Determining the Optimal Policies for Product Supply Chain Management System

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Abstract - This paper examines extended two-echelon newsvendor problem for a supplier with numerous distributors in a supply chain management system without service level agreements using additive demand model. The paper studies two different specific supply chain management systems: a restricted supply chain management system, where the supplier keeps separate inventory for each distributor and combined inventory system where the supplier holds one central inventory which is shared by different distributors. The single product producer who is also the supplier takes inventory decisions based on random demand of the distributors and the retailers. We observe that the profits of the supplier and the distributors increase for combined inventory system when the product wholesale price is a decision variable for normally distributed random demands.

Index terms: Combined Inventory, Concave function, Customer service level, Restricted Inventory, Supply chain.

I. BACKGROUND TO THE STUDY

Inventory management is about matching supply and demand in supply chain system. As observed in the myth of inventory management system, too much supply puts a significant strain on warehouse allocation, distribution centres and parking space with high cost of management leading to poor investment and irrelevant handling, routing and holding costs. On the other hand, too little supply of produced products could result in delayed deliveries and compromised service level agreements which in turn generate lost sales. This paper therefore studies two different supply management systems for a manufacturer who is also the supplier with several distributors servicing multiple retailers and consumers in supply chain system. The specific management systems are the traditional or restricted supply chain management systems; where the supplier holds separately, reserved inventories for the distributors; and combined inventory case where the supplier holds one central inventory or distribution centre which is shared by the numerous distributors.

For industrial application of the study, this paper examined a Cement Producing Factory (CPF) whose existing inventory decisions are to keep each distributors' inventory physically separate in their warehouse and distribution centres. Hence the paper determines inventory policy and optimal profits of the manufacturer and the distributors by examining the value and benefits of combined inventory over the restricted inventory policies using optimization techniques assuming random demand of the products are normally distributed. The paper therefore establishes, discuss, and compared analytically through suitable available industrial data the value and benefits of combined inventory over restricted inventory policies for supply chain management system.

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II. LITERATURE REVIEW

In the literature, [7] pioneered the benefits and value of inventory pooling system for profit maximisation and the effects of risk pooling on safety stocks due to shortened product life cycles, procurement trend and demand uncertainty. Product Component Commonality (PCC) involves products using same components to replace several distinguishable manufactured products so that safety stock costs are significantly reduced by combining together inventories of various distributors in single warehouse or distribution centres while still maintaining service level requirements. PCC is an outstanding supply chain strategy that can cope and cater for inventory challenges. Among the significant literature on PCC are the works of [4], [2], [1] and [5] who showed total reduction of number of units in stock when component commonality is applied. However, a major drawback of PCC is that it centred heavily on changes in safety stocks to the neglect of the value and usefulness of centralized inventory to the suppliers and distributors in a supply chain management system.

Inventory transhipment (IT) in single echelon supply chains system on the other hand involves transferring produced goods from one distributor, who has surplus stocks as left over, to other distributor of identical supply chain during stock-out, provided the cost of transferring the inventory is not too high. A significant amount of literature on IT believed in complete application of merged policy; [15], [3] and [13] suggested lateral transhipments without inventory pooling where their results only apply to determining transhipment rate among warehouses to the neglect of the benefits of combined inventory.

A good number of literature in supply chain management also investigated the value and usefulness of merging products having identical supply chain system with multiple echelon system. In their work [9] focussed on combining inventory products problem for substitutable products. Substitutable products occur when full substitution without stock-out is allowed. However, they do not give any result on the total inventory level of the items being merged but submits that optimal inventory levels of the substituted items increase for some items after pooling. In their research, [6] based their findings on the value and benefits of merging inventory over restricted inventory system using copulas to model dependence structure between identical

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demands; they assumed multivariate normal demands. [14] in their study of two-channel supply chain system with one supplier and two retailers established the superiority and benefits of pooled policy over the restricted inventory policy. In [12], they also determine a supply chain system consisting of the middlemen and the role of suppliers, Distribution Centres (DCs) and the retailers**.** Their objective was to minimize the location cost, transportation and routing costs and inventory costs through allocation and assignment process. [8] considered the location, production, distribution, and inventory system in order to determine facility locations and their capacity to minimize total network cost. Since product demands are assumed stochastic, the model considers risk pooling effect for both safety stock and running inventory. [11] considered a lost sale recapture model where they assume lost sales to be lost once and finally.

