

Thermodynamics Analysis of Radiative Hydromagnetic Couple Stress Fluid through a Channel

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Abstract— This work applies second law of thermodynamics to analyse the effect of radiation on electrically conducting couple stress fluid through a channel. A constant magnetic field is introduced across the flow channel and the resulting Navier-Stokes and energy equations are non-dimensionalized and solved using Adomian decomposition method (ADM) and differential transform method (DTM). The obtained velocity and temperature profiles are used to calculate the entropy generation rate and irreversibility ratio. The effects of radiation, magnetic field and couple stress parameters on the velocity, temperature, entropy generation rate and Bejan number are discussed with the aid of graphs. From the study, it is observed that increase in magnetic field and couple stress parameters reduces the fluid velocity while an increase in radiation parameter reduces the temperature of the fluid. Furthermore, radiation parameter increases entropy generation as heat transfer dominates irreversibility.

Index Terms— Thermal radiation, Entropy generation, Couple stress fluid, hydromagnetic, ADM, DTM

I. INTRODUCTION

The study energy transfer due to thermal radiation has received a boost in the past few decades due to its wide ranging applications in areas such as gas turbines, astrophysical flows, forest fire dynamics, fire spread buildings, nuclear power plants. Numerous research studies on radiative flows in porous and non-porous medium include: Mukhopadhyay [1] investigated radiation and variable fluid viscosity effects on flow and heat transfer and submitted that fluid temperature reduces with increasing value of both radiation parameter and Prandtl number. Olanrewaju et al. [2] considered the effect of radiation and viscous dissipation on the Blasius and Sakiadis flows. It was concluded that increase in Eckert number, Prandtl number and radiation parameter reduces the thermal boundary layer thickness along the plate resulting in fluid temperature reduction. Vyas et al. [3] investigated the effect of radiation on the entropy generation of an electrically conducting Couette flow inside a channel with naturally permeable base

and argued that there is an increase in entropy generation as the values of slip coefficient, magnetic field parameter, Brinkman number, radiation parameter and characteristic temperature ratio increase; while it reduces with the rising values of permeability parameter K . Moreover, Adesanya [4] discussed the effect of radiation on steady electrically conducting visco-elastic heat generating /absorbing slip flow through a porous medium, the results show that increase in both radiation and heat absorption parameters increases the fluid temperature and flow velocity of the non-Newtonian fluid within the channel. Other important studies on thermal radiation effects are in Refs. [5-11]

Investigations have been geared towards the analysis of entropy production in designing thermal systems; this is due to the translation of available energy for work to destruction. Currently, second law of thermodynamics is being applied in the field of heat transfer and thermal design, a method introduced by Bejan [12-13]. Thereafter, several investigations were carried out to examine entropy production under various flow configurations. Adesanya et al. [14] investigated the effects of couple stresses and convective heating on the entropy generation through porous channel. The results reveal that increase in couple stress parameter lowers entropy generation while an increase in Brinkman number registers a significant increase in entropy generation rate in the middle of the channel than at the walls. Das et al. [15] analyzed the effect of Navier slip on entropy generation of an electrically conducting viscous fluid in a porous channel and reported that entropy generation is enhanced with a rise in magnetic parameter. Jerry et al. [16] reported on the effect of an external oriented magnetic field on entropy generation in natural convection that, magnetic field lowers entropy generation and entropy generation due to viscous effects is the major contributor to irreversibility. Interested readers should see for more on entropy generation rate Refs. [17-20].

The Adomian decomposition and differential transform methods applied in this work have been found to be efficient, accurate and rapidly convergent. The methods have been used to obtain the solution of various linear and nonlinear boundary value problems [21-23]

To the best of our knowledge, entropy production due to the effect of thermal radiation on hydromagnetic couple

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stress fluid through a channel has not been reported. Therefore the main objective of this study is to analyze the effect of thermal radiation on entropy generation of MHD couple stress fluid flows through a channel.

