0=	Coal
	1=	Oil
	2=	Gas
	3=	Electricity
	4=	District or central heating
KITCHEN	2=	Kitchen in dwelling
	1=	Dwelling contains a kitchenette
	0=	else

the smell caused by the fuel, gas is less bothering. Electricity (FUEL=3) is the most convenient for the household.

The above assumptions are in accordance with the common perception of housing quality (Häussermann and Siebel, 1996).

Unfortunately, the data lacks any information on other characteristics that may be relevant to the household's decision; such as information on the layout of the dwelling, available balcony or terrace, carpeting, etc.

5.2 Ordinal data and DEA

For an appropriate description of the dwelling characteristics we have to include both cardinal and ordinal variables in the data envelopment analysis.

Various suggestions have been made to incorporate ordinal data into the CCR model (Cook, Kress and Seiford, 1996 and 1993) and into the BCC model (Banker and Morey 1986, Kamakura 1988, Rousseau and Semple 1993). For an input oriented model all the proposed procedures handle either ordinal input or both ordinal input and output variables. In these models for any of the t ordinal variables a q-dimensional vector of dichotomous dummy variables is added, where q denotes the number of values an ordinal variable can take. So, the number of variables in the analysis is enhanced by $t \cdot (q-1)$ variables. Every rank position for any ordinal variable is treated as a seperate input or output, respectively. Vassiloglu and Giorkas (1990) suggest that there be twice as much units as there are variables. Therefore the increased number of variables may cause problems when the number of analyzed units is small (see Table 3).

Our setup shows the convenient property, that we have ordinal variables only on the output side and that we have only one input variable in an input oriented model. This allows us to handle the ordinal output variables as if they were cardinal. Only the interpretation of the results has to adopt accordingly.

In equation (15) z denotes the vector of cardinal characteristics and B the consumption technology matrix. The ordinal characteristics' vector is z_t and the consumption technology matrix of the ordinal factors is B_t. The categories are coded by integer values, where a larger value means a preferred category (see section 5.1). s^+ and s_t^+ stand for the cardinal slacks and the ordinal slacks. The problem then reads:

$$\min \qquad \theta_0 - \varepsilon \vec{1} s_0^+ - \varepsilon \vec{1} s_{t0}^+ \tag{15}$$

subject to

$$\begin{aligned} Bx - s_0^+ &= z_0 \\ B_t x - s_{t0}^+ &= z_{t0} \\ \theta_0 \bar{p}_0 - px &= 0 \\ \vec{1}x &= 1 \\ x, s_0^+, s_{t0}^+ &\geq 0 \end{aligned}$$

The inclusion of ordinal variables causes – in contrast to problem (14) – a formulation basically calling for three steps to achieve efficiency.

1. It still holds true that the price has to be reduced to θ of the initial level as a first step for a unit to become efficient.

2. If positive slacks s^+ of the cardinal variables are recorded, these slacks have to be removed. Whether this can be achieved, is only a question of divisibility of the characteristics.[11]

3. Under the presence of positive slacks s_t^+ for the ordinal characteristics, it makes no sense to call for a removal of the slacks, if it yields a non-integer value, that has no meaning in the realm of an ordinal scale. Rather it follows that if a positive slack in the ordinal variable occurs, at least one unit of the composed reference unit contains a better value of this variable. To achieve efficiency for the unit in question, additionally to the removal of the cardinal slacks one has to lower the price of the unit further than θ.

6 Results

As there is legal regulation on rents for dwelling units, which were built with public subsidies, we have to perform the DEA on two subgoups: publicly subsidized and freely financed dwelling units.

Locational effects such as neighbourhood quality etc. influence the rent. We control for these effects by partitioning our analysis due to three census tracts, that compose the area of the analyzed locality, assuming that those locational effects do not vary tremendously within the tracts. So, publicly subsidized (freely financed) dwelling units from one census tract were only compared to publicly subsidized (freely financed) units from the same area.

Table 3 shows the distribution of the 418 dwelling units.

[11] It is a different issue, whether it is generally possible to remove slacks in the context of housing characteristics, that are actually seen as being fixed in the short run.

