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Dark radiation and observable gravity waves from string inflation

Relatore:
Prof. Michele Cicoli

Presentata da:
Gabriel Andres Piovano

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Abstract

In this thesis we work on a model of string inflation called *fibre inflation* where the inflaton is a 4D string modulus which parametrises the size of an internal 4-cycle in the six extra dimensions. We investigate whether this inflationary model can predict a tensor-to-scalar-ratio of order $r \approx 0.01$ which might be detectable in the near future. The main constraint comes from the amount of axionic dark radiation produced from the inflaton decay at reheating. Very light axions are a generic feature of 4D string models and behave as extra neutrino species which give rise to $\Delta N_{\rm eff} \neq 0$.
We first analyze the inflationary dynamics and derive the predictions for the spectral index n_s and the tensor-to-scalar ratio r as a function of the number of e-foldings N_e under the requirement of generating the correct measured amplitude of the density perturbations. We then focus on reheating. We start by computing the inflaton couplings to all particles in our model: MSSM fields in the visible sector (open strings living on D7-branes wrapping internal 4-cycles) and light axions belonging to the hidden sector (closed strings living in the bulk of the extra dimensions). This computation allows us to derive the dominant inflaton decay channels which are into Higgses, gauge bosons and light axions. In turn, these decay rates lead to a clear prediction for the reheating temperature $T_{\rm rh} \sim 10^{10}$GeV and the amount of dark radiation $\Delta N_{\rm eff} \sim 0.5 - 0.6$.
These values require $N_e \sim 57$ which can allow for $n_s \simeq 1.000$ and $r \simeq 0.01$ in agreement with Planck observations in the presence of extra dark radiation. We finally point out that, due to the high inflationary scale $M_{\rm inf} \sim 10^{16}$GeV, the supersymmetry breaking scale turns out to be too high to yield a correct Higgs mass around 125 GeV. This tension can be easily overcome if the visible sector is a simple extension of the MSSM like the NMSSM.

Sommario

In questo lavoro di tesi consideriamo un modello di inflazione di stringa chiamato fibre inflation, dove l'inflatone è un modulo di stringa 4D che parametrizza la dimensione di un 4-ciclo nelle sei dimensioni extra. Verificheremo se questo modello inflazionario può predire un tensor-to-scalar-ratio dell'ordine di $r \approx 0.01$, il quale potrebbe essere osservato in un prossimo futuro. Il vincolo principale proviene dall'ammontare di radiazione oscura assionica prodotta dal decadimento dell'inflatone durante il reheating. Assioni molto leggeri sono una proprietà generale di modelli di stringa 4D e si comportano in modo simile ai neutrini dando origine a $\Delta N_{\text{eff}} \neq 0$.

Per prima cosa analizzeremo la dinamica inflazionaria e deriveremo le predizioni per lo spectral index n_s e il tensor-to-scalar-ratio r come funzioni del numero di e-foldings N_e con il vincolo di generare la corretta ampiezza delle perturbazioni di densità. Ci concentreremo poi sul reheating. Inizieremo con il calcolare gli accoppiamenti dell'inflatone a tutte le particelle del nostro modello: campi del MSSM nel settore visibile (stringhe aperte che vivono su D7-brane che wrappano 4-cicli interni) e assioni leggeri che appartengono al settore nascosto (stringhe chiuse che vivono nel volume delle dimensioni extra). Questa calcolo ci consentirà di determinare i decadimenti dominanti dell'inflatone, i quali sono nei bosoni di Higgs, nei bosoni di gauge e negli assioni leggeri. Questi decadimenti conducono ad una chiara predizione per le temperature di reheating $T_{\text{rm}} \sim 10^{10}\,\text{GeV}$ e l'ammontare di radiazione oscura $\Delta N_{\text{eff}} \sim 0.5 - 0.6$.

Questi valori richiedono $N_e \sim 57$ i quali consentono $n_s \simeq 1.000$ e $r \simeq 0.01$ in accordo con le osservazioni di Planck in presenza di extra radiazione oscura. Notiamo infine che, a causa della elevata scala inflazionaria $M_{\text{inf}} \sim 10^{16}\,\text{GeV}$, la scala di energia della rottura della supersimmetria risulta essere troppo elevata per avere una corretta massa dell'Higgs intorno ai $125\,\text{GeV}$. Questo problema può essere facilmente risolto se il settore visibile è una semplice estensione del MSSM come il NMSSM.

Contents

Chapter 1

Introduction

The recent discovery of gravity waves gives us another spectacular confirmation of General Relativity and it starts a new epoch in the exploration of the Universe. In fact, gravitational radiation provides us new ways to look at our Universe. However, General Relativity is far from be a fundamental theory of gravitation, since it describes gravity only at the *classical level*. Moreover, the Standard Model of cosmology, which is based on General Relativity supplemented by the Standard Model of Particle Physic, has many flaws, both theoretical and experimental in nature, despite its undeniable success. *Inflation* provides a possible solution of the problems of the Standard Model of cosmology: inflationary models assume that the Universe has undergone a period of exponential expansion at its very early stages. This expansion is driven (in the simplest models) by a single scalar field, called *inflaton*, whose quantum perturbations lead to the formation of large scale structures and the anisotropies of the CMB observed nowadays. Actually, there are two types of quantum fluctuations around a homogeneous background that occur during inflation:

- inflaton $\delta\phi$, which is the source of scalar perturbations
- metric $\delta g_{\mu\nu}$, giving rise to tensor perturbations

The *tensor-to-scalar ratio* r is essentially the ratio between the power spectrum of tensor and scalar perturbations. Since gravity waves can be seen as perturbations of the metric tensor, a relatively large tensor-to-scalar-ratio acts as marker of gravity waves in the early stages of the Universe, i.e. primordial gravity waves. A tensor-to-scalar ratio of order $r \approx 0.01$ is in range of near future experiments and its detection would be a smoking gun for primordial gravity waves.

A useful observable related to r is the *spectral index* n_s, which essentially measures the scale invariance of the power spectrum of the density perturbations. It appears that the spectral index n_s is positively correlated with the number of neutrino-like species, ofter referred to as dark radiation. Hence the amount of dark radiation constraints the predictions for n_s and r.

Together with General Relativity, the Standard Model of particle physic represents one of the cornerstones of our current understanding of Nature down to 10^{-18} m, which is the smallest length actually probed with the LHC. The Standard Model of Particle Physic has been tested so far with great accuracy and the discovery of the Higgs boson in 2012 has signed the complete affirmation of this theory. Despite its outstanding success, it turns out to be only an *effective* field theory of a more fundamental one. A compelling theory which can accommodate gravity and quantum mechanics in an unified framework is instead *string theory*.
From a cosmological perspective, it includes many scalar particles which can play the rôle of the inflation, making possible to embed an inflationary model in the low-energy limit of string theory.

In this thesis we will work on a model of inflation derived from string, named *fibre inflation*, which can easily predict a high tensor to-scalar ratio. The aim of this work is to find out if fibre inflation can predict a tensor-to-scalar ratio of order $r \simeq 0.01$ which is consistent with current experimental constraints. This thesis has the following structure:

- Chapter two is a brief description of dark radiation
- Chapter three is an overview of global supersymmetry and the MSSM
- Chapter four is focused on the supergravity low-energy limit of string compactifications, supersymmetry breaking soft terms and moduli fields
- Chapter five illustrates the fibre inflation model. We shall perform a numerical evaluation so as to give a range of values for n_s and r consistent with the COBE normalisation of the density perturbations
- Chapter six contains the main results of this thesis. We will carry out our analysis by calculating the decay rates of inflaton, the reheating temperature and the number of extra neutrino species. All this information combined together allow us to conclude that $r \simeq 0.01$ is viable in fibre inflation.

Chapter 2

Dark Radiation, neutrinos and axions

2.1 Lambda-CDM model

The cosmological counterpart of the Standard Model is the ΛCDM model: it is a parametric version of the Hot Big Bang paradigm with Dark Energy and Dark Matter. More precisely, it assumes that the accelerated expansion of the Universe is due to a cosmological constant Λ i.e. from vacuum energy density, while non baryonic matter are Cold Dark Matter.

2.1.1 Review of standard cosmology

The Hot Big Bang theory is based under the assumption that the evolution of the Universe is described in term of General Relativity. We briefly recall here some concepts about cosmology. We start with the Einstein's field equations

$$\mathcal{R}^{\mu\nu} - \frac{1}{2}\mathcal{R}g^{\mu\nu} = 8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

Assuming that the Universe

- is isotropic and uniform
- is filled with a perfect fluid (in other words, timelike geodesics do not intersect except that in a singular point in the past and may be in a singularity in the future)

and using comoving coordinates, the solution of the Einstein field equation is the Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{2.1.1}$$

Here, the scale factor $a(t)$ characterizes the relative size of spacelike hypersurfaces Σ at different times. The curvature parameter k is $+1$ for positively curved Σ, 0 for flat Σ, and -1 for negatively curved Σ. It is possible to study the evolution of the homogeneous and isotropic Universe with only one function, the scale factor $a(t)$: the behaviour of the scale factor depends on the energy-momentum tensor and so it depends on the energy and matter content of the Universe. For a perfect fluid the energy momentum tensor is

$$T^{\mu}{}_{\nu} = (\rho + p)u^{\mu}u_{\nu} - p\delta^{\mu}_{\nu} \tag{2.1.2}$$

where ρ and p are respectively the proper energy density and pressure in the fluid rest frame and u^{μ} is the 4-velocity of the fluid. In a frame that is comoving with the fluid we may choose $u^{\mu} = (1,0,0,0)$ and so

$$T^{\mu}{}_{\nu} = (\rho + p)\delta^{\mu}_{0}\delta^{0}_{\nu} - p\delta^{\mu}_{\nu} \tag{2.1.3}$$

Thus, we can recast the Einstein equations in two coupled, non linear ordinary differential equations called the *Friedmann equations*

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2} + \frac{1}{3}\Lambda \tag{2.1.4a}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) \tag{2.1.4b}$$

where H is the *Hubble parameter* H, defined as

$$H := \frac{\dot{a}}{a} \tag{2.1.5}$$

Equations (2.1.4a) and (2.1.4b) can be combined into a continuity equation: in fact, if we derive (2.1.4a) respect to the time, we obtain

$$2H\dot{H} = \frac{1}{3}\dot{\rho} + 2k\frac{\dot{a}}{a^3}$$

and putting this expression into (2.1.4b) leads to

$$-\frac{H}{3}(\rho + 3p) - 2H^3 = \frac{1}{3}\dot{\rho} + 2\frac{kH}{a^2}$$

If we substitute the first Friedmann equation in the previous expression, we can write the following continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{2.1.6}$$

Using the fact that

$$\frac{a}{\dot{a}}\frac{\mathrm{d}}{\mathrm{d}t} = \frac{a}{\dot{a}}\frac{\mathrm{d}a}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}a} = a\frac{\mathrm{d}\ln(a)}{\mathrm{d}a}\frac{\mathrm{d}}{\mathrm{d}\ln(a)} = \frac{\mathrm{d}}{\mathrm{d}\ln(a)}$$

the continuity equation can also be written as

$$\frac{\mathrm{d}\ln(\rho)}{\mathrm{d}\ln(a)} = -3(1+\omega) \tag{2.1.7}$$

if we define the equation of state parameter

$$\omega = \frac{p}{\rho} \tag{2.1.8}$$

Eq. (2.1.7) may be integrated to give

$$\rho \propto a^{-3(1+\omega)} \tag{2.1.9}$$

We can now find the form of the scale factor by solving eq. (2.1.4a) with the energy density given by the previous expression. In the case of a flat Universe ($k = 0$) the solution of the Friedmann equation is

$$a(t) \propto \begin{cases} t^{\frac{2}{3(1+\omega)}} & \omega \neq -1 \\ e^{Ht} & \omega = -1 \end{cases} \tag{2.1.10}$$

The contributions to the overall energy density are mainly divided in *non-relativistic matter*, *relativistic matter (or radiation)* and *dark energy*. In general, the Universe may have non-vanishing *spatial curvature*. For a flat Universe dominated by one kind of matter, the energy density and the scale factor are

$$\rho \propto \begin{cases} a^{-3} & \textit{non-relativistic matter domination} \;\; \omega = 0 \\ a^{-4} & \textit{radiation domination} \;\; \omega = \frac{1}{3} \\ a^{0} & \textit{cosmological constant domination} \;\; \omega = -1 \end{cases} \tag{2.1.11}$$

$$a(t) \propto \begin{cases} t^{2/3} & \textit{non-relativistic matter domination} \;\; \omega = 0 \\ t^{1/2} & \textit{radiation domination} \;\; \omega = \frac{1}{3} \\ e^{Ht} & \textit{cosmological constant domination} \;\; \omega = -1 \end{cases} \tag{2.1.12}$$

In the presence of more matter species with non negligible energy densities, the first Friedmann equation (2.1.4a) becomes

$$H^2 = \frac{1}{3}(\rho_M + \rho_{\mathrm{rad}} + \rho_\Lambda + \rho_{\mathrm{curv}}) \tag{2.1.13}$$

where ρ_M, ρ_{rad}, ρ_Λ are respectively the energy densities of non-relativistic matter, relativistic matter ("radiation") and dark energy, and by definition

$$\frac{1}{3}\rho_{\text{curv}} = -\frac{k}{a^2} \tag{2.1.14}$$

is the contribution due to spatial curvature. Let us introduce the critical density ρ_c by

$$\rho_c := 3H_0^2 \tag{2.1.15}$$

Remember that in cosmology is customary to label with a subscript '0' a quantity evaluated at the present time t_0. Using the following parameters

$$\Omega_M = \frac{\rho_{M,0}}{\rho_c} \qquad \Omega_{\text{rad}} = \frac{\rho_{\text{rad},0}}{\rho_c} \qquad \Omega_\Lambda = \frac{\rho_{\Lambda,0}}{\rho_c} \qquad \Omega_{\text{curv}} = \frac{\rho_{\text{curv},0}}{\rho_c} \tag{2.1.16}$$

the first Friedmann (2.1.4a) takes the form

$$H^2 = \frac{1}{3}\rho_c\left[\Omega_{\text{matter}}\left(\frac{a_0}{a}\right)^3 + \Omega_{\text{radiation}}\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{\text{curv}}\left(\frac{a_0}{a}\right)^2\right] \tag{2.1.17}$$

If we evaluate the previous equation today, we get the consistency relation

$$1 = \Omega_{\text{matter}} + \Omega_{\text{radiation}} + \Omega_\Lambda + \Omega_{\text{curv}}$$

while the second Friedmann (2.1.4b) at $t = t_0$ becomes

$$\frac{1}{a_0 H_0^2}\frac{\mathrm{d}^2 a_0}{\mathrm{d}t^2} = -\frac{1}{2}(\Omega_{\text{matter}} + 2\Omega_{\text{radiation}} + -2\Omega_\Lambda) \tag{2.1.18}$$

2.1.2 Experimental evidence for dark radiation

Measurements of the Baryon Acoustic Oscillation (BAO) and of the Cosmic Microwave Background (CMB) combined with redshift and brightness measurements of supernovae (SNe) were able to drastically constrain the Ω_i. These observations working out that the Universe we live in is almost exactly *flat* i.e his spatial curvature is near to zero ($\Omega_k \approx 0$). Moreover, the Ω_i measured are [1]

$$\Omega_{\text{radiation}} \approx 5\cdot 10^{-5} \qquad \Omega_{\text{matter}} \approx 0.27 \qquad \Omega_\Lambda \approx 0.73 \tag{2.1.19}$$

with $\omega_\Lambda \approx -1$. Non relativistic matter is divided in *baryonic matter*, that is observable through electromagnetic radiation, and *dark matter*, which instead does not interact electromagnetically and strongly.

$$\Omega_{\text{baryons}} \simeq 0.046 \qquad \Omega_{\text{DM}} \simeq 0.23 \tag{2.1.20}$$

Dark matter can be firstly classified in *non thermal* and *thermal*. Thermal dark matter is originated via decoupling from a thermal bath and it can be further classified in *hot dark matter* and *cold dark matter*. Particle candidates which were relativistic at the time of decoupling constitute hot dark matter, while cold dark matter is formed of particles which were non relativistic.

From these data it is evident that the major contribution to the overall energy density comes from dark radiation with properties close to a cosmological constant Λ. Also, a significant contribution is due to *dark matter*, while known matter of the universe gives a negligible contribution to the energy density of the universe. These cosmological observations strongly suggests the search of new models beyond the Standard Model of particle physic.

2.2 Dark radiation and extra neutrino species

Dark radiation is usually quantified in terms of *extra neutrino species* ΔN_{eff}, i.e. the difference between the number of neutrino families of the Standard Model $N_{\text{eff, SM}} = 3$ and the effective number of neutrino specie N_{eff}. In order to understand how ΔN_{eff} can be related to the energy density of dark radiation and why it is useful doing so, we have to review some basic notions about thermodynamics and the thermal history of the universe. Assuming that early universe was in *local thermal equilibrium* (this hypothesis is suggest from the perfect black-body spectrum of the CMB) allows us to use the tools given by statistical mechanics. Remember that for a relativistic perfect gas the energy density is given by

$$\rho = \frac{\pi^2}{30} g(T) T^4 \begin{cases} 1 & \text{bosons} \\ \frac{7}{8} & \text{fermions} \end{cases} \tag{2.2.1}$$

where T is the temperature of the gas and g_* is the number of degree of freedom, i.e spin. Let us now consider a thermal bath of photons and other relativistic particle and let be T the temperature of the photon gas. We can define the *effective number of relativistic degrees of freedom* $g_*(T)$ as [2]

$$g_* = g_*^{\text{th}} + g_*^{\text{dec}} \tag{2.2.2}$$

where

$$g_*^{\mathrm{th}}(T) = \sum_{i=b} g_i + \frac{7}{8}\sum_{i=f} g_i \tag{2.2.3}$$

$$g_*^{\mathrm{dec}}(T) = \sum_{i=b} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{i=f} g_i \left(\frac{T_i}{T}\right)^4 \tag{2.2.4}$$

g_* has two contributions:

- g_*^{th} is given by the particles in thermal equilibrium with the photon gas i.e. $T_i = T \gg m_i$. When the temperature T drops below the mass m_i of a particle species, it becomes non relativistic and decouple from the photons, giving no more contributions to g_*
- g_*^{reh} is the contribution due to relativistic particle decouple from the photon. Neutrinos after $e^{\pm}$ annihilations are an example, within an excellent approximation, of relativistic particles decouple from photons

Neutrinos interact with the thermal plasma only through weak interactions and decouple from photons at $T \approx 1$ MeV. At this temperature, only photons, neutrinos and electrons (with the corresponding antiparticles) are still relativistic, so g_* is given by [2]

$$g_* = 2 + \frac{7}{8} 2 N_{\mathrm{eff,\,SM}} + \frac{7}{8} 4 \cdot 2 = 10.75 \tag{2.2.5}$$

with $N_{\mathrm{eff,\,SM}} = 3$ in the case of instantaneous neutrino decoupling. However, when the temperature T dropped below the electron mass starting e^+e^- annihilations neutrino decoupling was still occurring, so neutrinos got some energy and entropy from electron and positron decoupling. Taking this into account raises the effective number of neutrinos to $N_{\mathrm{eff,\,SM}} = 3.046$.

