Dynamic Response of Inclined Isotropic Elastic Damped Rectangular Mindlin Plate resting on Pasternak Foundation under a Moving Load

Agarana M.C., Gbadeyan J.A., and Ajayi O.O.

Abstract—**In this article, the dynamic behavior of inclined damped rectangular Mindlin plate under the influence of moving load along the mid-plate on the plate surface is considered. A numerical method is used to solve the non-dimensional form of the resulting coupled partial differential equations. The desired solutions are obtained with the aid of computer program developed in conjunction with MATLAB. It is observed that the response amplitude of the plate is affected significantly by the foundation moduli. Also, the effects of the shear deformation, rotatory inertia, damping and angle of inclination of the plate, to the horizontal, are noticeable.**

***Index Terms*—Pasternak foundation, Damped Inclined Mindlin plate, Moving Load, Dynamic response.**

I. Introduction

AN inclined rectangular Mindlin plate is a plate set at an angle, not perpendicular to a horizontal plane. However, the work done is the same: Work = Force × Distance, and the distance is increased, whereas the force is decreased [3,9]. In Elementary Physics, an object placed on a tilted surface (inclined plane) will often slide down the surface. The greater the tilt of the surface (i.e. the angle of inclination), the faster the rate at which the object will slide down it [10,11]. According to Newton's laws of motion, a moving load on an inclined plane will continue to slide down the plane if there is no applied force to balance the forces acting on it, especially if the surface is frictionless or with minimal friction. There are always, at least, two forces namely: the force of gravity and the normal force, acting upon the moving load positioned on an inclined plate [1,10]. The force of gravity acts in a downward direction, while the normal force acts in a direction perpendicular to the surface [3,8]. An inclined plane problem is in every way like any other net force problem with the sole exception that the surface has been tilted. An inclined plane therefore can be transformed into the form with which we are more comfortable, as illustrated in figure 2. After this transformation, we can ignore the force of gravity since it has been replaced by its two components [11]. We can now solve for the net force and the acceleration. For a load moving up the inclined plate, the applied force must be greater than the component of its weight moving down the inclined plate, to avoid sliding down [10,11]. Gbadeyan and Dada extended their works recently by considering the dynamic response of a Mindlin elastic rectangular plate subjected to distributed moving load, but neglected the effect of damping [6,7]. Also most author did not consider the possibility of the plate being inclined or resting on any elastic foundation.[1,2,3,6] The present paper consider the dynamic response of damped Mindlin elastic type of plates resting on a Pasternak foundation under the influence of a partially uniform distributed moving load [4,5,8,11,12,13]. Finite difference technique is used to solve the transformed non-dimensional form of the coupled differential equations governing the motion of such plates [13].

Manuscript received November 30, 2015; revised December 17, 2015. This work was supported by Covenant University.

M. C. Agarana is with the Department of Mathematics, Covenant University,Ota,OgunState,Nigeria.email:michael.agarana@covenantuniversity.edu.ng.

J. A. Gbadeyan is with the Mathematics Department. University of Ilorin, Kwara State, Nigeria. e-mail: j.agbadeyan @yahoo.com

O. O. Ajayi is with the Department of Mechanical Engineering, Covenent University Ota, Ogun State, Nigeria. e-mail: oluseyi.ajayi@covenantuniversity.edu.ng.

II. Governing Equations

The set of dynamic equilibrium equations which governs the behavior of damped inclined Mindlin plate supported by Pasternak foundation, and traversed by a partially distributed moving load can be written as follows [6,12,13]:

$$Q_x - \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = \frac{\rho h^3}{12}\frac{\partial^2 \psi_x}{\partial T^2} + \frac{\rho_L h_1^3}{12}\left[\begin{array}{l} \dfrac{\partial^2 \psi_x}{\partial T^2} + u\dfrac{\partial^2 \psi_x}{\partial x \partial T} + \dfrac{u}{D(\upsilon^2-1)}\left\{\dfrac{\partial M_x}{\partial T} + u\dfrac{\partial M_x}{\partial x}\right\} - \\ \dfrac{u\upsilon}{D(\upsilon^2-1)}\left\{\dfrac{\partial M_y}{\partial T} + u\dfrac{\partial M_y}{\partial y}\right\} \end{array}\right] B \tag{1}$$