However, this work determines the most profitable inventory policy for both the supplier and the distributors by examining two different inventory systems and exploring a cement factory who supplies several distributors servicing numerous retailers and consumers using optimisation techniques when product wholesale price is a decision variable for normally distributed random demands.

III. MODEL ASSUMPTIONS AND FORMULATION

Let D_i, $(i = 1, 2, \dots, N)$ be the random demand when the product wholesale price w_i is a decision variable. In this formulation, we employ additive demand function when product wholesale price is w_i. For restricted inventory case, we have

$$D_i(w_i) = y(w_i) + \varepsilon_i, \quad (i = 1, 2, \dots, N) \tag{1}$$

where $y(w_i)$ is a decreasing deterministic component of the demand function and ε_i is unpredictable demand component in the range $[A, B]$ having cumulative distribution function $G_i(.)$, probability density function $g_i(.)$ and mean value μ_i. We assume that the deterministic decreasing demand function $y(w_i)$ for the restricted inventory system is given by

$$y(w_i) = a - bw_i; \text{ where } a, b \geq 0, (i = 1, 2, \dots, N). \tag{2}$$

In a similar manner, we denote by $D_p(w_p)$ the combined inventory system for the joint demand of the distributors when the joint product wholesale price is w_p, i.e.,

$$D_p(w_p) = \sum_{i=1}^{N} D_i(w_p) = Ny(w_p) + \varepsilon_p \tag{3}$$

where,

$$\varepsilon_p = \sum_{i=1}^{N} \varepsilon_{i.} \tag{4}$$

The distributors' unpredictable random demand ε_p are continuous, the mean value of ε_p is

$$\mu_p = \sum_{i=1}^{N} \mu_i, \tag{5}$$

with cumulative distribution function $G_p(.)$ and probability density function $g_p(.)$. For nonnegative demands, the feasible distributor's wholesale cost price range $[c, w_{max}]$ is

$$y(w_{max}) + A = a - bw_{max} + A \geq 0 \tag{6}$$

For distributor's wholesale price c, the feasible distributor's wholesale cost price is given by

$$y(c) + A = a - bc + A \geq 0.$$

From the above basic assumptions, we explore a comprehensive and comparative study of restricted and combined inventory systems in order to determine the benefits and value of combined inventory system over restricted inventory system using optimisation techniques.

A. Model Formulation for Unconstrained Optimization Problem

Let $\Pi^{ri}(x_i, w_i)$ denote the supplier's expected profit for any chosen inventory level x_i at a wholesale unit cost price w_i for restricted inventory case. We define the profit function as

$$\Pi^{ri}(x_i, w_i) = \mathrm{E}[w_i \min(x_i, D_i) - v\,(x_i - D_i)^+ - cx_i], \quad (i = 1, 2, \dots, N) \tag{7}$$

where E is the expectation operator taken over the random variables D_i. The first term $\min(x_i, D_i)$ represents units sold at supplier's wholesale price w_i, the second term

$(x_i - D_i)^+ = \max(x_i - D_i; 0)$ corresponds to units salvaged at unit holding cost v and the third term is the unit manufacturing cost.

Since the random demand is continuous and separable in D_i (7) can be expressed as

$$\Pi^{ri}(x_i) = w_i \int_0^{\infty} \min(x_i, D_i) f_i(u)du - v \int_0^{x_i} \min(x_i - D_i) f_i(u)du - cx_i \tag{8}$$

In [10], they proved that (8) is concave and found the optimal quantities using specific distributions and parameters.

From the objective function of (7), the supplier unconstrained optimisation problem is

$$\max_{x_i \geq 0,\, w_i \in [c, w_{max}]} \Pi^{ri}(x_i, w_i), \; (i = 1, 2, \dots, N). \tag{9}$$

Since $\Pi^{ri}(x_i, w_i)$ in (9) are identical and separable, we then concentrate on $i = 1$ defined as

$$\max_{x_i \geq 0,\, w_1 \in [c, w_{max}]} \Pi^{r1}(x_1, w_1), (i = 1) \tag{10}$$

Since (7) is concave in $x_1$and w_1, it follows that the first order conditions for optimality of (10) is necessary and sufficient to determine the optimality value of the inventory level x_1. The optimal value x^{1*} is certainly nonnegative in $[A, B]$. For the existence of x^{1*}, we require specific additional well-defined distribution functions and parameters.

