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Performance Analysis and Modeling of MIMO Systems

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Abstract

In this paper, various channel estimation, interpolation and equalization techniques used in the analysis of MIMO configurations or formats are compared and the technique with the optimum performance determined. The channel estimation of these configurations were determined by modelling and simulating them in a wireless environment using MATLAB software. The figure of Merits used are the BER and MSE as a function of the SNR. The study revealed that MIMO is a more energy efficient technique since it achieved a good BER performance at lower transmit SNR, when compared to the MISO and SISO which requires higher SNR to achieve at same BER performance. This is as a result of the diversity and multiplexing gain experienced in the multiple antenna techniques using the STBC.

Keywords: Multiple Input Multiple Output (MIMO), Multiple Input Single Output (MISO), Single Input Single Output (SISO), Least Square Channel Estimation (LS), Minimum Mean Square Error Channel Estimation (MMSE), Space-Time Block Code (STBC)

INTRODUCTION

The high demand for bandwidth places a great responsibility on the shoulders of Communication Engineers to design antennas with high bandwidth. From antenna theory, there is a limitation to the bandwidth a single antenna can give, hence the need to deploy more complex techniques like MIMO. MIMO systems as the name implies, consists of multiple antennas at the input and output. It is a smart antenna technology which improves the performance in a communication system, without any extra cost on the communication resources. With MIMO, the capacity of a communication system increases linearly with the number of antennas, thereby achieving an increase in spectral efficiency, without requiring more resources in terms of bandwidth and power [8]. There are a number of different MIMO configurations or formats that can be used in antenna technology. These are termed SISO, SIMO, MISO and MIMO. These different MIMO formats offer different advantages and disadvantages - these can be balanced to provide the optimum solution for any given application [16].
MIMO technology has two main objectives which it aims to achieve: high spatial multiplexing gain and high spatial diversity [14]. To attain spatial multiplexing, the system is made to carry multiple data stream over one frequency, simultaneously-form multiple independent links (on same channel) between transmitter and receiver to communicate at higher data rates [15]. In low SNR environment, spatial diversity techniques are applied to mitigate fading and the performance gain is typically expressed as diversity gain (in dB) [5]; for higher SNR facilitates the use of spatial multiplexing (SM), i.e., the transmission of parallel data streams, and information theoretic capacity in bits per second per Hertz (bits/s/Hz) is the performance measure of choice [5]
The achievement of diversity and multiplexing is indeed very important for a reliable, high capacity and efficient MIMO system. However, studies have shown that the advantages of multiplexing and diversity cannot be realized fully simultaneously [8, 15]. There has to be a compromise, as one is achieved to its fullest at the expense of the other. Therefore, the aim of this research work is to reach that compromise, for optimum efficiency. Using bandwidth efficient modulation and coding techniques to achieve the benefits of diversity, there is an assumption that the data rate is constant, and the SNR increases as the BER decreases. While for multiplexing, a constant BER is assumed, and the data rate increases as the SNR increases.

MIMO CHANNEL MODEL

A wireless multipath fading channel can be modelled as shown in equation 1

$$h(t,\tau)=\sum_{k}\gamma_k(t)c(\tau-\tau_k) \qquad (1)$$

Where $h(t,\tau)$ is the baseband impulse response of the channel, τ_k represents the different delay paths, $c(t)$ represents the shaping pulse, $\gamma_k(t)$ represents the complex independent amplitude of the kth path. $\gamma_k(t)$ can be characterized using different statistical distributions, depending on the channel characteristics.

The MIMO transmission channel can be modelled as an N X M matrix as shown in equation (2); where $h_{NM}(t,\tau)$ is the time varying channel impulse response between the *Mth* transmit and *Nth* receive antennas [27].

$$\mathbf{h}(t,\tau)=\begin{bmatrix} h_{11}(t,\tau) & h_{12}(t,\tau) & \cdots & h_{1M}(t,\tau) \\ h_{21}(t,\tau) & h_{22}(t,\tau) & \cdots & h_{2M}(t,\tau) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1}(t,\tau) & h_{N2}(t,\tau) & \cdots & h_{NM}(t,\tau) \end{bmatrix} \qquad (2)$$

A 3X3 MIMO system with channel paths is shown in figure 1, for a better understanding.