II. MATHEMATICAL MODEL

Consider an incompressible, non-Newtonian couple stress fluid through a channel, the wall plates exchanges heat with the ambient in an axi-symmetrical manner and hot fluid is injected and sucked off at the lower and upper walls respectively with the same velocity. A uniformly transverse magnetic field, B_0 is applied in the direction of flow and the interaction of the induced magnetic field is assumed to be negligible compared to the interaction of the applied magnetic field. It is further assumed that the radiative heat flux in the energy equation follows Roseland approximation.

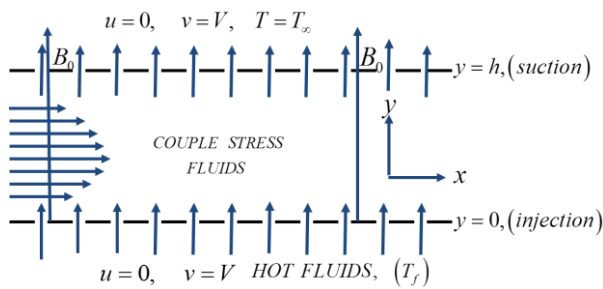


Fig.1 Schematic diagram of the problem

With these assumptions and configuration (see Fig. 1) the governing equations for the flow are written as: (Adesanya et al. [14])

$$\rho v_0 \frac{du'}{dy'} = -\frac{dp}{dx'} + \mu \frac{d^2u'}{dy'^2} - \eta \frac{d^4u'}{dy'^4} - \sigma B_0^2 u' \quad (1)$$

$$\rho c_p v_0 \frac{dT'}{dy'} = k \frac{d^2T'}{dy'^2} + \mu \left(\frac{du'}{dy'} \right)^2 + \eta \left(\frac{d^2u'}{dy'^2} \right)^2 + \sigma B_0^2 u'^2 - \frac{dq_r}{dy'} \quad (2)$$

$$E_G = \frac{k}{T_0^2} \left(\frac{dT'}{dy'} \right)^2 + \frac{16\sigma^c T_0^3}{3k^*} \left(\frac{d^2T'}{dy'^2} \right)^2 + \frac{\mu}{T_0} \left(\frac{du'}{dy'} \right)^2 + \frac{\eta}{T_0} \left(\frac{d^2u'}{dy'^2} \right)^2 + \frac{\sigma B_0^2 u'^2}{T_0} \quad (3)$$

The boundary conditions are:

$$k \frac{dT'}{dy'}(0) = -\gamma_1 (T_f - T'); k \frac{dT'}{dy'}(h) = -\gamma_2 (T' - T_0); u'(0) = \frac{d^2u'}{dy'^2}(0) = 0 = u'(h) = \frac{d^2u'}{dy'^2}(h) \quad (4)$$

Applying the Roseland approximation for radiation [3, 24, 25] yields

$$q_r = \frac{4\sigma^c}{3k^*} \frac{dT'^4}{dy'} \quad (5)$$

According to Raptis et al. [18] expressing the temperature function in equation (3) as a linear function of temperature and expanding it (T'^4) in a Taylor series about T_0 gives

$$T'^4 = T_0^4 + 4T_0^3 (T' - T_0) + 6T_0^2 (T' - T_0)^2 + 4T_0 (T' - T_0)^3 + (T' - T_0)^4 \quad (6)$$

If the higher order terms are neglected it yields

$$T'^4 \cong 4T_0^3 T' - 3T_0^3 \quad (7)$$

Substituting equations (5) and (7) in (2), we have

$$\rho c_p v_0 \frac{dT'}{dy'} = k \frac{d^2T'}{dy'^2} + \mu \left(\frac{du'}{dy'} \right)^2 + \sigma B_0^2 u'^2 + \frac{16\sigma^c T_0^3}{3k^*} \frac{d^2T'}{dy'^2} \quad (8)$$