ISBN: 978-988-14047-6-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

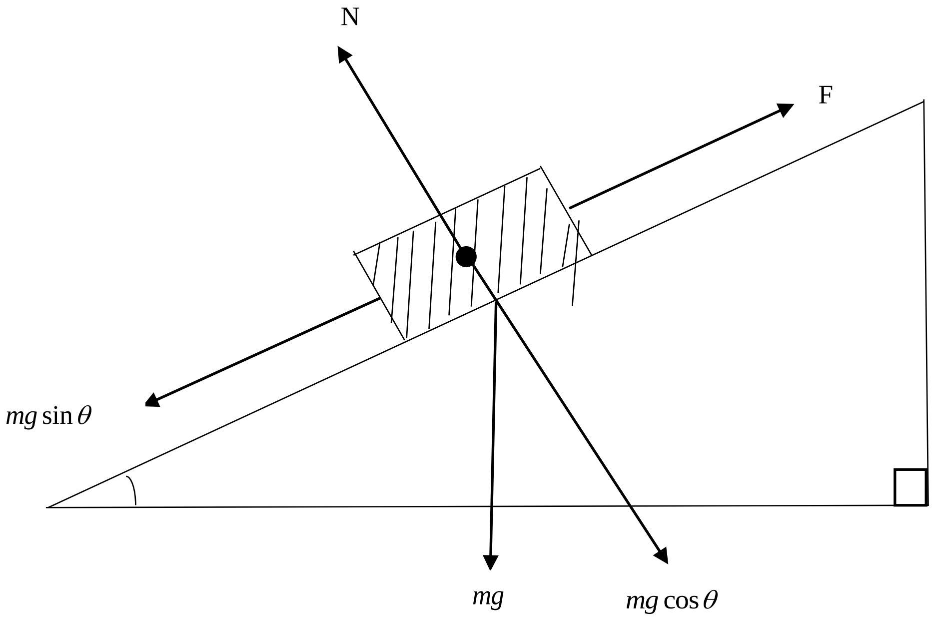

Fig. 1. Diagram of moving load on an inclined plane

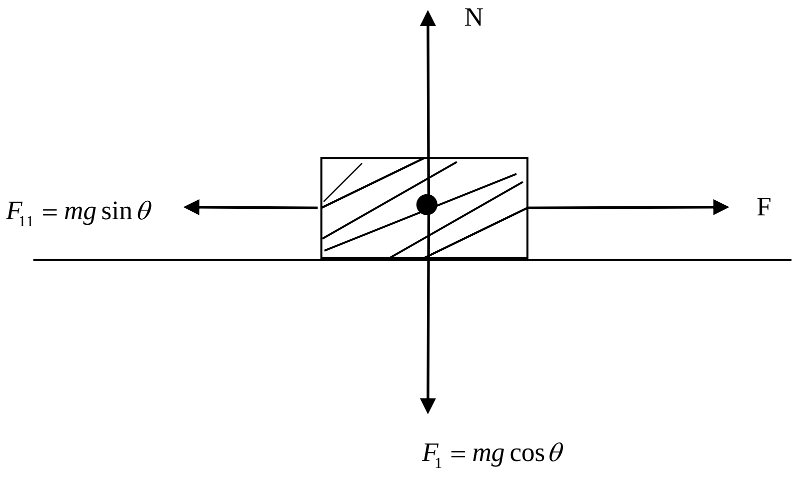

Fig 2.Diagram of a transformed inclined plane to a flat plane

ISBN: 978-988-14047-6-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$Q_y - \frac{\partial M_{xy}}{\partial y} - \frac{\partial M_y}{\partial y} = \frac{\rho h^3}{12}\frac{\partial^2 \psi_x}{\partial T^2} + \frac{\rho_L h_1^3}{12}\left[\begin{array}{l}\frac{\partial^2 \psi_y}{\partial T^2} + u\frac{\partial^2 \psi_x}{\partial x \partial T} + \frac{u}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial T} + u\frac{\partial M_x}{\partial x}\right\} - \\ \frac{u\upsilon}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial T} + u\frac{\partial M_x}{\partial x}\right\}\end{array}\right]B \tag{2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + kW + (M_f - \rho h)\gamma\frac{\partial \Delta_t}{\partial t} + \frac{M_L}{A}[g\cos\theta + \frac{\partial \Delta_t}{\partial t} + u\frac{\partial \Delta_t}{\partial t} + G_1\left(\frac{\partial \Delta_x}{\partial x} + \frac{\partial \Delta_y}{\partial y}\right) + u\left\{\frac{\partial \psi_x}{\partial t} + \frac{u}{D(\upsilon^2-1)}M_x - \frac{u\upsilon}{D(\upsilon^2-1)}M_y\right\} - \frac{u}{2Gh}\left\{\frac{\partial Q_x}{\partial t} + u\frac{\partial Q_x}{\partial x}\right\}B] = \rho h\frac{\partial^2 W}{\partial t^2} - M_L g\sin\theta \tag{3}$$

$$M_x = -D\left(\frac{\partial \psi_x}{\partial x} + \upsilon\frac{\partial \psi_y}{\partial y}\right) \tag{4}$$

$$M_y = -D\left(\frac{\partial \psi_y}{\partial y} + \upsilon\frac{\partial \psi_x}{\partial x}\right) \tag{5}$$

$$M_{xy} = \frac{-D(1-\upsilon)}{2}\left(\frac{\partial \psi_x}{\partial y} + \upsilon\frac{\partial \psi_y}{\partial x}\right) \tag{6}$$

$$Q_x = -\kappa^2 Gh\left(\psi_x - \frac{\partial W}{\partial x}\right) \tag{7}$$

