sustainability

MDPI

Article

# Distributed Optimisation Algorithm for Demand Side Management in a Grid-Connected Smart Microgrid

**Omowunmi Mary Longe[1], Khmaies Ouahada[1], Suvendi Rimer[1], Hendrick C. Ferreira[1] and A. J. Han Vinck[2]**

[1] Department of Electrical & Electronics Engineering Science, University of Johannesburg, South Africa; wunmigrace@yahoo.com, kouahada@uj.ac.za, suvendic@uj.ac.za, hcferreira@uj.ac.za
[2] University of Duisburg-Essen, Germany; han.vinck@uni-due.de
* Correspondence: wunmigrace@yahoo.com; Tel.: +27-78-552-8138

Academic Editor: name
Received: date; Accepted: date; Published: date

**Abstract:** The contributions of Distributed Energy Generation (DEG) and Distributed Energy Storage (DES) for Demand Side Management (DSM) purposes in a smart macrogrid or microgrid cannot be over-emphasized. If not well coordinated, standalone DEG and DES can lead to under-utilization of energy generation by consumers and financial investments, but a grid-connection mode of DEG and DES can offer arbitrage opportunities for consumers and utility provider(s). A grid-connected smart microgrid comprising heterogeneous (active and passive) smart consumers, electric vehicles and a large-scale energy storage device is considered in this work. Energy management by each smart entity is carried out by the proposed Microgrid Energy Management – Distributed Optimization Algorithm (MEM-DOA) installed distributively within the network according to consumer type. Each smart consumer optimizes its energy consumption, expenditure and trading for comfort and profit. The proposed model was observed to yield consumer satisfaction, financial benefits, grid reliability, resilience and sustainability, reduced investment on peaker plants, reduced Peak-to-Average-Ratio (PAR) demand and associated environmental benefits. The MEM-DOA also offers participating smart entities energy and tariff incentives so that passive smart consumers do not benefit more than active smart consumers like in some previous energy management algorithms.

**Keywords:** Distributed Energy Generation (DEG); Distributed Energy Storage (DES); Demand Side Management (DSM); Microgrid Energy Management – Distributed Optimization Algorithm (MEM-DOA); Smart microgrid

## 1. Introduction

Smart grid provides an enabling environment for the integration of Distributed Energy Generation (DEG) and Distributed Energy Storage (DES) for Demand Side Management (DSM) purposes with mutual benefits to electricity utility providers and consumers. The incorporation of DEGs and DES devices into the supply mix of the smart grid is expected to help in balancing energy demand and supply curves in (near) real time. These energy pockets maybe distributed within a smart grid in consumer premises or as microgrids. A microgrid can be a regional or communal energy system comprising distributed energy sources (renewable and/or non-renewable) often in order to optimize power quality, reliability, efficiency and sustainability with accompanying economic benefits (cheaper cost of energy, local employment generation and economic development) and environmental benefits (if renewable energy sources are used).

Microgrids would be a common feature in the smart grid either in standalone [1] or grid-connected [2] mode. Some literatures have shown contributions on DEG [3,4] and DES [5-7] in the smart grid. However, this work focuses on possible heterogeneous community of smart consumers with local DEG and DES in a grid-connected smart microgrid with a centralized large-scale Microgrid Energy Storage (MES) device for arbitrage opportunities. Each smart entity (smart consumers, MES device and utility) optimizes their benefits in the energy market through the proposed Microgrid Energy Management – Distributed Optimization Algorithm (MEM-DOA). The MEM-DOA is made up of energy consumption scheduling, storage and generation optimization algorithms. The MEM-DOA approach is proposed in order to enhance scalability of deployment, privacy and security in the smart microgrid. The proposed algorithm can be installed into the smart meters of consumers, and Energy Management Controller (EMC) of the MES device and Plug-in Hybrid Electric Vehicles (PHEVs) and the utility grid.

Another advantage of this work over existing literature [3-11] is that it guides against a situation where passive smart consumers can benefit more than active smart consumers in a smart grid as in [8]. Also, it encourages the penetration of DEG and DES devices in the future energy web. This type of architecture can additionally offer grid reliability and stability, financial benefits to all its smart entities, consumers' social welfare, reduction in Peak-to-Average-Ratio (PAR) demand and $CO_2$ emissions etc. Grid-connected DEG and DES [2-6,10,11] as will be presented in this work can offer grid resilience and stability in the face of energy imbalance with ever-growing demand and certain emergencies.

The rest of the paper is organized as follows: the smart microgrid model is described in Section II, while its mathematical formulation is presented in Section III. The MEM-DOA problems and the simulation results are presented in Sections IV and V respectively, while the conclusion is in Section VI.

## 2. Description of a Smart Microgrid Energy Management Model

This work proposes a model for a Smart Microgrid Energy Management (SMEM) system comprising heterogeneous consumers who are connected to the utility grid and a large scale Microgrid Energy Storage (MES) device. A sketch of the proposed grid-connected smart microgrid energy management architecture is presented in Figure 1. This type of architecture is envisaged as a possibility in the future, even among residential consumers, with increasing penetration of DEG and DES in the smart grid.

The smart microgrid has a large scale DES installed as a grid-connected-and-consumer-connected energy storage device providing an alternative centralised source of power to consumers in the smart microgrid. This MES device can be charged from the grid or by any active consumer in the microgrid with their excess energy generation or storage at low price periods and sell back the stored energy to the consumers and grid as the need arises at a higher price; thereby enhancing profitability in its energy trading.

Passive consumers are connected uni-directionally for energy flow with the grid and MES device, but bi-directionally for information and communication flow because they neither sell energy to the grid nor MES device. However, active consumers and PHEVs are connected bi-directionally with both MES device and the utility grid for energy, information and communication flows. Hence, a consumer is said to be passive in the SMEM network if it always buys all its energy consumption, or active if it has the ability to both buy and sell energy in the network.

Therefore, a consumer in the smart microgrid can either be passive (Type-A) consumer $a \in \mathcal{Z}_A, \mathcal{Z}_A \subset \mathbb{A}$ or an active consumer belonging to $\mathcal{W}_V$. The active consumers are further sub-divided into Type-B consumer $a \in \mathcal{Z}_B, \mathcal{Z}_B \subset \mathbb{A}$with DES e.g. in-Home Energy Storage (iHES) device only, Type-C consumer $a \in \mathcal{Z}_C, \mathcal{Z}_C \subset \mathbb{A}$ with DEG only, Type-D consumer $a \in \mathcal{Z}_D, \mathcal{Z}_D \subset \mathbb{A}$ with iHES device and DEG; and PHEVs$v \in \mathcal{V}$, where $\mathcal{W}_V = \mathcal{Z}_B \cup \mathcal{Z}_C \cup \mathcal{Z}_D \cup \mathcal{V}$.

