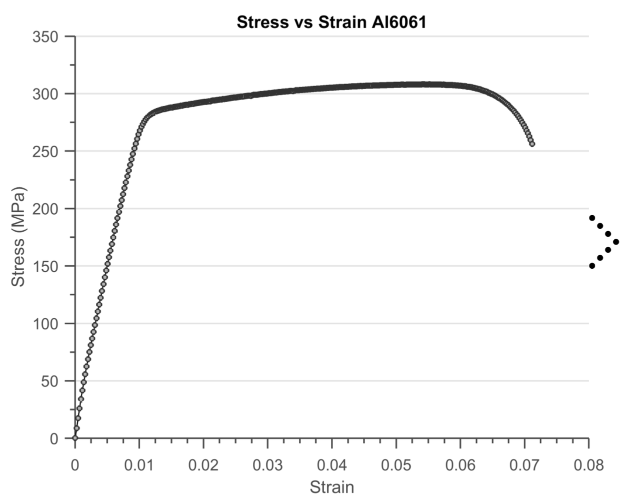
Identification of Physical Parameters Using Change-Point Kernels

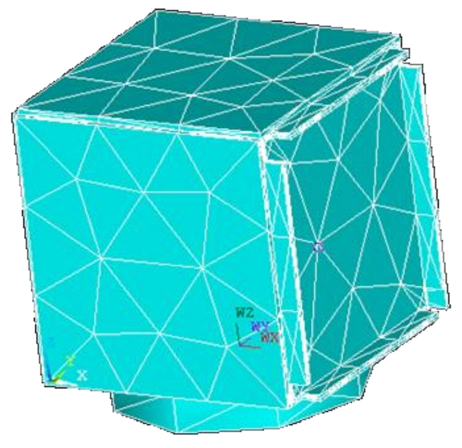
Ankit Chiplunkar (ankit.chiplunkar@airbus.com)
Emmanuel Rachelson, Michele Colombo, Joseph Morlier

1. Context

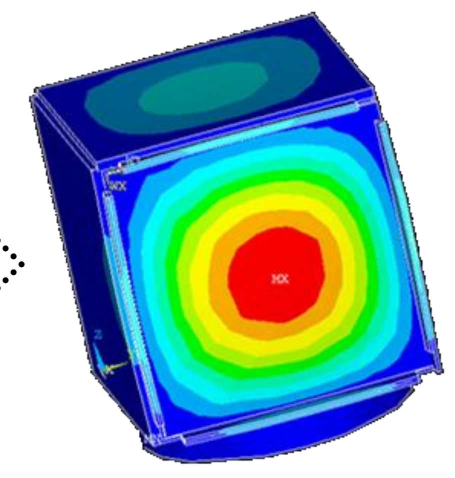
1. Engineering design is a costly exercise, primarily because gathering data for various design cases requires constructing and experimenting on that design point.
2. Hence engineers learn basic principles of the system and construct more detailed models based on the initial principles and simplifying assumptions. Eg. FEM in structural design or CFD in fluid simulations.



1.1 Stress-strain diagram for Aluminum AL6061 alloy.



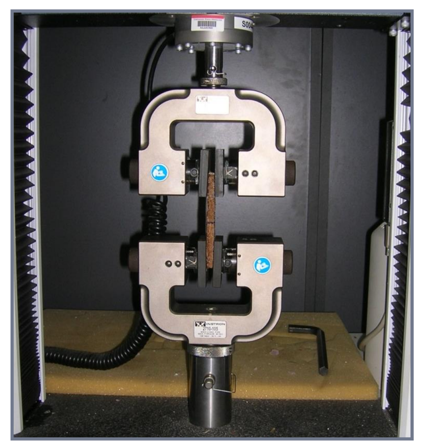
1.2 FEM of a small satellite, elements represent AL6061.



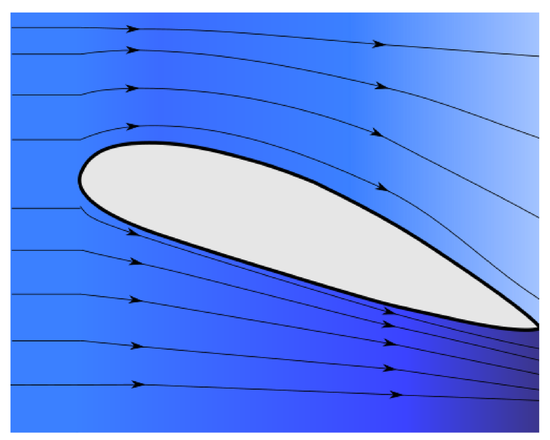
1.3 Displacements of the satellite under stress.

2. Objective

1. Various physical systems are approximated by a linear domain near the equilibrium and a non-linear domain where the approximations wear off. The values of slope and position of linear domain are important inputs in subsequent models. eg. Young's modulus and Plasticity
2. We want to estimate the physical parameters, experimental data for tensile test of AL6061 and lift diagrams for XFLR5 airfoil will be used to validate our method.