2.2.1 Entropy conservation

In the case of relativistic particles, we have that

$$\rho = \frac{\pi^2}{30} g_* T^4 \qquad\qquad p = \frac{1}{3}\rho$$

so, we get

$$\frac{\partial p}{\partial T} = \frac{4}{3}\frac{\pi^2}{30} g_* T^3 = \frac{\rho + p}{T} \tag{2.2.6}$$

Using the second law of thermodynamics: $TdS = dU + pdV$ combined with $U = \rho V$ and (2.2.6) , we obtain

$$\begin{aligned} dS &= \frac{1}{T}[d(\rho V) + d(pV) - V\,dp] \\ &= \frac{1}{T}\,d[(\rho+p)V] - \frac{V}{T^2}(\rho+p)\,dT \\ &= d\left[\frac{\rho+p}{T}V\right] \end{aligned} \tag{2.2.7}$$

For thermal process at equilibrium, we can show that entropy is conserved. In fact

$$\begin{aligned} \frac{dS}{dt} &= \frac{d}{dt}\left[\frac{\rho+p}{T}V\right] \\ &= \frac{V}{T}\left[\frac{d\rho}{dt} + \frac{1}{V}\frac{dV}{dt}(\rho+p)\right] + \frac{V}{T}\left[\frac{dp}{dt} - \frac{\rho+p}{T}\frac{dT}{dt}\right] \\ &= \frac{V}{T}\left[\frac{d\rho}{dt} + 3H(\rho+p)\right] + \frac{V}{T}\frac{dT}{dt}\left[\frac{dp}{dT} - \frac{\rho+p}{T}\right] = 0 \end{aligned} \tag{2.2.8}$$

This show the conservation of entropy at equilibrium, since the first term vanishes by the continuity equation (2.1.6) while the second is equal to zero due to equation (2.2.6).
It is more convenient to work with *entropy density* $s \equiv S/V$ and from (2.2.7) s is equal to

$$s = \frac{\rho+p}{T} \tag{2.2.9}$$

Using (2.2.6) the total entropy can be written as [2]

$$s = \sum_i \frac{\rho_i + p_i}{T_i} = \frac{2\pi^2}{45} g_{*S}(T)T^3 \tag{2.2.10}$$

where we have defined the *effective number of degrees of freedom in entropy*,

$$g_{*S} = g_{*S}^{\rm th} + g_{*S}^{\rm dec} \tag{2.2.11}$$

where

$$g_*^{\rm th}(T) = g_{*S}^{\rm th}(T) \tag{2.2.12}$$

$$g_{*S}^{\rm dec}(T) = \sum_{i=b} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{i=f} g_i \left(\frac{T_i}{T}\right)^3 \neq g_*^{\rm dec}(T) \tag{2.2.13}$$

Hence, g_{*S} is equal to g_* only when *all* relativistic species are in equilibrium at the same temperature. In the real universe this is true until the e^+e^- annihilation, when the neutrinos are decoupled from the photons but they can still be considered relativistic.

An important consequence of the entropy conservation is, via eq. (2.2.10), that

$$g_{*S}(T)T^3a^3 = \text{const} \implies T \propto g_{*S}^{-1/3}a^{-1} \tag{2.2.14}$$

Whenever a particle species becomes non-relativistic, its entropy is transferred to the other relativistic species still present in the thermal plasma, so T decrease slightly less slowly than a^{-1} thanks to the factor $g_{*S}^{-1/3}$. This is happening in e^+e^- annihilation, where electrons and positrons decouple from the thermal bath.

$$g_{*S}^{\text{th}} = \begin{cases} 2 + \frac{7}{8} \cdot 4 = \frac{11}{2} & T \gtrsim m_e \\ 2 & T < m_e \end{cases} \tag{2.2.15}$$

In equilibrium, the entropy of thermal bath and of the decoupled particles is separately conserved, so we have

$$g_{*S}^{\text{th}}(T_1) \cdot (a_1 T_1)^3 = g_{*S}^{\text{th}}(T_2) \cdot (a_2 T_2)^3 \implies (a_2 T_2) = \left(\frac{11}{4}\right)^{1/3} (a_1 T_1)$$

where T_1 and T_2 are the temperature of the thermal bath (i.e of the photons) respectively before and after e^+e^- annihilation. If the universe did not expand so much during the electron-positron decoupling then $a_1 \approx a_2$ and

$$T_2 = \left(\frac{11}{4}\right)^{1/3} T_1 \tag{2.2.16}$$

Therefore the photons are slightly reheated while the neutrinos temperature T_ν do not change because neutrinos are decoupled from photons. Before e^+e^- annihilations, $T_1 \approx T_\nu$ since $T_\nu \propto a^{-1}$ was still valid with an excellent approximation. Thus, defining $T_\gamma = T_2$ the photons temperature after e^+e^- annihilations, we get

$$T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_\nu \tag{2.2.17}$$

Equation (2.2.17) is valid up to now and it allows to link the total energy density of relativistic particles ρ_{total} at CMB time with N_{eff}:

$$\begin{aligned}\rho_{\text{total}} &= \rho_\gamma + \rho_\nu \\ &= \frac{\pi^2}{30} 2T_\gamma^4 + \frac{\pi^2}{30}\frac{7}{8} 2N_{\text{eff}} \left(\frac{T_\nu}{T_\gamma}\right)^4 T_\gamma^4 \\ &= \frac{\pi^2}{30} 2T_\gamma^4 \left(1 + \frac{7}{8} N_{\text{eff}} \left(\frac{4}{11}\right)^{4/3}\right)\end{aligned}$$

then

$$\rho_{\text{total}} = \rho_\gamma \left(1 + \frac{7}{8} N_{\text{eff}} \left(\frac{4}{11}\right)^{4/3}\right) \tag{2.2.18}$$

where ρ_γ and ρ_ν are respectively the energy density of radiation and neutrinos. Therefore, if $\Delta N_{\text{eff}} > 0$ dark radiation can be explained in terms of relativistic particles at CMB times not included in the Standard Model.

2.2.2 Current evidences for extra neutrino species

Extra relativistic degrees of freedom are mainly constrained by the measurements of CMB anisotropies. In fact, extra radiation implies a faster expansion of the Universe in the past and so a higher Hubble constant, leaving a footprint in the anisotropies of the CMB which is observable in the angular power spectrum of the CMB temperature.

Estimates from Hubble constant measurement

Estimates of the present value of the Hubble constant H_0 can be obtained through direct astrophysical measurements or from indirect measurements via CMB experiments. These two methods to measure H_0 explore different periods in the history of the Universe, thus any tension between them may be a smoking gun of the presence of new physics. In fact, CMB measurements are generally model dependent: the *Planck* collaboration fitted his data with the 6 parameter ΛCDM model taking $\Delta N_{\text{eff}} = 3.046$. On the contrary, direct observations of H_0 do not use ΔN_{eff} as a prior.
The Hubble Space Telescope (HST) [3] (hereafter R11) measured $H_0 = (73.8 \pm 2.4)\,\text{kms}^{-1}\text{Mpc}^{-1}$(68% CL). However, the author of [4] (hereafter E14) re-analyzed R11 Cepheid data and found a lower value $H_0 = (70.6 \pm 3.3)$ $\text{kms}^{-1}\text{Mpc}^{-1}$(68% CL).
On the CMB side, the *Planck* collaboration in 2013 and 2015 found respectively $H_0 = (67.3 \pm 1.3)\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$(68% CL) [5] and $H_0 = (67.3 \pm 1.0)$

km s^{-1} Mpc^{-1}(68% CL) [6]. Both of them are in tension at 2.5σ with R11 while they are within 1σ with respect to E14. Using the R11 value as a H_0 prior, *Planck* 2013 estimated $N_{\rm eff} = 3.62^{+0.50}_{-0.48}$(95% CL). By contrast, *Planck* 2015 found $\Delta N_{\rm eff} = 3.13 \pm 0.32$(68% CL) without using any prior.
So far a general consensus regarding the HST measure of the Hubble constant is missing, hence the *Planck* 2015 collaboration has also measured H_0 taking $\Delta N_{\rm eff} = 0.39$ as a prior and they have obtained $H_0 = (70.6 \pm 1.0)$ km s^{-1} Mpc^{-1}(68% CL), which is in good agreement with the H_0 value found by E14. But this is not the end of the story: the authors of [7] have performed new HST observations and they have also improved the previous analysis made by R11, giving $H_0 = (73.24 \pm 1.74)$ kms^{-1}Mpc^{-1}(68% CL).
The tension between HST and CMB measurements of H_0 could be solved or at least ameliorated if we take into account that H_0 and $N_{\rm eff}$ are positively correlated, as shown in fig. 2.1. So, a non zero $\Delta N_{\rm eff}$ leads to a value of H_0 higher than $N_{\rm eff} = 3.014$.

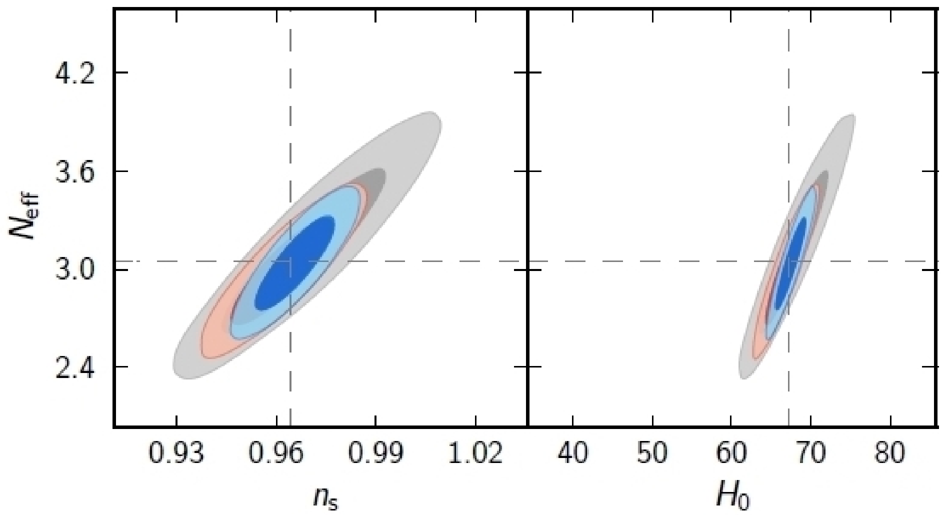

Figure 2.1: Adapted from [6, pag.33, fig.20]. 68% and 95% confidence regions on 1-parameter extensions of the base ΛCDM model for *Planck* TT+lowP (grey), *Planck* TT,TE,EE+lowP (red) and *Planck* TT,TE,EE+low P+BAO (blue). Horizontal dashes lines correspond to $N_{\rm eff} = 3.014$.

Relations between $\Delta N_{\rm eff}$, n_s and r

As we said previously, the presence of extra relativistic degrees of freedom affect the power spectrum of the CMB. The tensor-to-scalar-ratio r and the spectral index n_s measurements made by Planck assume the validity of the ΛCDM model with $N_{\text{eff}} = 3.014$. Taking into account an N_{eff} greater than 3.014 leads to different predictions with respect to the base ΛCDM model, as shown in fig. 2.2. In particular, for $\Delta N_{\text{eff}} = 0.39$ the spectral index

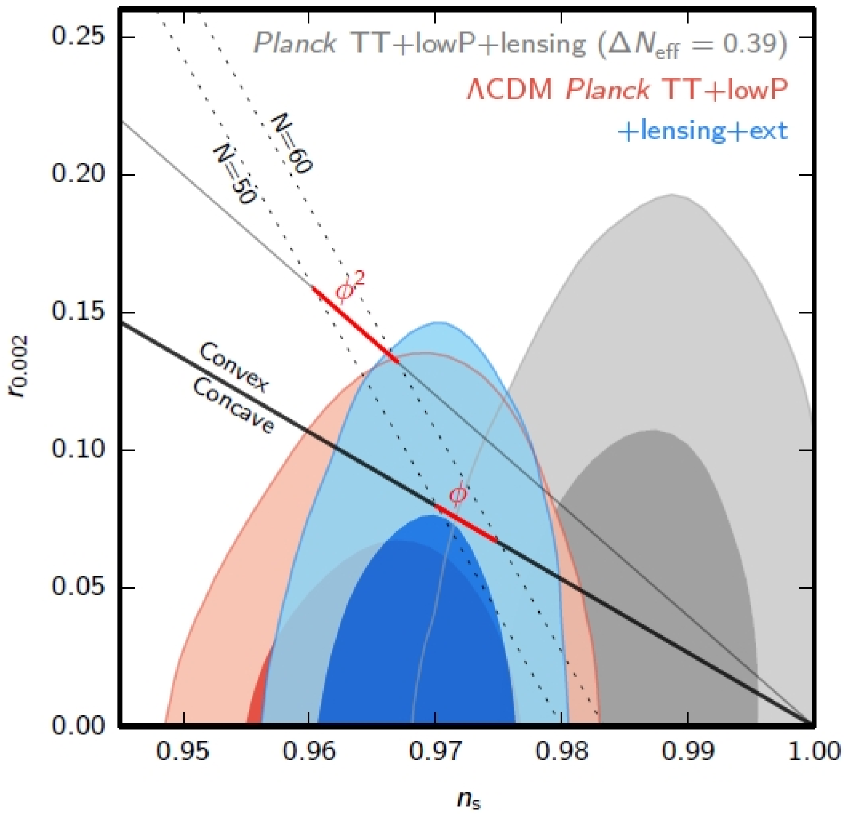

Figure 2.2: Adapted from ref. [6, pag.34, fig.21]. Constraints on the tensor-to-scalar ratio $r_{0.002}$ in the ΛCDM model, using *Planck* TT+lowP (red) and *Planck* TT+lowP+lensing+BAO+JLA+H_0. The result is model-dependent: for example, the grey contours show how the results change if there were $\Delta N_{\text{eff}} = 0.39$

is in the range $n_s \approx 0.98 - 0.99$ at one σ. We will use these experimental constraints in the next chapters in order to make a prediction for r in our model of fibre inflation. In fact, given a theoretical estimate for ΔN_{eff}, we can infer a reasonable value for n_s from fig. 2.1. Since n_s and r are not independent, we make a prediction for r given n_s, provided that inflation is viable for these values.

2.2.3 Why axions as dark radiation?

As we have seen previously, dark radiation is composed of particles not predicted by the Standard Model which are relativistic at CMB and BBN times, so these particles need to be extremely light and very weakly interacting. Axions and axion-like-particles (ALPs) are scalar particles which meet both the previous requirements since they enjoy an approximate Peccei Quinn shift symmetry that forbids perturbative mass terms, making them almost massless [8].
Moreover, ALPs often appear in low energy effective models derived from string theory since they are the imaginary part of *moduli fields*. Moduli are complex fields ubiquitous in the context of string compactification which interact only through gravity, so they are long lived and may had dominated the energy density of the Universe. For these reasons, we can build up a model of string inflation where the inflaton is the real part of a modulus with a non zero branching ratio for decays into light axions. This is possible if some of the moduli are stabilised in a perturbative way, since the real part of the moduli is not protected by any approximated shift symmetry and so it can takes a mass greater than that of the axions. In this situation the decay channels for one or more real parts of the moduli into axions are open, making axionic dark radiation an unavoidable feature [9].
There are other possible candidates as dark radiation particles, like hidden photons or right handed neutrinos, but dark radiation axions are, in principle, a general feature of moduli string inflation. So, axions can be produced in a natural way from the decay of the inflaton into hidden sectors of the theory.

2.2.4 Magnitude of dark radiation

We now derive a formula which expresses $\Delta N_{\rm eff}$ in terms of the *branching ratio* B_a of the inflaton decay into axions. Let be

$$\rho_{\nu 1} := \frac{\rho_\nu}{N_{\rm eff,SM}} = \frac{7}{8}\rho_\gamma \left(\frac{4}{11}\right)^{4/3} \tag{2.2.19}$$

the energy density of one neutrino species at neutrino decoupling. Given that the energy density of dark radiation at neutrino decoupling can be written as

$$\rho_{\rm DR} = \frac{7}{8}\rho_\gamma \Delta N_{\rm eff} \left(\frac{4}{11}\right)^{4/3} \tag{2.2.20}$$

it follows that

$$\Delta N_{\text{eff}} = \left.\frac{\rho_{\text{DR}}}{\rho_{\nu 1}}\right|_{\nu\,\text{decouple}} \tag{2.2.21}$$

In our model dark radiation is composed only of axions, which never thermalize during the history of universe with the thermal bath of SM particles because they interact only gravitationally. Therefore, the energy density of dark radiation and thermalized radiation scales as

$$\rho_{\text{DR}} = \rho_a \propto a^{-4} \tag{2.2.22}$$

$$\rho_{\text{SM}} \propto g_*(T)^{-1/3} a^{-4} \tag{2.2.23}$$

The ratio between ρ_{DR} and ρ_{SM} at T_{rh}, the *reheating temperature of the universe*, can be expressed in terms of B_a as (see [10])

$$\left.\frac{\rho_{\text{DR}}}{\rho_{\text{SM}}}\right|_{T=T_{\text{rh}}} \simeq \frac{B_a}{1-B_a} \tag{2.2.24}$$

Moreover, this ratio evolves with time as

$$\left.\frac{\rho_{\text{DR}}}{\rho_{\text{SM}}}\right|_{T=T_1} = \left(\frac{g_*(T_1)}{g_*(T_2)}\right)^{1/3} \left.\frac{\rho_{\text{DR}}}{\rho_{\text{SM}}}\right|_{T=T_2} \tag{2.2.25}$$

Putting all the previous equations together, we finally get

$$\begin{aligned}
\Delta N_{\text{eff}} = \left.\frac{\rho_{\text{DR}}}{\rho_{\nu 1}}\right|_{\nu\,\text{decouple}} &= \left.\frac{\rho_{\text{SM}}}{\rho_{\nu 1}}\right|_{\nu\,\text{decouple}} \left.\frac{\rho_{\text{DR}}}{\rho_{\text{SM}}}\right|_{\nu\,\text{decouple}} \\
&= \left.\frac{\rho_{\text{SM}}}{\rho_{\nu 1}}\right|_{\nu\,\text{decouple}} \left(\frac{10.75}{g_*(T_{\text{rh}})}\right)^{1/3} \left.\frac{\rho_{\text{DR}}}{\rho_{\text{SM}}}\right|_{T=T_{\text{rh}}} \\
&= \left.\frac{\rho_{\text{SM}}}{\rho_{\nu 1}}\right|_{\nu\,\text{decouple}} \left(\frac{10.75}{g_*(T_{\text{rh}})}\right)^{1/3} \frac{B_a}{1-B_a}
\end{aligned} \tag{2.2.26}$$

where $g_*(T_{\nu\,\text{decouple}}) = 10.75$. At the temperature of neutrino decoupling $T_{\nu\,\text{decouple}}$, the energy density of ρ_{SM} and $\rho_{\nu 1}$ are given by

$$\rho_{\text{SM}} = \frac{\pi^2}{30} g_*(T_{\nu\,\text{decouple}}) T^4_{\nu\,\text{decouple}}$$

$$\rho_{\nu 1} = \frac{\pi^2}{30}\left(\frac{7}{8}2\right) T^4_{\nu\,\text{decouple}}$$

Thus

$$\boxed{\Delta N_{\text{eff}} = \frac{43}{7}\left(\frac{10.75}{g_*(T_{\text{rh}})}\right)^{1/3} \frac{B_a}{1-B_a}} \tag{2.2.27}$$

Chapter 3

Supersymmetry and the MSSM

In this chapter we will briefly overview global supersymmetry and the MSSM. In a few words, supersymmetry relates particles with different statistic, i.e. bosons and fermions, into the same multiplet. The Minimal Supersymmetric Standard Model is an extension of the Standard Model of particle physics in a supersymmetric fashion. It is the low energy description of more fundamental theories like string theories, so it is certainly worth to study it.