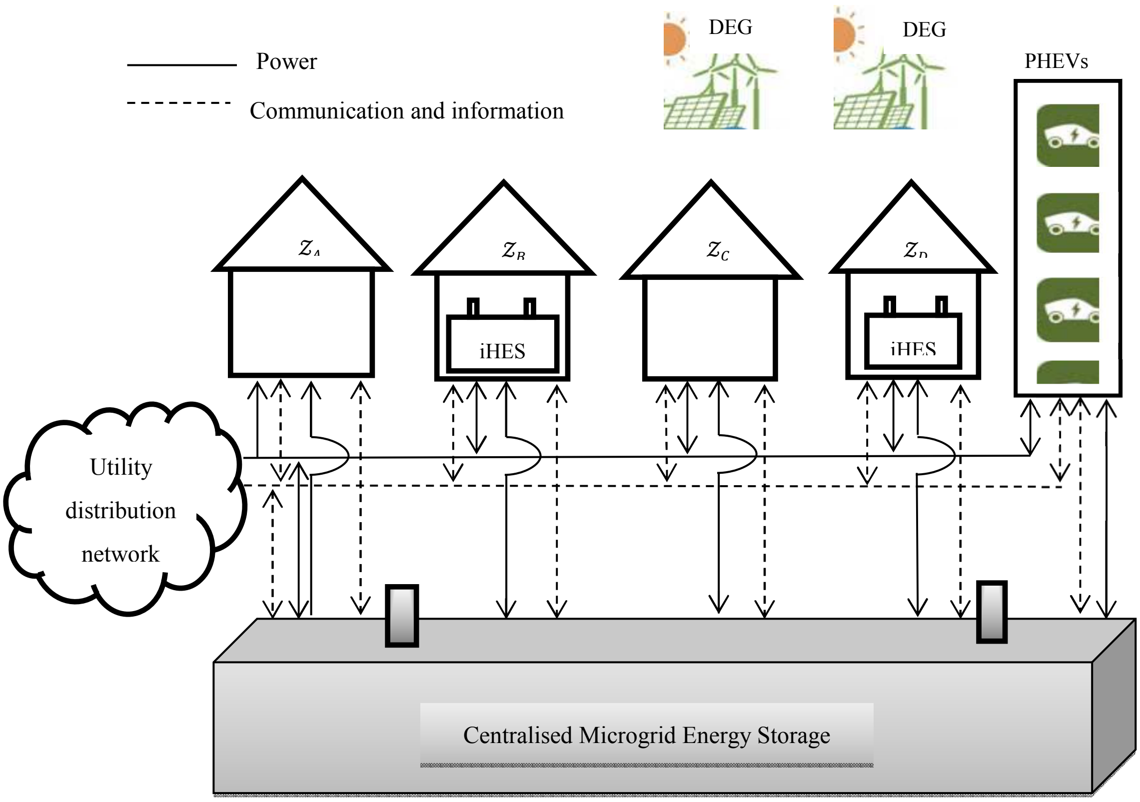


**Figure 1.** Proposed grid-connected smart microgrid architecture

These consumers' categories are chosen to reflect the possible different types of consumers that can exist in a smart grid or smart microgrid. An active consumer meets its local demand at every time $t \in \mathbb{T}$ from the energy generated by its local DEG and/or DES, utility grid or MES device depending on energy prices from these sources at the time that the energy is needed. If the active consumer demand is greater than the amount of energy available locally from its DEG and/or DES, then it purchases the difference from the cheaper seller between the grid and MES device. This can in a way reduce the need for investment on peaker plants by the utilities

## 3. MEM-DOA Problem Formulation

The proposed model will be made up of appliance consumption scheduling and dissatisfaction models for all residential consumers belonging to set $\mathbb{A} = \mathcal{Z}_A \cup \mathcal{Z}_B \cup \mathcal{Z}_C \cup \mathcal{Z}_D$; energy storage models for active consumers $\mathcal{Z}_B \cup \mathcal{Z}_D$, PHEVs $v$ and the microgrid $\mu$; and energy production models for active consumers $\mathcal{Z}_C \cup \mathcal{Z}_D$ and the grid $r$. Each model is mathematically formulated and presented in this section. A distributed optimisation approach is observed in this work so that each smart consumer can autonomously optimize its energy consumption and expenditure. Sub-sections 3.1, 3.2, and 3.3 in this section are adapted from our previous work in [7].

### *3.1. Appliance Energy Consumption Scheduling Model*

The consumer's load is categorized into non-shiftable, flexible, interruptible deferrable and uninterruptible deferrable smart appliances. Let every smart consumer $a \in \mathbb{A}$, $where\ \mathbb{A} = \mathcal{Z}_A \cup \mathcal{Z}_B \cup \mathcal{Z}_C \cup \mathcal{Z}_D$ in the smart microgrid, have non-shiftable appliances (e.g. lighting, cooking) $i \in \mathbb{I}$, flexible appliances (e.g. air-conditioner) $j \in \mathbb{J}$, uninterruptible deferrable appliances (e.g. dish washer) $f \in \mathbb{F}$and interruptible deferrable appliances (e.g. pool pump) $l \in \mathbb{L}$. The flexible and deferrable appliances would have their consumption shifted in power and time respectively. Therefore, all the smart appliances in a consumer premise belong to the set, $\mathbb{G} = \mathbb{I} \cup \mathbb{J} \cup \mathbb{F} \cup \mathbb{L} = \mathbb{I} \cup$

$\mathbb{H}$, where $\mathbb{H} = \mathbb{J} \cup \mathbb{F} \cup \mathbb{L}$. The total appliance load $x_{a,t}$ of consumer $a$ at any time $t \in \mathbb{T}$, where $\mathbb{T} = [1, 2, \dots, \mathfrak{t}]$ is given by [7]:

$$x_{a,t} = \sum_{i\in\mathbb{I}} x_{a,i,t} + \sum_{j\in\mathbb{J}} x_{a,j,t} + \sum_{f\in\mathbb{F}} x_{a,f,t} + \sum_{l\in\mathbb{L}} x_{a,l,t}. \quad (1)$$

The daily load vector for each consumer $a \in \mathbb{A}$ is $\boldsymbol{x}_a = [x_{a,1}, x_{a,2}, \dots, x_{a,\mathfrak{t}}]'$, while its total daily load $x_a$ is given as:

$$x_a = \sum_{t\in\mathbb{T}} x_{a,t}. \quad (2)$$

If the feasible period of operation $\mathcal{T}_{a,g}$ of any appliance $g$ in the household has a start time $t^s_{a,g}$ and end time $t^e_{a,g}$, where $\mathcal{T}_{a,g} = \{t | t^s_{a,g} \le t \le t^e_{a,g}\}$; and $g = \{i, j, f, l\}$, $\forall g \in \mathbb{G}$. Then, total energy $e_{a,g}$ consumed by any appliance $g$ in the smart home is given by:

$$e_{a,g} = \begin{cases} \sum_{t^s_{a,g}}^{t^e_{a,g}} x_{a,g,t}, & \forall t \in \mathbb{T}, g = \{i, j, f, l\}, \forall g \in \mathbb{G} \\ 0, & \forall t \in \mathbb{T} \backslash \mathcal{T}_{a,g},\ g = \{i, j, f, l\}, \forall g \in \mathbb{G} \end{cases}. \quad (3)$$

A power level constraint is set for each appliance such that:

$$x^{min}_{a,g} \le x_{a,g,t} \le x^{max}_{a,g}, g = \{i, j, f, l\}, \forall g \in \mathbb{G}, \forall t \in \mathcal{T}_{a,g}. \quad (4)$$

where $x^{min}_{a,g} \ge 0$, $x^{min}_{a,g}$ and $x^{max}_{a,g}$ are the minimum power level (OFF or standby mode) and maximum power level of each smart appliance respectively. The total energy $x_t$ consumed by all smart appliances owned by all the consumers in the smart microgrid at a time $t$ is given by:

$$x_t = \sum_{a\in\mathbb{A}} \sum_{g\in\mathbb{G}} e_{a,g}, g = \{i, j, f, l\}, \forall t \in \mathbb{T}. \quad (5)$$

### *3.2. Appliance Scheduling Dissatisfaction Model*

The dissatisfaction associated with appliance scheduling is modeled in this sub-section for the schedulable appliances.

#### 3.2.1. Power Shiftable (Flexible) Appliances

The dissatisfaction cost due to scheduling flexible smart appliances in a consumer's premise from its nominal load $u_{a,j,t}$ to an actual load $x_{a,j,t}$ with respect to energy tariff and given as [7]:

$$\bar{d}^t_{a,j} = \alpha_{a,j}\left(u_{a,j,t}\theta_t\left[1 - \left(\frac{x_{a,j,t}}{u_{a,j,t}}\right)^{\gamma_t}\right]\right),\ 0 \le \alpha_{a,j} \le 1, \quad (6)$$

where $\gamma_t < 1$, $\gamma_t\theta_t < 0$, $\gamma_t, \theta_t \in \mathbb{R}$ and $\alpha_{a,j}$ is the degree of dissatisfaction of a flexible load that is tolerable to the consumer. The first and second derivatives of (6) show results that are similar to satisfaction conditions in [11] and utility conditions in [12].

#### 3.2.2. Time Shiftable Appliances

The dissatisfaction cost incurred by shifting a consumers' load from its nominal usage period to an actual period in response to energy price is considered for deferrable loads.

Uninterruptible deferrable appliances can have their start times shifted although same duration of operation is maintained in the actual time. Hence, dissatisfaction in the start time of the operation is considered. The dissatisfaction cost function $\bar{d}^t_{a,f}$ for an uninterruptible deferrable load is:

$$\bar{d}^t_{a,f} = \alpha_{a,f}\left|t^{s,s}_{a,f} - t^s_{a,f}\right|,\ 0 \le \alpha_{a,f} \le 1, \forall f \in \mathbb{F}, \quad (7)$$

where $t_{a,f}^{s,s}$ and $t_{a,f}^{s}$ are the actual and nominal start times of the uninterruptible deferrable appliance $f \in \mathbb{F}$ respectively and $\alpha_{a,f}$ is the measure of tolerance of such delay/haste to the consumer by shifting the start time of task. Let the feasible operation period for schedulable uninterruptible deferrable appliances be $\mathcal{T}_{a,f}^{s} = \{t | t_{a,f}^{s,s} \leq t \leq t_{a,f}^{e,s}\}$.To ensure that the operation of an uninterruptible deferrable smart appliance continues once it starts without interruption, then the end time $t_{a,f}^{e,s}$ for the actual task is constrained as:

$$t_{a,f}^{e,s} \geq t_{a,f}^{s,s} + \eta_{a,f}, \ \forall f \in \mathbb{F}, \forall t \in \mathbb{T}, \forall a \in \mathbb{A}. \quad (8)$$

where nominal task duration $\eta_{a,f} = t_{a,f}^{e} - t_{a,f}^{s}$.

