Analytical Modelling of Human Arm Locomotion during Dance as a Dynamical Multiple Pendulum System

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Abstract— **During dance human arms move. The movement of the arms, to a large extent, determines how good a dance is. This paper attempts to model the locomotion of human arm, analytically, during a dance as a dynamical multiple pendulum system with three segments. The system is analyzed by considering some instances during the dance. The three angles the three segments make with the vertical are considered. The Euler-Lagrange equation is used in obtaining the equation of Motion. The resulted second order differential equation is solved analytically. Simulation results are presented. They are consistent with the ones in literature. Specifically the angular displacement values of the three segments are directly proportional to their respective angular acceleration. However, the novelty of this work is in the full analytical approach to 3-pendulum system and its application to dance.**
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Index Terms— Mathematical Modelling, Human Arm, Multiple pendulum, Euler-Lagrange, Analytical..

I. INTRODUCTION

Proper functioning of human arms results in a good dance. Human body movement modelling and analysis using mechanics and other Mathematical concepts is constantly expanding and becoming very important in human performance and rehabilitation studies [1]. Biomechanics of rhythmic movements and cycles are -to a large extent- the application of Newtonian mechanics to the physiology and neuromuscular skeletal systems [1].

As far as study of the actions like dancing is helpful to investigate the arm locomotion which is one of the most complicated motions of a human body [2,3]. The human dancing is characterized by excellent efficiency, stability and neural muscular control [3]. Dancing is a complex task with some branches of biomechanical minors that must be successfully performed including body support, forward propulsion, and arm swing. [4, 5]. An ordinary pendulum is one with the pivot at the top and the mass at the bottom[5,6,7]. A multiple pendulum can be referred to as a combination of several simple pendulums. The Lagrangian [6] is a mathematical function of the generalized coordinates, their time derivatives, and time, and contains the information about the dynamics of the system. Lagrangian mechanics is ideal for systems with conservative forces and for bypassing constraint forces in any coordinate system. [6] Dissipative and driven forces can be accounted for by splitting the external forces into a sum of potential and non-potential forces.[7,8]. Human body or part of the always strives to maintain balance. So during a dance the balancing of the arm ensures good and sustained dance, at least for a considerable long period of time.

Body mechanics involves the coordinated effort of muscles, bones, and the nervous system to maintain balance, posture, and alignment during moving, transferring, and positioning a body. This occurs during dance. Proper body mechanics allows individuals to carry out activities, such as dance, without excessive use of energy, and helps prevent injuries for dancers. During dance, a balance of the body is maintained through body mechanics. When a vertical line falls from the Centre of gravity through the wide base of support, body balance is achieved; otherwise the body will lose balance. Balance in this sense means an ability to maintain the line of gravity of a body within the base of support with minimal postural sway. The dynamic analysis of the different segments of the human arm during dance and the results obtained in this paper will go a long way in addressing the stability required in a dance.

Fig. 1 is the schematic of triple pendulum, representing the three segments of the human arm.

Manuscript received March 30,, 2017; revised April 10, 2017. This work was supported in full by Covenant University. Dynamic Modeling and Analysis of Inverted Pendulum using Lagrangian-Differential Transform Method

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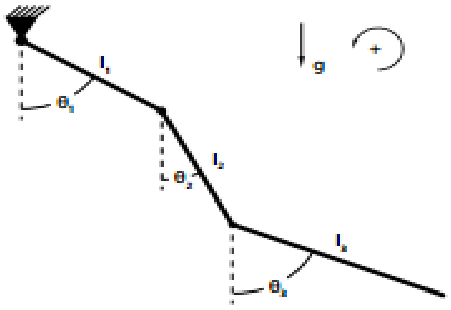

Fig.1. Schematic of Triple pendulum

II MODELING HUMAN ARM

For the purpose of modelling, the arm can be simulated as three links; where upper arm is considered as the first link that is jointed with elbow. Also, the lower part of the arm is assumed as the second link that is connected to the wrist. The palm is considered as the third link. To simply model the human arm the upper-link rotation angle from a vertical position is denoted by θ_1 and the corresponding rotation angle for the lower-links by θ_2 and θ_3 respectively. The length of link between shoulder and elbow joint is shown by l_1 and length of link between elbow and wrist joint is shown by l_2.[8] Friction and other dissipating forces are assumed negligible. Figure 1 shows a typical human arm in its downward position during a dance. Figure 2, on the other hand, shows the schematic of triple pendulum representing the three segments of human arm considered in this paper. The modelling of the dynamical system results in second order differential equation.[9,10,11]

A Mathematical Model Formulation

A n – pendulum, in this case n = 3, consists of one pendulum attached to another, then to another. This is an example of dynamical system which can exhibit chaotic behaviour. Consider human arm modelled as a triple bob pendulum with masses m_1, m_2 and m_3 attached by rigid massless wire of lengths l_1, $l_2$and l_3. The angles the wire make with the vertical are represented as θ_1, θ_2 *and* θ_3 respectively. The acceleration due to gravity is g and the positions of the bobs are given respectively as:

(x_1,y_1), (x_2,y_2) and (x_3,y_3)

where

$$x_1 = l_1 sin\theta_1 \quad (1)$$

$$x_2 = l_1 sin\theta_1 + l_2 sin\theta_2 \quad (2)$$

$$x_3 = l_1 sin\theta_1 + l_2 sin\theta_2 + l_3 sin\theta_3 \quad (3)$$

$$y_1 = -l_1 \cos\theta_1 \quad (4)$$

$$y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2 \quad (5)$$

$$y_3 = -l_1 \cos\theta_1 - l_2 \cos\theta_2 - l_3 \cos\theta_3 \quad (6)$$

It is assumed in this paper that the values of θ_1, θ_2 *and* θ_3 ranges from 0 to 90 degrees. This implies that none of the segments of the human arm during the dance should make more than 90 degrees with the vertical.

The potential energy of the system is then given by

$$V = m_1 g y_1 + m_2 g y_2 + m_3 g y_3 \quad (7)$$

Substituting (4), (5) and (6) into (7) gives:

$$\begin{aligned} V &= m_1 g - (-l_1 \cos\theta_1) + m_2 g(-l_1 \cos\theta_1 - l_2 \cos\theta_2) \\ &+ m_3 g(-l_1 \cos\theta_1 - l_2 \cos\theta_2 - l_3 \cos\theta_3) \\ &= -m_1 g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2 - m_3 g l_1 \cos\theta_1 \\ &- mgl \cos\theta - m_3 g l_2 \cos\theta_2 - m_3 g l_3 \cos\theta_3 \\ &= -(m_1 + m_2 + m_3) g l_1 \cos\theta_1 - (m_2 + m_3) g l_2 \cos\theta_2 \\ &- m_3 g l_3 \cos\theta_3 \end{aligned} \quad (8)$$

and the kinetic energy is given by:

$$T = \frac{1}{2}\mathrm{m_1 v_1^2} + \frac{1}{2}\mathrm{m_2 v_2^2} + \frac{1}{2}\mathrm{m_3 v_3^2} \quad (9)$$

$$T_1 = \frac{1}{2}\mathrm{m_1}\left(\dot{x}_1^2 + \dot{y}_1^2\right) \quad (10)$$

$$T_2 = \frac{1}{2}\mathrm{m_2}\left(\dot{x}_2^2 + \dot{y}_2^2\right) \quad (11)$$

$$T_3 = \frac{1}{2}\mathrm{m_3}\left(\dot{x}_3^2 + \dot{y}_3^2\right) \quad (12)$$

$$\begin{aligned} T &= T_1 + T_2 + T_3 = \frac{1}{2}\mathrm{m_1}\left(\dot{x}_1^2 + \dot{y}_1^2\right) \\ &+ \frac{1}{2}\mathrm{m_2}\left(\dot{x}_2^2 + \dot{y}_2^2\right) + \frac{1}{2}\mathrm{m_3}\left(\dot{x}_3^2 + \dot{y}_3^2\right) \end{aligned} \quad (13)$$

Differentiating equation (1) – (6) we have:

$$\dot{x}_1 = l_1\dot{\theta}_1 \cos\theta_1 \quad (14)$$

$$\dot{x}_2 = \dot{x}_1 + l_2\dot{\theta}_2 \cos\theta_2 = l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2 \quad (15)$$

ISBN: 978-988-14048-3-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\dot{x}_3 = \dot{x}_2 + l_3\dot{\theta}_3 \cos\theta_3 = \dot{x}_1 + l_2\dot{\theta}_2 \cos\theta_1 + l_3\dot{\theta}_3 \cos\theta_3$$
$$= l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_1 + l_3\dot{\theta}_3 \cos\theta_3 \quad (16)$$
$$\dot{y}_1 = l_1\dot{\theta}_1 \sin\theta_1 \quad (17)$$
$$\dot{y}_2 = \dot{y}_1 + l_2\dot{\theta}_2 \sin\theta_2 = l_1\dot{\theta}_1 \sin\theta_1 + l_2\dot{\theta}_2 \sin\theta_2 \quad (18)$$
$$\dot{y}_3 = \dot{y}_2 + l_3\dot{\theta}_3 \sin\theta_3 = \dot{x}_1 + l_2\dot{\theta}_2 \sin\theta_2 + l_3\dot{\theta}_3 \sin\theta_3$$
$$= l_1\dot{\theta}_1 \sin\theta_1 + l_2\dot{\theta}_2 \sin\theta_2 + \text{in}\,\theta_3 \quad (19)$$