3.1 Global supersymmetry

Supersymmetry (SUSY) arises as an extension of the Poincaré symmetry. In fact, a no-go theorem by Coleman and Mandula shows that, under mild and reasonable assumptions, the only possible ***Lie algebra*** related to the continuous symmetries of the S-matrix has the following generators:

- Poincarè generators P_μ, $M_{\mu\nu}$
- a finite number of Lorentz-scalars generators B_l which are related to some conserved quantum number (like electric charge, isospin, etc...)

Obliviously, the generator B_l must satisfied the following commutators relations

$$[P_\mu, B_l] = 0 \qquad\qquad [M_{\mu\nu}, B_l] = 0$$

Supersymmetries avoid the restrictions of Coleman-Mandula theorem by relaxing one of its assumptions. Indeed, a theorem by Haag, Lopuszanski and Sohnius shows that it is possible to generalised the Coleman Mandula theorem by considering a ***graded Lie algebra***, which are defined by commutators and anticommutators. The simplest extension of the Poincaré algebra can be obtained by including a Majorana spinor charge with 4 components $\mathcal{Q}_a$, $a = 1, \ldots, 4$ which satisfies the following (anti)commutation relations

$$\{\mathcal{Q}_a, \overline{Q}_b\} = 2(\gamma^\mu)_{ab} P_\mu \qquad [\mathcal{Q}_a, P_\mu] = 0 \qquad [\mathcal{Q}_a, M_{\mu\nu}] = (\sigma^{\mu\nu}{}_4)_{ab} \mathcal{Q}_b$$

where

$$(\sigma^{\mu\nu}{}_4) = \frac{i}{4}[\gamma^\mu, \gamma^\nu] \qquad\qquad \overline{Q}_a = (\mathcal{Q}^\dagger \gamma_0)_a$$

In terms of two component Weyl spinors $\mathcal{Q}_\alpha, \overline{\mathcal{Q}_{\dot\alpha}}$ the previous relations look like

$$\{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = 0 \qquad \{\overline{\mathcal{Q}}_{\dot\alpha}, \overline{\mathcal{Q}}_{\dot\beta}\} = 0 \qquad \{\mathcal{Q}_\alpha, \overline{\mathcal{Q}}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu$$

$$[\mathcal{Q}_\alpha, M^{\mu\nu}] = i(\sigma^{\mu\nu}{}_2)_\alpha{}^\beta \mathcal{Q}_\beta \qquad [\mathcal{Q}_\alpha, P_\mu] = 0 \qquad [\overline{\mathcal{Q}}_{\dot\alpha}, P_\mu] = 0$$

Majorana spinor charges with spin 1/2 are the only fermionic generators allowed by the theorem of Haag, Lopuszanski and Sohnius. In other words, there cannot be a consistent extension of the Poincaré algebra including generators transforming in dimensional representation higher than 1/2 under the Lorentz group. From now on we consider only one spinor charge ($N = 1$). The graded extension of the Poincaré algebra is often called *superalgebra*.

$N = 1$ supersymmetry representations

In ordinary quantum field theory massive and massless particles are irreducible representations of the Poincaré group and they are labels through the eigenvalues of the Casimir operators. The same things can be done for supersymmetry: it turns out that $C_1 = P^\mu P_\mu$ is still a good Casimir while $C_2 = W^\mu W_\mu$ is not. Thus, in the same supermultiplet there are particles with the same mass and different spin. It is possible to show that the operator $\overline{C}_2$ defined as:

$$B_\mu := W_\mu - \frac{1}{4}\overline{\mathcal{Q}}_{\dot\alpha}(\bar\sigma_\mu)^{\dot\alpha\beta}\mathcal{Q}_\beta \qquad C_{\mu\nu} := B_\mu P_\nu - B_\nu P_\mu \qquad \overline{C}_2 := C_{\mu\nu} C^{\mu\nu}$$

is a Casimir operator for the superalgebra. Thanks to C_1 and $\overline{C}_2$ each supermultiplet is labelled by the mass m (eigenvalue of C_1) and by the so called superspin y (eigenvalue of $\overline{C}_2$). In any supersymmetric multiplet, the number of bosonic degrees of freedom equals the number of fermionic degrees of freedom and for $N = 1$ supersymmetry the irreducible representations belongs to one of three kinds of supermultiplet:

- chiral multiplet. It consists of a complex scalar ϕ and a Weyl spinor ψ_α ,
- vector or gauge multiplet, which contain a vector A_μ and one Weyl spinor,
- gravity multiplet. It contain a spin 3/2 particle, the gravitino, and the graviton.

Internal symmetry generators commute with the SUSY generators, so particles within the same supermultiplet share the same quantum numbers.

Supersymmetric theories have many features that make them very appealing as (high energy) extension of the Standard Model. First of all, SUSY leads to cancellations between bosonic and fermionic corrections to the Higgs boson mass (the so called miraculous cancellation), solving the naturalness issue of the hierarchy problem.
From a more theoretical perspective, SUSY relates bosons with fermions since they are both contained in the same representation of the super Poincaré group. Furthermore, in SUSY theories it happens that the three gauge couplings unify at one single point at larger energies. This unification does not occur in the Standard Model. Furthermore SUSY theories emerge naturally as an effective low energy description of more fundamentals theories like string theory. Actually, it turns out that SUSY is required for making string theory consistent since it allows the absence of tachyonic scalars.
In the next subsection we will take a look at how to build up actions invariant under SUSY with the aid of superfields and superspace.

3.1.1 Superfields and superspace

In relativistic quantum field theories the formalism is chosen in such a manner that the physical equations are manifestly Lorentz covariant. This choice allows to easily build up actions which are invariant under Lorentz transformations. One may think to do the same for SUSY theory by introducing the notion of superspace and superfields.
Superspace is a generalisation of $4D$ Minkowski space (x^0, x^1, x^2, x^3) by including additional anticommuting spinorial coordinates θ_α , $\bar{\theta}_{\dot{\alpha}}$ on which SUSY transformations act. Thus, superspace is parametrised by coordinates $(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$. The anticommuting properties of the fermionic coordinates

imply

$$\theta_1^2 = \theta_2^2 = 0 \qquad \theta_1\theta_2 = -\theta_2\theta_1 \qquad \int \mathrm{d}\,\theta_\alpha\theta_\beta \equiv \frac{\partial}{\partial\theta_\beta}\theta_\alpha = \delta_\alpha^\beta$$

$$\int \mathrm{d}\,\theta_1 \int \mathrm{d}\,\theta_2 := \int \mathrm{d}^2\,\theta \qquad \int \mathrm{d}\,\theta_{\dot{1}} \int \mathrm{d}\,\theta_{\dot{2}} := \int \mathrm{d}^2\,\overline{\theta}$$

$$\int \mathrm{d}^2\,\theta(\theta\theta) = 1 \qquad \int \mathrm{d}^2\,\overline{\theta}(\overline{\theta}\overline{\theta}) = 1$$

Let be ϵ a Grassmann variable parameter: a translation in the superspace can be written as

$$x^\mu \mapsto x^\mu - i(\epsilon\sigma^\mu\overline{\theta}) + i(\theta\sigma^\mu\overline{\epsilon})$$

and the Majorana spinor $\mathcal{Q}_\alpha$ charges can be seen as generators of superspace translations

$$\mathcal{Q}_\alpha = -i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu)_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\frac{\partial}{\partial x^\mu} \tag{3.1.1}$$

$$\overline{\mathcal{Q}}_{\dot{\alpha}} = +i\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} - \theta^\beta(\sigma^\mu)_{\beta\dot{\alpha}}\frac{\partial}{\partial x^\mu} \tag{3.1.2}$$

Superfields unify the different components of a supermultiplet into a single mathematical object and they are functions of the superspace coordinates. It follows that superfields have a finite power expansion in $\theta_\alpha, \overline{\theta}_{\dot{\alpha}}$ leading to a finite number of components fields which belong to the supermultiplet. The simplest superfield is a scalar $S(x^\mu, \theta_\alpha, \overline{\theta}_{\dot{\alpha}})$ and its transformation under SUSY is given by

$$\delta S = i[S, \epsilon\mathcal{Q} + \overline{\epsilon}\overline{\mathcal{Q}}] = \left(\epsilon\mathcal{Q} + \overline{\epsilon}\overline{\mathcal{Q}}\right)S$$

A general scalar superfields $S(x^\mu, \theta_\alpha, \overline{\theta}_{\dot{\alpha}})$ is reducible with respects to SUSY transformations, however. In order to extract irreducible representations, a useful strategies is to impose extra constraints. For instance, they can be introduced by using the SUSY covariant derivatives

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu)_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu \qquad \overline{\mathcal{D}}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} - i\theta^\beta(\sigma^\mu)_{\beta\dot{\alpha}}\partial_\mu$$

Chiral superfields

Chiral superfields are characterised by the condition $\overline{\mathcal{D}}_{\dot\alpha}\Phi = 0$ and the constrained is solved by the superfield structure

$$\Phi(y^\mu,\theta) = \Phi(y^\mu) + \sqrt{2}\theta\psi(y^\mu) + \theta\theta F(y^\mu)$$

where $y = x + i\theta\sigma^\mu\overline{\theta}$. Here and in the following, we use the same symbol for the superfield Φ and its scalar component. F is an auxiliary non dynamical field, which can be integrated out through the equations of motion. Under SUSY transformation

$$\delta\Phi = i\left(\epsilon\mathcal{Q} + \overline{\epsilon}\overline{\mathcal{Q}}\right)$$

the fields components transform as

$$\delta\Phi = \sqrt{2}\epsilon\psi$$

$$\delta\psi = i\sqrt{2}\sigma^\mu\overline{\epsilon}\partial_\mu\Phi + \sqrt{2}\epsilon F$$

$$\delta F = i\sqrt{2}\overline{\epsilon}\,\overline{\sigma}^\mu\partial_\mu\psi$$

Notice that δF is a total derivative term.
Antichiral superfields $\Phi^\dagger$ are similarly defined by the constraint $D_\alpha\Phi^\dagger = 0$, and lead to the conjugate field content $(\Phi^*, \overline{\psi}_{\dot\alpha}, F^*)$.

Vector and gauge superfields

Vector superfields form another irreducible representations of the SUSY algebra and they satisfy the reality condition

$$V(x^\mu,\theta_\alpha,\overline{\theta}_{\dot\alpha}) = V^\dagger(x^\mu,\theta_\alpha,\overline{\theta}_{\dot\alpha})$$

Their (off shell) field content is

$$V \to \left(\lambda_\alpha\,, A^\mu, D\,; C\,, \chi_\alpha\,, N\right)$$

where $\lambda_\alpha\,, \chi_\alpha$ are Weyl spinors, N, D, C are real scalars and A^μ is a vector field . In the previous equation we have omitted gauge indices. Let Λ be a chiral superfield, then $i(\Lambda - \Lambda^\dagger)$ is a vector superfield and we can define a generalised gauge transformation to vector superfields via

$$V \mapsto V - i\left(\Lambda - \Lambda^\dagger\right)$$

which induces a standard gauge transformation for the vector component of V

$$V_\mu \mapsto V_\mu + \partial_\mu[\mathrm{Re}\,\Phi] \equiv V_\mu - \partial\alpha$$

The vector supermultiplet is subject to a generalised gauge invariance, so we can choose the components of Λ to gauge away some of the components of V. In the so called Wess-Zumino gauge the fields C, χ_α, N are set equal to zero. The remaining fields are a gauge boson A^μ, a Weyl spinor λ_α in the adjoint representation and a non propagating real auxiliary field D. The vector superfield expansion in the Wess-Zumino gauge is

$$V(x,\theta,\bar{\theta}) = (\theta\sigma^\mu\bar{\theta})A_\mu(x) + (\theta\theta)(\bar{\theta}\bar{\lambda}(x)) + (\bar{\theta}\bar{\theta})(\theta\lambda(x)) + \frac{1}{2}(\theta\theta)\bar{\theta}\bar{\theta}D(x)$$

It is possible to construct a gauge invariant quantity out of Φ and V by imposing the following transformation properties:

$$\left.\begin{array}{ll} \Phi \mapsto & \exp(iq\Lambda)\Phi \\ V \mapsto & V - i(\Lambda - \Lambda^\dagger) \end{array}\right\} \quad \Longrightarrow \quad \Phi^\dagger \exp(qV)\Phi \quad \text{gauge invariant}$$

Here, Λ is the chiral superfield defining the generalised gauge transformations. The usual gauge invariant field strength $F_{\mu\nu}$ can be shown to belong to a spinorial chiral superfield W_α (i.e. $\bar{D}_{\dot{\alpha}} W_\alpha = 0$) defined in the abelian case as

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V \qquad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}} V$$

and generalised in the non-abelian case to

$$W_\alpha = -\frac{1}{4}\bar{D}\,\bar{D}e^{-V}D_\alpha e^{V} \qquad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}DDe^{-gV}\bar{D}_{\dot{\alpha}}e^{gV}$$

where now $V = T_a V^a$, with T_a the hermitian gauge generators.

Supersymmetric actions for chiral multiplets

In ordinary field theories, the action is the integral of the Lagrangian over the spacetime

$$\mathcal{S} = \int \mathrm{d}^4 x\, \mathcal{L}$$

and it is invariant for spacetime translations because the Lagrangian changes by a total derivative under translations. Similarly, the action in a SUSY theory is given by an integral of the Lagrangian over the superspace

$$\mathcal{S}_{\mathrm{SUSY}} = \int \mathrm{d}^4 x \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \mathcal{L}_{\mathrm{SUSY}}$$

A supersymmetry transformation is nothing more than a translation in superspace, thus integrating the action variation over the full superspace gives zero.

$$\delta \int \mathrm{d}^4 x \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, h(\Phi, \Phi^\dagger, V) = \int \mathrm{d}^4 x \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta} \Big(\epsilon \mathcal{Q} + \bar{\epsilon}\bar{\mathcal{Q}}\Big) h(\Phi, \Phi^\dagger, V) = 0$$

where $h(\phi, \phi^\dagger, V)$ is a generic function of chiral and vector fields. For chiral fields, integrals over half of superspace are invariant. If $f(\Phi)$ is a function of chiral fields only, f itself is chiral, so

$$\bar{\mathcal{Q}}_{\dot{\alpha}} f \;\propto\; \theta^\beta (\sigma^\mu)_{\beta\dot{\alpha}} \partial_\mu f$$

and this implies

$$\delta \int \mathrm{d}^4 x \int \mathrm{d}^2\theta\, f(\Phi) = \int \mathrm{d}^4 x \int \mathrm{d}^2\theta \Big(\epsilon \mathcal{Q} + \bar{\epsilon}\bar{\mathcal{Q}}\Big) f(\Phi) = 0$$

In summary, in order to build up a SUSY invariant Lagrangian in components it is sufficient to look for the D terms and the F terms, since they are the only terms which are non vanishing under integration over respectively the entire superspace and half superspace.
The product of chiral superfields (with the same chirality) yields another chiral superfield, so the $4D$ spacetime integral of the F-term of an arbitrary polynomial of chiral superfields is SUSY invariant. In particular, the most general renormalizable supersymmetric couplings involving chiral superfields Φ_i is

$$\begin{aligned} \mathcal{L}_W &= \int \mathrm{d}^2\theta\, W(\Phi_i) + \text{ h.c.} \\ &:= \int \mathrm{d}^2\theta \left(\frac{1}{3} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} m^{ij} \Phi_i \Phi_j + \lambda^i \Phi_i \right) + \text{ h.c.} \end{aligned} \tag{3.1.3}$$

The holomorphic function $W(\Phi_i)$ is called *superpotential*, and integration over $\mathrm{d}^2\theta$ selects its F-term. Explicit integration over θ yields the interaction terms for the component fields, which include Yukawa couplings and fermion mass terms

$$\mathcal{L}_F = -\frac{1}{2} \frac{\partial^2 W}{\partial\Phi_i \partial\Phi_j} \psi_i \psi_j + \text{ h.c. } = -Y^{ijk} \Phi_k \psi_i \psi_j - \frac{1}{2} m^{ij} \psi_i \psi_j + \text{ h.c.}$$

Canonical kinetic terms for chiral multiplets are described in terms of superfields by

$$\int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \Phi_i^* \Phi_i$$

The product $\Phi_i^* \Phi_i$ is a real superfield: explicit integration over $\theta, \bar{\theta}$ in the previous equation produces the Lagrangian for the components fields. This Lagrangian includes the kinetic terms for fermions and bosons, as well as terms $|F_i|^2$ for the non-propagating auxiliary fields. Using a coupling

$$F_i \frac{\partial W}{\partial \Phi_i}$$

arising from (3.1.3), the equations of motion

$$F_i^* = -\frac{\partial W}{\partial \Phi_i}$$

allow us to eliminate the auxiliary fields. This introduces a F contribution to the scalar potential

$$V_F(\Phi_i) = \sum_i |F_i|^2 = \sum_i \left| \frac{\partial W}{\partial \Phi_i} \right|^2$$

Altogether, the interactions for chiral superfields give rise to masses, Yukawa couplings, and a scalar potential, with coupling uniquely determined by the superpotential.

SUSY gauge interactions

The kinetic terms and gauge interactions for gauge bosons and their superpartners arise from

$$\mathcal{L} = \frac{1}{4} \mathrm{Tr} \int \mathrm{d}^2 \theta \, W^\alpha W_\alpha + \text{ h.c. } = \mathrm{Tr} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i \lambda \sigma^\mu D_\mu \bar{\lambda} + \frac{1}{2} D^2 \right) \tag{3.1.4}$$

The interactions of a gauge multiplet V with a chiral superfield Φ are described by

$$\int \mathrm{d}^2 \theta \, \mathrm{d}^2 \bar{\theta} \, \Phi^\dagger e^{gV} \Phi \tag{3.1.5}$$

The terms (3.1.4), (3.1.5) are invariant under the following generalised gauge transformation, with gauge parameters in chiral superfield $\Lambda(x, \theta) = T^a \Lambda_a(x, \theta)$

$$\Phi(x, \theta) \to e^{-i\Lambda} \Phi(x, \theta) \qquad e^V \to e^{-i\Lambda^\dagger} e^{gV} e^{i\Lambda^\dagger}$$

with Φ transforming in some representation of the gauge group.