$$Q_y = -\kappa^2 Gh\left(\psi_y - \frac{\partial W}{\partial y}\right) \tag{8}$$

$$\frac{\partial W}{\partial t} = \Delta_t \tag{9}$$

$$\frac{\partial W}{\partial x} = \Delta_x \tag{10}$$

$$\frac{\partial W}{\partial y} = \Delta_y \tag{11}$$

where Equations. (4 – 8) are the equations for bending moments, twisting moments and shear force, ψ_x and ψ_y are local rotations in the $x-$ and $y-$ directions respectively. h and h_1 are the thickness of the plate and load respectively, γ is the viscous damping coefficient, θ is the angle of inclination of the plate with the horizontal, ρ and ρ_L are the densities of the plate and the load per unit volume respectively. $W(x,y,t)$ is the traverse displacement of the plate at time t, g is the acceleration due to gravity, θ is the angle of inclination of the plate, u is the velocity of the load (M_L) of rectangular dimension ξ by μ with one of its lines of symmetry moving along $Y = Y_1$, the plate is I_x by I_y in dimensions and $\xi = ut + \frac{\varepsilon}{2}$, $B = B_x B_y$ where

$$B_x = \begin{cases} 1 - H\left(x - \xi - \frac{\varepsilon}{2}\right), & 0 \le t \le \varepsilon/u \\ H\left(x - \xi + \frac{\varepsilon}{2}\right) - H\left(x - \xi - \frac{\varepsilon}{2}\right), & \frac{\varepsilon}{u} \le t \le \frac{L_x}{u} \\ H\left(x - \xi + \frac{\varepsilon}{2}\right), & \frac{L_x}{u} \le t \le \frac{(L_x+\varepsilon)}{u} \\ 0, & (L_x+\varepsilon)/u \le t \end{cases} \tag{12}$$

$$B_y = \left\{H\left(y - y_1 + \mu/2\right) - H\left(y - y_1 - \mu/2\right)\right\} \tag{13}$$

$$H(x) = \begin{cases} 1, & x > 0 \\ 0.5, & x = 0 \\ 0, & x < 0 \end{cases} \tag{14}$$

$H(x)$ is called Heaviside function.

G is the modulus of rigidity of the plate, D is the flexural rigidity of the plate defined by $D = \frac{1}{12}Eh^3(1-\upsilon^2)^{-1} = \frac{Gh^3}{6(1-\upsilon)}$ for isotropic plate, κ^2 is the shear correction factor and υ is the Poisson's ration of the plate.