The dimensionless parameters below are introduced in equations (1, 3, 4 and 8) to yield

$$y = \frac{y'}{h}, u = \frac{u'}{v_0}, \theta = \frac{T' - T_0}{T_f - T_0}, s = \frac{v_0 h}{\nu}$$

$$G = -\frac{h^2}{\mu v_0} \frac{dp}{dx}, Pr = \frac{\nu \rho c_p}{k}$$

$$Br = \frac{\mu v_0}{k(T_f - T_0)}, Ns = \frac{T_0^2 h^2 E_G}{k(T_f - T_0)^2},$$

$$\Omega = \frac{T_f - T_0}{T_0}, H^2 = \frac{\sigma B_0^2 h^2}{\mu}, N = \frac{4\sigma^c T_0^3}{kk^*}, \quad (9)$$

$$a^2 = \mu \frac{h^2}{\eta}$$

$$s \frac{du}{dy} = G + \frac{d^2u}{dy^2} - \frac{1}{a^2} \frac{d^4u}{dy^4} - H^2 u; u(0) = u(1) = \frac{d^2u(0)}{dy^2} = \frac{d^2u(1)}{dy^2} \quad (10)$$

$$\left(1 + \frac{4}{3} N \right) \frac{d^2\theta}{dy^2} = sp_r \frac{d\theta}{dy} -$$

$$Br \left[\left(\frac{du}{dy} \right)^2 + \frac{Br}{a^2} \left(\frac{d^2u}{dy^2} \right)^2 + Br H^2 u^2 \right];$$

$$\frac{d\theta(0)}{dy} = Bi_1 (\theta(0) - 1), \frac{d\theta(1)}{dy} = -Bi_2 \theta(1) \quad (11)$$

$$N_s = \left(1 + \frac{4}{3}N\right) \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega} \left\{ \left(\frac{du}{dy}\right)^2 + \frac{1}{a^2} \left(\frac{d^2u}{dy^2}\right)^2 + H^2u^2 \right\} \quad (12)$$

III. METHOD OF SOLUTION

Adomian decomposition and differential transform methods are applied to obtain the solution of the boundary value problems. However, owing to the magnitude of the symbolic solutions only graphical results are presented in Figures (2-10). To verify the accuracy of these computations the approximate solution obtained via ADM and DTM are validated by the exact solution. The numerical comparison is presented in Table 1.

Table I: Computation showing convergence of solution when $G = H = a = 1, s = 0.1$

u(y)	U_{EXACT}	ADM ABS ERROR	DTM ABS ERROR
0	0.0000000000	0.0000000000	0.0000000000
0.1	0.003680317	2.23528E-12	7.65E-13
0.2	0.006959623	4.332E-12	1.67E-12
0.3	0.009523962	6.1649E-12	2.83E-12
0.4	0.011151328	7.62074E-12	4.37E-12
0.5	0.011709275	8.59304E-12	6.38E-12
0.6	0.011153527	8.98038E-12	8.94E-12
0.7	0.009527596	8.67982E-12	1.22E-11
0.8	0.006963372	7.58431E-12	1.61E-11
0.9	0.003682714	5.57649E-12	2.08E-11
1	-7.00E-17	2.52331E-12	2.65E-11

IV. ENTROPY GENERATION

According to Bejan [11, 12] the expression for local entropy generation rate in (3) suggests five sources of entropy generation;

$$E_G = \frac{k}{T_o^2} \left(\frac{dT'}{dy'}\right)^2 + \frac{16\sigma T_o^3}{3k^*} \left(\frac{d^2T}{dy'}\right)^2 + \frac{\mu}{T_o} \left(\frac{du'}{dy'}\right)^2 + \frac{\eta}{T_o} \left(\frac{d^2u'}{dy'^2}\right)^2 + \frac{\sigma B_o^2 u'^2}{T_o}$$