Substituting equations (14) – (19) into equation (13) we obtain its polar coordinates expression as follows:

$$T = \frac{1}{2}m_1(l_1^2\dot{\theta}_1^2\cos^2\theta_1 + l_1^2\dot{\theta}_1^2\sin^2\theta_1) + \frac{1}{2}m_2[\{l_1^2\dot{\theta}_1^2\cos^2\theta_1$$
$$+l_2^2\dot{\theta}_2^2\cos^2\theta_2 + 2(l_1\dot{\theta}_1\cos\theta_1)(l_1\dot{\theta}_1\cos\theta_1)\} + \{l_1^2\dot{\theta}_1^2\sin^2\theta_1$$
$$+l_2^2\dot{\theta}_2^2\sin^2\theta_2 + 2(l_1\dot{\theta}_1\sin\theta_1)(l_2\dot{\theta}_2\sin\theta_2)\}] + \frac{1}{2}m_3[\{l_1^2\dot{\theta}_1^2\cos^2\theta_1$$
$$+l_2^2\dot{\theta}_2^2\cos^2\theta_2 + l_3^2\dot{\theta}_3^2\cos^2\theta_3) + 2(l_1\dot{\theta}_1\cos\theta_1)(l_2\dot{\theta}_2\cos\theta_2)$$
$$+2(l_2\dot{\theta}_2\cos\theta_2)(l_3\dot{\theta}_3\cos\theta_3) + 2(l_1\dot{\theta}_1\cos\theta_1)(l_3\dot{\theta}_3\cos\theta_3)\}$$
$$+\{l_1^2\dot{\theta}_1^2\sin^2\theta_1 + l_2^2\dot{\theta}_2^2\sin^2\theta_2 + 2(l_1\dot{\theta}_1\cos\theta_1)(l_3\dot{\theta}_3\cos\theta_3)\}] \quad (20)$$

Equation (20), after factorization, can be reduced to:

$$T = \frac{1}{2}[(m_1 + m_2 + m_3)(l_1^2\dot{\theta}_1^2\cos^2\theta_1 + l_1^2\dot{\theta}_1^2\sin^2\theta_1)$$
$$+(m_2 + m_3)(l_2^2\dot{\theta}_2^2\cos^2\theta_2 + l_2^2\dot{\theta}_2^2\sin^2\theta_2)$$
$$+m_3l_3^2\dot{\theta}_3^2] + m_2l_1l_2\dot{\theta}_1\dot{\theta}_2(\sin\theta_1\sin\theta_2 + \cos\theta_2\cos\theta_1)$$
$$+m_3l_1l_2\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)$$
$$+2m_3l_1l_3\dot{\theta}_1\dot{\theta}_3(\cos\theta_1\cos\theta_3)$$
$$+m_3l_2l_3\dot{\theta}_2\dot{\theta}_3(\cos\theta_2\cos\theta_3 + \sin\theta_2\cos\theta_3) \quad (23)$$

Finally, T can be written as

$$T = \frac{1}{2}[(m_1 + m_2 + m_3)l_1^2\dot{\theta}_1^2 + (m_2 + m_3)l_2^2\dot{\theta}_2^2$$
$$+m_3l_3^2\dot{\theta}_3^2] + m_2l_1l_2\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)$$
$$+2m_3l_1l_3\dot{\theta}_1\dot{\theta}_3(\cos\theta_1\cos\theta_3)$$
$$+m_3l_2l_3\dot{\theta}_2\dot{\theta}_3(\cos\theta_2\cos\theta_3 + \sin\theta_2\cos\theta_3) \quad (24)$$

The Lagrange equation for θ_1, θ_2 and θ_3 is given as follows[1]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_i}\right) - \frac{\partial L}{\partial\theta_i} = 0 \quad (25)$$

Where L is the Lagrangian given as

$$L = T - V \quad (26)$$

$$\therefore L = equation(24) - equation(8)$$
$$L = (m_1 + m_2 + m_3)(\frac{1}{2}l_1^2\dot{\theta}_1^2 - gl_1\cos\theta_1)$$
$$+(m_2 + m_3)(l_2^2\dot{\theta}_2^2 - gl_2\cos\theta_2)$$
$$+m_3(\frac{1}{2}l_3^2\dot{\theta}_3^2 - gl_3\cos\theta_3)$$
$$+m_3l_1l_2\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)$$
$$+2m_3l_1l_3\dot{\theta}_2\dot{\theta}_3(\cos\theta_2\cos\theta_3 + \cos\theta_3\sin\theta_2) \quad (27)$$