The interruptible deferrable appliances can have their task being interrupted during operation and continued at a later time within the actual feasible period $\mathcal{T}_{a,l}^{s} = \{t | t_{a,l}^{s,s} \leq t \leq t_{a,l}^{e,s}\}$ and $\mathcal{T}_{a,l}^{s_1} + \mathcal{T}_{a,l}^{s_2} + \ldots + \mathcal{T}_{a,l}^{s_q} = \mathcal{T}_{a,l}^{s}$, where $\mathcal{T}_{a,l}^{s_1}, \mathcal{T}_{a,l}^{s_2}, \ldots, \mathcal{T}_{a,l}^{s_q}$ are possible operation periods of q number of scheduled sub-tasks within the actual feasible period, $\mathcal{T}_{a,l}^{s}$. The dissatisfaction cost of an interruptible deferrable appliance $\bar{d}_{a,l}^{t}$ is measured as a function of difference between the nominal duration $\eta_{a,l}$ and the actual duration $\eta_{a,l}^{s}$ taken to complete the entire task and is given as:

$$\bar{d}_{a,l}^{t} = \alpha_{a,l} \, |\eta_{a,l} - \eta_{a,l}^{s}|, \ \ 0 \leq \alpha_{a,l} \ \leq 1, \forall l \in \mathbb{L}. \quad (9)$$

where$\alpha_{a,l}$ is the tolerance factor to measure the degree to which the consumer can tolerate changes in the duration taken to complete a task, $\eta_{a,l} = |t_{a,l}^{e} - t_{a,l}^{s}|, \eta_{a,l}^{s} = |t_{a,l}^{e,s} - t_{a,l}^{s,s}|, \forall t \in \mathbb{T}$. Therefore, applying (6), (7) and (9), the total dissatisfaction cost $\overline{d_a}$ in a consumer's premise, can be defined as the summation of the load dissatisfaction costs of all shiftable appliances and is given as [7]:

$$\overline{d_a} = \sum_{j \in \mathbb{J}, t \in \mathbb{T}} \bar{d}_{a,j}^{t} + \sum_{f \in \mathbb{F}, t \in \mathbb{T}} \bar{d}_{a,f}^{t} + \sum_{l \in \mathbb{L}, t \in \mathbb{T}} \bar{d}_{a,l}^{t}, \forall a. \quad (10)$$

The values of $\alpha_{a,j}, \alpha_{a,f}, \alpha_{a,l}, \gamma_t$ and $\theta_t$ can be varied to model different levels of consumer dissatisfaction.

### *3.3. Local Distributed Energy Storage Model*

The local DES (e.g. battery) model as in [7] applies only to Type-B active consumer $a \in \mathcal{Z}_B, \mathcal{Z}_B \subset \mathbb{A}$ and Type-D active consumer $a \in \mathcal{Z}_D, \mathcal{Z}_D \subset \mathbb{A}$ in this smart microgrid model. If $b_{a,t}$ is the energy stored in the battery at time $t \in \mathbb{T}$for consumer $a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}$, then, the battery daily energy storage scheduling vector $\boldsymbol{b}_a = [b_{a,1}, b_{a,2}, \ldots, b_{a,t}, \ldots, b_{a,\mathfrak{t}}]'$. Therefore, $b_{a,t}$ can be expressed in terms of the energy charging profile $b_{a,t}^{+}$ and energy discharging profile $b_{a,t}^{-}$ as $b_{a,t} = b_{a,t}^{+} - b_{a,t}^{-}$, where $b_{a,t}^{+}, b_{a,t}^{-} \geq 0$. The charging efficiency $\beta_a^{+}$ and discharging efficiency $\beta_a^{-}$ fulfil conditions $0 < \beta_a^{+} \leq 1$ and $\beta_a^{-} \geq 1$ respectively. Therefore, the battery is only effectively charged and discharged with $\beta_a^{+} b_{a,t}^{+}$ and $\beta_a^{-} b_{a,t}^{-}$ amount of energy respectively. The charging and discharging efficiency vector $\boldsymbol{\beta}_a = [\beta_a^{+}, -\beta_a^{-}]'$ and per-timeslot storage scheduling vector is $\boldsymbol{b}_{a,t} = [b_{a,t}^{+}, b_{a,t}^{-}]'$. This implies that $\boldsymbol{\beta}'_a \boldsymbol{b}_{a,t}$ is the energy charged/discharged at time $t \in \mathbb{T}$. Since the maximum charging/discharging rate $b_a^{max}$ of the battery cannot be exceeded at any charging/discharging time then, the constraint (11) is introduced:

$$\boldsymbol{\beta}'_a \boldsymbol{b}_{a,t} \leq b_a^{max}, a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}, \ \forall t \in \mathbb{T}. \quad (11)$$

The energy leakage rate $\lambda_a$ of the battery is constrained as $0 < \lambda_a \leq 1$. If $q_{a,t-1}$ is the charge level of the battery at time $t-1$, which was reduced at $\lambda_a$ leakage rate then, the present time $t$charge level can be expressed as: $q_{a,t} = q_{a,t-1}(1 - \lambda_a) + \boldsymbol{\beta}'_a \boldsymbol{b}_{a,t}, a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}, \ \forall t \in \mathbb{T}$. Also, the charge level $q_{a,t}$ of the battery is bounded as $0 \leq q_{a,t} \leq b_{a,cap}$, where $b_{a,cap}$ is the battery capacity. Therefore, for every $a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}$, smart consumer [7]:

$$-q_{a,t-1}(1 - \lambda_a) \leq \boldsymbol{\beta}'_a \boldsymbol{b}_{a,t} \leq b_{a,cap} - q_{a,t-1}(1 - \lambda_a). \quad (12)$$

Also, $q_{a,t}$ and initial charge level $q_{a,t_0}$ are related by:

$$q_{a,t} = q_{a,t_0}(1 - \lambda_{a,t}) + \sum_{t=t_0}^{t} \lambda_{a,t-t_0} \boldsymbol{\beta}'_a \boldsymbol{b}_{a,t}, a \in \{\mathcal{Z}_B, \mathcal{Z}_D\}. \quad (13)$$

The storage device can go through integer number of charging and discharging cycles, which oppose fluctuations in the daily energy demand of the consumer. Therefore, $q_{a,t_0}$ and daily final charge level $q_{a,t_t}$ can be related by:

$$|q_{a,t_t} - q_{a,t_0}| \le \mho_a,\ \mho_a \in \mathbb{R}^+, a \in \{\mathcal{Z}_B, \mathcal{Z}_D\},, \forall t \in \mathbb{T}, \quad (14)$$

where$\mho_a$ is sufficiently a small positive constant. Each battery is assumed to be sufficiently small compared to the aggregate load so as not to influence tariffs during charging and discharging periods. Examples of possible local DES devices include lithium-ion batteries, lead-acid batteries etc.

### *3.4. Consumer Distributed Energy Generation Model*

A consumer's DEG can be either dispatchable or non-dispatchable energy generator. Dispatchable generators include micro-turbines, internal combustion engines etc., while non-dispatchable generators include solar panels, wind turbines etc. Only non-dispatchable generators are considered in this work due to their associated environmental friendliness and ease of deployment.

For a non-dispatchable generator owned by consumers $a \in \{\mathcal{Z}_C, \mathcal{Z}_D\}$, the DEG production at time $t$ is $\mathrm{g}_{a,t}$. The non-dispatchable generators produce energy based on available intermittent resources e.g. solar radiation. A consumer can sell its excess local generation to the grid or MES device and buy back later again at periods when these resources are naturally not available or less than the quantity required to meet consumer's demand.