2.1 Tensile test setup for stress strain diagrams.

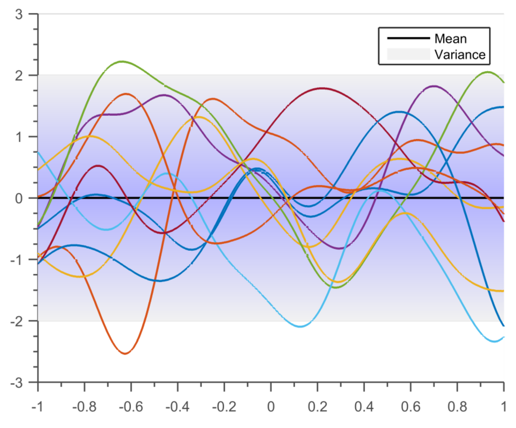


2.2 Flow over an airfoil, the lift depends on the angle of air

3. Gaussian Process Regression

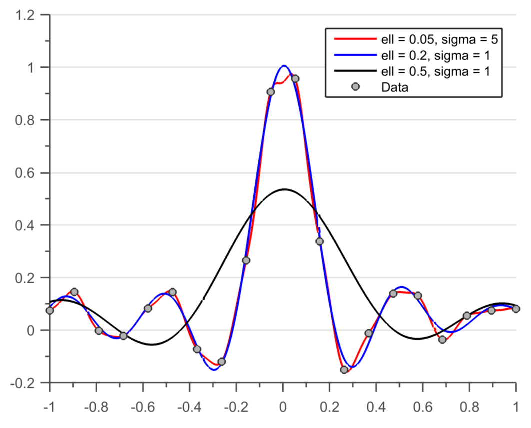
1. Gaussian Process is a flexible, non-linear, prior over functions.
2. It enables to tractably compute the posterior distribution which is consistent with prior belief and observed data.
3. The prior can be easily manipulated to encode a hypothesis space or family of functions.

Prior



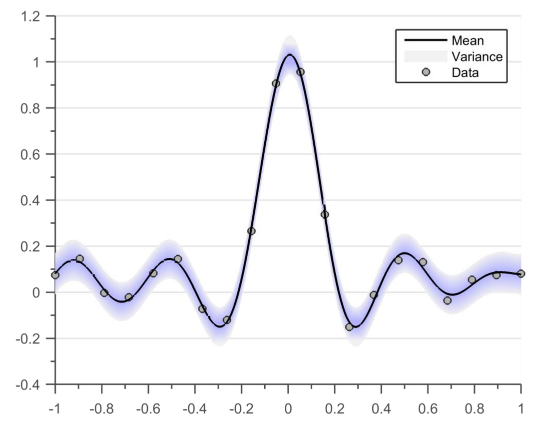
$GP\big(m(x), \mathbf{k}(\mathbf{x}, \mathbf{x}')\big)$
3.1 Random draws from a Standard Exponential (SE) prior.

Likelihood



$\frac{1}{2} y^T K^{-1} y - \frac{1}{2} \log|K| - \frac{n}{2} \log(2\pi)$
3.2 Mean values from three different priors, maximizing likelihood gives best prior.

Posterior

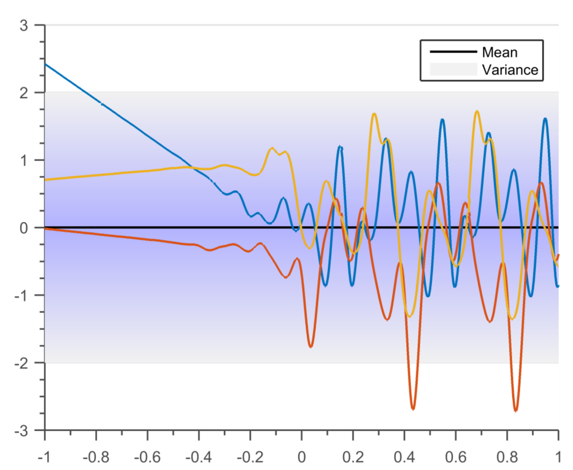


$GP\big(\boldsymbol{k}\boldsymbol{K}^{-1}\boldsymbol{y},\ \boldsymbol{k}(*,*) - \boldsymbol{k}\boldsymbol{K}^{-1}\boldsymbol{k}\big)$
3.3 Posterior eliminates all functions non-coherent with data

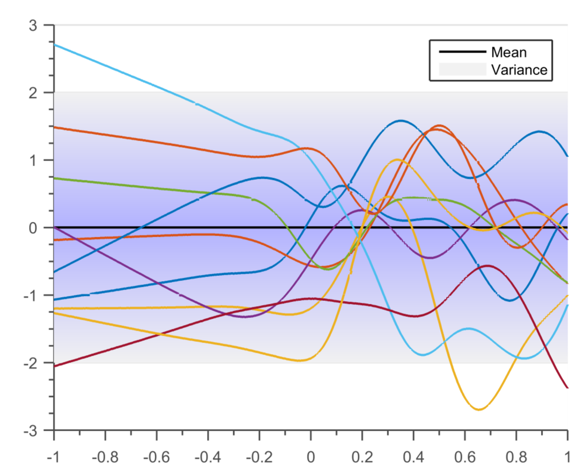
4. Method

1. We will use a changepoint kernel to encode the approximation of change from a linear to non-linear domain.
2. Changepoint kernels can be defined through multiplication with sigmoidal functions.
3. We maximize marginal likelihood to estimate the position of the changepoint.