Therefore for θ_1:

$$\frac{\partial L}{\partial\dot{\theta}_1} = (m_1 + m_2 + m_3)(l_1^2\dot{\theta}_1) + m_3l_1l_2\dot{\theta}_2(\cos\theta_1\cos\theta_2$$
$$+\cos\theta_1\sin\theta_2) + 2m_2l_1l_3\dot{\theta}_3(\cos\theta_1\cos\theta_3) \quad (28)$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right) = l_1^2(m_1 + m_2 + m_3)\ddot{\theta}_1$$
$$+m_3l_1l_2\ddot{\theta}_2(\cos\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)$$
$$+m_3l_1l_2\dot{\theta}_2[-\cos\theta_1\sin\theta_2 - \cos\theta_2\sin\theta_1$$
$$+\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2]$$
$$+2m_2l_1l_3\ddot{\theta}_3\cos\theta_1\cos\theta_3$$
$$+2m_2l_1l_3\dot{\theta}_3(-\cos\theta_1\sin\theta_3 - \cos\theta_3\sin\theta_1 \quad (29)$$
$$\frac{\partial L}{\partial\theta_1} = (m_1 + m_2 + m_3)(l_1g\sin\theta_1)$$
$$+m_3l_1l_2\dot{\theta}_1\dot{\theta}_2(-\sin\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$$
$$-2m_3l_1l_3\dot{\theta}_1\dot{\theta}_3\sin\theta_1\cos\theta_3 \quad (30)$$

So the Euler-Lagrange differential equation for θ_1 is gotten by subtracting equation (30) from equation (29) and equate it to zero:

$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right) - \frac{\partial L}{\partial\theta_1} = 0$, after dividing through by l, gives

$$(m_1 + m_2 + m_3)(l_1\ddot{\theta}_1 - g\sin\theta_1) + m_3l_2[\ddot{\theta}_2\cos\theta_1\cos\theta_2$$
$$+\ddot{\theta}_2\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1 + \sin\theta_1\sin\theta_2$$
$$-\dot{\theta}_2\cos\theta_1\sin\theta_2 - \dot{\theta}_2\cos\theta_2\sin\theta_1 + \dot{\theta}_2\cos\theta_1\cos\theta_2$$
$$-\dot{\theta}_2\sin\theta_1\sin\theta_2] + 2l_2\dot{\theta}_2[m_2\ddot{\theta}_3\cos\theta_1\cos\theta_3$$
$$+m_3\dot{\theta}_1\dot{\theta}_3\cos\theta_3\sin\theta_1 - m_2\dot{\theta}_3\cos\theta_1\sin\theta_3$$
$$-m_2\dot{\theta}_3\sin\theta_1\cos\theta_3] = 0 \quad (31)$$

Similarly for θ_2:

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_2}\right) - \frac{\partial L}{\partial\theta_2} = 0$ divided through by l_2 gives

$$(m_2 + m_3)(l_2\ddot{\theta}_2 + g\sin\theta_2) + m_3l_1[\ddot{\theta}_1\cos\theta_1\cos\theta_2$$

ISBN: 978-988-14048-3-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$
\begin{aligned}
&+\ddot{\theta}_1 \cos\theta_1 \sin\theta_2 - \dot{\theta}_1 \cos\theta_1 \sin\theta_2 - \dot{\theta}_1 \sin\theta_1 \cos\theta_2 \\
&+\dot{\theta}_1 \cos\theta_1 \cos\theta_2 - \dot{\theta}_1 \sin\theta_2 \sin\theta_1 \\
&+\dot{\theta}_1\dot{\theta}_2 \cos\theta_1 \sin\theta_2 - \dot{\theta}_1\dot{\theta}_2 \cos\theta_1 \cos\theta_2] \\
&+m_3 l_3[\ddot{\theta}_3 \cos\theta_2 \cos\theta_3 + \ddot{\theta}_3 \cos\theta_3 \sin\theta_2 \\
&-\dot{\theta}_3 \cos\theta_2 \sin\theta_3 - \dot{\theta}_3 \sin\theta_2 \cos\theta_3 \\
&+\dot{\theta}_3 \cos\theta_2 \cos\theta_3 - \dot{\theta}_3 \sin\theta_3 \sin\theta_2 \\
&+\dot{\theta}_2\dot{\theta}_3 \sin\theta_2 \cos\theta_3 - \dot{\theta}_2\dot{\theta}_3 \cos\theta_3 \cos\theta_2] = 0 \qquad (32)
\end{aligned}
$$

Also for θ_3:

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_3}\right) - \frac{\partial L}{\partial \theta_3} = 0$ divided through by l_3 gives

$$
\begin{aligned}
&m_3(l_3\ddot{\theta}_3 - g\sin\theta_3) + 2m_2 l_1(\ddot{\theta}_1 \cos\theta_1 \cos\theta_3 \\
&-\dot{\theta}_1 \cos\theta_1 \sin\theta_3 - \dot{\theta}_1 \cos\theta_3 \sin\theta_1) \\
&+m_3 l_2(\ddot{\theta}_2 \cos\theta_3 \cos\theta_2 + \ddot{\theta}_2 \sin\theta_2 \cos\theta_3 \\
&+\dot{\theta}_2\dot{\theta}_3 \cos\theta_2 \sin\theta_3 + \dot{\theta}_2\dot{\theta}_3 \sin\theta_2 \sin\theta_3 \\
&+\dot{\theta}_2 \cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3 \\
&-\cos\theta_2 \sin\theta_3 - \cos\theta_3 \sin\theta_2) \\
&+2m_3 l_1 \dot{\theta}_1\dot{\theta}_3 \cos\theta_1 \sin\theta_3 = 0 \qquad (33)
\end{aligned}
$$