### *3.5. Microgrid Energy Storage Model*

The MES device is modelled similarly to consumers' DES devices and applies same explanations and formulations. Therefore, if daily energy storage scheduling vector $\boldsymbol{b}_\mu = [b_{\mu,1}, b_{\mu,2}, \dots, b_{\mu,t}, \dots, b_{\mu,t}]$ for the MES device then, (11) – (13) can be adopted and re-written for the MES device as follows:

$$\boldsymbol{\beta}'_\mu \boldsymbol{b}_{\mu,t} \le b_\mu^{max}, 0 < \beta_\mu^+ \le 1, \beta_\mu^- \ge 1, b_{\mu,t}^+, b_{\mu,t}^- \ge 0, \forall t, \quad (15)$$

$$-q_{\mu,t-1}(1-\lambda_\mu) \le \boldsymbol{\beta}'_\mu \boldsymbol{b}_{\mu,t} \le b_{\mu,cap} - q_{\mu,t-1}(1-\lambda_\mu), \quad (16)$$

$$q_{\mu,t} = q_{\mu,t_0}(1-\lambda_{\mu,t}) + \textstyle\sum_{t=t_0}^{t} \lambda_{\mu,t-t_0} \boldsymbol{\beta}'_\mu \boldsymbol{b}_{\mu,t}\, , \forall t \in \mathbb{T}, \quad (17)$$

The energy charged/discharged by the MES device $\boldsymbol{\beta}'_\mu \mathbf{b}_{\mu,t}$ at time $t$ is further simplified as:

$$\boldsymbol{\beta}'_\mu \mathbf{b}_{\mu,t} = (\boldsymbol{\beta}'_\mu \mathbf{b}_{\mu,t})^r + (\boldsymbol{\beta}'_\mu \mathbf{b}_{\mu,t})^{\mathcal{W}_V} + ({\boldsymbol{\beta}'_\mu}^- {\mathbf{b}_{\mu,t}}^-)^{\mathcal{Z}_A},\ \mathcal{W}_V = \{\mathcal{Z}_B, \mathcal{Z}_C, \mathcal{Z}_D, \mathcal{V}\}, \quad (18)$$

where$(\boldsymbol{\beta}'_\mu \mathbf{b}_{\mu,t})^r$ and $(\boldsymbol{\beta}'_\mu \mathbf{b}_{\mu,t})^{\mathcal{W}_V}$ are the charged/discharged energy by the grid and active consumers respectively, and $({\boldsymbol{\beta}'_\mu}^- {\mathbf{b}_{\mu,t}}^-)^{\mathcal{Z}_A}$ is the MES discharging profile towards consumer $\mathcal{Z}_A$, $({\mathbf{b}_{\mu,t}}^+)^{\mathcal{Z}_A} = 0$ since consumer $a \in \mathcal{Z}_A$ does not sell energy to the MES device. The quantity of charge $q_{\mu,t}$ in the MES device at any time $t$ is the aggregate of the charges stored in it by the grid and active consumers and is given as:

$$q_{\mu,t} = q_{\mu,t}^r + q_{\mu,t}^{\mathcal{Z}_B} + q_{\mu,t}^{\mathcal{Z}_C} + q_{\mu,t}^{\mathcal{Z}_D} + q_{\mu,t}^v, \quad (19)$$

where $q_{\mu,t}^r$, $q_{\mu,t}^{\mathcal{Z}_B}$, $q_{\mu,t}^{\mathcal{Z}_C}$, $q_{\mu,t}^{\mathcal{Z}_D}$ and $q_{\mu,t}^v$ are the quantities of charge stored in the MES device by the grid, consumers $a \in \mathcal{Z}_B$, $a \in \mathcal{Z}_C$ and $a \in \mathcal{Z}_D$, and PHEVs respectively.

Some storage devices that can serve as MES devices include Compressed-Air Energy Storage (CAES), Pumped-Storage Hydroelectric (PSH) etc. The MES device is a form of large-scale energy storage that can be owned by a private operator or utility provider.

### *3.6. Plug-in Hybrid Electric Vehicle Battery Model*

The PHEVs in the smart microgrid shall be modeled with respect to its battery characteristics only, and not driving pattern. Let $b_{v,t}$ be a PHEV charging/discharging profile at time $t$; then, the daily storage vector for every PHEV battery $v \in \mathcal{V}$ can be denoted as $\mathbf{b}_v = [b_{v,1}, b_{v,2}, \dots, b_{v,t}, \dots, b_{v,\mathrm{t}}]$. Then, storage profile for the PHEV can be modeled as follows:

$$\boldsymbol{\beta}_v' \mathbf{b}_{v,t} \le b_v^{max}, v \in \mathcal{V}, \forall t \in \mathbb{T}, \tag{20}$$

$$q_{v,t} = q_{v,t-1}(1 - \lambda_v) + \boldsymbol{\beta}_v' \mathbf{b}_{v,t}, v \in \mathcal{V}, \forall t \in \mathbb{T}, \tag{21}$$

$$-q_{v,t-1}(1 - \lambda_v) \le \boldsymbol{\beta}_v' \mathbf{b}_{v,t} \le b_{v,cap} - q_{v,t-1}(1 - \lambda_v), \forall v, \tag{22}$$

$$q_{v,t} = q_{v,t_0}(1 - \lambda_{v,t}) + \textstyle\sum_{t=t_0}^{t} \lambda_{v,t-t_0} \boldsymbol{\beta}_v' \mathbf{b}_{v,t}, v \in \mathcal{V}, \forall t \in \mathbb{T}, \tag{23}$$

and

$$|q_{v,t_{\mathrm{t}}} - q_{v,t_0}| \le \mho_v, v \in \mathcal{V}, \forall t \in \mathbb{T}, \tag{24}$$

In order to prevent the PHEVs from increasing peak demand beyond grid and MES capacities, their charging/discharging profiles $b_{v,t}$ and hence the load $\mathcal{R}_{v,t} = b_{v,t}$ are centrally scheduled within the microgrid and is constrained by:

$$0 \le \textstyle\sum_{v \in \mathcal{V}} \mathcal{R}_{v,t} \le \bar{b}_{v,t}, \forall v \in \mathcal{V}, \forall t \in \mathbb{T}, \tag{25}$$

where$\bar{b}_{v,t} = (\bar{b}_{v,t})^r + (\bar{b}_{v,t})^\mu$ is the maximum energy the PHEVs can draw from the utility grid and MES device at any timeslot respectively. The PHEVs are modeled as separate aggregate load in the microgrid without attachment to any particular consumer, although they could also play similar roles as iHES device in consumer premises depending on their configurations.

## 4. MEM-DOA Optimization Problems

### *4.1. Microgrid Energy Storage Cost Model*

The MES device buys energy from the grid and active consumers during low price periods and sells energy back to them at a higher price than purchasing price in order to maximize its profit. If the charging/discharging load of the MES device $\mathcal{R}_{\mu,t} = b_{\mu,t}, \forall t \in \mathbb{T}$, then, the MES daily cost function $C_\mu(\boldsymbol{\mathcal{R}}_\mu)$ is given as:

$$C_\mu(\boldsymbol{\mathcal{R}}_\mu) = \textstyle\sum_{t \in \mathbb{T}} \left( P_{\mu \to y,t}^{SP} \boldsymbol{b}_{\mu,t}^{-} - P_{\mu,t}^{BP} \boldsymbol{b}_{\mu,t}^{+} - P_{\mu,t}^{O} b_{\mu,t} \right), y = \{r, \mathcal{Z}_A, \mathcal{Z}_C, \mathcal{Z}_D\}, \forall t \in \mathbb{T}, \tag{26}$$

where $P_{\mu,t}^{SP}$ and $P_{\mu,t}^{BP} = min(P_{r,t}^{SP}, P_{a,\mathcal{Z}_B,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP})$ are the respective selling and buying prices of the MES and $P_{\mu,t}^{O} b_{\mu,t}$ is its charging/discharging operating cost. Type-A consumers are passive energy generators in the microgrid and hence, would always buy energy from the MES device with a penalty price. For instance, the selling price of energy from the MES device to any buyer $P_{\mu \to y,t}^{SP}$ is given by:

$$P_{\mu \to y,t}^{SP} = \begin{cases} \omega_{\mu,t} P_{\mu,t}^{BP}, & \text{if} q_{\mu,t}^{y} \ge b_{\mu \to y,t}^{\mu-}, \forall t \in \mathbb{T} \\ \omega_{\mu,t} P_{\mu,t}^{BP} \mathcal{P}_{y,t}, & \text{if} q_{\mu,t}^{y} < b_{\mu \to y,t}^{\mu-}, \forall t \in \mathbb{T}' \end{cases} \tag{27}$$

where $y = \{r, \mathcal{Z}_A, \mathcal{Z}_C, \mathcal{Z}_D, v\}, \omega_{\mu,t}$ is a preset MES provider coefficient of profit in order to maximize reasonable profit for the MES device provider, $\mathcal{P}_{y,t}$ is the buyer's price penalty for requesting more energy than contributed to the MES present charge level, $q_{\mu,t}^{y}$ is the energy contribution by a buyer $y$ to the MES charge level and $b_{\mu \to y,t}^{\mu-}$ is the amount of energy to be discharged from the MES device to buyer $y$ at time $t$. The MES selling price (SP) to the passive consumers would be the highest at every time $t \in \mathbb{T}$ since they do not have contribution to the energy stored in the MES device. Also, $P_{\mu,t}^{BP} = min(P_{r,t}^{SP}, P_{a,\mathcal{Z}_B,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP})$, where $P_{r,t}^{SP}, P_{a,\mathcal{Z}_B,t}^{SP} P_{a,\mathcal{Z}_C,t}^{SP}$ and $P_{a,\mathcal{Z}_D,t}^{SP}$ are selling prices for grid and active consumers $\mathcal{Z}_B, \mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively. The value of $\omega_{\mu,t}$ is constrained as $\omega_{\mu,t} > 1$ to ensure compliance with rate-of-return on investment regulations. This would help the MES device provider to set a SP or tariff that is high enough to attract further capital investment and also low enough so as not to negatively affect customers' welfare. In this work, a buyer's price penalty $\mathcal{P}_{y,t}$ is given by:

$$\mathcal{P}_{\mathcal{Y},t} = \frac{max(P_{-\mathcal{Y},t}^{SP})}{min(P_{-\mathcal{Y},t}^{SP})}, q_{\mu,t}^{\mathcal{Y}} < b_{\mu \to \mathcal{Y},t}^{\mu-}, \mathcal{Y} = \{r, \mathcal{Z}_A, \mathcal{Z}_B, \mathcal{Z}_C, \mathcal{Z}_D\}. \quad (28)$$

where$P_{-\mathcal{Y},t}^{SP}$ is the SP of other buyers excluding $\mathcal{Y}$ at time $t$. The MEM-DOA for the MES device is formulated as a linear program and solved using simplex method [14,15]:

$$\min_{\boldsymbol{\mathcal{R}}_\mu \in \mathbb{R}} C_\mu(\boldsymbol{\mathcal{R}}_\mu)$$

$$s.t.\;\; (15) - (19), (26), (27), (28)$$

$$P_{\mu,t}^{BP} = min(P_{r,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP}), \forall t \in \mathbb{T}. \quad (29)$$