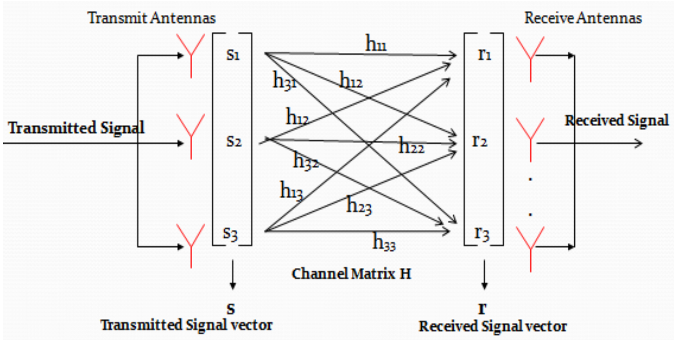

Figure 1: 3X3 MIMO System Showing Channel Paths

METHODS OF CHANNEL ESTIMATION

(i) Pilot Symbol Aided Channel Estimation

Known pilot symbols are transmitted, and at the receiver, the received signal and the known transmitted pilot symbols are used to generate an estimate of the channel. The channel can be accurately estimated using pilot symbols, yielding good performance. However, pilots occupy bandwidth, thereby making the system less bandwidth efficient. Also there is an overhead cost required in transmitting pilots or training symbols along with data symbols. Assuming an OFDM system with orthogonal subcarriers, without inter channel interference, the training symbols (or pilots) for K subcarriers can be represented by a diagonal matrix $\mathbf{X}$ as shown in equation (3) [10];

$$\mathbf{X} = \begin{bmatrix} X[0] & 0 & \cdots & 0 \\ 0 & X[1] & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & X[K-1] \end{bmatrix} \quad (3)$$

Where k = 0, 1, 2… N-1, $X[k]$ represents a pilot tone at the kth subcarrier, with mean $E\{X[k]\}$ = 0 and variance $Var\{X[k]\} = \sigma_x^2$, and equation 4 is shown as.

$$\mathbf{Y} \triangleq \begin{bmatrix} Y[0] \\ Y[1] \\ \vdots \\ Y[K-1] \end{bmatrix} = \begin{bmatrix} X[0] & 0 & \cdots & 0 \\ 0 & X[1] & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & X[K-1] \end{bmatrix} \begin{bmatrix} H[0] \\ H[1] \\ \vdots \\ H[K-1] \end{bmatrix} + \begin{bmatrix} N[0] \\ N[1] \\ \vdots \\ N[K-1] \end{bmatrix} \quad (4)$$

Where $\mathbf{Y}$ is the received signal vector, $\mathbf{H}$ is the channel response vector, and $\mathbf{N}$ is the noise vector with mean $E\{N[k]\}$=0 and variance $Var\{N[k]\}=\sigma_z^2$ [10].

(ii) Least Square Channel Estimation

The least Square Estimator has the best linear unbiased channel estimation in the presence of Additive White Gaussian Noise (AWGN). The least square channel estimation method calculates the channel estimate $\hat{\mathbf{H}}$ by minimizing the cost function as shown in equation (5) [10]

$$J(\hat{\mathbf{H}}) = \left\| \mathbf{Y} - \mathbf{X}\hat{\mathbf{H}} \right\|^2$$
$$J(\hat{\mathbf{H}}) = (\mathbf{Y} - \mathbf{X}\hat{\mathbf{H}})^{\mathbf{H}}(\mathbf{Y} - \mathbf{X}\hat{\mathbf{H}})$$
$$J(\hat{\mathbf{H}}) = \mathbf{Y}^{\mathbf{H}}\mathbf{Y} - \mathbf{Y}^{\mathbf{H}}\mathbf{X}\hat{\mathbf{H}} - \hat{\mathbf{H}}^{\mathbf{H}}\mathbf{X}^{\mathbf{H}}\mathbf{Y} + \hat{\mathbf{H}}^{\mathbf{H}}\mathbf{X}^{\mathbf{H}}\mathbf{X}\hat{\mathbf{H}} \quad (5)$$

Differentiating the function $J(\hat{\mathbf{H}})$ with respect to $\hat{\mathbf{H}}$, and equating to zero, we have

$$\frac{\partial J(\hat{\mathbf{H}})}{\partial \hat{\mathbf{H}}} = -2(\mathbf{X}^{\mathbf{H}}\mathbf{Y})^* + 2(\mathbf{X}^{\mathbf{H}}\mathbf{X}\hat{\mathbf{H}})^* = 0$$
$$\mathbf{X}^{\mathbf{H}}\mathbf{X}\hat{\mathbf{H}} = \mathbf{X}^{\mathbf{H}}\mathbf{Y} \quad (6)$$
$$\hat{\mathbf{H}}_{LS} = (\mathbf{X}^{\mathbf{H}}\mathbf{X})^{-1}\mathbf{X}^{\mathbf{H}}\mathbf{Y} = \mathbf{X}^{-1}\mathbf{Y}$$