$$K_{cp}(k_1, k_2) = \sigma(x) k_1(x, x') \sigma(x') + \big(1 - \sigma(x)\big) k_2(x, x') \big(1 - \sigma(x')\big)$$

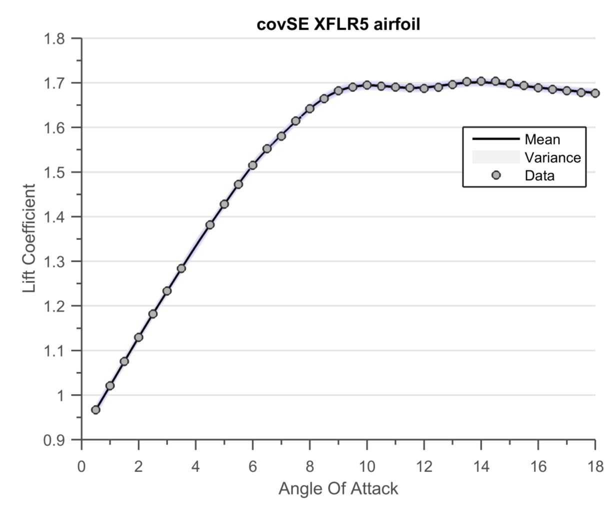


4.1 Random draws from Linear to Periodic changepoint kernel

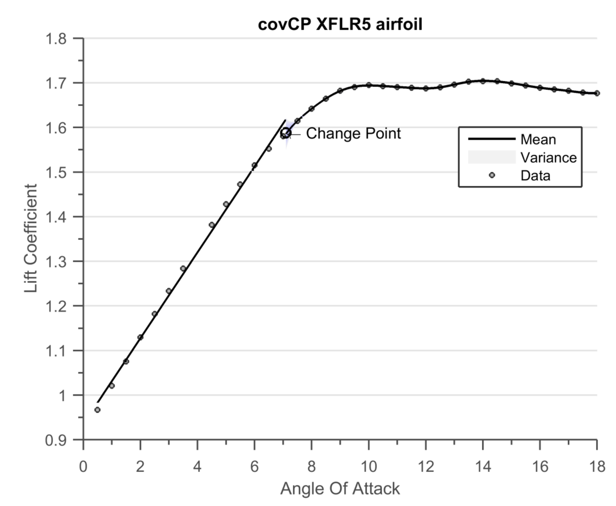


4.2 Random draws from Linear to SE changepoint kernel

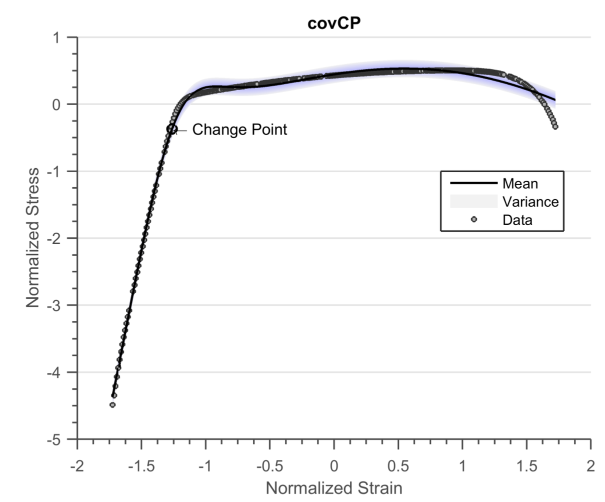
5. Results and Discussion



5.1 Gaussian Process regression using a SE kernel for XFLR5 airfoil



5.2 Regression using a Linear to SE changepoint kernel



State of art method Young's modulus	38,4 GPa
Changepoint Young's Modulus	38,6+- 2,3 GPa
ChangePoint	0,94% of strain
Marginal likelihood	372.3145 +-5,57

5.1 GP regression for AL6061 data, using the changepoint kernel. The Young's modulus calculated using engineering judgement was 38,4 GPa

1. The marginal likelihood of CP(linear, SE) kernel is rigged with many local minima's. The algorithm tries to put a changepoint at every observation point. Hence either using a constrained optimization or a global optimizer is advised.
2. The changepoint kernel is also numerically unstable hence cross-validating the learners improves performance.
3. Osborne et al perform changepoint estimation by placing a prior over them and performing a MAP estimate. We would like use this method in the future for changepoint estimation.

6. References

1. Garnett, R., Osborne, M. A., & Roberts, S. J. (2009). Sequential Bayesian prediction in the presence of changepoints.
2. Carl Edward Rasmussen and Christopher K. I. Williams. Gaussian Processes for Machine Learning (2005).
3. Abbott, Ira H., Albert E. Von Doenhoff, and Louis Stivers Jr. "Summary of airfoil data." (1945).
4. Jhbdel, Airfoil flow, 2010