*4.2. Utility Cost Model*

Let $\mathrm{g}_{r,t}$be the energy generation by the electricity utility provider at time $t$and bounded by the utility grid maximum energy production capacity $\mathrm{g}_r^{max}$ be given as:

$$0 \le \mathrm{g}_{r,t} \le \mathrm{g}_r^{max}, \forall t \in \mathbb{T}. \quad (30)$$

The constrain in (30) ensures that all the load from all devices connected to the grid does not exceed grid capacity at any given time. Also, the load balance on the grid at any time $t$ can be given as:

$$\mathrm{g}_{r,t} = \mathcal{R}_{a,\mathcal{Z}_A,t} + \mathcal{R}_{a,\mathcal{Z}_B,t} + \mathcal{R}_{a,\mathcal{Z}_C,t} + \mathcal{R}_{a,\mathcal{Z}_D,t} + \mathcal{R}_{\mu,t}. \quad (31)$$

Where $\mathcal{R}_{a,\mathcal{Z}_A,t}, \mathcal{R}_{a,\mathcal{Z}_C,t}$ and $\mathcal{R}_{a,\mathcal{Z}_D,t}$ are the total grid loads from consumers $\mathcal{Z}_A, \mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively. The utility cost function, $C_t(\mathrm{g}_{r,t})$ is the cost to the utility for providing $\mathrm{g}_{r,t}$supply and can be modeled as a non-decreasing convex function using the energy cost function for thermal generators [3,4,9]:

$$C_t(\mathrm{g}_{r,t}) = c_1^t(\mathrm{g}_{r,t})^2 + c_2^t\mathrm{g}_{r,t} + c_3^t, \forall t \in \mathbb{T}, \quad (32)$$

Where $c_1^t > 0$ and $c_2^t, c_3^t \ge 0$. Also, in accordance with rate-of-return on investment regulations, $P_{r,t}^{SP}$and utility buying price $P_{r,t}^{BP}$is modified from [3] and given as:

$$P_{r,t}^{SP} = \omega_{r,t}\frac{C_t(\mathrm{g}_{r,t})}{\mathrm{g}_{r,t}} = \omega_{r,t}P_{r,t}^{BP},\; \forall t \in \mathbb{T}, \quad (33)$$

Where $\omega_{r,t} > 1$ is a preset utility profit coefficient. This ensures mutual financial benefits between utility, consumers and MES provider. The total daily cost of electricity vector to the utility $\boldsymbol{C}_r$is the total cost of generation to meet its load and cost of energy purchases from the active consumers and MES device, and it is given as:

$$\mathbf{C}_r = \sum_{t\in\mathbb{T}}\Big(C_t\big(\mathrm{g}_{r,t}\big) + P_{r,t}^{BP}\big(b_{a,\mathcal{Z}_B,t} - x_{a,\mathcal{Z}_B,t} - ({b_{\mu,t}}^{+})^{\mathcal{Z}_B}\big)^{+} + P_{r,t}^{BP}\big(\mathrm{g}_{a,\mathcal{Z}_C,t} - x_{a,\mathcal{Z}_C,t} - ({b_{\mu,t}}^{+})^{\mathcal{Z}_C}\big)^{+} +$$

$$P_{r,t}^{BP}\big(\mathrm{g}_{a,\mathcal{Z}_D,t} + b_{a,\mathcal{Z}_D,t} - x_{a,\mathcal{Z}_D,t} - ({b_{\mu,t}}^{+})^{\mathcal{Z}_D}\big)^{+} + P_{r,t}^{BP}\sum_{v\in\mathcal{V}}({b_{v,t}}^{-})^{r} + P_{r,t}^{BP}({b_{\mu,t}}^{-})^{r}\Big), \forall t \in \mathbb{T}. \quad (34)$$

where$({b_{\mu,t}}^{+})^{\mathcal{Z}_B}, ({b_{\mu,t}}^{+})^{\mathcal{Z}_C}$ and $({b_{\mu,t}}^{+})^{\mathcal{Z}_D}$ are energy sold to the MES device by consumers $\mathcal{Z}_C$ and $\mathcal{Z}_D$ respectively and $({b_{\mu,t}}^{-})^{r}$ is energy bought from the MES device by the grid. The MEM-DOA for the utility grid is formulated as a convex programming problem [16] and solved using interior-point method [17] as follows:

$$\min_{\mathrm{g}_{r,t}\in\mathbb{R}} \boldsymbol{C}_r$$

$$s.t.\quad (30) - (34),$$

$$P_{r,t}^{BP} = min(P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP}, P_{\mu\rightarrow r,t}^{SP}). \quad (35)$$

*4.3. Type-A Consumer Cost Model*

Since the Type-A consumer is a passive consumer its cost function is basically the cost of meeting its local demand from the grid or MES device depending on their energy selling prices. Therefore, the daily cost $C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A})$ of Type-A smart consumer $a \in \mathcal{Z}_A, \mathcal{Z}_A \subset \mathbb{A}$, is given as:

$$C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A}) = \boldsymbol{P}_{a,\mathcal{Z}_A}^{BP}\boldsymbol{x}_{a,\mathcal{Z}_A} + \bar{\boldsymbol{d}}_{a,\mathcal{Z}_A}^{t}, a \in \mathcal{Z}_A, \quad (36)$$

where $\boldsymbol{P}_{a,\mathcal{Z}_A}^{BP} = min(\boldsymbol{P}_r^{SP}, \boldsymbol{P}_{\mu\rightarrow\mathcal{Z}_A}^{SP}) = min([P_{r,1}^{SP}, P_{r,2}^{SP}, \dots, P_{r,\mathfrak{t}}^{SP}], [P_{\mu\rightarrow\mathcal{Z}_A,1}^{SP}, P_{\mu\rightarrow\mathcal{Z}_A,2}^{SP}, \dots, P_{\mu\rightarrow\mathcal{Z}_A,\mathfrak{t}}^{SP}]) = [P_{a,\mathcal{Z}_A,1}^{BP}, P_{a,\mathcal{Z}_A,2}^{BP}, \dots, P_{a,\mathcal{Z}_A,\mathfrak{t}}^{BP}]$ is consumer $a \in \mathcal{Z}_A$ purchasing or buying price and $\boldsymbol{x}_{a,\mathcal{Z}_A} = [x_{a,\mathcal{Z}_A,1}, x_{a,\mathcal{Z}_A,2}, \dots, x_{a,\mathcal{Z}_A,\mathfrak{t}}]$ is the total appliance load for consumer $a \in \mathcal{Z}_A$ at time $t$. The MEM-DOA for Type-A passive consumer shall be formulated as a convex programming problem [16] solved using interior-point method [17] and is given as:

$$\min_{\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_A}, \bar{d}_a^t \in \mathbb{R}} C_{a,\mathcal{Z}_A}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_A})$$

$$s.t. \quad (1) - (10),$$

$$\boldsymbol{P}_{a,\mathcal{Z}_A}^{BP} = min(\boldsymbol{P}_r^{SP}, \boldsymbol{P}_{\mu\rightarrow\mathcal{Z}_A}^{SP}), a \in \mathcal{Z}_A, \forall t \in \mathbb{T}. \quad (37)$$

*4.4. Type-B Consumer Cost Model*

The daily cost function $C_{a,\mathcal{Z}_B}(\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_B})$ for each Type-B consumer is given as:

$$C_{a,\mathcal{Z}_B}(\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_B}) = \sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_B,t}^{BP}(x_{a,\mathcal{Z}_B,t} - b_{a,\mathcal{Z}_B,t})^{+} - \sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_B,t}^{SP}(b_{a,\mathcal{Z}_B,t} - x_{a,\mathcal{Z}_B,t})^{+} + \sum_{t\in\mathbb{T}} \bar{d}_{a,\mathcal{Z}_B}^{t}, a \in \mathcal{Z}_B, \quad (38)$$