Table 3: Distribution of dwellings

Census tract	Publicly subsidized no	yes
no. 25	120	19
no. 26	105	25
no. 27	107	42

6.1 Results of the DEA computation

Model (15) has been solved for the 418 dwelling units in the six partitions. In a first step we want to discuss the results of the models. For the discussion we do not distinguish between the six partitions, rather we concentrate on the whole 418 dwelling units.

The mean for the efficiency scores θ is 0.8119 and for ι it is 0.8054, the standard deviation is 0.20. The minimum result for θ is 0.2700 and for ι it is 0.2697. As displayed in table 4 about 32.1% of all 418 analyzed units, that is 134 units, were scored $\theta = 1$; 30.9% were rendered efficient with $\iota = 1$.

Table 4: Efficiency scores and inefficiency categories

θ	ι	slacks	number of dwellings	percentage
$\theta = 1$	$\iota = 1$	no slacks	129	30.9
$\theta = 1$	$\iota < 1$	slacks	5	1.2
$\theta < 1$	$\iota = \theta$	no slacks	32	7.7
$\theta < 1$	$\iota < \theta$	slacks	252	60.3

The distribution of the efficiency scores θ and ι are displayed in Table 5.

Observing the distribution of the slacks for the ordinal variables in Table 6 one can assert that most dwelling units were compared to reference units of approximately the same ordinal quality. Especially, concerning `YEARBLT`, more than 70% of all units were compared to reference units built in the same decade, more than 90% did not differ more than one decade. Over 90% of the units were compared to units of an equal provision of toilet, bathroom and kitchen and more than 70% were compared to units of equivalent heating facilities.

About 38.5% of all units have no positive component in their vector of slacks. This means 161 units showed a θ efficiency score that was equal to the

[12] This means for expample, that 10 percent of all dwelling units have a θ smaller than 0.478 and that 20 percent of all dwellings have a θ smaller 0.625.

Table 5: Distribution of DEA scores

	Value	
Percentile[12]	θ	ι
10.00	0.478	0.474
20.00	0.625	0.615
30.00	0.728	0.714
40.00	0.808	0.799
50.00	0.879	0.874
60.00	0.936	0.926
70.00	1.000	1.000
80.00	1.000	1.000
90.00	1.000	1.000

Table 6: Distribution of ordinal slacks

	value of the cardinal slacks		value of the ordinal slacks					
Percentile	SIZE	ROOMS	YEARBLT	BATH	TOILET	HEATING	FUEL	KITCHEN
10.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
30.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
40.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
60.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
70.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
80.00	1.000	0.172	0.384	0.000	0.000	0.202	0.676	0.000
90.00	7.000	0.500	1.003	0.000	0.000	0.741	1.330	0.000
Mean	1.59	0.12	0.27	0.01	0.01	0.16	0.31	0.07

ι score. Only three units revealed a total of five variables with positive slacks relative to the reference unit (see Table 7).

Table 7: Number of positive components in slack vector

Number of pos. components	Frequency	Percent
0	161	38.5
1	92	22.0
2	76	18.2
3	69	16.5
4	17	4.1
5	3	0.7
	418	100.0

6.2 Explaining the inefficiency

Discrimination was defined in section 1 as systematic differential treatment. Due to the decision model, the computed inefficiencies were caused by landlords, who withheld dwelling units from certain households.

Certainly, a household would never have the opportunity to choose from the whole range of appartments represented by the consumption technology. Additionally, certain inefficiencies on housing market are inevitable, as the landlords cannot adapt to shortrun changes in demand. But, on a housing market where there is no discriminatory behavior, one would expect to find the inefficiencies being equally distributed among the various household types.

Now, we want to test, whether the differential treatment systematically depends on personal characteristics of the households. If we find a statistically significant relationship, we can reject the hypothesis, that the inefficiencies and therefore the treatment is independent of the personal household characteristics.

One way of finding such a systematic relationship is to interpret the efficiency scores as the dependent variable that is determined by independent personal household characteristics. Let θ_q denote the efficiency score of a dwelling, inhabited by household q, $(q = 1, \ldots, n)$, and the type of the household can be described by the v-dimensional vector of personal characteristics y_q.