III. Numerical Examples

Using the following parameters values for the purpose of simulation; $l_1 = 4$, $l_2 = 3$, $l_3 = 2$; $m_1 = 3$, $m_2 = 2$, $m_3 = 1$; and g=10,and integrating with respect to time t afterword, the resulting equations from equation (31), (36) and (41) are as follows, respectively:

$$
\begin{aligned}
&2\ddot{\theta}_3 - 10\sin\theta_3 + 16(\ddot{\theta}_1 \cos\theta_1 \cos\theta_3 \\
&-\dot{\theta}_1 \cos\theta_1 \sin\theta_3 - \dot{\theta}_1 \cos\theta_3 \sin\theta_1) \\
&+3(\ddot{\theta}_2 \cos\theta_3 \cos\theta_2 + \ddot{\theta}_2 \sin\theta_2 \cos\theta_3 \\
&+\dot{\theta}_2\dot{\theta}_3 \cos\theta_2 \sin\theta_3 + \dot{\theta}_2\dot{\theta}_3 \sin\theta_2 \sin\theta_3 \\
&+\dot{\theta}_2 \cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3 - \cos\theta_2 \sin\theta_3 \\
&-\cos\theta_3 \sin\theta_2) + 8\dot{\theta}_1\dot{\theta}_3 \cos\theta_1 \sin\theta_3 = 0 \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
&3(3\ddot{\theta}_2 + 10\sin\theta_2) + 4[\ddot{\theta}_1 \cos\theta_1 \cos\theta_2 \\
&+\ddot{\theta}_1 \cos\theta_1 \sin\theta_2 - \dot{\theta}_1 \cos\theta_1 \sin\theta_2 - \dot{\theta}_1 \sin\theta_1 \cos\theta_2 \\
&+\dot{\theta}_1 \cos\theta_1 \cos\theta_2 - \dot{\theta}_1 \sin\theta_2 \sin\theta_1 \\
&+\dot{\theta}_1\dot{\theta}_2 \cos\theta_1 \sin\theta_2 - \dot{\theta}_1\dot{\theta}_2 \cos\theta_1 \cos\theta_2] \\
&+2[\ddot{\theta}_3 \cos\theta_2 \cos\theta_3 + \ddot{\theta}_3 \cos\theta_3 \sin\theta_2 \\
&-\dot{\theta}_3 \cos\theta_2 \sin\theta_3 - \dot{\theta}_3 \sin\theta_2 \cos\theta_3 \\
&+\dot{\theta}_3 \cos\theta_2 \cos\theta_3 - \dot{\theta}_3 \sin\theta_3 \sin\theta_2 \\
&+\dot{\theta}_2\dot{\theta}_3 \sin\theta_2 \cos\theta_3 - \dot{\theta}_2\dot{\theta}_3 \cos\theta_3 \cos\theta_2] = 0 \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
&(2\ddot{\theta}_3 - g\sin\theta_3) + 16(\ddot{\theta}_1 \cos\theta_1 \cos\theta_3 \\
&-\dot{\theta}_1 \cos\theta_1 \sin\theta_3 - \dot{\theta}_1 \cos\theta_3 \sin\theta_1) \\
&+3(\ddot{\theta}_2 \cos\theta_3 \cos\theta_2 + \ddot{\theta}_2 \sin\theta_2 \cos\theta_3 \\
&+\dot{\theta}_2\dot{\theta}_3 \cos\theta_2 \sin\theta_3 + \dot{\theta}_2\dot{\theta}_3 \sin\theta_2 \sin\theta_3 \\
&+\dot{\theta}_2 \cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3 - \cos\theta_2 \sin\theta_3 \\
&-\cos\theta_3 \sin\theta_2) + 8\dot{\theta}_1\dot{\theta}_3 \cos\theta_1 \sin\theta_3 = 0 \qquad (36)
\end{aligned}
$$