$\frac{k}{T_o^2} \left(\frac{dT'}{dy'}\right)^2$ is irreversibility due to heat transfer,

$\frac{16\sigma T_o^3}{3k^*} \left(\frac{d^2T}{dy'}\right)^2$ is the entropy generation due to thermal

radiation, $\frac{\mu}{T_o} \left(\frac{du'}{dy'}\right)^2$ is entropy generation as a result of fluid friction, $\frac{\eta}{T_o} \left(\frac{d^2u'}{dy'^2}\right)^2$ is entropy generation due to couple stresses and $\frac{\sigma B_o^2 u'^2}{T_o}$ is due to the effect of MHD.

The dimensionless form is given in equation (12) as

$$N_s = \left(1 + \frac{4}{3}R\right) \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega} \left\{ \left(\frac{du}{dy}\right)^2 + \frac{1}{a^2} \left(\frac{d^2u}{dy^2}\right)^2 + H^2u^2 \right\}$$

Investigating entropy generation within the flow, we let

$$N_1 = \left(1 + \frac{4}{3}R\right) \left(\frac{d\theta}{dy}\right)^2, \quad (13)$$

$$N_2 = \frac{Br}{\Omega} \left\{ \left(\frac{du}{dy}\right)^2 + \frac{1}{a^2} \left(\frac{d^2u}{dy^2}\right)^2 + H^2u^2 \right\}$$

The Bejan number $Be = 0$ is the irreversibility due to viscous dissipation, couple stress effect and magnetic effect and $Be = 1$ is when the irreversibility due to heat transfer dominates the flow. $Be = \frac{1}{2}$ indicates that both contribute equally to entropy generation. Writing Bejan number as

$$Be = \frac{N_1}{N_s} = \frac{1}{1 + \Phi}, \quad \Phi = \frac{N_2}{N_1} \quad (14)$$

V. RESULTS AND DISCUSSION

This study considers the effect of thermal radiation on the entropy generation rate of a hydromagnetic couple stress fluid through porous channel. The velocity and temperature profiles are solved using Adomian decomposition and differential transform methods, and the results showing the effects of some pertinent parameters on the velocity, temperature, entropy generation and Bejan number are discussed with the aid of graphs in Figs. 2-10.

EFFECTS OF PARAMETERS VARIATION ON VELOCITY AND TEMPERATURE PROFILES

Fig.2 depicts the effect of increasing magnetic field on fluid velocity. The plot shows that fluid velocity reduces with a rise in the values of magnetic field parameter. This is because the magnetic field applied clumps fluid particles together leading to reduction in fluid velocity. In Fig.3 effect of couple stress inverse parameter on fluid velocity is displayed, from the figure it is shown that fluid velocity is enhanced as couple stress inverse parameter increases. This

implies that couple stresses will eventually retard the flow velocity due to the size dependent effect introduced by the increased couple stresses which increases fluid viscosity.

In Fig. 4, effect of variation in radiation parameter on fluid temperature is displayed. The plot indicates that temperature is lowered as thermal radiation parameter increases in value; the reduction can be attributed to the fact that increasing radiation parameter N has an increasing effect on the absorption parameter k^* which results to a decrease in temperature. Fig. 5 displays the implication of increasing the values magnetic field parameter on temperature; it is revealed that the temperature is enhanced across the channel as the value of Hartman's number increases. The presence of Ohmic heating in the flow corresponds to the rise in fluid temperature. However in Fig. 6, variation in couple stress inverse parameter corresponds to a reduction in fluid temperature. This means that couple stresses enhance fluid temperature; the rise in temperature is due to increased size of fluid particles which increases flow resistance as fluid layers slide over the other resulting in temperature rise.