The MEM-DOA for Type-B active smart consumer is formulated as a convex programming problem [16] as follows:

$$\min_{\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_B}, \bar{d}_a^t \in \mathbb{R}} C_{a,\mathcal{Z}_B}(\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_B})$$

$$\text{s.t.} \quad (1) - (10), (38),$$

$$P_{a,\mathcal{Z}_B,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu,t}^{SP}), a \in \mathcal{Z}_B, \forall t \in \mathbb{T},$$

$$P_{a,\mathcal{Z}_B,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP}), a \in \mathcal{Z}_B, \forall t \in \mathbb{T},$$

$$\mathcal{P}_{\mathcal{Z}_B,t} = \frac{max(P_{r,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP}, P_{v,t}^{SP})}{min(P_{r,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP}, P_{v,t}^{SP})}, \text{if} q_{\mu,t}^{\mathcal{Z}_B} < b_{\mu\rightarrow\mathcal{Z}_B,t}^{\mu-}, \quad (39)$$

Solving (39) for each Type-B consumer ensures minimised energy consumption and expenditure from the utility grid at peak times with accompanying consumer maximised satisfaction.

*4.5. Type-C Consumer Cost Model*

A Type-C smart consumer $a \in \mathcal{Z}_C$ possesses non-dispatchable DEG locally. Since the consumer does not have a storage device, it would have to sell out its excess generation during the day to the grid or MES device. Therefore, the per timeslot load $\mathcal{L}_{a,\mathcal{Z}_C,t}$ and daily cost function $C_{a,\mathcal{Z}_C}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C})$ for Type-C consumer are given by (40) and (41) respectively:

$$\mathcal{L}_{a,\mathcal{Z}_C,t} = x_{a,\mathcal{Z}_C,t} - \mathrm{g}_{a,\mathcal{Z}_C,t}, a \in \mathcal{Z}_C, \mathcal{Z}_C \subset \mathbb{A}, \quad (40)$$

$$C_{a,\mathcal{Z}_C}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C}) = \sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_C,t}^{BP}(x_{a,\mathcal{Z}_C,t} - \mathrm{g}_{a,\mathcal{Z}_C,t})^{+} - \sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_C,t}^{SP}(\mathrm{g}_{a,\mathcal{Z}_C,t} - x_{a,\mathcal{Z}_C,t})^{+} + \sum_{t\in\mathbb{T}} \bar{d}_{a,\mathcal{Z}_C}^{t}, a \in \mathcal{Z}_C, \quad (41)$$

where $P_{a,\mathcal{Z}_C,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu,t}^{SP})$ and $P_{a,\mathcal{Z}_C,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP})$ are buying and selling prices respectively, $x_{a,\mathcal{Z}_C,t}$ is total appliances demand and $\mathrm{g}_{a,\mathcal{Z}_C,t}$ is generation by consumer $a \in \mathcal{Z}_C$ at time $t$. Each Type-C smart consumer also has its MEM-DOA formulated as a convex programming problem [16] and solved using interior-point method [17] is given as:

$$\min_{\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_C},\bar{d}_a^t \in \mathbb{R}} C_{a,\mathcal{Z}_C}\left(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_C}\right)$$

$$s.t. \quad (1)-(10),(41),$$

$$P_{a,\mathcal{Z}_C,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu\to\mathcal{Z}_C,t}^{SP}), a \in \mathcal{Z}_C,\ \forall t \in \mathbb{T},$$

$$P_{a,\mathcal{Z}_C,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP}), a \in \mathcal{Z}_C, \forall t \in \mathbb{T}. \qquad (42)$$

*4.6. Type-D Consumer Cost Model*

The Type-D consumer $a \in \mathcal{Z}_D$is the active consumer that possesses both non-dispatchable DEG and DES device in its premise. Its total load $\mathcal{L}_{a,\mathcal{Z}_D,t}$at any time $t$ is given by:

$$\mathcal{L}_{a,\mathcal{Z}_D,t} = x_{a,\mathcal{Z}_D,t} + b_{a,\mathcal{Z}_D,t} - \mathrm{g}_{a,\mathcal{Z}_D,t}, a \in \mathcal{Z}_D, \mathcal{Z}_D \subset \mathbb{A}, \qquad (43)$$

where $x_{a,\mathcal{Z}_D,t}$ is the consumer's total appliances demand, $b_{a,\mathcal{Z}_D,t}$ is the energy charging/discharging profile for its DES device and $\mathrm{g}_{a,\mathcal{Z}_D,t}$ is the generation from its DEG at time $t$. Therefore, the daily cost function $C_{a,\mathcal{Z}_D}\left(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D}\right)$ for each Type-D consumer is given as:

$$C_{a,\mathcal{Z}_D}\left(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D}\right) = \sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_D,t}^{BP}\left(x_{a,\mathcal{Z}_D,t} + b_{a,\mathcal{Z}_D,t} - \mathrm{g}_{a,\mathcal{Z}_D,t}\right)^+ - \sum_{t\in\mathbb{T}} P_{a,\mathcal{Z}_D,t}^{SP}\left(\mathrm{g}_{a,\mathcal{Z}_D,t} - b_{a,\mathcal{Z}_D,t} - x_{a,\mathcal{Z}_D,t}\right)^+ + \sum_{t\in\mathbb{T}} \bar{d}_{a,\mathcal{Z}_D}^t. \qquad (44)$$

Where $P_{a,\mathcal{Z}_D,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu,t}^{SP})$ and $P_{a,\mathcal{Z}_D,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP})$ are consumer $a \in \mathcal{Z}_D$ buying and selling prices respectively at time $t$. Finally, the MEM-DOA for Type-D active smart consumer is formulated as a convex programming problem [16] and solved using interior-point method [17] as follows:

$$\min_{\boldsymbol{\mathcal{R}}_{a,\mathcal{Z}_D},\bar{d}_a^t \in \mathbb{R}} C_{a,\mathcal{Z}_D}(\boldsymbol{\mathcal{L}}_{a,\mathcal{Z}_D})$$

$$s.t. \quad (1)-(10),(44),$$

$$P_{a,\mathcal{Z}_D,t}^{BP} = min(P_{r,t}^{SP}, P_{\mu\to\mathcal{Z}_D,t}^{SP}), a \in \mathcal{Z}_D, \forall t \in \mathbb{T},$$

$$P_{a,\mathcal{Z}_D,t}^{SP} = max(P_{r,t}^{BP}, P_{\mu,t}^{BP}), a \in \mathcal{Z}_D, \forall t \in \mathbb{T}. \qquad (45)$$

The solutions to (37), (39), (42) and (45) offer the smart consumers optimized satisfaction, energy consumption and expenditure with financial savings.

*4.7. Plug-in Hybrid Electric Vehicle Battery Storage Cost Model*

The MEM-DOA for the PHEVs is centralised within the PHEVs community network, but distributed in relation with other smart entities in the smart microgrid and is formulated as a linear programming problem which can be solved using simplex method [14,15]:

$$\min_{\boldsymbol{\mathcal{R}}_v \in \mathbb{R}} C_v(\boldsymbol{\mathcal{R}}_v)$$

$$\text{s.t. } (20)-(25),$$

$$\mathcal{R}_{v,t} = b_{v,t}, \forall t \in \mathbb{T},$$

$$\mathcal{P}_{v,t} = \frac{max(P_{r,t}^{SP}, P_{a,\mathcal{Z}_B,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP})}{min(P_{r,t}^{SP}, P_{a,\mathcal{Z}_B,t}^{SP}, P_{a,\mathcal{Z}_C,t}^{SP}, P_{a,\mathcal{Z}_D,t}^{SP})}, \text{if} q_{\mu,t}^{v} < b_{\mu\to v,t}^{\mu-},$$

$$b_{v,t} \le \bar{b}_{v,t}, \bar{b}_{v,t} = (\bar{b}_{v,t})^r + (\bar{b}_{v,t})^\mu, 0 \le \sum_{v\in\mathcal{V}} \mathcal{R}_{v,t} \le \sum_{v\in\mathcal{V}} \bar{b}_{v,t}, \forall t. \qquad (46)$$

Peak-to-Average-Ratio (PAR) demand from the grid can be found using (46) and solved using simplex method [14,15]:

$$PAR = \frac{Peak\ demand}{Average\ demand} = \frac{\max_{t\in\mathbb{T}} \sum_{a\in\mathbb{A}} Y_{a,r}^{t}}{\frac{1}{t}\sum_{a\in\mathbb{A},t\in\mathbb{T}} Y_{a,r}^{t}}\ . \quad (47)$$

## 5. Numerical Results and Discussions

The MEM-DOA simulation was considered for three hundred consumers (one hundred households in each category of consumer) with residential data obtained from [18] and Time-of-Use (TOU) pricing tariffs in South Africa adopted [19].