This gives the least square channel estimation equation. Where $\mathbf{X}$ represents the pilots, and $\mathbf{Y}$ is the received signal. According to [10], assuming $\mathbf{X}$ to be diagonal, and due to the cancellation of inter-channel interference, the least square channel estimates for each subcarrier can be given as shown in equation(7)

$$\hat{H}_{LS}(k) = \frac{Y(k)}{X(k)}, \quad (7)$$

Where k = 0, 1, 2… N-1

The complexity increases as number of transmit antennas and pilot symbols increases. Therefore, to account for the BER, more pilot symbols are required as the number of transmit antenna increases.

The MSE of the Least Square channel estimate is given as shown in equation (8) [10];

$$MSE_{LS} = E\{(\mathbf{H} - \hat{\mathbf{H}}_{LS})^{\mathbf{H}}(\mathbf{H} - \hat{\mathbf{H}}_{LS})\}$$
$$MSE_{LS} = E\{(\mathbf{H} - \mathbf{X}^{-1}\mathbf{Y})^{\mathbf{H}}(\mathbf{H} - \mathbf{X}^{-1}\mathbf{Y})\}$$
$$MSE_{LS} = E\{(\mathbf{X}^{-1}\mathbf{Z})^{\mathbf{H}}(\mathbf{X}^{-1}\mathbf{Z})\}$$
$$MSE_{LS} = E\{\mathbf{Z}^{\mathbf{H}}(\mathbf{X}\mathbf{X}^{\mathbf{H}})^{-1}\mathbf{Z}\}$$
$$MSE_{LS} = \frac{\sigma_z^2}{\sigma_x^2} \quad (8)$$

The MSE is inversely proportional to the SNR, which could result to noise enhancements, when channel is in deep nulls. The LS method is the most widely used channel estimation technique due to its simplicity [3].

(iii) Minimum Mean Square Error Channel Estimation

As indicated in [4], the MMSE channel estimation technique performs better than the LS, giving about 10 to 15dB gain in SNR, for the same MSE of channel estimate. Although the MMSE has a good performance, it still has a disadvantage, which is its system complexity, which can result in high cost and power consumption. To reduce this system complexity, a low rank approximation can be applied, using the Singular Value Decomposition (SVD) as discussed in [4, 29]. As depicted in [24], estimation techniques in the frequency domain produces errors in time varying channels. Hence, time domain techniques could be employed. The MMSE can exploit the time diversity of the time varying channel, but with some degree of interference in higher modulation orders.

Therefore, to reduce the interference, the MMSE with Successive Detection could be employed.

In the MMSE channel estimation, the mean square error between the exact channel response and the estimated channel response is minimized [3, 24].The MMSE estimate can be calculated by employing the weight matrix **W**, where the MMSE estimate is $\hat{\mathbf{H}} \triangleq \mathbf{W}\tilde{\mathbf{H}}$. The MSE between the actual channel **H** and the channel estimate $\hat{\mathbf{H}}$ is given as [10]:

$$J(\hat{\mathbf{H}}) = E\left\{\|\mathbf{e}\|^2\right\} = E\left\{\left\|\mathbf{H} - \hat{\mathbf{H}}_{LS}\right\|^2\right\} \quad (9)$$

The MMSE actually minimizes the MSE in equation (9). According to the principle of orthogonality, the estimation error vector $e = H - \hat{H}$ has to be orthogonal to $\hat{H}$, for the error to be minimized.