Let $z_1 = x_1 - y(w_1)$ be the stocking factor for $i = 1$. We define by $\Lambda_1(z_1)$ the expected excess stocks and $\Theta_1(z_1)$ the expected shortage stocks with mean value μ_1 having probability density function $g_1(z_1)$ and the cumulative density function $G_1(z_1)$ in $[A, B]$ i.e.

$$\Lambda_1(z_1) = \int_A^{z_1} (z_1 - u) g_1(u)\, du \quad (11)$$

and

$$\Theta_1(z_1) = \int_{z_1}^{B} (u - z_1) g_1(u)\, du \quad (12)$$

In addition, $\Theta_1(z_1)$ and $\Lambda_1(z_1)$ satisfy the equation below

$$\Theta_1(z_1) = \Lambda_1(z_1) - z_1 + \mu_1 \quad (13)$$

Thus, the supplier's expected profit function $\Pi^{r1}(x_{1,}w_1)$ from distributor 1 is given by

$$\Pi^{r1}(x_1, w_1) = \int_A^{x_1 - y(w_1)} [w_1(y(w_1) + u) - v(x_1 - y(w_1) - u)] g_1(u)\, du + \int_{x_1 - y(w_1)}^{B} w_1 x_1 g_1(u) du - c x_1$$

$= (w_1 - c)(y(w_1) + \mu_1) - (c + v)\Lambda_1(x_1 - y(w_1)) - (w_1 - c)\Theta_1(x_1 - \quad y(w_1))$. Therefore,

$$\Pi^{r1}(x_1, w_1) = \mathrm{I}(w_1) - \mathrm{L}(x_1, w_1) \quad (14)$$

where $\mathrm{I}(w_1) = (w_1 - c)(y(w_1) + \mu_1)$ and the loss function $\mathrm{L}(x_1, w_1)$ is given by

$$\mathrm{L}(x_1, w_1) = (c + v)\Lambda_1(x_1 - y(w_1)) + (w_1 - c)\Theta_1(x_1 - y(w_1)).$$

Hence, the expected profit for restricted inventory system is the sum of the riskless profit and the expected loss function due to uncertainty.

[14] invoke the properties of $\Pi^{r1}(x_1, w_1)$ to find the optimal quantity using specific distributions and parameter. In addition, the distributor's wholesale price w_1 is given by

$$x_1^*(w_1) = y(w_1) + G_1^{-1}(t_1), \text{ where } t_1 := \frac{w_1 - c}{w_1 + v} \quad (15)$$

For the uniqueness of optimal solution, we define and impose the failure rate defined by

$$\lambda(z_1) = \frac{g(z_1)}{1 - G(z_1)} \quad (16)$$

where $1 - G(z_1)$ is the reliability function of the stocking component.

Given the mark-up price for the distributors to be $p_1 = w_1 + m_1$, the distributor's profit is

$$\pi^{r1}(x_1) = E[m_1 \min(x_1, D_1)] \quad (17)$$

For combined inventory management system, the company's' expected joint profit for x_p at w_p is given by

$$\Pi^p(x_p, w_p) = E\left[w_p \min\left(x_p, D_p(w_p)\right) - v\left(x_p - D_p(w_p)\right)^+ - c x_p\right] \quad (18)$$

We define additional distribution functions $\Lambda_p(z_p)$ and $\Theta_p(z_p)$ with mean μ_p for combined inventory system with inventory stock z_p and the unpredictable random demand is ε_p. Thus

$$\Pi^p(x_P, w_p) = I(w_p) - L(x_P, w_p) \quad (19)$$

Due to non-negativity property of demands, supplier unconstrained optimisation problem is

Maximize $\Pi^p(x_p, w_p)$

$x_p, w_p \in [c, w_{max}]$

For distributor's wholesale cost price w_p and inventory level x_p, $\Pi^p(x_p, w_p)$ is concave in x_p and w_p. Thus given the distributor's wholesale price w_p, then

$$x^{p*}(w_p) = N y(x_p) + G_p^{-1}(t_p), \text{ where } t_p := \frac{w_p - c}{w_p + v} \quad (21)$$

For the uniqueness of optimal solution of problem (20), we define increasing failure rate function as $\lambda(z_p) = \frac{g(z_p)}{1 - G(z_p)}$, where $1 - G(z_p)$ is the reliability function of z_p.