Since the inertia effect of the load is considered, the uniform partially distributed applied load takes on the form [6]:

$$P(x,y,t) = \frac{-M_L}{A}\left[g\sin\theta + \frac{d^2W}{dt^2}\right]B - M_L g\sin\theta \tag{15}$$

Acceleration $\frac{d^2W}{dt^2}$ is defined as

$$\frac{d^2W}{dt^2} = \frac{\partial^2 W}{\partial t^2} + 2u\frac{\partial^2 W}{\partial x \partial t} + u^2\frac{\partial^2 W}{\partial x^2} \tag{16}$$

Similarly,

$$\frac{d^2\psi_x}{dt^2} = \frac{\partial^2 \psi_x}{\partial t^2} + 2u\frac{\partial^2 \psi_x}{\partial x \partial t} + u^2\frac{\partial^2 \psi_x}{\partial x^2} \tag{17}$$

and

$$\frac{d^2\psi_y}{dt^2} = \frac{\partial^2 \psi_y}{\partial t^2} + 2u\frac{\partial^2 \psi_y}{\partial x \partial t} + u^2\frac{\partial^2 \psi_y}{\partial x^2} \tag{18}$$

A. *Initial Conditions*

$$W(x,y,0)=\frac{\partial W}{\partial T}(x,y,0) \tag{19}$$

B. *Boundary Conditions*

$$W(x,y,t)=M_x(x,y,t)=\psi_y(x,y,t)=0, \text{ for } x=0 \text{ and } x=a$$
$$W(x,y,t)=M_x(x,y,t)=\psi_x(x,y,t)=0, \text{ for } y=0 \text{ and } y=b \tag{20}$$

III. . Problem Solution

The set of partial differential Equations. (1) - (11)**,** are the partial differential equations to be solved for the following eleven dependent variables M_x, M_y, M_{xy}, Q_x, Q_y, ψ_{xt}, ψ_{yt}, W, Δ_t, Δ_x and Δ_y.

A numerical procedure, finite difference method, can be used to solve the system of Equations. (1) - (11). Rearranging them in matrix form results in

$$R_{i,j+1}S'_{i,j+1}+P_{i+1,j+1}=-T_{i,j}S'_{i,j}-Y_{i+1,j}S_{i+1,j}+Z_k \tag{21}$$

$$i=1,\ 2,\ 3\ldots N-1; \quad j\ =\ 1,\ 2,\ 3\ldots M-1$$

Where *N* and *M* are the number of the nodal points along *x* and *y* axes respectively. Z_k is a matrix representing the right hand side of the transformed Equations. (1) – (11) defined by

$$Z_k=A_{i,j}\,S_{i,j}+P_{i,j+1}S^0{}_{i,j+1}+G_{i+1,j}\,S^0{}_{i+1,j}+D_{i+1,j+1}\,S^0{}_{i+1,j+1}+E_1 \tag{22}$$

Effect Of Angle Of Inclination On Deflection Of The Inclined Plate

For the purpose of this paper let $B=0,$ which implies $B_x=0$ and $L_x+\varepsilon/2\le t$. Also, $M_f-\rho h=M$ (mass); and $0\le\theta\le\pi/2$. For $\gamma=1$, equation (3) becomes:

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+kW+M\gamma\frac{\partial\Delta_t}{\partial t}=\rho h\frac{\partial^2 W}{\partial t^2}-M_L g\sin\theta \tag{23}$$

$$\rho h\frac{\partial^2 W}{\partial t^2}-\left[\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+M\gamma\frac{\partial\Delta_t}{\partial t}\right]-kW=M_L g\sin\theta \tag{24}$$

since $\Delta_t=\dfrac{\partial W}{\partial t}$, $\dfrac{\partial\Delta_t}{\partial t}=\dfrac{\partial^2 W}{\partial t^2}$ and ρh is a mass.
Therefore, Eq. (24) becomes

since $\Delta_t=\dfrac{\partial W}{\partial t}$, $\dfrac{\partial\Delta_t}{\partial t}=\dfrac{\partial^2 W}{\partial t^2}$ and ρh is a mass.
Therefore, Eq. (24) becomes

$$-\left(\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}\right)-kW=M_L g\sin\theta \tag{25}$$

When $\theta=0^\circ$,

$$W=-\frac{1}{k}\left(\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}\right) \tag{26}$$