Expansion in components shows that generalised gauge transformations restricted to the Wess-Zumino gauge yield ordinary gauge transformations, and that the physical fields ψ, Φ, and A^μ, λ transform as usual under them. The term (3.1.5) includes kinetic terms for the chiral multiplet fermions and scalars, and their usual gauge invariant interactions with gauge bosons. In addition, it produces a linear term in the auxiliary fields D_a, which together with the $|D|^2$ term in (3.1.4) yields the equation of motion

$$D^a = -g\Phi_i^*(T^a)_{ij}\Phi_j$$

where i, j run over gauge indices. Elimination of the auxiliary field leads to a D-term contribution to the scalar potential

$$V_D = \sum_a \frac{1}{2}|D^a|^2 = \sum_a \frac{g^2}{2}|\Phi_i(T^a)_{ij}\Phi_j|^2$$

Finally, for a $U(1)$ gauge field there is an additional SUSY invariant term that one may add to the Langrangian, the Fayet Illiopoulos (FI) term

$$\mathcal{L}_{\text{FI}} = \xi \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta} V$$

The integration selects the D-term, whose transformation under supersymmetry is a total derivative, thus producing a SUSY invariant term upon $4D$ spacetime integration. The only effect of the FI term is to modify the $U(1)$ D-term scalar potential, which in a theory of chiral multiplets Φ_k with $U(1)$ charges q_k reads

$$V_{U(1)} = \frac{1}{2}|D|^2 = \frac{g^2}{2}\left|\sum_k q_k|\Phi_k|^2 + \xi\right|^2$$

It is also possible to introduce field dependent gauge kinetic terms, with structure

$$\frac{1}{4}\int \mathrm{d}^2\theta f(\Phi_i)\mathrm{Tr}(W^\alpha W_\alpha) + \text{ h.c.}$$

where $f(\Phi)$ is the *gauge kinetic function*.

Global SUSY breaking

Experimental observations clearly state that SUSY must be a broken symmetry. In fact, SUSY implies that all fields in a supermultiplet must have the same mass. We may expect SUSY to be broken spontaneously for energies not too far from the electroweak energy scale. In spontaneous symmetry breaking the vacuum state is no more invariant under a certain symmetry while the Langrangian still does, that is in the case of SUSY

$$\mathcal{Q}_\alpha |0\rangle \neq 0$$

In theories with global SUSY the order parameter for SUSY breaking is the ground state energy, as follows. From the anticommutation relation $\{\mathcal{Q}_\alpha, \overline{\mathcal{Q}}_{\dot{\alpha}}\}$ we obtain that the ground state energy is positive semi definite

$$\langle 0| H |0\rangle \geq 0$$

which is equal to zero if and only if the vacuum is invariant $\mathcal{Q}_\alpha |0\rangle = 0$. Notice that this is consistent with the form of the scalar potential seen previously, i.e.

$$\langle 0| H |0\rangle = \langle 0| V |0\rangle = \sum_i |F_i|^2 + \frac{1}{2} \sum_a |D^a|^2 \geq 0$$

Hence SUSY breaking requires non zero vevs for some of the auxiliary fields, i.e.

$$\langle 0| F_i |0\rangle \neq 0 \quad \text{and/or} \quad \langle 0| D^a |0\rangle \neq 0 \quad \text{for some } \ i, a.$$

The broken generator $\mathcal{Q}_\alpha$ is a spinor with spin 1/2, so the associated Goldstone particle is a Weyl spinor, the goldstino ψ_G, which is characterised by

$$\langle 0| \delta_\theta \psi_G |0\rangle \neq 0$$

The goldstino is in general the fermionic partner of the non-vanishing F or D auxiliary fields breaking SUSY, or a combination thereof.

3.2 Minimal SUSY extension of the SM

The simplest way to extend the Standard Model to a phenomenologically viable supersymmetric theory is by introducing all the superpartners needed to fill out the supermultiplets with $N = 1$ SUSY generator. If we take into account only renormalizable interactions, the irreducible representations are only given by chiral and vector supermultiplets [11].

Particle content of chiral superfield

By construction SUSY generators commute with the gauge generators, so particles belonging to the same supermultiplets must transform in the same way under $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Standard Model gauge bosons must trasforms as the adjoint representation of the gauge group, which is always its own conjugate. Thus, vector multiplets can only contain fermions whose left handed and right handed components transform in the same way under gauge group. The known quarks and leptons are chiral fermions instead, so they must belong to chiral supermultiplets.

As consequence, the bosonic superpartners of quark and leptons are spin-0 bosons and they are respectively called *squarks* and *sleptons*. Describing the Standard Model fermions through Dirac spinors, it means that each left handed and right handed part has its own scalar complex partners. In fact, they are separate two component Weyl fermions with different properties under gauge trasformation. For example, the electron Dirac field e has two superpartners called selectrons and denoted by $\tilde{e}_L$ and $\tilde{e}_R$. Necleting their masses, neutrinos can be treated as left handed Weyl spinors. The gauge interactions of each of the these squark and slepton fields are the same as for the corresponding Standard Models fermions: for instance, the left-handed squarks $\tilde{u}_L$ and $\tilde{d}_L$ couple to the W boson, while $\tilde{u}_R$ and $\tilde{d}_R$ do not.

The Higgs boson $\mathrm{SU}(2)_L$-doublet has spin 0 so it is embedded into a chiral supermultiplet, that is usually denoted by H_u. However, one chiral Higgs supermultiplet it is not sufficient in order to generate all fermions masses, because the superpotential must be holomorphic. In fact, H_u couples to up-type quarks and generates their masses after electroweak breaking. Down-type quark masses instead requires coupling containing H_u^* terms, which are forbidden because the superpotential must be holomorphic. Moreover, if we consider only one Higgs chiral multiplet the electroweak gauge symmetry is plague with a gauge anomaly, making the theory inconsistent. All the previous observations enforce the presence of another Higgs chiral multiplet with opposite $\mathrm{U}(1)_Y$ quantum number respect to H_u. This supermultiplet is usually label as H_d and it must transform in same way under gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2)_L$ as H_u. Following the usual nomenclature where spin 1/2 superpartners are labelled by appending "-ino" to the name of the Standard Model particles, fermionic superpartners of the Higgs bosons are called higgsinos. These particles are denoted by $\tilde{H}_d, \tilde{H}_u$ for the $\mathrm{SU}(2)_L$ doublet left-handed Weyl spinor.

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L$	$U(1)_Y$	$U(1)_{em}$
squarks, quarks (x 3 families)	Q	$\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}$	$\begin{pmatrix}u_L\\ d_L\end{pmatrix}$	$(\mathbf{3},2)$	1/6	$\begin{pmatrix}2/3\\ -1/3\end{pmatrix}$
	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{\mathbf{3}},\mathbf{1})$	−2/3	−2/3
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{\mathbf{3}},\mathbf{1})$	1/3	−1/3
sleptons, leptons (x 3 families)	L	$\begin{pmatrix}\tilde{\nu}_L\\ \tilde{e}_L\end{pmatrix}$	$\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$	$(\mathbf{1},2)$	−1/2	$\begin{pmatrix}0\\ -1\end{pmatrix}$
	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1},\mathbf{1})$	1	1
Higgs, higgsinos	H_u	$\begin{pmatrix}H_u^+\\ \tilde{H}_u^0\end{pmatrix}$	$\begin{pmatrix}\tilde{H}_u^+\\ \tilde{H}_u^0\end{pmatrix}$	$(\mathbf{1},2)$	1/2	$\begin{pmatrix}1\\ 0\end{pmatrix}$
	H_d	$\begin{pmatrix}H_d^0\\ H_d^-\end{pmatrix}$	$\begin{pmatrix}\tilde{H}_d^0\\ \tilde{H}_d^-\end{pmatrix}$	$(\mathbf{1},2)$	−1/2	$\begin{pmatrix}0\\ -1\end{pmatrix}$

Table 3.1: Chiral supermultiplets in the MSSM. The spin 0 fields are complex scalars, and the spin 1/2 fields are left handed two component Weyl fermions.

Table 3.1 summarizes all of the chiral supermultiplets necessary for a minimal phenomenologically viable extension of the Standard Model, classified according to their transformation properties under the gauge group $SU(3) \times SU(2)_L$. It is customary to defined chiral multiplets in terms of left-handed Weyl spinors, so we have taken into account the charge-conjugation of all right-handed fields.
In the second column of 3.1 we indicate a symbol for the whole supermultiplet. Thus, for example:

- Q is a $SU(2)_L$-doublet chiral supermultiplet containing $\tilde{u}_L$, u_L and $\tilde{d}_L$, d_L
- $\bar{u}$ is a $SU(2)_L$-singlet chiral supermultiplet containing $\tilde{u}_R^*$ and $u_R^\dagger$

For each of the quark and lepton supermultiplets there are three families and table 3.1 just lists first families representative. The nomenclature for the other families is similar to that of first family. When needed, a family index $i = 1, 2, 3$ can be affixed to the chiral supermultiplets names $(Q_i, \bar{u}_i \ldots)$, for example $(\bar{e}_1, \bar{e}_2, \bar{e}_3) = (\bar{e}, \bar{\mu}, \bar{\tau})$. The bar on $\bar{u}, \bar{d}, \bar{e}$ field is part of the name, and does not denote any kind of conjugation.

Particle content of vector supermutliplet

The gauge bosons of the Standard Model are contained in vector supermultiplets together with their fermionic superpartners which are generically called gauginos. We have that

- $SU(3)_C$ color gauge interaction of QCD have the gluons as strong force mediators. The corresponding fermionic superpartners are spin 1/2 color octet called gluinos.
- gauge bosons vector W^+, W^-, W^0 and B^0 mediate electroweak interactions. Their supersymmetric partner with spin 1/2 are $\tilde{W}^+, \tilde{W}^-, \tilde{W}^0$ and $\tilde{B}^0$ named winos and bino.

Electroweak symmetry breaking cause W^0 and B^0 gauge eigenstates to mix giving mass eigenstates Z_0 and γ. The corresponding gaugino mixtures of $\tilde{W}^0$ and $\tilde{B}^0$ are called zino $(\tilde{Z})$ and photino $(\tilde{\gamma})$. In the case of unbroken supersymmetry they have respectively masses m_Z and 0.
Tables 3.1 and 3.2 summarize the particle content of the MSSM.

Names	spin 1/2	spin 1	$SU(3)_C, SU(2)_L$	$U(1)_Y$	$U(1)_{em}$
gluino, gluon	$\tilde{g}$	g	$(\mathbf{8}, \mathbf{1})$	0	0
winos, W bosons	$\tilde{W}^0, \tilde{W}^\pm$	$W^0, W^\pm$	$(\mathbf{1}, \mathbf{3})$	0	$(0, \pm 1)$
bino, B boson	$\tilde{B}^0$	B^0	$(\mathbf{1}, \mathbf{1})$	0	0

Table 3.2: Vector supermultiplets in the MSSM

3.2.1 MSSM superpotential

The superpotential for the MSSM is

$$W_{\text{MSSM}} = \bar{u} Y_u Q H_u - \bar{d} Y_d Q H_d - \bar{e} Y_e L H_d + \mu H_u H_d \qquad (3.2.1)$$

where Y_u, Y_d, Y_e are 3 matrices in family space. All color, weak isospin and family indices are understood. For instance, we denote

$$\mu H_u H_d \equiv \mu \epsilon^{\alpha\beta} (H_u)_\alpha (H_d)_\beta \qquad \bar{u} Y_u Q H_u \equiv \bar{u}^{ia} (Y_u)_{ij} Q^j_{a\alpha} \epsilon^{\alpha\beta} (H_u)_\beta$$

with

- $\alpha, \beta = 1, 2$ are $SU(2)_L$ weak isospin index,
- $a = 1, 2, 3$ is a $SU(3)_C$ color index,
- $i = 1, 2, 3$ is a family index.

Color index is lowered (raised) in the $\mathbf{3}(\bar{\mathbf{3}})$ of $SU(3)_C$. The parameter μ has mass-dimension one and gives a mass to the component of chiral supermultiplets H_u and H_d. It is in general required to obtain a phenomelogically viable electroweak breaking and Higgs spectrum.

Matter parity

The superpotential (3.2.1) is not the most general superpotential compatible with gauge invariance and renormalizability. There are other possible terms for the superpotential but they violate either baryon number (n_B) or lepton number (n_L). Thus these extra terms induce dangerous couplings which contribute to the proton decay, leading to an extremely short proton lifetime.

Therefore, in the MSSM one adds a new symmetry, which has the effect of eliminating the possibility of n_B and n_L violating terms in the superpotential. This new symmetry is called *matter parity*: it is a multiplicatively conserved quantum number defined as

$$P_M = (-1)^{3(B-L)}$$

for each particle in the theory. It follows that

- all quarks, leptons and sfermions have $P_M = -1$
- Higgs bosons and higgsinos have $P_M = 1$
- gauge boson and gauginos do not carry any baryon or lepton number, so $P_M = +1$

Each term in the Lagrangian (or in the superpotential) is allowed only if the product of P_M for all fields in it is +1. It is easy to see that the terms in (3.2.1) are allowed.

3.2.2 MSSM soft term Lagrangian

A naive way to break SUSY is by introducing explicit breaking terms in the effective MSSM Lagrangian. These terms must take a particular form in order not to induce quadratic divergence for Higgs mass. In this case they are called *soft terms* and satisfy the following properties:

1. the couplings must be of positive mass dimension (hence the name soft)
2. the soft term Lagrangian is gauge invariant
3. the soft term Lagrangian is compatible with matter parity

The first condition ensures a natural solution of the hierarchy problem if the scale of the soft terms is not too far from the TeV scale. Having in mind the previous conditions, we can write down the soft term Lagrangian for the MSSM

$$\begin{aligned}\mathcal{L}^{\text{MSSM}}_{\text{soft}} = &\frac{1}{2}\left(M'_a\lambda^a\lambda^a + \text{h.c.}\right) - (m'^2)_{\alpha\bar{\beta}}C^\alpha C^{*\bar{\beta}} + \\ &-\left(\frac{1}{6}A'_{\alpha\beta\gamma}C^\alpha C^\beta C^\gamma + B'H_uH_d + \text{h.c.}\right)\end{aligned} \tag{3.2.2}$$

where λ^a are gaugino fields lists in table (3.2). The unnormalized soft terms are given by:

- mass terms M'_a for the gauginos.
- mass terms $(m'^2)_{\alpha\overline{\beta}}$ for sfermions (i.e. squarks and sleptons) and the Higgs scalars. They are 3×3 hermitian matrices in family space.
- mass term B' required in order to have the correct electroweak breaking energy scale
- trilinear scalar couplings $A'_{\alpha\beta\gamma}$

It has been shown rigorously that the MSSM with a softly broken supersymmetry Lagrangian $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$ is indeed free of quadratic divergences in quantum corrections to scalar masses, to all orders in perturbation theory. However, soft terms are rather ad hoc parameters and they seem not well motivated on theoretical grounds. Actually, we will see in the next chapter that soft terms appear naturally in the low-energy limit of supergravity theories with spontaneously broken SUSY.

Chapter 4

Four dimensional string theories

In this chapter we will give an overview of four dimensional effective models derived from string theory. First of all, we will just introduce four dimensional supergravity, which is a quantum field theory of gravity in a supersymmetric fashion. Supergravity is a "bridge" between the MSSM (or some extension of it) and the high energy physics that could be hopefully described by string theory. We will see how supergravity is linked with the MSSM through gravity mediation, a mechanism that naturally generate the soft terms needed for the theory to be consistent with experiments. Actually, supersymmetry can be spontaneously broken only in a hidden sector and it is transmitted to the visible sector through some "messenger fields". Moduli fields coming from string theory belong to the hidden sector and can play role of these messengers in gravity mediation. The last section of this chapter briefly describes the moduli fields and how they arise in the four dimensional low energy limit string theory.

4.1 A supersymmetric theory of gravity

We have seen that one missing piece of the puzzle is a quantum theory of gravity. Three of four fundamental interactions of Nature are correctly described in terms of (renormalizable) quantum field theories so the perturbation theory is viable and consistent for them. But if we try to quantize gravity in the same way, the resulting quantum field theory of gravity is not renormalizable. In fact, they appear an infinite number of infinities in all order of perturbation theory. So, we have look for a new path.
Generally, quantum field theories with more symmetry are more convergent in the perturbative expansion. An example is global supersymmetry, where all divergences in the Higgs sector are cancelled. So, it is tempting to

introduce supersymmetry in the general theory of gravitation. We expect that this new theory, the so-called *supergravity* (SUGRA in short), has a better high energy behaviour than pure gravity. This is basically true but unfortunately, still (super)symmetry is not enough to cancel all divergences in the theory.
Things go better when we consider more than one supersymmetry generator i.e. extended supergravity but divergences are still present in perturbation theory. Moreover, supergravity with $N > 1$ generators is not chiral, because their appear right handed fermions in the same supermultiplet. For this reason extended supergravity in four dimensions is not interesting from a phenomenological viewpoint [12].
Despite supergravity is not a renormalizable theory, it is an essential ingredient in the framework of *superstring theory*. In fact, the supergravity Lagrangian can be seen as an *effective Lagrangian* which comes as a low-energy limit of a supersymmetric string theory. More precisely, supergravity in $D = 10$ is the effective theory of superstring for energy below the string scale $M_s = 1/l_s$, where massive string modes cannot be produced. For energy below the Kaluza-Klein scale M_{KK}, only the massless modes of the Kaluza-Klein tower are relevant, therefore it is possible to integrate out the 6 extra dimensions, leaving as approximate theory an $N = 1$, $D = 4$ supergravity theory. The breaking of supergravity leads to the so-called soft supersymmetry-breaking terms which determine the spectrum of supersymmetric particles.
From the previous argument it is now clear how supergravity is crucial in order to connect low energy supersymmetric theories with the possible final theory of elementary particles.

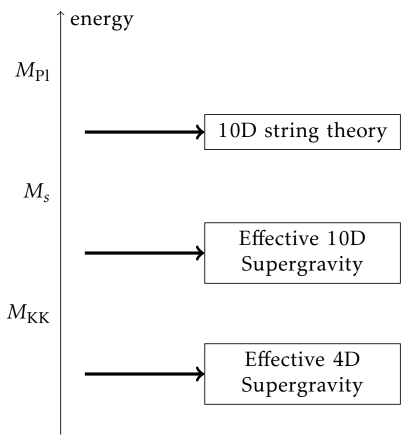

Figure 4.1: Energy scales relevant in string phenomenology. Supergravity is an *effective* field theory which is valid up to the string scale M_s.