Therefore;

$$\begin{aligned} E\{\mathbf{e}\tilde{\mathbf{H}}^H\} &= E\left\{(\mathbf{H} - \hat{\mathbf{H}})\tilde{\mathbf{H}}^H\right\} \\ &= E\left\{(\mathbf{H} - \mathbf{W}\tilde{\mathbf{H}})\tilde{\mathbf{H}}^H\right\} \\ &= E\{\mathbf{H}\tilde{\mathbf{H}}^H\} - \mathbf{W}E\{\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\} \\ &= \mathbf{R}_{\mathbf{H}\tilde{\mathbf{H}}} - \mathbf{W}\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}} = 0 \end{aligned} \quad (10)$$

Where $\tilde{\mathbf{H}}$ is the LS channel estimate, $\mathbf{H}$ is the actual channel response vector, $\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}}$ is the auto-correlation of $\tilde{\mathbf{H}}$, and $\mathbf{R}_{\mathbf{H}\tilde{\mathbf{H}}}$ is the cross-correlation matrix between $\mathbf{H}$ and $\tilde{\mathbf{H}}$. The correlation matrix of the channel is assumed to be unknown at the receiver. Equating to zero and solving for **W**.

Therefor;

$$\mathbf{W} = \mathbf{R}_{\mathbf{H}\tilde{\mathbf{H}}}\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}}^{-1} \quad (11)$$

Therefore, MMSE channel estimate $\hat{\mathbf{H}}$, can be given as [12];

$$\begin{aligned} \hat{\mathbf{H}} &= \mathbf{W}\tilde{\mathbf{H}} = \mathbf{R}_{\mathbf{H}\tilde{\mathbf{H}}}\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}}^{-1}\tilde{\mathbf{H}} \\ \hat{\mathbf{H}}_{\mathbf{MMSE}}(\mathbf{k}) &= \mathbf{R}_{\mathbf{HH}}(\mathbf{R}_{\mathbf{HH}} + \sigma^2\mathbf{I}_{\mathbf{N}\times\mathbf{N}})^{-1}\hat{\mathbf{H}}_{\mathbf{LS}}(\mathbf{k}) \end{aligned} \quad (12)$$

Where $\mathbf{R_{HH}}$ is the autocorrelation or covariance matrix of **H.**

Given by

$$\mathbf{R_{HH}} = E[\mathbf{HH}^*] \quad (13)$$

As shown in the equation, the MMSE requires prior knowledge of the channel covariance matrix and noise variance. This is a drawback to the MMSE because the receiver also needs to estimate these since they are not available a prior.

METHODOLOGY

The basic steps taken in the modelling and simulation of a wireless communication system, with channel estimation in this paper are described in this section. To run the simulations, a transmitter, channel and receiver are required. The figures of merit used are the BER and MSE as a function of the SNR. The BER versus the SNR was plotted on a two dimensional graph using the plot tool in Matlab. Table 1 shows system initialization parameters employed in this paper.

Table 1: System Simulation Parameters and Values

System parameters	**Value**
Simulation Runs	100000
Data-Length	128
Frame-Length	64 for QPSK
SNR Values(dB)	0 to 30
Channel Type	Multipath Channel
Number of Channel Taps	5
Cyclic Prefix	10
Pilot-Data Ratio	1:1, 1:3, 1:7 and 1:15
Modulation Techniques	QPSK
Antenna Configurations	SISO, MISO(2X1), MIMO (2X2)

RESULTS AND ANALYSIS

BER Comparison for SISO LS with Different Pilot-Data Ratios

The interpolated SISO LS channel estimates, with different pilot-data ratio are used for equalization and detection of the transmitted data. The BER is plotted against the SNR as shown in Figure 2.

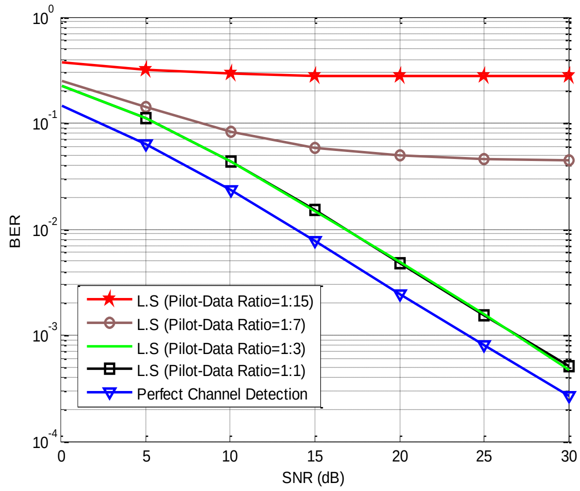

Figure 2: BER Comparison for LS Channel Estimation with Different Pilot-Data Ratio.