A. *Application of Product-to-sum identities:*

Applying product-to-sum identities to equations (43). (44) and (45) gives the following:

$$
\begin{aligned}
&2\ddot{\theta}_3 - 10\sin\theta_3 + 8\ddot{\theta}_1\{\cos(\theta_1-\theta_3) + \cos(\theta_1+\theta_3)\} \\
&-8\dot{\theta}_1\{\sin(\theta_1+\theta_3) - \sin(\theta_1-\theta_3)\} - 8\dot{\theta}_1\{\sin(\theta_3+\theta_1) \\
&-\sin(\theta_3-\theta_1)\} + 1.5\ddot{\theta}_2\{\cos(\theta_3-\theta_2) + \cos(\theta_3+\theta_2\} \\
&+1.5\ddot{\theta}_2\{\sin(\theta_3+\theta_2) - \sin(\theta_3-\theta_2)\} \\
&+1.5\dot{\theta}_2\dot{\theta}_3\{\sin(\theta_2+\theta_3) - \sin(\theta_2-\theta_3)\} \\
&+1.5\dot{\theta}_2\dot{\theta}_3\{\cos(\theta_2-\theta_3) - \cos(\theta_2+\theta_3)\} + 1.5\dot{\theta}_2\{\cos(\theta_2-\theta_3) \\
&+\cos(\theta_2+\theta_3)\} - 1.5\{\cos(\theta_2-\theta_3) - \cos(\theta_2+\theta_3)\} \\
&-1.5\{\sin(\theta_2+\theta_3) - \sin(\theta_2-\theta_3\} - 1.5\{\sin(\theta_3+\theta_2) \\
&-\sin(\theta_3-\theta_2\} + 4\dot{\theta}_1\dot{\theta}_3\{\sin(\theta_1+\theta_3) - \sin(\theta_1-\theta_3)\} = 0 \quad (37)
\end{aligned}
$$

$$
\begin{aligned}
&3(3\ddot{\theta}_2 - 10\sin\theta_2) + 2\ddot{\theta}_1\{\cos(\theta_1-\theta_2) + \cos(\theta_1+\theta_2)\} \\
&+2\ddot{\theta}_1\{\sin(\theta_1+\theta_2) - \sin(\theta_1-\theta_2)\} - 2\dot{\theta}\{\sin(\theta_1+\theta_2) \\
&-\sin(\theta_1-\theta_2)\} - 2\dot{\theta}\{\sin(\theta_1+\theta_2) + \sin(\theta_1-\theta_2)\} \\
&+2\dot{\theta}\{\cos(\theta_1-\theta_2) + \cos(\theta_1+\theta_2)\} - 2\dot{\theta}\{\cos(\theta_2-\theta_1) \\
&-\cos(\theta_2+\theta_1)\} + 2\dot{\theta}_1\dot{\theta}_2\{\sin(\theta_1+\theta_2) - \sin(\theta_1-\theta_2)\} \\
&-2\dot{\theta}_1\dot{\theta}_2\{\cos(\theta_1-\theta_2) + \cos(\theta_1+\theta_2)\}] \\
&+\ddot{\theta}_3\{\cos(\theta_2-\theta_3) + \cos(\theta_2+\theta_3)\} + \ddot{\theta}_3\{s\,\mathrm{in}(\theta_3+\theta_2) \\
&-\sin(\theta_3-\theta_2)\} - \dot{\theta}_3\{s\,\mathrm{in}(\theta_2+\theta_3) \\
&-\sin(\theta_2-\theta_3)\} - \dot{\theta}_3\{s\,\mathrm{in}(\theta_2+\theta_3) + \sin(\theta_2-\theta_3)\} \\
&+\dot{\theta}_3\{\cos(\theta_2-\theta_3) + \cos(\theta_2+\theta_3)\} - \dot{\theta}_3\{\cos(\theta_3-\theta_2) \\
&-\cos(\theta_3+\theta_2)\} + \dot{\theta}_2\dot{\theta}_3\{\sin(\theta_2+\theta_3) + \sin(\theta_2-\theta_3)\ \} \\
&-\dot{\theta}_2\dot{\theta}_3\{\cos(\theta_3-\theta_2) + \cos(\theta_3+\theta_2)\}] = 0 \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
&(2\ddot{\theta}_3 - 10\sin\theta_3) + 8\ddot{\theta}_1\{\cos(\theta_1-\theta_3) \\
&+\cos(\theta_1+\theta_3)\} - 8\dot{\theta}_1\{\sin(\theta_1+\theta_3) - \sin(\theta_1-\theta_3)\}
\end{aligned}
$$