When $\theta=30^\circ$,

$$W=-\frac{M_L g}{2k}-\frac{1}{k}\left(\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}\right) \tag{27}$$

When $\theta=60^\circ$,

$$W=-\frac{\sqrt{3}M_L g}{2k}-\frac{1}{k}\left(\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}\right) \tag{28}$$

When $\theta=90^\circ\ \left(=\pi/2\right)$,

$$W=-\frac{M_L g}{k}-\frac{1}{k}\left(\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}\right) \tag{29}$$

From Equation. (15), if $B=0$, the applied load becomes

$$P(x,y,t)=-M_L g\sin\theta \tag{30}$$

When $\theta=0^\circ$,

$$P=0 \tag{31}$$

When $\theta=30^\circ$,

$$P=-\frac{1}{2}M_L g \tag{32}$$

When $\theta=60^\circ$,

$$P=-\frac{\sqrt{3}}{2}M_L g \tag{33}$$

When $\theta=90^\circ$,

$$P=-M_L g \tag{34}$$

Now for $\gamma>1$, equation 24 becomes

$$M(1-\gamma)\frac{\partial^2 W}{\partial t^2}-\left[\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}\right]-kW=M_L g\sin\theta \quad (35)$$

$$M(1-\gamma)\frac{\partial^2 W}{\partial t^2}-\frac{\partial Q_x}{\partial x}-\frac{\partial Q_y}{\partial y}-kW=M_L g\sin\theta \quad (36)$$

$$W=\frac{1}{k}[M(1-\gamma)\frac{\partial^2 W}{\partial t^2}-\frac{\partial Q_x}{\partial x}-\frac{\partial Q_y}{\partial y}+M_L g\sin\theta] \quad (37)$$

From equation (37); as the angle of inclination of the plate, θ,increases and $\gamma<1$, the magnitude of deflection, W, increases.

Each term in Equations. (21) and (22) is an 11 x 11 matrix.

IV. RESULT DISCUSSION

The numerical calculations were carried out for a simply supported rectangular inclined plate resting on a Pasternak foundation and subject to a moving load. Damping effect was considered. The values of the damping ratios are taken to be 0, 1, 100 and 150 respectively. In Fig.3 the deflection of Mindlin, non-Mindlin,, at different values of time and foundation modulus, were shown. It is obvious that the maximum amplitude of Mindlin plate is higher than that of non-Mindlin plate. We notice, also, from figure 4 that the higher the damping ratio, the lower the deflection amplitude, at a particular time. We can deduce from equation (31) - (34) that for $\gamma=1$, we need to apply more force to be able to pull the load uphill as θ increases. Also, for $\gamma=1$, we can deduce, from equations (27) – (29), that the magnitude of the deflection (W) increases as the angle of inclination θ increases. This shows that damping affects both the applied force and the deflection of the inclined plate. From equation (37), it can be seen that, for a particular value of k, the magnitude of W increases as the angle of inclination θ increases. In Fig. 6, the deflection of the plate for different values of the foundation modulus (G) is presented. It is observed that the foundation stiffness have effect on the deflection of the plate. The highest value of the foundation modulus, produces the maximum response amplitude. Fig. 5 shows the effect of the velocity on the deflection of the inclined plate. From the figure it can be seen that the higher the velocity the higher the response amplitude

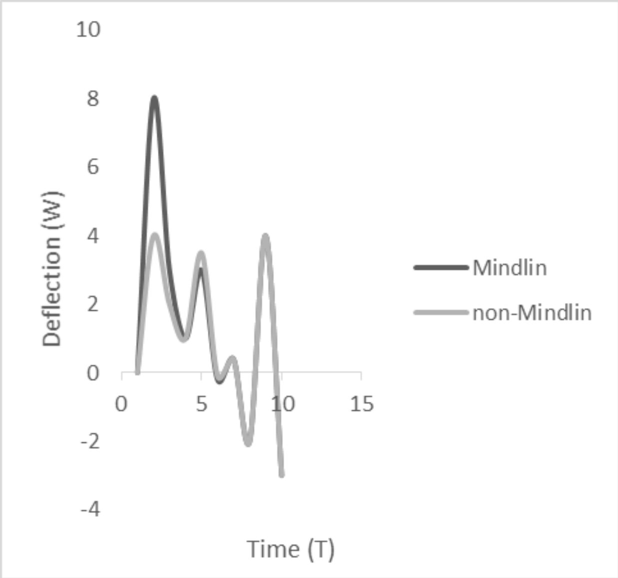

Figure 3: Deflection of Mindlin and non-Mindlin plates for different values of time