A general regression model can be set up as:

$$\theta_q = f(y_q, \beta) + e_q, \qquad q = 1, \ldots, n \tag{16}$$

where e_q denotes an error term and β denotes the parameter vector to be estimated. For estimation, a priori knowledge about the functional form of $f(.)$ has to be supplied. Often a linear relationship is assumed. Analogously the ι score could be regressed.

Certain properties of the efficiency scores computed by DEA models like (15) pose serious problems on linear OLS regression models (Holvad and Hougaard 1993; Lovell, Walters and Wood 1994).

We want to impose a transformation on the efficiency scores as to reflect the efficiency categories discussed above.

$$\bar{\theta}_q = \begin{cases} 1 & \text{if } \theta_q = 1 \\ 0 & \text{else} \end{cases} \tag{17}$$

$$\bar{\iota}_q = \begin{cases} 1 & \text{if } \iota_q = \theta_q \\ 0 & \text{else} \end{cases} \tag{18}$$

So $\bar{\theta}_q$ takes the value 1, if no reduction of the price is necessary to establish efficiency for dwelling x_q, whereas $\bar{\iota}_q$ takes the value 0, if slacks have to be removed in order to achieve efficiency for the unit. Hence, all four efficiency categories can be represented by the two transformed variables. How the variables relate to the efficiency categories can be seen from Table 8.

Table 8: Transformed efficiency scores and efficiency categories

	θ	
$\bar{\iota}$	=1	=0
=1	1.	2.
=0	3.	4.

As the dependent variables are dichotomous variables after the transformation, we apply a logistic regression instead of the linear OLS regression.[13]

The logistic regression model can be formulated as:

$$\Pr(\bar{\theta} = 1|y) = F(y\beta) \tag{19}$$

$$F(s) = \frac{1}{1 + e^{-s}} \tag{20}$$

where y is the $v+1$-dimensional vector of personal household characteristics, that includes $y_0 = 1$ to allow for a constant parameter β_0. Therefore

$$s = \beta_0 + \beta_1 y_1 + \beta_2 y_2 + \ldots + \beta_v y_v \tag{21}$$

Analogous to equatios (19) and (20) a regression model can be established for the second efficiency score $\bar{\iota}$:

$$\Pr(\bar{\iota} = 1|y) = F(y\beta) \tag{22}$$

[13] For a discussion of logsistic regessions see e.g. van Houwelingen and le Cessie (1988).

$$F(s) = \frac{1}{1+e^{-s}} \tag{23}$$

The logistic regression in (19) and (20) estimates the probability of $\bar{\theta}_q = 1$ and in (22) and (23) it estimates that of $\bar{\iota}_q = 1$ dependent on the personal characteristics y_q of household q.

The personal household characteristics that we include in the logistic regression are household size, presence of dependent children in the household, and nationality, gender, age, educational level[14] of the household head, as well as information on whether or not the household was a single parent household (see Table 9).

Additionally, a variable has been included to indicate public subsidies for the dwelling. As can be seen from Table 3, the number of units analyzed in any partition of the sector of freely financed dwellings is significantly higher than in the sector of publicly subsidized dwelling units. Therefore, the proportion of efficient units is significantly higher among subsidized dwellings. The variable `PUBLSUB` is included to capture indirect effects caused by the differing proportions of efficient units.

If gender discrimination existed, one would expect the variables that indicating the gender of the household head to have a negative sign, significantly different from zero. If discrimination of Non-German headed households existed, one would anticipate a positive sign for the nationality variable. A negative sign for the age variables would also indicate discrimination of the corresponding groups.

6.3 Result of the logistic regression

6.3.1 $\bar{\theta}$ as the dependent variable

Table 10 reports the regression results explaining the transformed DEA efficiency score $\bar{\theta}$. The regression of the computed and transformed DEA scores against ten variables, characterizing the household, succeeds in explaining a significant portion of the inefficiency.

To test for $\beta_i = 0$, we employ the Wald statistic, which is χ^2-distributed under the null-hypothesis. The corresponding degrees of freedom are displayed in column 'df' and the level of significance in column 'Sig.'.[15]

The variable indicating a household head, younger than 30 years, is negative and significant at $\alpha = 0.05$. The education variable as well as the nationality variable are positive and significant, also at the $\alpha = 0.05$ level.