4.1.1 Local supersymmetry

Supergravity can be seen as a *gauge theory* of global supersymmetry, likewise gravity is a "gauge theory" of global space-time transformation. Remember that a superfield Φ transforms under global supersymmetry like

$$\delta\Phi = i(\epsilon\mathcal{Q} + \bar{\epsilon}\overline{\mathcal{Q}})\Phi$$

From the anticommutator of the Majorana spinor generator $\mathcal{Q}$ it is evident that supersymmetry is an extension of the Poincarè space-time symmetry

$$\{\mathcal{Q}, \overline{\mathcal{Q}}\} = 2\gamma^{\mu}P_{\mu}$$

Thus, promoting global supersymmetry to *local* i.e. taking $\epsilon = \epsilon(x)$, we obtain space-time dependent translation that differ point to point or, in other words, we make a *general coordinate transformations* [12]. As consequence local supersymmetry necessarily implies gravity and by obvious reasons it is also called supergravity. In order to keep the action invariant under local supersymmetric transformations a gauge field has to be introduced, as in the case of ordinary gauge symmetry. In supergravity, the gauge field is a Majorana spinor field with spin 3/2, the so-called *gravitino* Ψ_a^{μ}, which carries both a vector index μ and a spinor index a.

4.2 Scalar potential

4.2.1 F-term potential

In our inflationary model the potential is given by the scalar potential of a four dimensional, $N = 1$ supergravity theory. Let us concentrate first on the chiral supergravity Lagrangian. It turns out to depend only on a single *arbitrary* real, dimensionless function of the scalar fields ϕ_i^* and ϕ_j with $i, j = 1, \dots n$, the Kähler function [11]

$$G(\phi^*, \phi) = \frac{1}{M_{\text{Pl}}^2} K(\phi^*, \phi) + \ln\left(\frac{|W(\phi)|^2}{M_{\text{Pl}}^6}\right) \tag{4.2.1}$$

where K is the Kähler potential and W is the superpotential. Remember that K is a real function which has both perturbative and non- perturbative corrections, while W is an analytic function and it can only receive non perturbative correction, since the superpotential is not renormalizable.

Equation (4.2.1) expresses the fact that the scalar-field space in supersymmetry is a Kähler manifold and the scalar field should be thought of as the coordinate of the manifold. From now on we use the following notation

$$\partial^i G = \frac{\delta G}{\delta \phi_i} \qquad \partial_i G = \frac{\delta G}{\delta \phi^i} \qquad \partial^{\bar{i}} G = \frac{\delta G}{\delta \phi^*_i} \qquad \partial_{\bar{i}} G = \frac{\delta G}{\delta \phi^{*i}}$$

In particular, the Kähler metric $K_{i\bar{j}}$ is a hermitian matrix given by

$$G_{i\bar{j}} \equiv \partial^2_{i\bar{j}} G = \frac{\delta^2 G}{\delta \phi^i \delta \phi^{*j}} = \frac{1}{M^2_{\text{Pl}}} \frac{\delta^2 K}{\delta \phi^i \delta \phi^{*j}} = \frac{1}{M^2_{\text{Pl}}} K_{i\bar{j}} \tag{4.2.2}$$

where $K_{i\bar{j}}^* = K_{\bar{j}i}$ and $*$ indicates the conjugate transpose. The inverse of this matrix is denoted $G^{i\bar{j}} = M^2_{\text{Pl}} K^{i\bar{j}}$ so that

$$G^{i\bar{k}} G_{\bar{k}j} = G^{\bar{i}k} G_{k\bar{j}} = \delta^i_j = \delta^{\bar{i}}_{\bar{j}} \tag{4.2.3}$$

An important property of G is its invariance under the transformations

$$K \to K + h(\phi) + h(\phi^*)$$
$$W \to e^{-h(\phi)} W$$

with h an arbitrary analytic function. This property is the Kähler invariance and it makes the chiral Lagrangian invariant under the previous transformation.

We are only interested in the scalar potential so it is sufficient for our purpose to consider the following simplified action in the flat spacetime limit [13]:

$$S = -3 \int \mathrm{d}^4 x\, \mathrm{d}^4 \theta\, \varphi_w \overline{\varphi}_w \exp\left\{ -\frac{K}{3M^2_{\text{Pl}}} \right\} + \left(\int \mathrm{d}^4 x\, \mathrm{d}^2 \theta\, \varphi^3_w W + \text{ h.c.} \right) \tag{4.2.4}$$

The field φ_w is an auxiliary non physical field know as *Weyl compensator* field, introduced in such a way that it makes the action invariant under scale and conformal transformations. This fictitious symmetry makes easier to derive the action in components. The field φ_w has to be fixed in order to break the artificial conformal invariance and its value is chosen such that the physical fields are properly normalized:

$$\varphi_w \overline{\varphi}_w \exp\left\{ -\frac{K}{3M^2_{\text{Pl}}} \right\} = M^2_{\text{Pl}}$$

It can be shown that the F-term scalar potential is given by (sum over repeated indexes is understood)

$$V_F = M_{\rm Pl}^4 e^G \Big[G^{i\bar{j}} \partial_i G \, \partial_{\bar{j}} G - 3 \Big] \tag{4.2.5}$$

This implies that, contrary to global supersymmetry, K and W are not totally independent since the action depends only on the invariant combination G. In terms of K and W the F-term scalar potentials reads

$$V_F = \exp\left\{ \frac{K}{M_{\rm Pl}^2} \right\} \left[D_i W K^{i\bar{j}} D_{\bar{j}} W^* - 3 \frac{|W|^2}{M_{\rm Pl}^2} \right] \tag{4.2.6}$$

$$D_k W := \partial_k W + \frac{1}{M_{\rm Pl}^2} W \partial_k K \qquad D_{\bar{k}} W^* := \partial_{\bar{k}} W^* + \frac{1}{M_{\rm Pl}^2} W^* \partial_{\bar{k}} K$$

The F-terms are given by

$$F^i = -M_{\rm Pl} \, e^{G/2} \, G^{i\bar{k}} \partial_{\bar{k}} G = -M_{\rm Pl} \, e^{G/2} \, \partial^i G = -\exp\left\{ \frac{K}{2M_{\rm Pl}^2} \right\} \frac{W^*}{|W|} K^{i\bar{k}} D_{\bar{k}} W^* \tag{4.2.7}$$

$$F^{\bar{i}} = -M_{\rm Pl} \, e^{G/2} \, G^{\bar{i}k} \partial_k G = -M_{\rm Pl} \, e^{G/2} \, \partial^{\bar{i}} G = -\exp\left\{ \frac{K}{2M_{\rm Pl}^2} \right\} \frac{W}{|W|} K^{\bar{i}k} D_k W \tag{4.2.8}$$

and we can rewrite the scalar potential (4.2.6) as

$$V_F = K_{i\bar{j}} F^i F^{\bar{j}} - 3 \exp\left\{ \frac{K}{M_{\rm Pl}^2} \right\} \frac{|W|^2}{M_{\rm Pl}^2} \tag{4.2.9}$$

For $M_{\rm Pl} \to \infty$ we recover the expression for the scalar potential of a global supersymmetry:

$$V_F \sim K_{i\bar{j}} \partial^i W \partial^{\bar{j}} W^* \qquad \text{for } M_{\rm Pl} \to \infty$$

where the F - terms are given by $F^i = \partial^{\bar{i}} W^*$

4.2.2 D-term potential

So far we have considered only pure supergravity. If we consider supersymmetric matter and Yang-Mills coupled with supergravity, we must take into account D-terms contributions to the scalar potential, which are given by [11]

$$V_D = \frac{1}{2}\mathrm{Re}[f_{ab}\widehat{D}^a\widehat{D}^b] \tag{4.2.10}$$

with

$$\widehat{D}^a = f_{ab}^{-1}D^b \quad \text{where} \quad D^a = -G^i(T^a)_i{}^j\phi_j = -\phi^{*\bar{i}}(T^a)_{\bar{i}}{}^{\bar{j}}G_{\bar{j}} \tag{4.2.11}$$

where the label a, b are gauge groups indexes and $(T^a)_i{}^j$ are the group generators in the same representation as the chiral matter. The analytic function $f_{ab}(\phi)$ is called *gauge kinetic function*, which transforms as a symmetric product of adjont representations of the chiral matter gauge group.
Note that both D-terms (4.2.10) and F-terms (4.2.7) are linear combination of G_i. Thus, we can recast the D-terms in the following way

$$D^a = \frac{e^{-G/2}}{M_{\mathrm{Pl}}^2}F^i(T^a)_i{}^j\phi_j$$

and this show that D-terms can be written as linear combinations of F-terms [13]. As consequence, in supergravity there is no single D-terms supersymmetry breaking, since a non zero D-term requires at least a non vanishing F-term.
If gauge singlet scalar fields acquire expectation values, supersymmetry may be broken only by F-terms; in case of gauge non-singlets, both F- and D-terms lead to supersymmetry breaking. The full scalar potential is

$$V = V_F + V_D$$

and it is completely determined once the functions G and f_{ab} are known. Also the supergravity Lagrangian turns out to depend only on G and f_{ab}.

4.3 Minimum of the scalar potential

In our model of string inflation we take into account as tree-level inflationary potential only the F-part of scalar potential i.e. we assume that V_F dominates during inflation over V_D. D-term scalar potential is treated as a small correction which may uplift the minimum [14]. Thus, from now on we focus only on V_F.
In global supersymmetry the scalar potential is non-negative, so there is supersymmetric breaking if and only if the vacuum energy is positive. Things are drastically different in the case of supergravity, since the scalar

potential V_F (4.2.6) may be negative because of the term $-3|W|^2/M_{\rm Pl}^2$. Thus, we can have supergravity breaking with the minimum of the potential positive, negative or equal to zero and these three cases correspond to:

1. positive vacuum energy implies a de Sitter space
2. zero vacuum energy implies a Minkowski space
3. negative vacuum energy implies an anti-de Sitter space.

Recent cosmological observations point out a positive vacuum energy of the observable universe [11],

$$\rho_{\rm vac} = \Lambda^4 M_{\rm Pl}^4 \approx (2.3 \times 10^{-12}\,{\rm GeV})^4 \approx 10^{-120} M_{\rm Pl}^4$$

which is clearly tiny compared to the scale Λ_S usually associated with supersymmetric breaking. In fact, requiring a supersymmetric particle mass of about $m_{\rm soft} \approx 1\,{\rm TeV}$ sets Λ_S to

- $\Lambda_S \approx 10^{10}\,{\rm GeV}$ for gravity mediated supersymmetry breaking
- $\Lambda_S \approx 10^{4}\,{\rm GeV}$ for gauge-mediated supersymmetry breaking

In fact, a naive estimate would give a supersymmetry breaking vacuum $\langle V \rangle$ of order $|\langle F \rangle|^2$, so at least roughly [11]

- $\langle V \rangle \approx (10^{10}\,{\rm GeV})^4$ for gravity mediated supersymmetry breaking
- $\langle V \rangle \approx (10^{4}\,{\rm GeV})^4$ for gauge-mediated supersymmetry breaking

Thus, it is unclear *why* the terms in the scalar potential in a supersymmetric breaking minimum should be such that $\langle V \rangle \approx 0$ at the minimum. Without a clear explanation for the tiny value of $\rho_{\rm vac}$, it is questionable to set $\langle V \rangle \approx 0$. Nevertheless, taking $\langle V \rangle = 0$ as constraint, we obtain from eq. (4.2.7) and eq. (4.2.9)

$$\left\langle K_{i\bar{j}} F^i F^{\bar{j}} \right\rangle = 3 \exp\left\{ \frac{K}{M_{\rm Pl}^2} \right\} \frac{|\langle W \rangle|^2}{M_{\rm Pl}^2} = 3 M_{\rm Pl}^4 e^{\langle G \rangle} \tag{4.3.1}$$

We will see later on that, in our model, the tree level Kähler potential and superpotential coming from string theory are such that eq. (4.3.1) is satisfied for all Kähler moduli.

4.3.1 Super Higgs effect

When one or more F_i acquire a VEV, supergravity is broken and the gravitino obtain a mass in similar fashion the gauge vectors take mass during electroweak symmetry breaking. In supergravity the evidence of spontaneous breaking of supersymmetry is the appearence of Goldstone fermions or goldstinos, which are linear combinations of the spinors ψ_i in the same multiplets of the auxiliary fields F_i. These Goldstone fermions can be "gauged away" [15] through a gauge transformation of the so far massless gravitino Ψ^μ. As a result the two helicity degrees of freedom $h = \pm 1/2$ of the goldistino are swallowed by the gravitino to give a massive spin 3/2 particle with mass

$$m_{3/2}^2 = \exp\left\{\frac{K}{M_{\rm Pl}^2}\right\}\frac{|\langle W\rangle|^2}{M_{\rm Pl}^4} \tag{4.3.2}$$

This is the so called *super-Higgs effect*: the two degrees of freedom of the goldstino become the longitudinal component of the gravitino. From eq. (4.3.1) we can infer that if we want a Minkowski vacuum i.e. $\langle V\rangle = 0$, the gravitino mass can be recast as

$$m_{3/2}^2 = \frac{1}{3M_{\rm Pl}^2}\left\langle K_{i\bar{j}}F^iF^{\bar{j}}\right\rangle$$

showing that the gravitino mass can be seen as the order parameter of the spontaneous symmetry breaking of supergravity.
In realistic models supersymmetry has to be broken spontaneously, as we will see in next section, in order to generate the so called *"soft" supersymmetry breaking terms*, soft terms in short. These terms must not spoil the solution of the hierarchy problem, hence their name.

4.4 SUSY breaking and gravity mediation

As we said so far, SUSY breaking must be parametrized using the so-called *soft terms*. In fact, there is a general (tree level) constraint among boson and fermions masses, given by [16]

$$\sum m_{J=0}^2 - 2\sum m_{J=1/2}^2 + 3\sum m_{J=1}^2 = 0 \tag{4.4.1}$$

This equation ensures the absence of quadratic divergences and it holds separately for particles of a given charge, since SUSY generators commute with eletric charge. For instance, the sum of d-type squark masses cannot exceed $2m_b^2$, with m_b the b quark mass, clearly contradicting experimental

constraints on sparticles masses. So, this is the ultimately reason why we consider soft breaking terms.
A supersymmetry breaking order parameter cannot belong to any of the MSSM supermultiplets, thus supersymmetry can be spontaneously broken *only* in a separate sector, named *hidden sector*,and SUSY breaking is transmitted to the SM *visible sector* by some mediating particles. There are several proposal for the mediation mechanism: we consider the case where gravitational interaction act as the messenger, i.e. the so called *gravity mediation* [16].
In this context we consider two types of chiral multiplets:

- matter supermultiplets C^α which correspond to MSSM chiral multiplets list in table 3.1
- hidden sector supermultiplets h_m with gravitational strength couplings to the MSSM.

In our model, the hidden sector is given by the *moduli* coming (together with their fermionic superpartners, the modulini) from the embedding Type IIB string theory. One or more of these moduli acquires a non vanishing vev and so do some auxiliary fields $\langle F_m \rangle \neq 0$, breaking supergravity. Thus, the gravitino takes mass and gravitational interactions induced the soft terms in the observable sector, which feels the SUSY breaking indirectly.

Kähler potential and superpotential

By expanding in powers of chiral matters fields C^α the superpotential and Kähler potential of the MSSM we obtain [16]

$$\mathcal{W} = W(h_m) + \frac{1}{2}\mu(h_m)H_u H_d + \frac{1}{6}Y_{\alpha\beta\gamma}(h_m)C^\alpha C^\beta C^\gamma + \dots \tag{4.4.2}$$

$$\mathcal{K} = K(h_m, h_m^*) + K_{\alpha\overline{\beta}}(h_m, h_m^*)C^\alpha C^{*\overline{\beta}} + \left[Z(h_m, h_m^*)H_u H_d + \text{h.c.}\right] + \dots \tag{4.4.3}$$

where the term μ in the superpotential is the supersymmetric version of the Higgs boson mass in the Standard model.
As to ensure the correct Higgs vev of order 174GeV without miraculous cancellations, the SUSY mass term μ has to be in range of m_{soft}. However, we can already see a puzzle: the μ term is SUSY preserving while B' (the soft term which appears in Higgs potential) is not. Since they are dimensionful parameters that are conceptually quite distinct, why μ should be of the same order as the soft SUSY breaking terms? This is the so called μ problem[11].

We consider one of the several solutions proposed in the literature: the Giudice-Masiero mechanism [17]. The proposal of Giudice and Masiero is to introduce a non-vanishing Z term in the Kähler potential which can *naturally* generate a μ term of the order of m_{soft}. This solution fits well in superstring models, since a bilinear term likes $Z_{\alpha\beta}C^{\alpha}C^{\beta}$ in the observable fields often appears in the Kähler potential.

Derivation of the soft terms

The soft terms can be derived in the context of gravity mediation through the following procedure: first of all, we need to insert equations (4.4.2) and (4.4.3) in the supergravity lagrangian and then we have to replace h_m and their auxiliary fields F^m by their vevs. Finally, by taking the limit $M_{\text{Pl}} \to \infty$ keeping $m_{3/2}$ fixed (the so-called *flat limit*) we formally eliminate the non-renormalizable gravity corrections and we decouple from gravity. Thus, we are left with a global SUSY Lagrangian plus a Lagrangian with SUSY breaking terms.

$$\mathcal{L}_{\text{sugra}}(\langle h_m \rangle, \langle F^m \rangle) \xrightarrow[M_{\text{Pl}}\to\infty]{m_{3/2}\text{ fixed}} \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{SUSY breaking terms}}$$

We can infer the soft terms by comparing the general soft terms Lagrangian $\mathcal{L}^{\text{MSSM}}_{\text{soft}}$ (3.2.2) with the effective one $\mathcal{L}_{\text{SUSY breaking terms}}$ coming from supergravity.