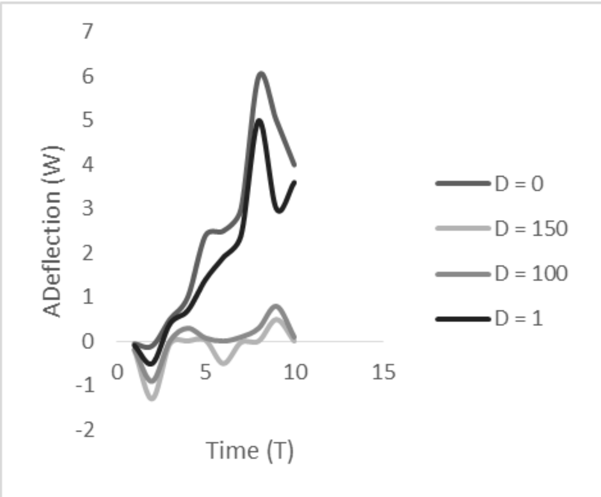

Fig.4. Effect of damping on deflection of the plate

ISBN: 978-988-14047-6-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

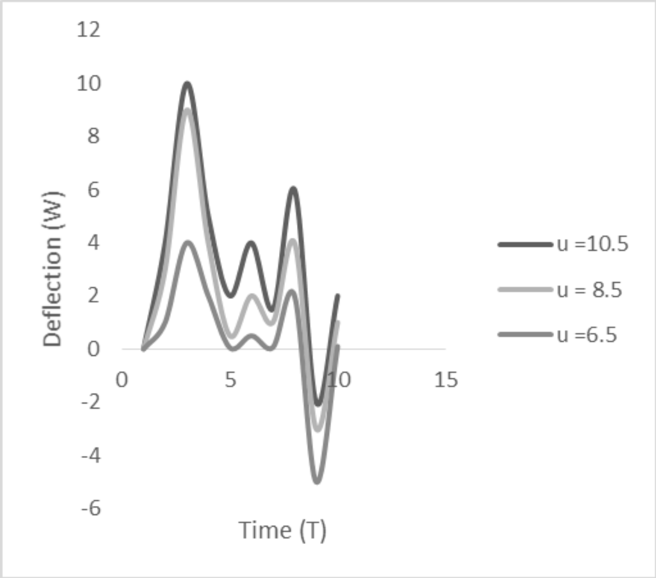

Fig.5. Deflection of plate at different velocities (u) and time(T)