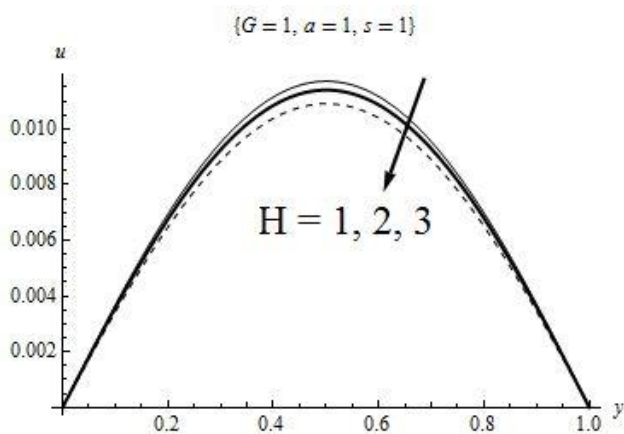


Fig 2 Velocity profile for magnetic field parameter

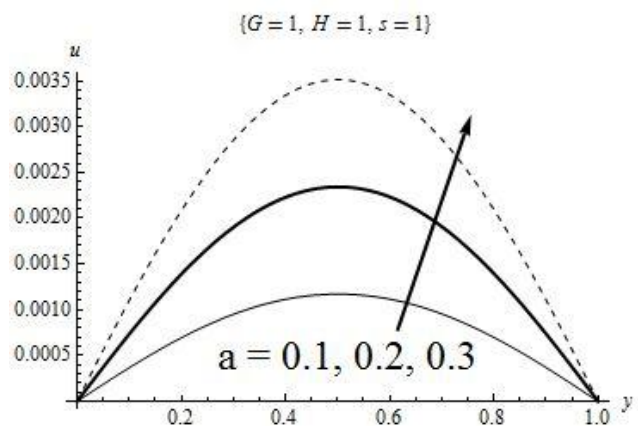


Fig 3 Velocity profile for couple stress inverse parameter

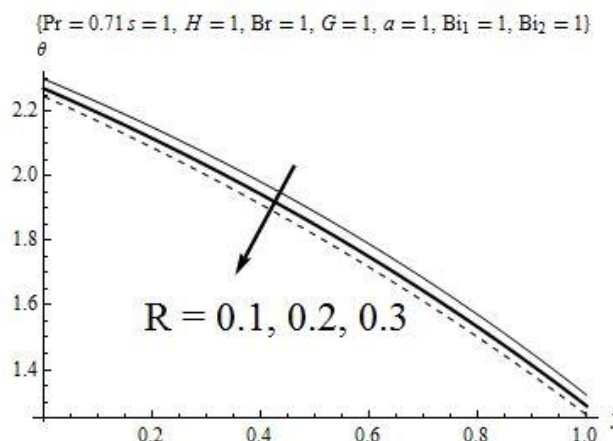


Fig 4 Temperature profile for radiation parameter

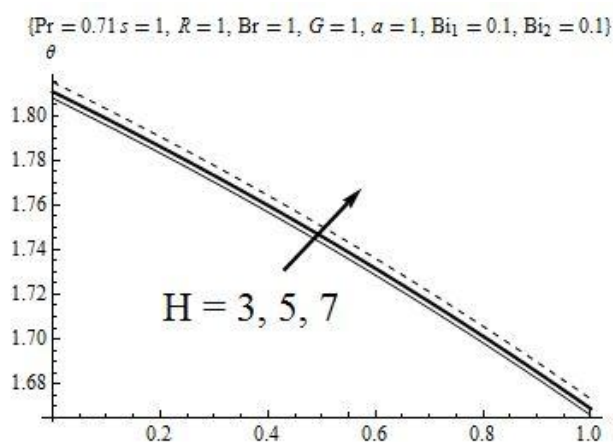


Fig 5 Temperature profile for magnetic field

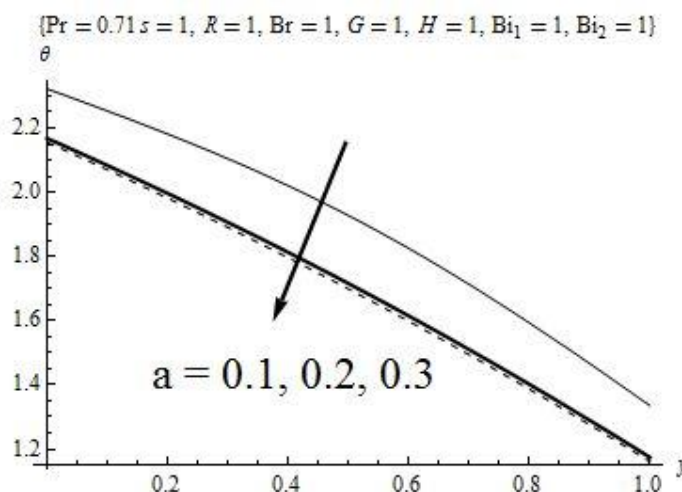


Fig 6 Temperature profile for couple stress inverse parameter

EFFECTS OF PARAMETERS VARIATION ON ENTROPY GENERATION RATE

In this section, effects of parameters variation on the entropy generation are displayed in Figs. 7-8. Fig.7 represents the effect of variation in thermal radiation parameter on entropy generation rate; it is clearly revealed in that thermal radiation encourages entropy production due to increase in the emission rate within the flow channel. Fig.8 is the variation of couple stress inverse on entropy generation rate. It is obvious from the plot