ISBN: 978-988-14048-3-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$-8\dot{\theta}_1\{\sin(\theta_3+\theta_1)-\sin(\theta_3-\theta_1)\}+\frac{3}{2}\ddot{\theta}_2\{\cos(\theta_3-\theta_2)$$
$$+\cos(\theta_3+\theta_2)\}+\frac{3}{2}\ddot{\theta}_2\{\sin(\theta_2+\theta_3)+\sin(\theta_2-\theta_3)\}$$
$$+\frac{3}{2}\dot{\theta}_2\dot{\theta}_3\{\sin(\theta_2+\theta_3)-\sin(\theta_2-\theta_3)\}$$
$$+\frac{3}{2}\dot{\theta}_2\dot{\theta}_3\{\cos(\theta_2-\theta_3)-\cos(\theta_2+\theta_3)\}$$
$$+\frac{3}{2}\dot{\theta}_2\{\cos(\theta_2-\theta_3)+\cos(\theta_2+\theta_3\}-\frac{3}{2}\{\cos(\theta_2-\theta_3)$$
$$-\cos(\theta_2+\theta_3)\}-\frac{3}{2}\{\sin(\theta_2+\theta_3)-\sin(\theta_2-\theta_3)\}$$
$$-\frac{3}{2}\{\sin(\theta_3+\theta_2)-\sin(\theta_3-\theta_2)\}$$
$$+4\dot{\theta}_1\dot{\theta}_3\{\sin(\theta_1+\theta_3)-\sin(\theta_1-\theta_3)\}=0 \qquad (39)$$

Equations (1),(2) and (3) respectively give:

$$\frac{x_1}{l_1}=\sin\theta_1 \Rightarrow \theta_1=\sin^{-1}(\frac{x_1}{l_1})$$

$$\frac{x_2-x_1}{l_2}=\sin\theta_2 \Rightarrow \theta_2=\sin^{-1}(\frac{x_2-x_1}{l_2})$$

$$\frac{x_3-x_2}{l_3}=\sin\theta_3 \Rightarrow \theta_3=\sin^{-1}(\frac{x_3-x_2}{l_3})$$

Assuming the three bobs positions at the beginning of the dance are as follows:

$(x_1,y_1) = (3,6)$ (40)
$(x_2,y_2)=(4,3)$ (41)
$(x_3,y_3)= (5,1)$ (42)
Then

$$\theta_1=\sin^{-1}(\frac{x_1}{l_1})=\sin^{-1}\left(\frac{3}{4}\right)=49^\circ \qquad (43)$$

$$\theta_2=\sin^{-1}(\frac{x_2-x_1}{l_2})=\sin^{-1}\left(\frac{1}{3}\right)=19^\circ \qquad (44)$$

$$\theta_3=\sin^{-1}(\frac{x_3-x_2}{l_3})=\sin^{-1}\left(\frac{1}{2}\right)=30^\circ \qquad (49)$$

Using these values, equation (37), (38) and (39) were reduced to:

$$2\ddot{\theta}_3+2.48\ddot{\theta}_2+9.12\ddot{\theta}_1-15.68\dot{\theta}_1+1.89\dot{\theta}_2\dot{\theta}_3$$
$$+2.46\dot{\theta}_2+4.68\dot{\theta}_1\dot{\theta}_3 = 7.73 \qquad (50)$$
$$9\ddot{\theta}_2-9.77+2.48\ddot{\theta}_1+0.85\ddot{\theta}_1-2.14\dot{\theta}_1-1.63\dot{\theta}_1\dot{\theta}_2$$
$$+2.2\ddot{\theta}_3-0.2\dot{\theta}_3-1.08\dot{\theta}_2\dot{\theta}_3=0 \qquad (51)$$
$$2\ddot{\theta}_3-5+9.09\ddot{\theta}_1-15.71\dot{\theta}_1+3.31\ddot{\theta}_2$$
$$+0.95\dot{\theta}_2\dot{\theta}_3+2.46\dot{\theta}_2-2.75+2.64\dot{\theta}_1\dot{\theta}_3= 0 \qquad (52)$$

In this paper we also considered the behaviour of an arm segment, assuming the other two at a particular time are of constant velocities. This happens at some instances during a dance.

Firstly, assuming the velocities of the first and second segments of the arm; l_1 and l_2 are constants, that is in equilibrium state, which implies their accelerations are zeros too, then solving equations (50), (51) and (52) simultaneously, give:

$$2.2\ddot{\theta}_3 = 4.48 \qquad (53)$$

Similarly, assuming velocities of the first and third segments of the arm; l_1 and l_3 are constants, gives:

$$9.83\ddot{\theta}_2 += 4.48 \qquad (54)$$

Now, assuming the velocities of the second and third segments of the arm; l_2 and l_3 are zero, gives:

$$3.4\ddot{\theta}_1-2.31\dot{\theta}_1 = 4.48 \qquad (55)$$

Integrating equation (53), (54) and (55) with respect to time t gives: (54)

$$\dot{\theta}_3 = 2.04t + C \qquad (56)\ (55)$$
$$\dot{\theta}_2 = 0.46t + C \qquad (57)$$
$$3.4\dot{\theta}_1-2.31\theta_1 = 4.48t + C \qquad (58)\ (56)$$

Where C is a constant.