Diagonal Kähler matter metric

The Kähler matter metric $K_{\alpha\overline{\beta}}$ is in general non diagonal but string compactifications often lead to a diagonal metrics, which is a welcome feature since it obviously simplify the soft terms:

$$K_{\alpha\overline{\beta}}(h_m, h_m^*) = \delta_{\alpha\overline{\beta}}\tilde{K}_{\alpha}(h_m, h_m^*)$$

From now on we label with C^{α} the corresponding scalar particles of the chiral matter supermultiplets (i.e. squarks and sleptons) and likewise h_m are used to denote the scalar fields belonging to the hidden sector chiral supermultiplet. The SUSY breaking soft terms in the effective action for a diagonal matter metric is

$$\begin{aligned}\mathcal{L}^{\text{MSSM}}_{\text{soft}} = &\frac{1}{2}\Big(M_a \widehat{\lambda}^a \widehat{\lambda}^a + \text{h.c.}\Big) - (m^2)_{\alpha} \widehat{C}^{\alpha} \widehat{C}^{*\bar{\alpha}} + \\ &-\left(\frac{1}{6} A_{\alpha\beta\gamma} \widehat{Y}_{\alpha\beta\gamma} \widehat{C}^{\alpha} \widehat{C}^{\beta} \widehat{C}^{\gamma} + B\widehat{\mu} \widehat{H}_{\text{u}} \widehat{H}_{\text{d}} + \text{h.c.}\right)\end{aligned} \tag{4.4.4}$$

where $\widehat{C}^\alpha$ and $\widehat{\lambda}^\alpha$ are the canonically normalized scalars fields (sfermions and Higgs boson) and gauginos

$$\widehat{C}^\alpha = K_\alpha^{1/2} C^\alpha \qquad \widehat{\lambda}^\alpha = (\mathrm{Re}\, f_\alpha)^{1/2} \lambda^\alpha$$

while M_a, $(m^2)_\alpha$, $A_{\alpha\beta\gamma}$ and B are the *normalized* soft terms. In the previous expressions $\widehat{Y}_{\alpha\beta\gamma}$ and $\widehat{\mu}$ are respectively the *physical Yukawa couplings* and the rescaled parameter

$$\widehat{Y}_{\alpha\beta\gamma} = (\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{-1/2} \frac{W^*}{|W|} e^{K/2} Y_{\alpha\beta\gamma} \tag{4.4.5}$$

$$\widehat{\mu} = (\tilde{K}_{H_u} \tilde{K}_{H_d})^{-1/2} \mu' = \left(\frac{W^*}{|W|} e^{K/2} \mu + m_{3/2} Z - F^{\overline{m}} \partial_{\overline{m}} Z \right) \tag{4.4.6}$$

where

$$\partial_{\overline{m}} Z \equiv \frac{\delta Z}{\delta h_m^*}$$

In the context of gravity mediation, the overall scale of the soft terms are roughly set by the mass of the gravitino: $M_{\mathrm{soft}} \approx m_{3/2}$. In our model the mass of the gravitino is of the order

$$m_{3/2} \approx \left(10^{14} - 10^{15}\right) \mathrm{GeV}$$

which makes the mass of the soft terms very high [18].

4.5 Moduli fields

As we said, supergravity theories can be seen as low energy description of superstring theories. For us it sufficient to mention that our model of string inflation is derived from a type IIB string theory, which is a ten dimensional $N = 2$ chiral superstring theory. The six extra dimensions are described by compact Calabi-Yau 3-folds, a very special class of 3 dimensional Kähler manifolds. By taking a particular projection on the Calabi Yau three fold, named *orientifold projection*, it is possible to obtain an $N = 1$ chiral theory (remember that SUSY theories with $N > 1$ cannot be chiral).
Making contact with our four dimensional world requires a compactification of extra dimensions. We assume the ten dimensional $\mathcal{M}_{10}$ to take the form

$$\mathcal{M}_{10} = \mathcal{M}_4 \times X_6$$

where X_6 is the Calabi-Yau manifold (with six real dimension) [19].

This is referred to as *compactification* of string theory on X_6. In order to grasp the essentials concepts of the derivation of an effective low energy 4D supergravity theory from a string theory, lets write the Einstein-Hilbert action [19]

$$\mathcal{S}_{\mathrm{EH}}^{(10)} = \frac{M_s^8}{2\pi}\int_{\mathcal{M}_{10}} \mathrm{d}^{10}x\sqrt{-\hat{g}}\,R_{10} \tag{4.5.1}$$

where R_{10} is the ten dimensional Ricci scalar while $\hat{g}$ is the metric tensor determinant of the ten dimensional metric $\hat{g}_{MN}$. The integral can be factorized in the following way

$$\mathcal{S}_{\mathrm{EH}}^{(10)} = \frac{M_s^8}{2\pi}\int_{\mathcal{M}_4} \mathrm{d}^4x\sqrt{-g}\,R_4\int_{X_6} \mathrm{d}^6x\sqrt{-\hat{g}_6} \tag{4.5.2}$$

where R_4 and g are respectively the four dimensional Ricci scalar and metric tensor determinant. We know that the second integral must give the volume of the extra dimensions:

$$\mathrm{vol}(X_6) = \mathcal{V}l_s^6 = \frac{\mathcal{V}}{M_s^6} = \int_{X_6} \mathrm{d}^6x\sqrt{-\hat{g}_6}$$

Thus, the $4D$ Einstein-Hilbert action for energies below the Kaluza Klein scale M_{kk} looks like

$$S_{\mathrm{EH}}^{(4)} = \frac{M_s^2}{2\pi}\mathcal{V}\int_{\mathcal{M}_4} \mathrm{d}^4x\sqrt{-g}\,R_4 = \frac{M_{\mathrm{Pl}}^2}{2\pi}\int_{\mathcal{M}_4} \mathrm{d}^4x\sqrt{-g}\,R_4$$

and after compactification and dimensional reduction i.e. once we have integrated out the extra dimensions we are left with a four dimensional supergravity effective action. In the second equation we used

$$M_s \sim M_{\mathrm{Pl}}/\sqrt{\mathcal{V}}$$

which expresses the string scales in terms of the volume $\mathcal{V}$ of the Calabi Yau in string length natural unit $l_s = 1$.
A by product of string compactification are scalar particles called ***moduli***. So far we have only mentioned these particles, now we give some details of them. Moduli arise from non trivial perturbations of the ten dimensional metric $\hat{g}_{MN}$ such that the Calabi Yau condition $R_{MN} = 0$ (vanishing Ricci tensor) is preserved. These fields essentially parametrise the shape and the size of extra dimensions and there are three kinds of moduli:

- the axion dilaton S whose VEV gives the string coupling,
- complex structure moduli U^a, $a = 1,\dots,h^{1,2}$ parametrising the shape of the extra dimensions
- Kähler moduli T_i, $i = 1,\dots,h^{1,1}$ parametrising the size of the extra dimensions

All these moduli are uncharged massless scalar fields with gravitational couplings to all ordinary particles. Since massless scalars can mediate long range forces, moduli would lead to an unobserved fifth forces, so it is mandatory to find a mechanism which give mass to them. This procedure is call *stabilisation* of moduli. Finding a non zero vacuum expectation value (vev) for the moduli may be useful for understanding the main features of the low-energy field theory, since the gauge and Yukawa couplings turns out to depend on the moduli fields.
Let us now focus on the axio-dilaton modulus S and on the Kähler moduli T_i. The former can be written as

$$S = e^{-\phi} + iC_0 \qquad\qquad e^{-\langle\phi\rangle} = 1/g_s$$

As the string coupling is a parameter which appears in all string theories, the dilaton represents an universal feature in string compactifications. Its imaginary part, C_0, is an axion-like field.
Kähler moduli are given by integration of differential p-form over cycles [20]:

$$T_i = \tau_i + ib_i = \tau_i + i\int_{D_i} C_4$$

where C_4 is a 4-form while D_i is a 4-cycle. We have that $\tau_i = \mathrm{vol}(D_i)$ in string length natural unit ($l_s \equiv 1$) while b_i is an axion-like field which enjoy a Peccei- Quinn shift symmetry $b_i \to b_i + i\epsilon$.
As this shift symmetry is only broken by non perturbative corrections, the perturbative expansions terms in both α' and g_s cannot be functions of $\mathrm{Im}(T_i)$ [21]. On one hand, the Kähler moduli T_i must be enter in the perturbative action as $T_i + T_i^*$, leading to a Kähler potential of the form $K = K(T_i + T_i^*)$. On the other hand, the non-renormalisation theorem forbids perturbative corrections to the superpotential, and since $T_i + T_i^*$ is non-holomorphic, W cannot depend on T_i. This implies that the T_i does not appear in the Yukawa couplings $Y_{\alpha\beta\gamma}$ to any order in perturbation theory, because $Y_{\alpha\beta\gamma}$ are determined by the superpotential [21]. Therefore, it is

possible to show that at the leading order in α' and g_s, the Kähler potential can be written as [20]

$$K_{\text{tree}} = K(U,T,S) = -2\ln[\mathcal{V}(T+T^*)] - \ln[S+S^*] - \ln\left[-i\int_X \Omega(U)\wedge\overline{\Omega}(U^*)\right]$$

where Ω is the Calabi Yau holomorphic $(3,0)$ form. The tree level superpotential is given by

$$W = W_{\text{tree}}(U,S) \sim \int_{X_6} G_3(S)\wedge\Omega(U)$$

where $G_3(S)$ is a three form.

4.5.1 No-scale structure

One of the main consequences of the shift symmetry enjoy by the Kähler moduli is the no-scale structure of the scalar potential i.e. the moduli T_i are flat directions of leading order scalar potential. In order to understand this important feature it is worth to mention that the volume $\mathcal{V}$ is a homogeneous function of degree 3/2 in the $\tau_i = \text{Re}(T_i)$ [20]

$$\mathcal{V}(\lambda\tau) = \lambda^{3/2}\mathcal{V}(\tau) \qquad \text{for all } \lambda\in\mathbb{R}$$

We have that

$$\frac{\partial K_{\text{tree}}}{\partial T_i} = \frac{1}{2}\frac{\partial K_{\text{tree}}}{\partial \tau_i} = -\frac{1}{\mathcal{V}(\tau)}\frac{\partial \mathcal{V}}{\partial \tau_i}$$

so, thanks to the Euler theorem for homogeneous function we can write

$$\sum_{i=1}^{h^{1,1}} \tau_i \frac{1}{2}\frac{\partial K_{\text{tree}}}{\partial \tau_i} = -\frac{1}{\mathcal{V}(\tau)}\sum_{i=1}^{h^{1,1}}\tau_i\left(\frac{\partial \mathcal{V}}{\partial \tau_i}\right) = -\frac{1}{\mathcal{V}(\tau)}\frac{3}{2}\mathcal{V}(\tau) = -\frac{3}{2} \qquad (4.5.3)$$

Using the previous equation it can be proved that [20]

$$\sum_{i,\bar{j}=1}^{h^{1,1}} K^{i\bar{j}}K_iK_{\bar{j}} = 3 \qquad \text{(no-scale structure)} \qquad (4.5.4)$$

Eq (4.5.4) is usually indicated as the no-scale structure equation. We have now all the necessary ingredients for calculating the tree level scalar potential:

$$V = e^{K(U,T,S)}\left(K^{i\bar{j}}D_iWD_{\bar{j}}W^* - 3|W|^2\right)_{i,\bar{j}=U,S,T}$$

Taking into account that

$$D_T W = W K_T$$

we readily obtain

$$V_{\text{tree}} = e^K \left(\sum_{i,\bar{j}=1}^{h^{1,2}} K^{i\bar{j}} D_i W D_{\bar{j}} W^* + K^{S\bar{S}} |D_S W|^2 \right) + e^K \left(\sum_{p,\bar{q}=1}^{h^{1,1}} K^{p\bar{q}} K_p K_{\bar{q}} \right) |W|^2$$

$$= e^K \left(\sum_{i,\bar{j}=1}^{h^{1,2}} K^{i\bar{j}} D_i W D_{\bar{j}} W^* + K^{S\bar{S}} |D_S W|^2 \right) \geq 0 \qquad (4.5.5)$$

where in the second line we used eq. (4.5.4). Since the tree level scalar potential is positive semidefinite, we can locate its minimum at $V_{\text{tree}} = 0$. This condition leads to a set of differential equations

$$D_S W = 0 = D_U W \qquad (4.5.6)$$

whose solutions allow to fix supersymmetrically the dilaton and the complex structure moduli at tree level [20]. Therefore we can integrate out the dilaton and the complex structure moduli by setting them equal to their vev.
By contrast, Kähler moduli cannot be stabilised by the tree level scalar potential because the leading contributions for the T_i fields identically vanish. Thus, Kähler moduli are flat directions for the potential. This implies that we must consider subleading perturbative and non perturbative corrections if we want to stabilise these moduli.
It is worth to notice that Kähler moduli already break supersupersymmetry at this level of approximation, since $D_T W = W K_T \neq 0$. In the context of type IIB string compactification with supergravity mediation, Kähler moduli can lead to the spontaneous symmetry breaking of supergravity in the hidden sector. As consequence, the supersymmetry is also broken in the visible sector thanks to the soft terms induced by the Kähler moduli.

In conclusion of this chapter, we just mention that the scalar potential corrections needed to stabilise Kähler moduli may also generate a potential suitable for inflation, where the inflaton could be one Kähler modulus. This is the case for many different Calabi Yau set up and our model of fibre inflation is one of them.

Chapter 5

A string inflationary model

5.1 Fibre inflation

We finally discuss our model of string inflation, called ***Fibre inflation***. It is a $4D$ string model embedded in a type IIB string theory whose Calabi Yau is endowed with a $K3$ fibration structure (so the name fibre inflation)[22]. We can express the volume of the $K3$ Calabi-Yau as a function of three τ_i fields:

$$\mathcal{V} = \alpha\left(\sqrt{\tau_1}\tau_2 - \gamma\tau_3^{3/2}\right) \tag{5.1.1}$$

where

- τ_1 is the fiber modulus,
- τ_2 is the base modulus,
- τ_3 is a blow-up mode. It is an "auxiliary" field required to stabilise the volume $\mathcal{V}$ at its minimum

α and γ can be regarded as real constants, with the constant γ taken to be positive and order unity.
As we said in the previous chapter, all moduli need to be stabilised and this can be achieved in the context of LARGE volume scenario. We do not enter in the technical details here. We just mention that by taking the volume $\mathcal{V}$ of the Calabi Yau large and positive respect to l_s^6, we can obtain a scalar potential which admits an Anti de Sitter non supersymmetric minimum. In particularly, we will work in the parameter regime

$$\mathcal{V}_0 := \alpha\sqrt{\tau_1}\tau_2 >> \alpha\gamma\tau_3^{3/2} >> 1$$

All moduli can be stabilised at the minimum of this LARGE volume scenario potential, which receives both perturbative and non perturbative corrections:

$$V \approx V_{\text{tree}} + \delta V_{(\alpha')} + \delta V_{(\text{sp})} + \delta V_{(g_s)}$$

where V_{tree} is given by (4.5.5). We consider three kind of corrections to the scalar potential:

- $\delta V_{(g_s)}$ are given by non perturbative corrections to the superpotential
- $\delta V_{(\alpha')}$ and $\delta V_{(g_s)}$ arise from perturbative corrections to the Kähler potential, where $\delta V_{(g_s)}$ are string loop corrections

In Fibre inflation the inflationary potential are due to string loop corrections to the Kähler potential, which is a characteristic feature of the model together with the prediction of observable primordial gravity waves (i.e. large tensor-to-scalar-ratio).

Corrections to the Kähler potential and superpotential

If we take into account leading order corrections in α', the Kähler potential reads

$$K \simeq K_0 + \delta K_{(\alpha')} = -2M_{M_{\text{Pl}}} \ln\left[\mathcal{V} + \frac{\hat{\xi}}{2}\right] \tag{5.1.2}$$

where ξ is a model dependent constant which controls the size of the α' corrections

$$\hat{\xi} \equiv \frac{\xi}{g_s^{3/2}} \qquad\qquad \xi \in \mathbb{R}$$

and K_0 is defined as

$$K_0 = -2\ln\mathcal{V}$$

Including non perturbative corrections to the superpotential leads to

$$W = W_0 + \sum_{i=1}^{3} A_i e^{-a_i T_i} \tag{5.1.3}$$

W_0 and g_s are fixed after the stabilisation of the dilaton S and of the complex moduli U_a to

$$W_0 = \left\langle \int_X G_3(S) \wedge \Omega(U) \right\rangle$$

$$g_s = \langle \text{Re}\,(S) \rangle$$

A_i, $i = 1,2,3$ corresponds to threshold effects and can depend on the complex structure moduli and on the details the model, while the constants a_i depend on the source of the non perturbative corrections. For our purpose we treat both of them as numerical constants, with the constraint $a_i\tau_i \gg 1$, $i = 1,2,3$. We can neglect the contributions given by the moduli T_1 and T_2 at first approximation, since we consider the large volume regime [14]:

$$W \approx W_0 + Ae^{-aT_3} \tag{5.1.4}$$

5.1.1 Inflationary potential

Scalar potential without string loop corrections

In the aim of work out some predictions for the spectral index n_s and the tensor-to-scalar-ratio r we need first to calculate the scalar potential. Neglecting in a first stage the string loop corrections, the potential can be written as [14]

$$\begin{aligned} V &\approx V_{\text{tree}} + \delta V_{(\alpha')} + \delta V_{(\text{sp})} = \\ &= 8a_3^2|A_3|^2 \frac{\sqrt{\tau_3}}{3\alpha\gamma\mathcal{V}} e^{-2a_3\tau_3} - 4a_3|A_3||W_0|\frac{\tau_3}{\mathcal{V}^2}e^{-a_3\tau_3} + \frac{3\hat{\xi}|W_0|^2}{4\mathcal{V}^3} + V_{\text{up}} \end{aligned} \tag{5.1.5}$$

where the phases of W_0 and A_3 are absorbed in the stabilisation of the axion b_3. V_{up} is the uplift term in the scalar potential. It enables the tuning of the minimum near to zero in the presence of string loop corrections. It can be seen that the tree level scalar potential depends on the fields T_1 and T_2 only through the volume $\mathcal{V}$. As consequence, it is not possible to stabilise both the moduli τ_1 and τ_2 at this level of approximation and there is one modulus (or combination of) τ_1 and τ_2 which remains completely flat.