Given combined distributor mark-price $p_p = w_p + m_p$, the joint profit of the distributors is

$$\pi^p(x_p) = E[m_p \min(x_p, D_p)].$$

B. Comparison of Inventory Systems for Unconstrained Optimization Problem

To compare the results of combined versus restricted inventory systems, we invoke the work of [14].

Thus, under certain basic properties and assumptions on ε_i and ε_p the supplier and the distributors will always prefer combined inventory policy rather than the restricted inventory system. Hence it is beneficial and of high advantage for the supplier and the distributors to operate combined inventory system rather than restricted inventory system while using additive demand model.

IV. NUMERICAL APPLICATION OF THE MODEL TO A PRODUCTION COMPANY

This section gives practical application of the model. The data used in this study represent daily demand of cements by three major distributors who buy from the factory directly and sell to retailers or the consumers on weekly basis within a year. The data cover a period of 52 weeks. The total production capacity of the factory as at the time of the study is 600 bags of cements per truck with expected trucks of 500 per day. Table1 below gives the base parameters.

Table 1: The Base Parameters and Decision Variables

Total stock of inventory level of cements	x
Random demand of cements	D
Manufacturing cost price per bag of cement	c
Distributor wholesale price per bag of cement	w
Distributor retail price per bag of cement	p
Distributor holding cost per bag of cement	v

Analysing the data, we let $R_i, (\, i = 1, 2, 3)$ represents the three major distributors in the factory for the restricted inventory and R_p represents combined inventory case for the joint distributors. From the data, we obtain the following classifications through MATLAB 2010a programme.

Table 2: Summary of Base Parameters and Decisions Variables from the Data

Parameter	R_1	R_2	R_3	R_p
N	52	52	52	R
x	101000	151000	51000	300000
D	80038	120057	40019	240115
Min.	70200	105300	35100	210600
Max.	89800	134700	44900	269400
μ	80038.46	120057.69	40019.23	240115.38
σ	5879.447	8819.17	2939.72	17638.34
Profit	7756360	12079540	3433180	25940300

V. DISCUSSION OF RESULTS

The unit of measurement used in this study are naira and kobo. From Table 2 above, the total profit for the combined inventory system is higher than the profit from joint restricted inventories case. From Table 2 we then discuss and compare profits of the suppliers against other decision variables and parameters.

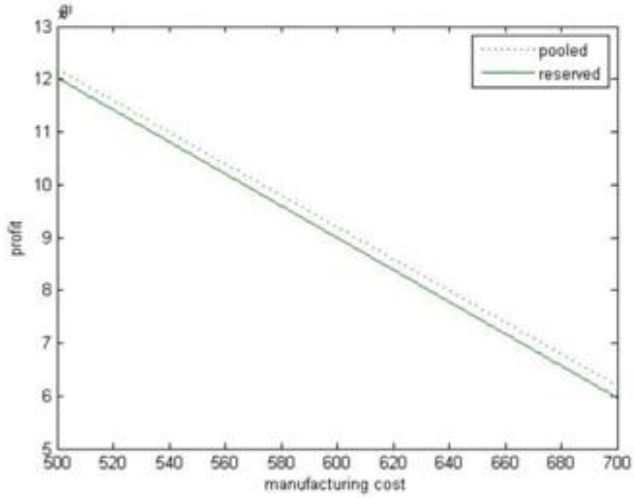

Fig. 1: Graph of Profit Function for Inventory Systems Versus the Manufacturing Cost.

Fig. 1 shows the relationship between the profits and manufacturing cost for the two-inventory management systems. From the graph above, the profit from combined inventory of the distributors is slightly higher than the profit from the restricted inventory. The total safety stock for the distributors in the restricted inventory case is higher than total safety stock for the combined inventory case. Hence, it is beneficial and of high advantage for the supplier to operate combined inventory system rather than the restricted inventory system.