For the SISO LS channel estimation shown in Figure 2, it can be seen that performing interpolation of the channel estimates with pilot-data ratio of 1:1 (32 pilots) and 1:3 (16 pilots), the BER performance is good. Therefore the pilot-data ratios are sufficient to interpolate the channel estimates. But with a pilot-data ratio of 1:7 (8 pilots) and 1:15 (4 pilots), there is a severe degradation in the BER even with increase in SNR. This could be as a result of the pilot-data ratio being insufficient to interpolate the channel estimate, thereby giving inaccurate results when used for equalization and detection.

BER Comparison for SISO MMSE with Different Pilot-Data Ratios

The interpolated SISO MMSE channel estimates, with different pilot-data ratio are used for equalization and detection of the transmitted data. The BER is plotted against the SNR as shown in Figure 3.

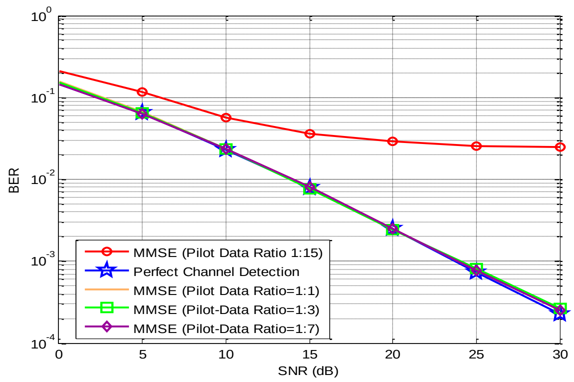

Figure 3: BER Comparison for SISO MMSE Channel Estimation with Different Pilot-Data Ratio.

For the MMSE channel estimation shown in Figure 3, it can be seen tha5 performing interpolation of the channel estimates with pilot-data ratio of 1:1 (32 pilots), 1:3 (16 pilots), and 1:7 (8 pilots), the BER performance is better than the LS. Therefore it could be said that with the MMSE, the pilot-data ratio of 1:1, 1:3, and 1:7 is sufficient to interpolate the channel estimate (better than the LS where only 1:1 and 1:3 is sufficient). But with a pilot-data ratio of 1:15 (4 pilots), there is a severe degradation in the BER even with increase in SNR. This could be as a result of the pilot-data ratio of 1:15 (only 4 pilots) being insufficient to interpolate channel estimate, thereby giving inaccurate results when used for equalization and detection.

BER Comparison for MISO LS Channel Estimation with Different Pilot-Data Ratio

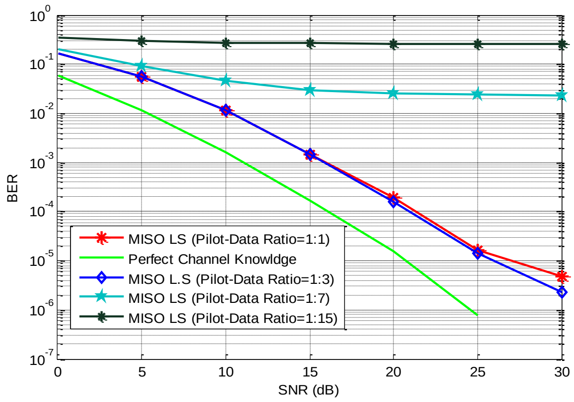

Figure 4: BER Comparison for MISO (2X1) LS Channel Estimation with Different Pilot-Data Ratio.

For the MISO LS channel estimation shown in Figure 4, it can be seen that the pilot-data ratio of 1:1, and 1:3, are sufficient to interpolate the channel estimate accurately. But with a pilot-data ratio of 1:7 and 1:15, due to errors in the interpolated channel estimates, the system begins to experience a severe degradation in the BER even with increase in SNR, as a result, the pilot-data ratio of 1:7 and 1:15 can be said to be insufficient to interpolate the LS channel estimate accurately (like observed in the SISO).

BER Comparison for MIMO LS Channel Estimation with different Pilot-Data Ratio

The BER for 2X2 MIMO STBC using LS channel estimation is shown in Figure 5. It can be seen from Figure 5 that for pilot-data ratio of 1:1 and 1:3, the BER performance is very good, achieving an allowable BER threshold of 10^{-3} with a low SNR value of about 9dB. But for pilot-data ratio of 1:7 and 1:15, the performance is poor. This further confirms (like in SISO and MISO 2X1) that the pilot-data ratio of 1:3 is the optimum pilot-data insertion ratio for the LS, and 1:7 and 1:15 are insufficient to interpolate the channel estimates without errors.