For simplicity we consider the modulus τ_1 as flat direction. In fact, as we will see in the next susbsection, string loop corrections to the potential depends on τ_1, so this flat direction can be easily lift, giving a mass to τ_1. This is a good candidate for the inflaton, since we expect that τ_1 remains lighter than $\mathcal{V}$ and τ_3 after stabilisation. Indeed, the inflaton is usually the lighter modulus in string inflation which is displaced from his minimum. During inflation it rolls down (slowly) towards his minimum while the others heavier moduli stay (approximately) at their minima.
The potential completely stabilises the fields τ_3 and $\mathcal{V}$ and the minimum

satisfying $a_3\tau_3 \gg 1$ is given by [14]

$$\langle\tau_3\rangle = \left(\frac{\hat{\xi}}{2\alpha\gamma}\right)^{2/3} \quad \text{and} \quad \langle\mathcal{V}\rangle = \left(\frac{3\alpha\gamma}{4a_3|A|_3}\right)|W_0|\sqrt{\langle\tau_3\rangle}e^{a_3\langle\tau_3\rangle} \tag{5.1.6}$$

String loops corrections to the potential

The scalar potential receives the following string loop corrections

$$\delta V_{(g_s)} = \frac{|W_0|^2}{\mathcal{V}^2}\left(g_s^2\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}\sqrt{\tau_1}} + g_s^2\frac{C\tau_1}{\mathcal{V}^2}\right) \tag{5.1.7}$$

where A, B and C are unknown coefficients. Here the use the same notation adopted by ref. [18] which is slightly different respect to the one use by ref. [14]. Label by A', B' and C' the parameters used in ref. [14] we have that

$$A' = g_s^2 A \qquad B' = B \qquad C' = g_s^2 C \tag{5.1.8}$$

So, from the values taken from ref. [14, table 2, pag.19] by setting $g_s \simeq 0.301$ we expect the following ranges for the parameters A, B and C

$$A \approx \mathcal{O}(10^{-1}) - \mathcal{O}(10^{-2}) \qquad B \approx \mathcal{O}(1) - \mathcal{O}(10) \qquad C \approx \mathcal{O}(10^{-3}) - \mathcal{O}(10^{-4})$$

In either case, we expect that the string loop corrections stabilise τ_1 at a minimum. Indeed, minimizing $\delta V_{(g_s)}$ with respect to τ_1 with $\mathcal{V}$ and τ_3 fixed at their minima gives

$$\langle\tau_1\rangle^{3/2} = \left(\frac{8g_s^2 A\mathcal{V}}{B}\right)\left(1 + \frac{B}{|B|}\sqrt{1 + 32g_s^4\frac{AC}{B^2}}\right)^{-1} \tag{5.1.9}$$

When $g_s^4 \gg 1$, the previous expression reduces to

$$\langle\tau_1\rangle^{3/2} \simeq \begin{cases} g_s^2\left(\frac{4A\mathcal{V}}{B}\right) & \text{if } B > 0 \\ g_s^{-2}\left(\frac{|B|\mathcal{V}}{2C}\right) & \text{if } B < 0 \end{cases} \tag{5.1.10}$$

which require $A > 0$ for $B > 0$ and $C > 0$ for $B < 0$. Rewriting these minima in terms of the original fields τ_1 and τ_2 leads to

$$\langle\tau_1\rangle \simeq \begin{cases} g_s^2\left(\frac{4A}{B}\right)\langle\tau_2\rangle << \langle\tau_2\rangle & \text{if } B > 0 \\ g_s^{-2}\left(\frac{|B|}{2C}\right)\langle\tau_2\rangle >> \langle\tau_2\rangle & \text{if } B < 0 \end{cases} \tag{5.1.11}$$

For large enough $\mathcal{V}$ we can safely suppose that the fields $\mathcal{V}$ and τ_3 are not displaced from their minima during inflation, making a single field dynamic an excellent approximation. Since the inflaton will turn out to be mostly τ_1 after canonical normalisation, we have a simple pictorial view of the inflationary process. For $B > 0$ inflation start with the $K3$ fibre τ_1 much larger than the base and then the situation evolves in such a way that $K3$ decreases his size while the τ_2 being larger keeping the volume $\mathcal{V}$ approximately constant. At the end of inflation the base has became larger than the $K3$ fibre.

5.1.2 Single-field inflation

When $\tau_3 = \langle\tau_3\rangle$ and $\mathcal{V} = \langle\mathcal{V}\rangle$ are fixed at their τ_1-independent minima, so that $\partial_\mu\tau_3 = \partial_\mu\mathcal{V} = 0$, the relevant dynamics reduces to [14]

$$\mathcal{L}_{\text{inf}} = -\frac{3}{8}\frac{1}{\tau_1^2}\partial_\mu\tau_1\partial^\mu\tau_1 - V_{\text{inf}}(\tau_1) \tag{5.1.12}$$

with the scalar potential given by

$$V_{\text{inf}} = V_0 + \left(g_s^2\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}\tau_1} + g_s^2\frac{C\tau_1}{\mathcal{V}^2}\right)\frac{|W_0|^2}{\mathcal{V}^2} \tag{5.1.13}$$

The τ_1 independent constant V_0 consists of

$$V_0 = \frac{8a_3^2|A_3|^2\sqrt{\langle\tau_3\rangle}}{3\alpha\gamma\langle\mathcal{V}\rangle}e^{-2a_3\langle\tau_3\rangle} - \frac{4|W_0|a_3|A_3|\langle\tau_3\rangle}{\langle\mathcal{V}\rangle^2}e^{-a_3\langle\tau_3\rangle} + \frac{3\hat{\xi}|W_0|^2}{4\langle\mathcal{V}\rangle^3} + \delta V_{\text{up}} \tag{5.1.14}$$

Once $\mathcal{V}$ is fixed at this minimum this term does not depend at all on τ_1. The canonical inflaton is therefore given by (we set $M_{\text{Pl}} = 1$)

$$\varphi = \frac{\sqrt{3}}{2}\ln\tau_1 \quad \text{and so} \quad \tau_1 = e^{\kappa\varphi} \quad \text{with} \quad \kappa = \frac{2}{\sqrt{3}} \tag{5.1.15}$$

The potential becomes

$$\begin{aligned} V_{\text{inf}} &= V_0 + \frac{W_0^2}{\mathcal{V}^2}\left(g_s^2 A e^{-2\kappa\varphi} - \frac{B}{\mathcal{V}}e^{-\kappa\varphi/2} + g_s^2\frac{C}{\mathcal{V}^2}e^{\kappa\varphi}\right) = \\ &= \frac{1}{\langle\mathcal{V}\rangle^{10/3}}\left(\mathcal{C}_0 e^{\kappa\widehat{\varphi}} - \mathcal{C}_1 e^{-\kappa\widehat{\varphi}/2} + \mathcal{C}_2 e^{-2\kappa\widehat{\varphi}} + \mathcal{C}_{\text{up}}\right) \end{aligned} \tag{5.1.16}$$

where we shift $\varphi = \langle\varphi\rangle + \widehat{\varphi}$ by its vacuum value and adjust

$$V_0 = \frac{\mathcal{C}_{\rm up}}{\langle\mathcal{V}\rangle^{10/3}} \qquad \text{with} \quad \mathcal{C}_{\rm up} = \mathcal{C}_1 - \mathcal{C}_0 - \mathcal{C}_2$$

In the regime $g_s^4 << 1$ we have

$$\langle\varphi\rangle = \frac{1}{\sqrt{3}} \ln(\zeta\mathcal{V})$$

with

$$\zeta \simeq \begin{cases} g_s^2 \dfrac{4A}{B} & \text{if } B > 0 \\ g_s^{-2} \dfrac{|B|}{2C} & \text{if } B < 0 \end{cases}$$

The coefficients $\mathcal{C}_i$ are independent on $\langle\mathcal{V}\rangle$ and they are given by

$$\mathcal{C}_0 = g_s^2 C|W_0|^2\zeta^{2/3} \qquad \mathcal{C}_1 = B|W_0|^2\zeta^{-1/3} \qquad \mathcal{C}_2 = g_s^2 A|W_0|^2\zeta^{-4/3}$$

From now on we focus only in the case with $B > 0$, since the case $B < 0$ leads to a prediction for r ruled out by observations [18]. Notice that we have

$$\frac{\mathcal{C}_0}{\mathcal{C}_1} = \frac{\zeta C}{B} = 4g_s^4 \frac{|A|C}{B^2} \tag{5.1.17}$$

$$R := \frac{\mathcal{C}_0}{\mathcal{C}_2} = \zeta^2 \frac{C}{A} = 16 g_s^4 \frac{|A|C}{B^2} \tag{5.1.18}$$

$$\frac{\mathcal{C}_1}{\mathcal{C}_2} = \frac{\zeta B}{g_s^2 A} = 4 \tag{5.1.19}$$

Thus, we can finally write the scalar potential as

$$V \simeq \frac{g_s e^{-K_{\rm cs}}}{8\pi} \frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}} \left[(3-R) - 4e^{-\kappa\widehat{\varphi}/2} + e^{-2\kappa\widehat{\varphi}} + Re^{\kappa\widehat{\varphi}}\right] \tag{5.1.20}$$

where the prefactor $(g_s e^{-K_{\rm cs}})/8\pi$ is the correct overall normalisation obtained from dimensional reduction [22]. Hereafter we set $e^{-K_{\rm cs}} = 1$ since [20]

$$e^{-K_{\rm cs}} = \left\langle -i \int_{X_6} \Omega \wedge \overline{\Omega} \right\rangle \approx \mathcal{O}(1)$$

Therefore, from now on we consider

$$\boxed{V \simeq \frac{g_s}{8\pi}\frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}}\left[(3-R) - 4e^{-\kappa\widehat{\varphi}/2} + e^{-2\kappa\widehat{\varphi}} + Re^{\kappa\widehat{\varphi}}\right]} \tag{5.1.21}$$

with

$$\mathcal{C}_2 = |W_0|^2\left(\frac{|B|^4}{256 g_s^2|A|}\right)^{1/3} = \mathcal{C}_{\text{tuning}}\left(\frac{1}{256 g_s^2}\right)^{1/3} \tag{5.1.22}$$

where we define

$$\mathcal{C}_{\text{tuning}} := |W_0|^2\left(\frac{|B|^4}{|A|}\right)^{1/3}$$

Fig. 5.1 shows the inflationary potential for different values of R. The potential in the graphic is unnormalised, i.e. we have set the prefactor

$$\frac{g_s}{8\pi}\frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}} = 1 \qquad \text{in graphic 5.1}$$

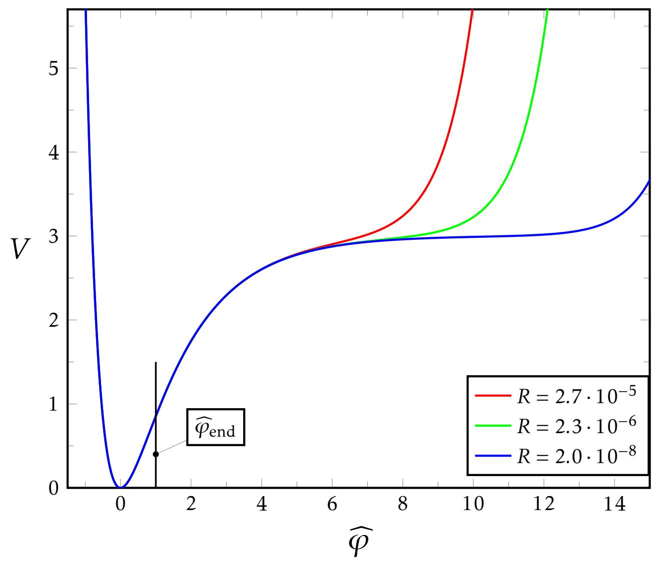

Figure 5.1: Inflationary potential for different values of the parameter R. The potential in the graphics is unnormalised, so the unit of measure is arbitrary. The graphics also show the end point of inflation used in the following calculations

Notice in passing that the mass of the inflaton field $\widehat{\varphi}$ can be readily calculated by taking the second derivative of (5.1.21) evaluated at the minimum:

$$m_\varphi^2 = V''(0) = \frac{g_s}{8\pi}\frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}} \tag{5.1.23}$$

5.2 Fibre inflation predictions

In this section we will find out some predictions for the spectral index n_s and the tension-to-scalar ratio r. These two observables turn out to depend only on the slow roll parameters ϵ and η evaluated at the horizon exit [14]

$$n_s = 1 + 2\eta_* - 6\epsilon_* \quad \text{and} \quad r = 16\epsilon_* \tag{5.2.1}$$

Recalling the definitions of the slow roll parameters

$$\epsilon = \frac{M_p^2}{2V^2}\left(\frac{\partial V}{\partial \widehat{\varphi}}\right)^2 \qquad \eta = \frac{M_p^2}{V}\left(\frac{\partial^2 V}{\partial \widehat{\varphi}^2}\right)$$

we can readily calculate ϵ and η from our inflationary potential (5.1.21):

$$\epsilon \approx \frac{8}{3}\left(\frac{e^{-k\widehat{\varphi}/2} - e^{-2k\widehat{\varphi}} + \frac{1}{2}Re^{k\widehat{\varphi}}}{3 - 4e^{-k\widehat{\varphi}/2} + e^{-2k\widehat{\varphi}} + Re^{k\widehat{\varphi}}}\right)^2 \tag{5.2.2}$$

$$\eta \approx -\frac{4}{3}\left(\frac{e^{-k\widehat{\varphi}/2} - 4e^{-2k\widehat{\varphi}} - Re^{k\widehat{\varphi}}}{3 - 4e^{-k\widehat{\varphi}/2} + e^{-2k\widehat{\varphi}} + Re^{k\widehat{\varphi}}}\right) \tag{5.2.3}$$

Thus, in order to give a predictions for n_s and r we have to estimate the value of inflaton field at the horizon exit $\widehat{\varphi}_*$.
The horizon exit is tied up with the number of e-foldings N_e, which measure the slow roll inflation life span. So, in this section we will make the following analysis:

1. we first calculate the horizon exit in the range of [50, 62] e-foldings,
2. given $\widehat{\varphi}_*$ we estimate n_s and r,
3. we finally taking into account experimentally observations that constraint the parameter space for inflation.

In fact, inflation has to last long enough for solving the horizon problem: for instance, a typical value is about 60 e-foldings [2]. Moreover, the energy scale at which inflation occurs must be sufficient so as to generate the observed density perturbation. This requirement involve the so called COBE normalisation: it is a quite stringent experimental constraint which allows to fix the energy at which inflation occurs, thereby restricting the accessible parameter space.

Actually, the number of e-foldings is set by the post inflationary thermal history and by the inflationary energy scale. In particular, N_e depends on the reheating epoch through the equation of state $p = w\rho$ describing this period and the reheating temperature $T_{\rm rh}$. The number of e-folding can be written as [14]

$$N_e \simeq 62 + \ln\left(\frac{M_{\rm inf}}{10^{16}}\right) - \frac{1-3w}{3(1+w)}\ln\left(\frac{M_{\rm inf}}{T_{\rm rh}}\right) \tag{5.2.4}$$

where $M_{\rm inf}$ is the energy scale of inflation.
In the next chapter we will calculate the reheating temperature and $\Delta N_{\rm eff}$ due to the decay of the inflaton using the parameters inferred in this section. Through the reheating temperature we can narrow the range of e-foldings and so the predictions for n_s and r. Finally, the number of extra neutrino species allow us to select a spectral index n_s and so a tensor-to-scalar r by using fig. 2.1.

5.2.1 Number of e-foldings

The number of e-folding is given by (setting $M_{\rm P} = 1$)

$$N_e = \int_{\widehat{\varphi}_{end}}^{\widehat{\varphi}_*} \frac{V}{V'}\, d\widehat{\varphi} \approx \int_{\widehat{\varphi}_{end}}^{\widehat{\varphi}_*} \frac{1}{\sqrt{2\epsilon}}\, d\widehat{\varphi} \tag{5.2.5}$$

where $\widehat{\varphi}$ is the value of the inflaton field at horizon exit, while $\widehat{\varphi}_{\rm end}$ set the end of inflation. From now on we consider $\widehat{\varphi}_{\rm end} = 1$, following [14] (see also fig. 5.1). We have written a Matlab code so as to evaluating $\widehat{\varphi}_*$ numerically for the number of efoldings listed in table 5.1 and table 5.2.
Once the horizon exit values are known, we can readily calculate both n_s and r: table 5.1 and table 5.2 report our predictions with $R = 2.3 \cdot 10^{-6}$ and $R = 2.7\cdot 10^{-5}$ respectively. Notice that the predictions for both n_s and r with $R = 2.7 \cdot 10^{-5}$ are greater than $R = 2.3 \cdot 10^{-6}$ for equal number of e-foldings. Fig. 5.2 show r versus n_s for two different values of the parameter R. The green curve represents the approximate relation $r \approx 6(n_s - 1)^2$ for $R \to 0$, which is a good one for n_s in the range $[0.96, 0.98]$.
From fig. 2.1 it is possible to infer that a $\Delta N_{\rm eff} \approx 0.6$ requires a spectral index in the range of approximately $[0.97, 1.00]$. By comparing fig. 2.1 with the graphic 5.2 it seems that for $R = 2.3 \cdot 10^{-6}$ it is quite difficult to obtain a tensor-to-scalar-ratio in range of $r \approx 0.01$. By contrast, for $R = 2.7 \cdot 10^{-5}$ the tensor-to-scalar-ratio is never below $r = 0.01$ for every n_s.

N_e	n_s	r	M_{inf}
50	0.965	$7.14 \cdot 10^{-3}$	$7.4 \cdot 10^{15}$
54	0.968	$6.27 \cdot 10^{-3}$	$7.1 \cdot 10^{15}$
57	0.970	$5.73 \cdot 10^{-3}$	$7.0 \cdot 10^{15}$
60	0.972	$5.26 \cdot 10^{-3}$	$6.8 \cdot 10^{15}$
62	0.973	$4.99 \cdot 10^{-3}$	$6.7 \cdot 10^{15}$

Table 5.1: $R = 2.3 \cdot 10^{-6}$, $g_s = 0.301$

N_e	n_s	r	M_{inf}
50	0.987	$1.06 \cdot 10^{-2}$	$8.1 \cdot 10^{15}$
54	0.993	$1.03 \cdot 10^{-2}$	$8.0 \cdot 10^{15}$
57	0.998	$1.02 \cdot 10^{-2}$	$8.0 \cdot 10^{15}$
60	1.004	$1.02 \cdot 10^{-2}$	$8.0 \cdot 10^{15}$
62	1.008	$1.04 \cdot 10^{-2}$	$8.0 \cdot 10^{15}$

Table 5.2: $R = 2.7 \cdot 10^{-5}$, $g_s = 0.301$

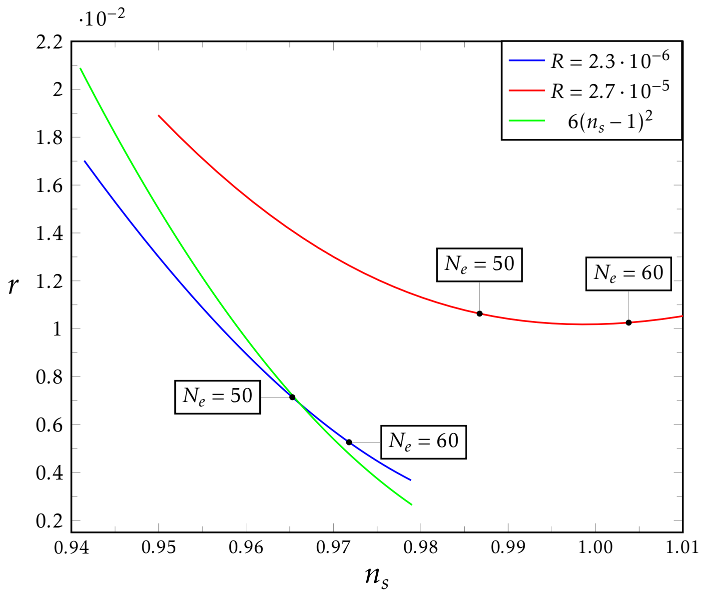

Figure 5.2: r versus n_s for different parameter R. The green curve represents the approximate equation $r \approx 6(n_s - 1)^2$ for $R = 2.3 \cdot 10^{-6}$