The results of the simulations are presented in Figure 2 (for Type-A and Type-B smart consumers) and Figure 3 (for Type-C and Type-D smart consumers). Since Type-A smart consumers are passive smart consumers their MEM-DOA optimises the source of energy supply in consumer premises, energy consumption and expenditure. For Type-B smart consumers with an iHES device, consumer load, battery charging/discharging and energy expenditure were optimised. Type-B MEM-DOA ensures that the battery is only charged at low price/off-peak periods, but discharged primarily to meet consumer demand at peak/high price periods.

Active smart Type-C and Type-D consumers have their respective local generations and storage sources of power supply prioritized in the consumer premises. However, since solar resource is only available in the day and the generation mostly exceeded consumer demand, then the excess generation was sold mostly to the MES device due to the incentive on energy price available to it from the MES provider when it wanted to purchase energy from it in the future.

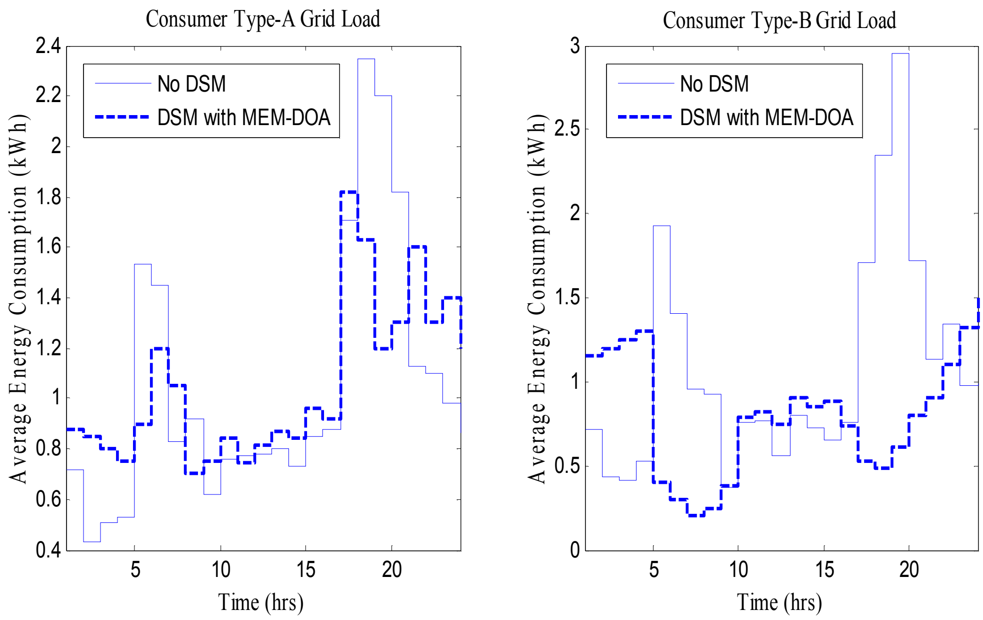


**Figure 2.** Average load profiles for Type-A and Type-B smart consumers

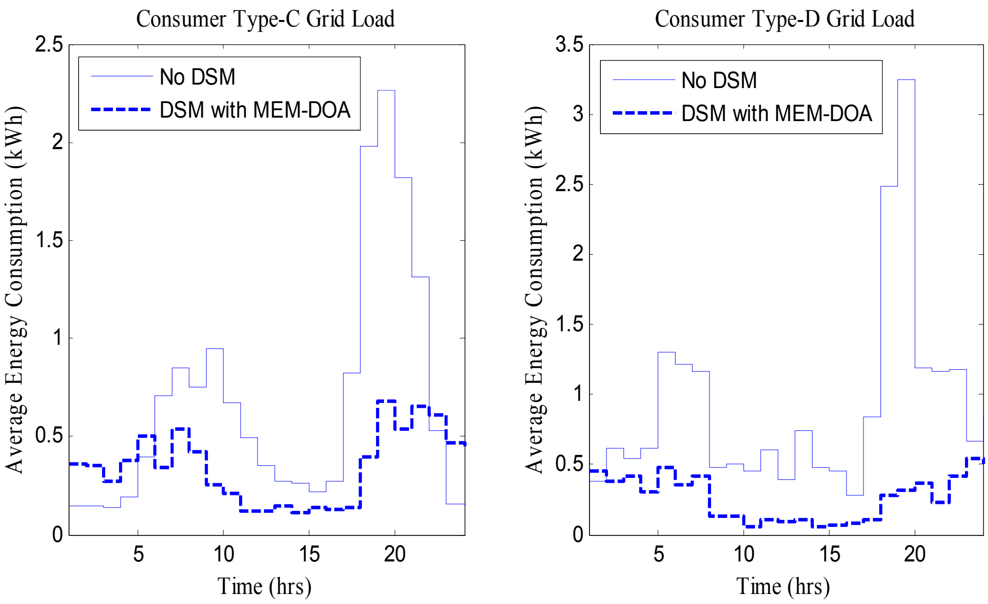


**Figure 3.** Average load profiles for Type-C and Type-D smart consumers

Type-D smart consumers could store their excess electrical energy generation in their iHES and use the stored charge at peak times and only energy request from the MES or utility when their demand exceeds their total local generation and storage levels. The iHES device could be charged from either the solar panel locally or externally from the grid or MES device and hence offer consumers more satisfaction and financial savings. The MEM-DOA model has shown to offer reduction in grid peak demand in all considered scenarios with increasing penetration of DES and DEG in consumer premises as shown in Figures 2 and 3. There was a lower reduction in grid peak demand for Type-B than for Type-C smart consumers because Type-C smart consumers do not have local storage for their excess generation during the day and would have to purchase from the grid or MES device at peak periods.

Also, the centralised MEM-DOA for the PHEVs ensured that only limited PHEVs were scheduled to be charged from the grid and MES device at peak periods, while most of the PHEV loads were scheduled for charging at night (low price period). The aggregate battery charging/discharging load profile for the hundred PHEVs considered is presented in Figure 4. The load profile had more consumption at non-peak periods than peak periods as compared to the nominal (No DSM) scenario where individual PHEVs owner could decide to charge its PHEV in the evenings especially, upon arrival at home. This MEM-DOA load profile offers the PHEV owners an average of 18% savings on energy expenditure.

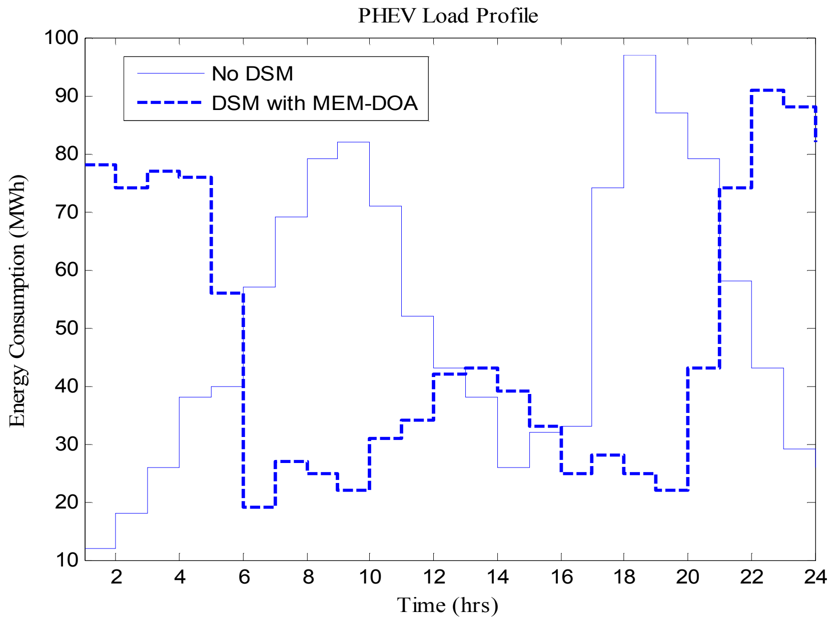


**Figure 4.** PHEV aggregate load profile

Also, box plots are presented in Figure 5 for smart residential consumers Types A to D and aggregate energy consumption (including PHEV demand) respectively in the smart microgrid showing the relationship in the consumption distribution between its average and peak values. The MEM-DOA plots are seen to be better that the initial unscheduled consumption.

It can also be noticed that the aggregate peak demand in the MEM-DOA reduced by 68% compared to the traditional peak demand. The utility and MES device providers also benefitted from the proposed MEM-DOA technique by 28% and 33% increase in revenue respectively. The higher increase in revenue by the MES provider could be due to the consumers preferring most times to buy from the MES device than the grid due to the price incentive received. Also, the aggregate PAR demand reduced by 46% from 2.9 to 1.56.