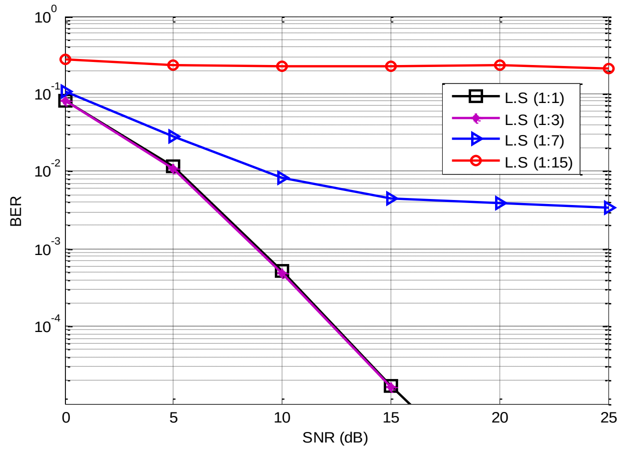

Figure 5: BER for MIMO (2X2) LS Channel Estimation with Different Pilots-Data Ratio

BER Comparison for MIMO MMSE Channel Estimation with different Pilot-Data Ratio

The BER for 2X2 MIMO STBC using MMSE channel estimation is shown in Figure 6. It can be seen from Figure 6 that for pilot-data ratio of 1:1, 1:3 and 1:7, the BER performance is very good, achieving an allowable BER threshold of 10^{-3} with a low SNR value of about 4dB. But for pilot-data ratio of 1:15, the performance is poor. This further confirms that the pilot-data ratio of 1:7 is the optimum pilot insertion ratio for the MMSE, and 1:15 is insufficient to interpolate the channel estimates without errors. Therefore the MMSE (with optimum pilot-data ratio of 1:7) could be said to be more bandwidth efficient than the LS (with optimum pilot-data ratio of 1:3).

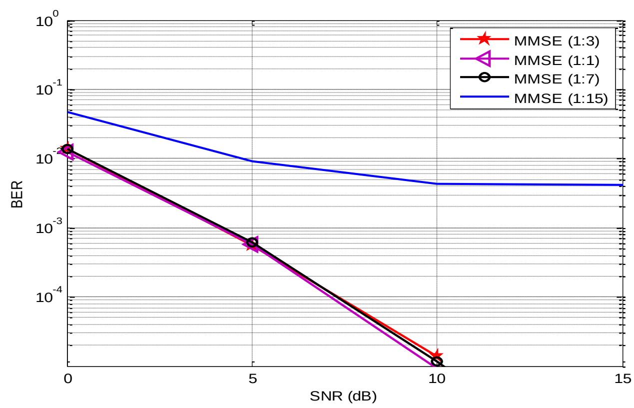

Figure 6: BER for MIMO (2X2) MMSE Channel Estimation with Different Pilots-Data Ratio

CONCLUSION

In this paper, various channel estimation, interpolation and equalization techniques are compared, and the technique with the optimum performance is determined. The LS estimation and MMSE channel estimation techniques are compared. The LS is computationally less complex because of the fewer mathematical operations required, than the MMSE which has more computational complexity. The LS gives a good MSE and BER performance, but requires more SNR (transmit power) to achieve the same performance as the MMSE. The MMSE on the other hand is more resistance to noise than the LS, and gives a better performance than the LS. On system complexity and operational cost, the MMSE requires a higher operational cost than the LS. The MMSE also requires a prior knowledge of the noise variance. The different pilot-data insertion ratios are examined, the minimum amount of pilots that are sufficient to accurately interpolate the channel estimates are determined. Using a pilot-data ratio of 1:1 is bandwidth inefficient, because it gives similar performance with a pilot-data ratio of 1:3. For the LS, the optimum pilot data ratio is 1:3, because with pilot-data ratio of 1:7, the LS degrade in performance. The MMSE on the other hand is able to minimize the errors of the LS, by giving a good performance with a pilot-data ratio of 1:7. Therefore the MMSE is more bandwidth efficient than the LS. Channel estimation in SISO, MISO and MIMO are compared. The MIMO is a more energy efficient technique, achieving a good BER performance at lower transmit SNR, when compared to the MISO and SISO which requires higher SNR to achieve same BER performance. The MIMO gives the optimum performance, followed by the MISO and SISO. This is as a result of the diversity and multiplexing gain experienced in the multiple antenna techniques using the STBC.

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