But θ_1 = 49, equation (58) becomes

$$\dot{\theta}_1 = 1.32t + 33.29 + C \qquad (59)$$

Applying Newton's second law of motion at t=1 and assuming C=0, equations (56), (57), (59) gives:

$$m_1\ddot{\theta}_1 = 3 \text{ x } 25.74 = 77.22 \qquad (87)$$
$$m_2\ddot{\theta}_2 = 2 \text{ x } 0.46 = 0.92 \qquad (88)$$
$$m_3\ddot{\theta}_3 = 1\text{x } 2.04 = 2.04 \qquad (89)$$

IV. RESULT DISCUSSION

At a particular time t, say t = 2, when the angle the arm segments make with the vertical are 49, 19, and 30 degrees respectively, taking C to be zero, the corresponding angular velocities are 35.93, 0.92 and 4.08 respectively. Also their corresponding accelerations are 25.74 ,0.46, and 2.04 respectively. From the results, at that particular instance of the dance that informed the position of the arm at that time, the arm segment with the highest angle with the vertical have the highest velocity and acceleration, and vice versa. This shows there is a positive correlation between

ISBN: 978-988-14048-3-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

the size of the angle the arm segments make with the vertical and velocity and acceleration of the segment. Also from the table below it can be seen that the higher the angular displacement the higher the angular acceleration. The same explanation goes for the angular displacement and angular velocity. From the table again, the angular velocities increase as time (t) increases. This implies that the upper arm should move faster than both the lower arm and the wrist to ensure good balance and by extension a good dance. However, the wrist should move faster than the lower arm. The angle the upper segment makes the vertical should be the highest, followed by that of the wrist. The angle the lower segment makes with the vertical should be the lowest in order to maintain a good balance during a dance. Finally, in other to maintain balance during dance, using the given parameter values, the resultant force needed to apply at segment 1, segment 2, and segment 3 of the arm are 77.22units, 0.92 units and 2.04 units respectively

IV. CONCLUSION

This paper sets out to, analytically; model the movement of human arm during dance as a 3-pendulum system. With the aid of the Lagragian and Euler - lagrange equation, the dynamics of the three segments of the human arm is analysed. At given values of the length of the different segments the values of the angular displacement of the three segment were computed. It is observed that there is a positive relationship between the computed angular displacements and the respective angular acceleration of the arm segments. The analytical results show that the higher the angular displacement the higher the angular velocity and the angular acceleration. To have a good dance therefore, the upper arm with highest angular displacement, compared to the other sections of the human arm, should move fastest, followed by the wrist section then the lower arm, with the least angular displacement. This supports stability and balance of the whole arm movement during a dance.

Reference

[1] Loay A. Al-Zu’be et al.(2012) Modelling and Simulating Human Arm Movement using a 2 Dimensional 3 Segments Coupled Pendulum System. World Academy of Science, Engineering and Technology International Journal of Medical, Health, Biomedical, Bioengineering and Pharmaceutical Engineering Vol:6, No:11.

[2] Y Bazargan-Lari et al (2014) Dynamics and regulation of Locomotion of a Human Swing Leg as a Double-pendulum Considering self- impact Joint Constraint. Journal of biomedical and physics engineering. 4(3), p. 91- 102

[3] Drillis, R., R. Contini, and M. Bluestein, Body Segment Parameters; a Survey of Measurement Techniques. Artif Limbs, 1964. 25: p. 44-66.

[4] Nick Eyre and Jeff Holzgrafe, Modelling of a Damped Triple Pendulum. Franklin W. Olin College of Engineering. ENGR 2340: Dynamics. https://www.nickeyre.com/images/triplepend ulum.pdf

[5] Winter, D.A. Biomechanics of human movement with applications to study of human locomotion. Crit Rev Biomed Eng, 1984. 9(4,) p. 287-314

[6] Lagrangian mechanics. From Wikipedia, the freeencyclopaedia https://en.wikipedia.org/wiki/Lagrangian mechanic

[7] M.C. Agarana and Agboola,(2015),Dynamic Analysis of Damped driven pendulum using Laplace transform method, 26(3, 98-109)

[8] Bannister, Ross. "The Double Pendulum." The Double Pendulum. University of Reading, June 2001. Web. 09 Dec. 2012.

[9] M.C Agarana and S.A Iyase, 2015, Analysis of Hermite’s Equation Governing the motion of Damped Pendulum with small displacement, International Journal of Physical Sciences,10(2), 364 – 370 [10] Agarana M.C..and Bishop S.A, 2015, Quantitative Analysis of Equilibrium Solution and Stability for Non-linear Differential Equation Governing Pendulum Clock, International Journal of Applied Engineering Research, Vol. 10 No.24, pp.44112-44117

[11] M.C..Agarana and M.E. Emetere, (2016), Solving non-linear damped driven simple pendulum with small amplitude using a semi analytical method, ARPN Journal of Engineering and Applied Sciences, Vol 11, No. 7, 2016