The negative dissatisfaction experienced by all the active consumers (Type-B, Type-C and Type-D) showed that integration of DES and DEG into consumer premises with centralised energy storage would offer satisfaction to consumers. The financial savings can also serve as a form of compensation for the initial investments incurred by the active consumers on DEG and DES devices. All the passive consumers (Type-A) would be slightly dissatisfied by an average of 0.121 kWh energy consumption daily, but can reduce their dissatisfaction by trading off financial savings. However, the financial savings observed by all consumers as enhanced by the presence of the centralised MES, DES and DEG devices in the smart microgrid. For instance, the dissatisfaction for Type-B consumers is less than for ESDS consumers in [7] due to the inclusion of the MES device and arbitrage opportunities in this model, although both consumers possess only iHES devices locally. Active consumers can through financial savings obtain a pay-back in the long-run on their investments on DES devices and DEG. Consumer dissatisfaction was not considered for the PHEVs however, their charging/discharging profile can affect the residential consumers dissatisfaction in amount of energy to be purchased from the MES device, energy prices from grid and MES device, and price penalties.

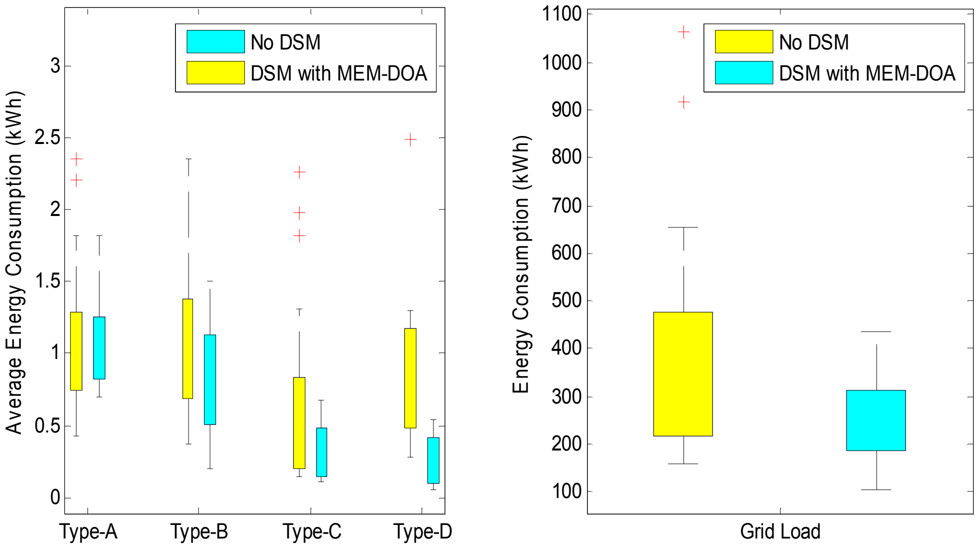


**Figure 5.** Comparison of consumer type consumption and aggregate grid load

**Table 1.** Average financial savings and dissatisfaction for the MEM-DOA smart residential consumers

| Consumer Type | Average Financial Savings | Average Dissatisfaction |
|---|---|---|
| Consumer Type-A | 18% | 0.121 kWh |
| Consumer Type-B | 35% | -1.289 kWh |
| Consumer Type-C | 32% | -0.874 kWh |
| Consumer Type-D | 56% | -2.935 kWh |

The utility and MES device providers also benefitted from the proposed DSM technique with MEM-DOA by 34% and 37% increases in revenue respectively. The higher increase in revenue by the MES provider could be due to consumers preferring at most times to buy from the MES device rather than the grid because of the price incentive received from contributed storage. In a competitive energy market that the smart grid would become, more incentives are likely to be experienced, which could lead to lower tariffs from electricity utility providers.

## 6. Conclusion

In this work, a DSM technique employing a price-incentivized energy trading in a grid-connected smart microgrid among smart consumers, a centralized MES and utility grid was presented. The smart consumers were either passive (no local DEG or DES) or active (with at least one of DEG and DES locally) consumers. Distributed optimization algorithm was employed to enhance scalability, consumer privacy and security. The proposed distributed algorithm called MEM-DOA for each type of participating smart entity resides within consumers' smart meters, and EMC for utility and MES providers. The results of the simulations showed financial savings for all participating entities. It further offered a reduced PAR demand and peak demand when compared with the traditional aggregate residential load profile. This algorithm ensures that the active consumers benefit more from the energy trading than passive consumers so they could have faster returns on investment. This consequentially would encourage the DES and DEG manufacturing industry as more consumers will become willing to partronise them. Commercial and industrial consumers can be included in future work.

**Author Contributions:** Omowunmi M. Longe and Khmaies Ouahada conceived and designed the problem formulation; Omowunmi M. Longe designed the simulation software, carried out data analysis and wrote the paper; while all the authors were involved in result validation and editing of the paper.

**Conflicts of Interest:** The authors declare no conflict of interest.

**References**


1. Longe, O. M.; Ouahada, K.; Ferreira, H. C.; Rimer, S. Renewable Energy Sources Microgrid Design for Rural Area in South Africa. In Proc. IEEE ISGT - North America; **2014**, 1-6.
2. Jang, Q.; Xue, M.; Geng, G. Energy management of microgrid in grid-connected and stand-alone modes. IEEE Trans. on Smart Grid **2013**, 28, 3, 3380-3389.
3. Yang, P.; Chavali, P.; Gilboa, E.; Nehorai, A. Parallel load schedule optimization with renewable distributed generators in smart grids. IEEE Trans. on Smart Grid **2013**, 4, 3, 1431-1441.
4. Delfino, F.; Minciardi, R.; Pampararo, F.; Robba, M. A multilievel approach for the optimal control of distributed energy resources and storage. IEEE Trans. on Smart Grid **2014**, 5, 4, 2155-2162.
5. Jayasekara, N.; Masoum, M. A. S.; Wolfs, P. J. Optimal operation of distributed energy storage systems to improve distribution network load and generation hosting capability. IEEE Trans. on Sust. Energy **2016**, 7, 250-261.
6. Khani, H.; Zadeh, M. R. D.; Hajimiragha, A. H. Transmission congestion relief using privately owned large-scale energy storage systems in a competitive electricity market. IEEE Trans. on Power Sys. **2016**, 31, 2, 1449-1458.
7. Longe, O. M.; Ouahada, K.; Rimer, S.; Harutyunyana, A. N.; Ferreira, H. C. Distributed Demand Side Management with Battery Storage for Smart Home Energy Scheduling. Sustainability **2017**, 9, 120, 1–13.
8. Vytelingum, P., Voice, T.D. Agent-based micro-storage management for the smart grid. In Proc. 9th International Conference on Autonomous Agents and Multiagent Systems; 2010, 39-46.
9. Mohsenian-Rad, A.; Wong, V.; Jatskevich, J.; Schober R.; LeonGarcia, A. Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid. IEEE Trans. on Smart Grid **2010**, 1, 3, 320-331.
10. Hussain, A.; Bui, V-H.; Kim H-M. Fuzzy logic-based operation of Battery Energy Storage Systems (BESSs) for enhancing the resilience of hybrid microgrids. Energies **2017**, 10, 271, 1-19.
11. Baurzhan, S.; Jenkins, G. P. On-grid solar PV versus diesel electricity generation in sub-Saharan Africa: Economics and GHG emissions. Sustainability 2017, 9, 372, 1-15.
12. Yang, P.; Tang G.; Nehorai, A. A game-theoretic approach for optimal time-of-use electricity pricing. IEEE Trans. on Power Sys. **2013**, 28, 2, 884-892.
13. Samadi, P.; Mohsenian-Rad, A.-H.; Schober, R.; Wong, V. W.; Jatskevich, J. Optimal real-time pricing algorithm based on utility maximisation for smart grid. In Proc. International Conference on Smart Grid Communications, Gaithersburg, MD, 2010; 415-420.
14. Dantzig G. B.; Thapa, M. N. *Linear programming 1: Introduction*. Springer, **1997**.
15. Dantzig G. B.; Thapa, M. N. *Linear programming 2: Theory and Extensions*. Springer, **2003**.
16. Boyd S., Vandenberghe, L. *Convex Optimisation.* Cambridge University Press, New York, United States of America, 7th Edition, **2009**.
17. Nesterov Y.; Nemirovski, A. *Interior-point polynomial algorithms in convex programming.* Studies in Applied Mathematics, SIAM Publications, Philadelphia, **1994**.
18. Eskom, Consumption Data, Data Acquisition Department, Pietermaritzburg, South Africa, 2015 – 2016.
19. Eskom, South Africa, "Eskom Tariffs and charges 2016/2017," Available at: http://www.eskom.co.za. Retrieved: 17-08-2016.