[14] Unfortunately the data set did not supply any data on the household's income. We tried to include data on the occupation of the household head as a proxy for income. But as about 25% of this data was 'missing value', we included a variable indicating the educational level of the household, as a rough proxy for the occupation and therefore for income situation of the household.

[15] For one degree of freedom, the Wald statistic is the squared quotient of the estimated parameter and its standard error.

Table 9: Variables for the logistic regression

variable	value		meaning
AGE_L_30	1	=	Household head younger than 30 years
	0	=	else
AGE_G_50	1	=	Houshold head older than 50
	0	=	else
EDUC_HI	1	=	High educational level, household head finished middle school or grammar school
	0	=	else
CPLNOKI	1	=	Couple, no dependent children
	0	=	else
CPLWIKI	1	=	Couple with dependent children
	0	=	else
FEMNOKI	1	=	Single woman, no dependent children
	0	=	else
FEMWIKI	1	=	Single woman with dependent children
	0	=	else
MALENOKI	1	=	Single man, no dependent children
	0	=	else
NATIONAL	1	=	Household head is of german nationality
	0	=	else
NUPEOP	1-9	=	Number of people in household
PUBLSUB	1	=	Dwelling was publicly subsidized
	0	=	else
THETAEFF			Transformed efficiency score $\bar{\theta}$
IOTAEFF			Transformed efficiency score $\bar{\iota}$

Table 10: Results of the logistic regression (dependent variable: THETAEFF)

Variable	β	S.E	Wald	df	Sig.
AGE_L_30	-0.5801	0.2610	4.9410	1	0.0262
AGE_G_50	-0.4093	0,3744	1.1955	1	0.2742
EDUC_HI	0.6111	0.2377	6.6105	1	0.0101
CPLWIKI	-0.0631	0.4228	0.0223	1	0.8813
FEMNOKI	-0.1631	0.3520	0.2146	1	0.6432
FEMWIKI	-0.3201	0.6078	0.2774	1	0.5984
MALENOKI	0.3121	0.3342	0.8722	1	0.3503
NATIONAL	0.8312	0.3629	5.2453	1	0.0220
NUPEOP	0.2221	0.1537	2.0871	1	0.1485
PUBLSUB	0.8484	0.2773	9.3587	1	0.0022
Constant	-1.9489	0.5670	11.8158	1	0.0006

Concerning the household age, this result means that households with a household head under 30, are more likely to pay more for their dwelling units than others. Taking the educational variable as a proxy for the income, one can assert, that with increasing income the probability of an efficient decision rises. With respect to the nationality of the household head we can conclude that German headed households are more likely to come to an overall efficient decision than others. So households with heads of other than German nationality have a greater probability of renting too expensive.

As expected, the variable for public subsidies is positive and highly sigificant.

Taking a closer look at the results for the nationality variable rises the question, whether the obtained results are artificially created by neglection of the magnitude of the inefficiency through the transformation in equation (17). So we want to further examine the results of the data envelopment computations in two directions.

First, we observe the distribution of the original efficiency scores θ for both groups of households; German headed households and others. If the distribution of the latter is shifted left relative to the German headed households, we can conclude that Non-German households tend to have a smaller θ than German headed ones. Since it can not be reasonably assumed, that the efficiency scores show a normal distribution, instead of a t-test we employ the non-parametric Wilcoxon rank sum test to test for identity of the distributions. The test shows that the distribution of θ in the group NATIONAL=0 is significantly ($\alpha = 0.05$) shifted left relative to the distribution in the group NATIONAL=1 (Wilcoxon rank sum W=10724; Z=-2.1676).[16]

Second, we want to analyze the composition of the reference units.

[16] Qualitatively, the same holds true for the distribution of θ in the groups indicated by the age variable and the education variable. Not significantly for the education variable, though.

About 89% of the efficient units are inhabited by German headed households. If all inefficient units were placed randomly under the frontier, one would expect to find the same proportion of German headed households in the reference sets.