that entropy generation rises as couple stress inverse increases in values. The implication of this is that increase in couple stresses decreases entropy generation rate. The reason is shown in Fig.2, the plot indicates that fluid velocity reduces with increase in couple stresses due to increased fluid viscosity. The effect of this on the flow is that it lowers the randomness of fluid particles and consequently the reduction in entropy production rate.

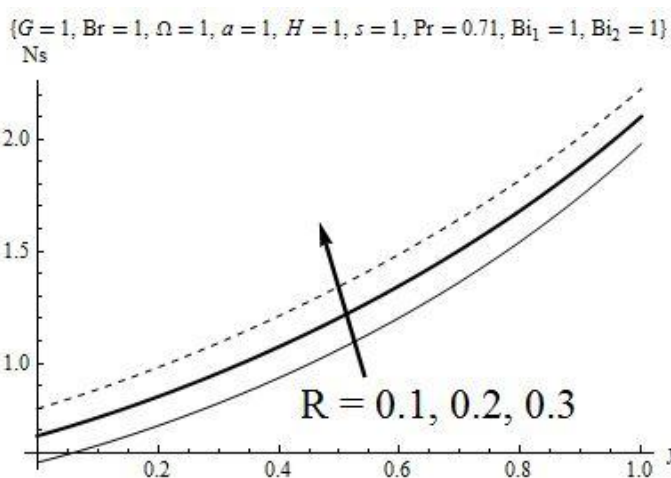


Fig 7 Entropy generation profile for radiation parameter

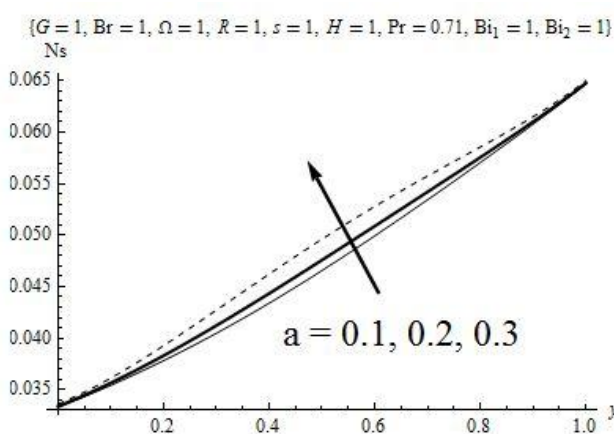


Fig 8 Entropy generation profile for couple stress inverse parameter

EFFECTS OF PARAMETERS VARIATION ON BEJAN NUMBER

Figs.9 and 10 present the effect of variation in thermal radiation and magnetic field parameters on Bejan number. The plots show that Bejan number increases as the values of these parameters increased. This is an indication that heat transfer dominates entropy generation rate