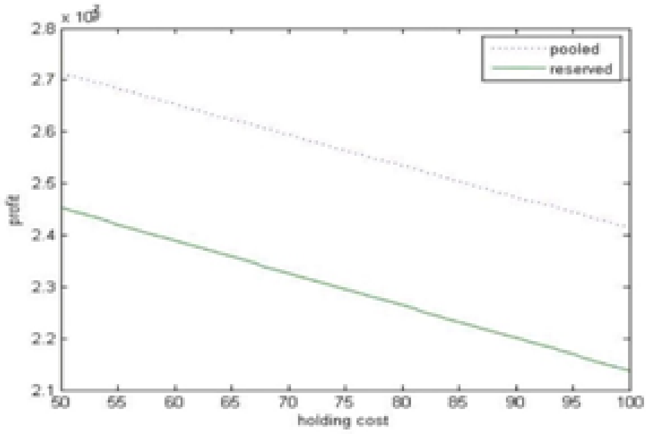

Fig. 2: The profit Function for the Inventory Systems Versus the Holding Cost.

The profit of the distributors from combined inventories is much higher than the profit from separate inventories when profits are compared to supplier holding cost. Because of the advantage of inventory pooling, the factory does not need to keep as much stock as in the restricted inventory case. For equal distributors' wholesale price, the distributors will prefer combined rather than restricted inventory case.

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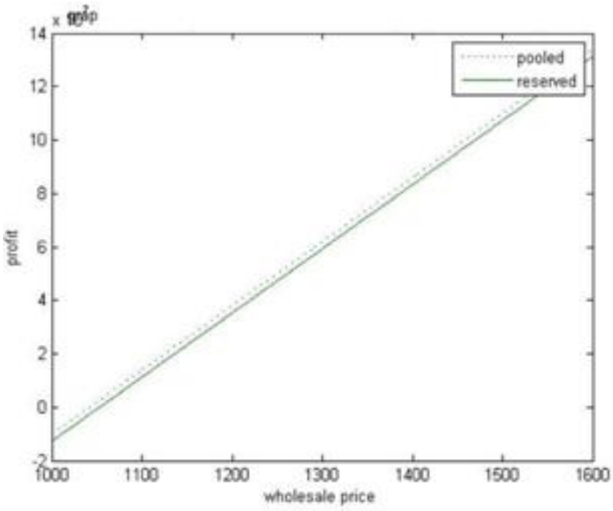

Fig. 3: Graph of profit Function of the Inventory Systems Versus the Wholesale Price.

From the graph above, the profit function is slightly higher in combined inventory than in the restricted system. For equal distributors' wholesale price, the distributors will prefer combined rather than restricted inventory case. Hence it is beneficial and of high advantage for the factory which is the supplier, and the distributors to operate combined inventory system rather than restricted inventory system.

VI. CONCLUSION AND RECOMMENDATIONS

The graphs above showed the relationship between profits of the supplier when other decision variables are held constant for normally distributed random demands. As observed from the graphs, the profit function for combined inventory is higher than the profit for restricted inventory. Thus, for both inventory systems, the number of standard deviation depends on the relationship between w and v. For equal distributors' wholesale prices, the distributors prefer combined rather than restricted inventory system. Therefore, because of combine inventory, the supplier does not need to keep as much stock as observed in the restricted inventory case. Hence, in all cases, the graphs showed that in single product supplier, the cement factory, and the distributors will always prefer combined inventory system to restricted inventory system. However, for identically and normally distributed random variables, with same service level requirement, the single supplier gets more benefit from combined inventory than restricted inventory system. In the absence of service level requirement, the total optimal inventory level and the mean demand is decreased when inventory is combined and increased when inventory is restricted. Due to benefits of inventory pooling, the distributors and the supplier will always prefer combined inventory policy.

From the analyses and graphs of the functions above, a lot of results were encountered in establishing the superiority and benefits of combined inventory policy over restricted inventory policy. A more complete simulation study of the two supply chain systems is required in future to capture full understanding of supply chain management system policies.

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