Let the reference index set $R(i)$ contain all indexes of the dwelling units in the reference set of unit i. Then the mean weighted[17] sum of German households in the reference set Γ is

$$\Gamma = \frac{\sum_{i \in D} \sum_{j \in R(i)} x_j \cdot y_{mj}}{|D|} \tag{24}$$

where y_{mj} is the characteristic of household j, that is 1 for a German household head and 0 otherwise; D contains the indexes of all units, that are chosen for aggregation and $|D|$ denotes the number of elements in D.

If D contains all 284 dwelling units, whose θ-score is less than one, then Γ is 0.9404, which significantly differs (with $\alpha = 0.005$) from the expected value of 0.89. So we find more German headed households in the reference sets than expected. Thus German headed households tend to dominate the units more often. With D constisting of the indexes of only the German headed households the underrepresentation of Non-German households in the reference sets is revealed by Γ being 0.9397. $\Gamma = 0.9441$ is obtained by only taking account of the Non-German households.

The interpretation of this result is, that the reference sets of the units with $\theta < 1$ consists of about 94% of German headed households and of about 6% of Non-German headed households. This shows that inefficiency is more severe for Non-German headed households than the sheer percentage of 11% of efficient Non-German households relative to 14.4% of Non-German households in the sample might suggest. [18]

6.3.2 $\bar{\iota}$ as the dependent variable

The overall results, displayed in Table 11, does not differ tremendously from the results obtained by regressing $\bar{\theta}$. Only the variable indicating the age under 30 looses significance. This result is an expected one because as Table 6 and Table 7 show, slacks are not a major determinant for the overall inefficiency ι.

A striking result of this regression is that the household size variable turns out to be positive and significant. But a positive correlation coefficient of the number of rooms and the size of the household shows that, not surprisingly, larger families tend to rent larger dwelling units. So the positive sign of the parameter estimated for the household size variable can be due to the fact that there are less large dwellings relative to small dwellings. Hence the proportion of efficient (ι=1) dwellings is larger among large units.

[17] As weights we use the x_j.

[18] The distribution of the characteristics of the units with $\theta = 1$, shows, that the units are quite homogenously distributed on the frontier. So large facets spanned by only a few units, are not the cause for this result.

Table 11: Results of the logistic regression (dependent variable: IOTAEFF)

Variable	β	S.E	Wald	df	Sig.
AGE_L_30	-0.3546	0.2574	1.8982	1	0.1683
AGE_G_50	-0.0206	0,3663	1.0032	1	0.9551
EDUC_HI	0.6881	0.2321	8.7911	1	0.0030
CPLWIKI	-0.0968	0.4327	0.0501	1	0.8230
FEMNOKI	0.2307	0.3548	0.4229	1	0.5155
FEMWIKI	-6686	0.6129	1.1902	1	0.2753
MALENOKI	0.4122	0.3432	1.4430	1	0.2296
NATIONAL	0.8243	0.3515	5.4995	1	0.0190
NUPEOP	0.4519	0.1778	6.4570	1	0.0111
PUBLSUB	0.9052	0.2777	10.6262	1	0.0011
Constant	-2.3874	0.5951	16.0922	1	0.0001

We excluded all dwellings larger than three rooms and ran the regression again. The results basically confirm the findings obtained for the whole sample in sign and magnitude of the estimated parameters, not sigificantly for the household size, though.

7 Conclusion

In the previous sections we argued that inefficiencies in consumption of dwelling units can be induced by landlords' behavior. We identified inefficiencies by employing a data envelopment analysis on housing market data.

Section 6.3.1 and section 6.3.2 showed that personal household characteristics have a certain explanatory power concerning inefficiencies. So we can reject the hypothesis that inefficiency is not systematically influenced by personal household characteristics. A large part of inefficiency can be attributed to personal household characteristics and so can differential treatment, causing the inefficiency. Hence, we can conclude that the differential treatment is discriminatory in the above sense.

Households with a Non-German, low-income or young household head were found to be subject to discriminatory differential treatment by landlords. Further, we can assert that the consequences of discriminatory behaviour are higher rents for the discriminated group rather than a direct loss of utility. Discriminatory behaviour is more likely to influence the second step of the Lancasterian consumption decision than the first step.

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