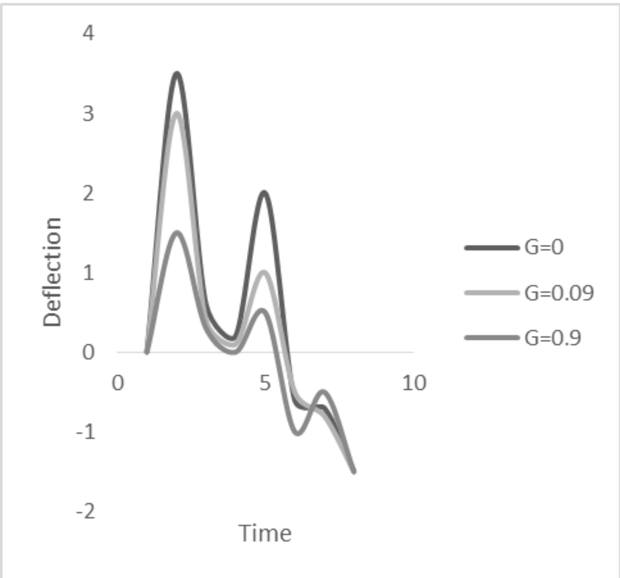

Fig. 6. Deflection of plate at various foundation modulus and different times

V. CONCLUSION

The dynamic response of a damped inclined Mindlin plate, carrying a uniform partially distributed moving load , supported by a Pasternak foundation, has been analysed. The non-dimensional equations of motion were transformed into equivalent finite difference ones, and then solved. Results have been presented not only for the deflection but also for the effect of velocity on the deflection of the inclined plate. Also the effects of both the damping and angle of inclination of the plate was examined. Hence most of the components composing the dynamic response of the system have been obtained. A numerical example of simply supported rectangular plate is presented. It is shown that the elastic subgrade, on which the damped inclined Mindlin plate rests has a significant effect on the dynamic response of the plate to a partially distributed load. The effects of the angle of inclination and the damping coefficient were very evident. For Mindlin plate, both the effect of rotatory inertia and shear deformation, on the dynamic response of the damped inclined Mindlin plate, to the moving load are considered. This gives a more realistic result for practical application, especially when such plate is considered to rest on a foundation.

REFERENCES

[1] M. Abu-Hilal, “Dynamic response of a double Euler-Bernoulli beam due to a moving constant load.” *Journal of Sound and Vibration*, Vol. 297, pp 471-491. 2006

[2] J.V. Amiri, A. Nikkho, M.R. Dnvoodi, and M.E. Hassanabadi, “Vibration analysis of a Mindlin elastic plate under a moving mass excitation by eigenfunction expansion method.”. *Thin-Walled Structure*, Vol. 62, pp 53-64, 2013

[3] O. Civalek, “Large deflection static and dynamic analysis of thin circular plates resting on two parameter elastic foundation HDQ/FO couple methodology approaches”. *International Journal of Computational Mechanics*, Vol. 2, no. 2, pp 271-291, 2005.

[4] J.A. Gbadeyan and M.C. Agarana, 2014. “Dynamic analysis of railway bridges supported by Winkler foundation under uniform partially distributed moving railway vehicle”. WIT Transactions on The Built Environment, Vol.135. WIT Press, 2014.

[5] J.A. Gbadeyan, and M.S. Dada, “Dynamic response of plates on Paternak foundation to distributed moving load.”. *Journal of the Nigerian Association of Mathematical Physics*, Vol. 5, pp 185-200, 2001

[6] J.A. Gbadeyan and M.S. Dada, “Dynamic response of a Mindlin elastic rectangular plate under a distributed moving mass”. *International Journal of Mechanical Science*, Vol. 48 pp 323-340, 2006

[7] R.D. Mindlin, “Influence of rotatory to inertia and shear on flexural motions of isotropic elastic plates”. *Journal of Applied Mechanics*, Vol. 18, 1957.

[8] T. Nguyan-Thoi, H. Luong-Van, P. Phung-Van, T. Rabczuk, and D. Tran- Trung, “Dynamic responses of composite plates on the paternak foundation subjected to a moving mass by a cell-based smoothed discrete shear gap (CS-FEM-DSG3) method.”. *International Journal of Ccomposite Materials*, Vol. 3 Issue 6A, pp 19-27. 2013

[9] T. Zhang, T. and G. Zheng, “Vibration analysis of an elastic beam subject to a moving beam with flexible connections.”. *American Association of Civil Engineers*, Vol. 130, no.1, pp 120-130, 2010.

[10] Khan Academy, 2014,’’ forces on inclined plane’’. www.khanacademy.org.

[11] M. C. Agarana, J. A. Gbadeyan, Olasumbo O. A. T. A. Anake and O. J. Adeleke, ‘‘ Dynamic Response of an inclined Railway bridge supported by Winkler Foundation under a Moving Railway Vehicle.’’ Australian journal of basic and applied sciences, Vol. 9, no.11, pp 355-361, 2015

[12] E.B. Are, A.S. dowu and J.A. Gbadeyan, ‘‘Vibration of damped simply supported orthotropic rectangular plate resting on elastic winkler foundation, subjected to moving loads’’ Advances in Applied science research, Vol. 4 No.5, pp387-393, 2013.

[13] M.C. Agarana and J.A. Gbadeyan, Finite difference dynamic analysis of railway bridges supported by Pasternak foundation under uniform partially distributed moving Railway vehicle, International conference on systems Engineering and Engineering Management 2015, IAENG. 21-23 October, 2015.