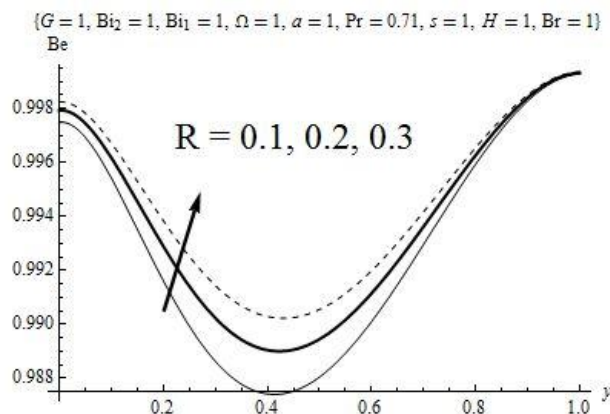


Fig 9 Bejan number for radiation parameter

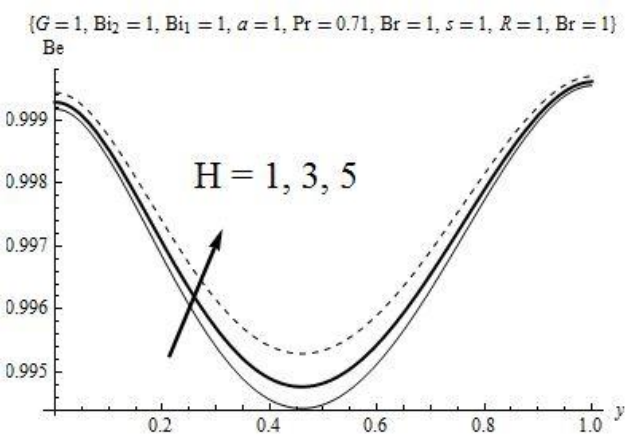


Fig 10 Bejan number for magnetic field

VI. CONCLUSION

In this study, effect of thermal radiation on the entropy generation rate of hydromagnetic couple stress fluid through porous channel is investigated. The velocity and temperature profiles obtained are solved via Adomian decomposition method and differential transform method. The results showing the effects of some pertinent parameters are discussed graphically.

- Increase in magnetic field parameter and couple stresses inhibits the fluid flow.
- Increase in thermal radiation parameter reduces the temperature while increase in magnetic field and couple stresses enhances the temperature.
- Radiation parameter encourages entropy generation while increase in couple stress parameter decreases the entropy generation rate.
- Heat transfer dominates entropy generation rate.

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NOMENCLATURE

u'	fluid velocity
μ	dynamic viscosity
ρ	fluid density
T'	fluid temperature
T_0	initial fluid temperature
T_f	final fluid temperature
k	thermal conductivity of the fluid
c_p	specific heat at constant pressure
v_0	constant velocity of fluid suction/injection
ν	kinematic viscosity
σ	electrical conductivity of the fluid
η	fluid particle size effect due to couple stresses
E_G	local volumetric entropy generation rate
$\gamma_{1,2}$	heat transfer coefficients
B_0	uniform transverse magnetic field
q_r	radiative heat flux.
u	dimensionless velocity
s	suction/injection parameter
θ	dimensionless temperature
a	couple stress parameter
Pr	Prandtl number
Br	Brinkman number
Ω	parameter that measures the temperature difference between the two heat reservoirs
H	magnetic field parameter
N_s	dimensionless entropy generation rate
Be	Bejan number
$Bi_{1,2}$	Biot numbers respectively
G	axial pressure gradient
N	thermal radiation parameter
σ^c	Stefan-Boltzman constant
k^*	mean absorption coefficient for thermal radiation.