5.2.2 Scale of inflation and COBE normalisation

We now evaluate the inflationary scale using the following expression [14]

$$M_{\rm inf} = V_{\rm end}^{1/4} \simeq \left(\frac{g_s}{8\pi}\mathcal{C}_2\right)^{1/4} \frac{M_{\rm Pl}}{\langle\mathcal{V}\rangle^{5/6}} \tag{5.2.6}$$

It is possible to invert eq. (5.2.6) and express $\mathcal{C}_2$ as a function of $M_{\rm inf}$:

$$\mathcal{C}_2 = \frac{8\pi}{g_s}\left(\frac{M_{\rm inf}}{M_{\rm Pl}}\right)^4 \langle\mathcal{V}\rangle^{10/3} \tag{5.2.7}$$

The COBE normalisation is given by the requirement that inflation must reproduce the observed primordial scalar density perturbations of $\delta_H = 1.92 \cdot 10^{-5}$. This condition can be expressed as [14]

$$A_{\rm COBE} \equiv \frac{1}{M_{\rm Pl}^6}\left(\frac{V_*^{3/2}}{V_*'}\right)^2 = \frac{1}{M_{\rm Pl}^4}\frac{V_*}{2\epsilon_*} \simeq 2.7 \cdot 10^{-7} \tag{5.2.8}$$

where the potential V and ϵ are evaluated at the horizon exit. We rewrite the previous expression in a more convenient way:

$$A_{\rm COBE} = \frac{1}{2\epsilon_*}\frac{g_s}{8\pi}\frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}}\widehat{V}_* \qquad \text{with} \quad V = \frac{g_s}{8\pi}\frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}}\widehat{V}_*$$

where $\widehat{V}_*$ is the scalar potential (5.1.21) without the prefactor $g_s\mathcal{C}_2/\left(8\pi\langle\mathcal{V}\rangle^{10/3}\right)$. By inserting eq. (5.2.7) in the previous expression we obtain:

$$M_{\rm inf} = M_{\rm Pl}\left(\frac{2\epsilon_*}{\widehat{V}_*}2.7 \cdot 10^{-7}\right)^{1/4} \tag{5.2.9}$$

Eq. (5.2.9) allow to fix the scale of inflation once the horizon exit is known using the constraint given by the COBE normalisation. If we want to trust our effective field theory $M_{\rm inf}$ should not be greater than 10^{16} GeV. Table 5.1 and table 5.2 show that the scale of inflation is always below 10^{16} GeV for the range of N_e we considered.
Now we can estimate $\mathcal{V}$ using eq. (5.2.7) and (5.1.22):

$$\frac{8\pi}{g_s}\left(\frac{M_{\rm inf}}{M_{\rm Pl}}\right)^4 \langle\mathcal{V}\rangle^{10/3} = \mathcal{C}_2 = |W_0|^2\left(\frac{|B|^4}{256 g_s^2|A|}\right)^{1/3} \Longrightarrow$$

$$\Longrightarrow\ 8\pi(256)^{1/3}\left(\frac{M_{\rm inf}}{M_{\rm Pl}}\right)^4\left(\frac{\langle\mathcal{V}\rangle^{10}}{g_s}\right)^{1/3} = |W_0|^2\left(\frac{|B|^4}{|A|}\right)^{1/3} \equiv \mathcal{C}_{\rm tuning} \tag{5.2.10}$$

Assuming $M_{\text{inf}} = 7.5 \cdot 10^{15}\,\text{GeV}$ and $g_s = 0.301$ as reference values, we readily obtain

$$\left(2.1 \cdot 10^{-8}\right)\langle\mathcal{V}\rangle^{10/3} = \mathcal{C}_{\text{tuning}}$$

So, a Calabi Yau volume $\mathcal{V} \approx 10^4$ requires

$$\mathcal{C}_{\text{tuning}} := |W_0|^2 \left(\frac{B^4}{|A|}\right)^{1/3} \approx 4.6 \cdot 10^5 \quad \text{with} \quad M_{\text{inf}} = 7.5 \cdot 10^{15}\,\text{GeV}$$

For $B \approx \mathcal{O}(10)$ and $|A| \approx \mathcal{O}(10^{-2})$ we have that $|W_0| \approx \mathcal{O}(100)$ and this shows that our model of fibre inflation require little tuning. In fact, a superpotential of about $|W_0| \approx \mathcal{O}(100)$ appears quite naturally during compactifications. Larger volumes require a considerable amount of fine tuning of the parameters $|W_0|$, A, B, C, so a Calabi Yau volume of about $\mathcal{V} \approx 10^4$ can be considered as an upper bound.
Actually, this condition can be stated in an equivalent and clearier way. Through the COBE normalisation we set the scale of inflation, which has to be $M_{\text{inf}} \approx 10^{16}\text{GeV}$. Let us consider $|W_0| \approx 100$, which is a "natural" order of magnitude for the superpotential and let be $g_s = 0.301$. For these values $\mathcal{C}_2 \approx 10^5$ and by recasting eq. (5.2.6) we can infer that

$$\langle\mathcal{V}\rangle = \left(\frac{g_s}{8\pi}\mathcal{C}_2\right)^{3/10} \left(\frac{M_{\text{Pl}}}{M_{\text{inf}}}\right)^{6/5} \approx 10^4$$

Thus, in order to match the COBE normalisation the volume of the Calabi Yau can not be larger than 10^4. From now on we take $\mathcal{V} = 10^4$ as benchmark value.

Mass of the inflaton

We have seen that the mass of the inflaton can be written as eq. (5.1.23)

$$m_\varphi \equiv \sqrt{\frac{g_s}{8\pi}\mathcal{C}_2 \frac{M_{\text{Pl}}}{\langle\mathcal{V}\rangle^{5/3}}} \tag{5.2.11}$$

Using equation eq. (5.2.7) and eq. (5.2.9) the inflaton mass becomes

$$m_\varphi = \left(\frac{M_{\text{inf}}}{M_{\text{Pl}}}\right)^2 M_{\text{Pl}} = \left(\frac{2\epsilon_*}{\widehat{V}_*} 2.7 \cdot 10^{-7}\right)^{1/2} M_{\text{Pl}} \tag{5.2.12}$$

It can be seen from the previous expression that the mass of the inflaton is set by the energy scale of inflation, which depends logarithimicaly on the

number of e-foldings (5.2.4). Thus, in our case the mass of the inflaton is essentially fixed by the amplitude of the primordial density perturbations and by the "steepness" of the potential, where the former is an experimental constraint while the latter is a geometric feature of the inflationary potential. It is important to remark that the right hand side of eq. (5.2.12) is evaluated at the horizon exit, which can be considered as the starting point of inflation, while the left handed side is of course calculated in the minimum of the potential.
If the assume that $M_{\rm inf} = 7.5 \cdot 10^{15}$, the inflaton mass is about

$$m_\varphi \simeq 2.3 \cdot 10^{13}\,\text{GeV}$$

in agreement with [18]. m_φ is significantly lower than the mass of the gravitino $m_{3/2} \sim M_{\rm Pl}/\mathcal{V} \approx (10^{14} - 10^{15})\,\text{GeV}$.
To conclude this section, we report in table 5.3 our parameters benchmark that we will use throughout the next chapter:

	$M_{\rm inf}\,(\text{GeV})$	$\langle\mathcal{V}\rangle$	g_s	$\mathcal{C}_{\rm tuning}$
$R = 2.3 \cdot 10^{-6}$	$7.0 \cdot 10^{15}$	$1.0 \cdot 10^{4}$	0.301	$3.5 \cdot 10^{5}$
$R = 2.7 \cdot 10^{-5}$	$8.0 \cdot 10^{15}$	$1.0 \cdot 10^{4}$	0.301	$6.0 \cdot 10^{5}$

Table 5.3: Benchmark values for some parameters

As we can see in table 5.1 and table 5.2, the scale of inflation is slowly varying with respect to the number of e-foldings, so it is an excellent approximation taking a reference value for $M_{\rm inf}$. In table 5.3 we have calculated $C_{\rm tuning}$ using the formula (5.2.10) and the parameters listed in the same table.

$$\mathcal{C}_{\rm tuning} = 8\pi(256)^{1/3}\left(\frac{M_{\rm inf}}{M_{\rm Pl}}\right)^4\left(\frac{\langle\mathcal{V}\rangle^{10}}{g_s}\right)^{1/3}$$

Chapter 6

Reheating after fibre inflation

In this concluding chapter, we will finally give some predictions for the reheating temperature of the Universe and the extra neutrino dof. This task requires the branching ratio for the inflaton decays into the visible and the hidden sector, so the first half of this chapter is devoted to study of the possible couplings and decay channels for the inflaton. Once we have the decay rates, we are able to calculate the reheating temperature after inflation and ΔN_{eff}. The former allow us to give a prediction for n_s and r which basically depends only on the parameter R, while from the latter we can conclude if fibre inflation can accommodate $r \approx 0.01$.

6.1 Canonical normalization and axionic couplings

It is of primary importance to canonically normalize the Lagrangian so as to determine the correct couplings of the inflaton to the others fields and the mass spectrum. In this section we determine the fields redefinitions necessary for this purpose. Neglecting the string loop corrections to the Kähler potential, by taking the fields derivative of eq. (5.1.2), we obtain the following Kähler metric in the large volume regime

$$K^0_{i\bar{j}}(\tau_i) = \frac{1}{4\tau_2^2}\begin{pmatrix} \frac{\tau_2^2}{\tau_1^2} & \gamma\left(\frac{\tau_3}{\tau_1}\right)^{3/2} & -\frac{3\gamma}{2}\frac{\sqrt{\tau_3}}{\tau_1^{3/2}}\tau_2 \\ \gamma\left(\frac{\tau_3}{\tau_1}\right)^{3/2} & 2 & -3\gamma\sqrt{\frac{\tau_3}{\tau_1}} \\ -\frac{3\gamma}{2}\frac{\sqrt{\tau_3}}{\tau_1^{3/2}}\tau_2 & -3\gamma\sqrt{\frac{\tau_3}{\tau_1}} & \frac{3\alpha\gamma}{2}\frac{\tau_2^2}{\mathcal{V}\sqrt{\tau_3}} \end{pmatrix} \tag{6.1.1}$$

where all the terms subleading respect to $\sqrt{\tau_3/\tau_2}$ has been dropped. In particularly, from now on we consider $\mathcal{V} \approx \alpha\sqrt{\tau_1}\tau_2$.
The kinetic Lagrangian to the leading order for the Kähler moduli is given by (sum over repeated indexes is understood)

$$-\frac{\mathcal{L}_{\text{kin}}}{\sqrt{-g}} = K^0_{l\overline{m}}(T_i + T_{\overline{i}})\partial^\mu T^l \partial_\mu T^{*\overline{m}} =$$

$$= \left(\frac{\delta\tau_i}{\delta T_l}\right)\left(\frac{\delta\tau_j}{\delta T^*_{\overline{m}}}\right)\frac{\delta^2 K^0(\tau_i)}{\delta\tau_i\delta\tau_j}\left(\partial^\mu\tau_l\partial_\mu\tau_m + \partial^\mu b_l\partial_\mu b_m\right) =$$

$$= \frac{1}{4}\frac{\delta^2 K^0(\tau_i)}{\delta\tau_i\delta\tau_j}\left(\partial^\mu\tau_i\partial_\mu\tau_j + \partial^\mu b_i\partial_\mu b_j\right)$$

Therefore, we can split the kinetic Lagrangian into two parts: one for the real part of the Kähler moduli ($\mathcal{L}_{\text{kin},\tau}$) and one for the axions ($\mathcal{L}_{\text{kin},\psi}$), so we can separately diagonalise them.

$$-\frac{\mathcal{L}_{\text{kin}}}{\sqrt{-g}} = -\frac{\mathcal{L}_{\text{kin},\tau}}{\sqrt{-g}} - \frac{\mathcal{L}_{\text{kin},b}}{\sqrt{-g}}$$

Moduli canonical normalization

In this subsection we just report the canonically normalized moduli fields. For the extended calculations, see Appendix A. The relations between the moduli τ_1, τ_3 and $\mathcal{V}$ and their canonical normalized counterpart are

$$\tau_1 = \exp\left(\frac{2}{\sqrt{3}}\frac{\chi_1}{M_{\text{Pl}}} + \sqrt{\frac{2}{3}}\frac{\chi_2}{M_{\text{Pl}}} + \frac{3}{2}\frac{\Phi^2}{M_{\text{Pl}}^2}\right) \tag{6.1.2}$$

$$\mathcal{V} = \exp\left(\sqrt{\frac{3}{2}}\frac{\chi_2}{M_{\text{Pl}}} + \frac{9}{4}\frac{\Phi^2}{M_{\text{Pl}}^2}\right) \tag{6.1.3}$$

$$\tau_3 = \left(\frac{3\mathcal{V}}{4\alpha\gamma}\right)^{2/3}\left(\frac{\Phi}{M_{\text{Pl}}}\right)^{4/3} \tag{6.1.4}$$

We start off by neglecting the field Φ which is involved in subleading corrections to the canonical normalization, so we can write

$$\tau_1 \approx \exp\left(\frac{2}{\sqrt{3}}\frac{\chi_1}{M_{\text{Pl}}} + \sqrt{\frac{2}{3}}\frac{\chi_2}{M_{\text{Pl}}}\right) \tag{6.1.5}$$

$$\mathcal{V} \approx \exp\left(\sqrt{\frac{3}{2}}\frac{\chi_2}{M_{\text{Pl}}}\right) \tag{6.1.6}$$

A Taylor expansion around the vev of the canonically normalized $\mathcal{X}_1$ and $\mathcal{X}_2$ fields leads to

$$\tau_1 \approx \exp\left(\frac{2}{\sqrt{3}}\frac{\langle\mathcal{X}_1\rangle}{M_{\rm Pl}} + \sqrt{\frac{2}{3}}\frac{\langle\mathcal{X}_2\rangle}{M_{\rm Pl}}\right)\exp\left(\frac{2}{\sqrt{3}}\frac{\delta\mathcal{X}_1}{M_{\rm Pl}} + \sqrt{\frac{2}{3}}\frac{\delta\mathcal{X}_2}{M_{\rm Pl}}\right)$$

$$\mathcal{V} \approx \exp\left(\sqrt{\frac{3}{2}}\frac{\langle\mathcal{X}_2\rangle}{M_{\rm Pl}}\right)\exp\left(\sqrt{\frac{3}{2}}\frac{\delta\mathcal{X}_2}{M_{\rm Pl}}\right)$$

The vev can be written as

$$\langle\tau_1\rangle \approx \exp\left(\frac{2}{\sqrt{3}}\frac{\langle\mathcal{X}_1\rangle}{M_{\rm Pl}} + \sqrt{\frac{2}{3}}\frac{\langle\mathcal{X}_2\rangle}{M_{\rm Pl}}\right)$$

$$\langle\mathcal{V}\rangle \approx \exp\left(\sqrt{\frac{3}{2}}\frac{\langle\mathcal{X}_2\rangle}{M_{\rm Pl}}\right)$$

and so we have

$$\tau_1 \approx \langle\tau_1\rangle\exp\left(\frac{2}{\sqrt{3}}\frac{\delta\mathcal{X}_1}{M_{\rm Pl}} + \sqrt{\frac{2}{3}}\frac{\delta\mathcal{X}_2}{M_{\rm Pl}}\right)$$

$$\mathcal{V} \approx \langle\mathcal{V}\rangle\exp\left(\sqrt{\frac{3}{2}}\frac{\delta\mathcal{X}_2}{M_{\rm Pl}}\right)$$

Expanding τ_1 around his vev, i.e. $\tau_1 \approx \langle\tau_1\rangle + \delta\tau_1$, we find that

$$\tau_1 \approx \langle\tau_1\rangle\exp\left\{\frac{2}{\sqrt{3}}\frac{\delta\mathcal{X}_1}{M_{\rm Pl}} + \sqrt{\frac{2}{3}}\frac{\delta\mathcal{X}_2}{M_{\rm Pl}}\right\} \approx \langle\tau_1\rangle\left[1 + \frac{2}{\sqrt{3}}\frac{\delta\mathcal{X}_1}{M_{\rm Pl}} + \sqrt{\frac{2}{3}}\frac{\delta\mathcal{X}_2}{M_{\rm Pl}}\right]$$

hence

$$\frac{\delta\tau_1}{\langle\tau_1\rangle} \approx \left[\frac{2}{\sqrt{3}}\frac{\delta\mathcal{X}_1}{M_{\rm Pl}} + \sqrt{\frac{2}{3}}\frac{\delta\mathcal{X}_2}{M_{\rm Pl}}\right]$$

We are only interested in the possible decays of $\delta\mathcal{X}_1$, that turns out to be mostly the inflaton. Thus, we can neglect $\delta\mathcal{X}_2$

$$\frac{\delta\tau_1}{\langle\tau_1\rangle} \approx \frac{2}{\sqrt{3}}\frac{\delta\mathcal{X}_1}{M_{\rm Pl}} \tag{6.1.7}$$

The previous equation can be derived in the same way from eq. (5.1.15):

$$\tau_1 = \exp\left\{\frac{\kappa\varphi}{M_{\rm Pl}}\right\} \approx \exp\left\{\frac{\kappa\langle\varphi\rangle}{M_{\rm Pl}}\right\}\exp\left\{\frac{\kappa\widehat{\varphi}}{M_{\rm Pl}}\right\} \implies$$

$$\implies \tau_1 \approx \langle\tau_1\rangle\left[1 + \kappa\frac{\widehat{\varphi}}{M_{\rm Pl}}\right] \implies \frac{\delta\tau_1}{\langle\tau_1\rangle} \approx \kappa\frac{\widehat{\varphi}}{M_{\rm Pl}}$$

So, we can finally write

$$\boxed{\frac{\delta\tau_1}{\langle\tau_1\rangle} \approx \frac{2}{\sqrt{3}}\frac{\delta\chi_1}{M_{\rm Pl}} \equiv \kappa\frac{\widehat{\varphi}}{M_{\rm Pl}}} \qquad \kappa = \frac{2}{\sqrt{3}} \tag{6.1.8}$$

Couplings between moduli and axions

From the canonical normalization of $\mathcal{L}_{{\rm kin},b}$ we can derive the Lagrangian of the couplings between the field $\widehat{\varphi}$ and the axions. The canonically normalized axions $\hat{b}_1$ and $\hat{b}_2$ are given by [23]

$$b_1 = \sqrt{2}\,\hat{b}_1 \qquad\qquad b_2 = \frac{\mathcal{V}}{\alpha}\hat{b}_2$$

and the canonically normalized kinetic Lagrangian for the axions reads

$$-\frac{\mathcal{L}_{{\rm kin},b}}{\sqrt{-g}} \supset \frac{1}{2\tau_1^2}\partial_\mu\hat{b}_1\partial^\mu\hat{b}_1 + \frac{1}{2}\tau_1\partial_\mu\hat{b}_2\partial^\mu\hat{b}_2 \tag{6.1.9}$$

Using eq. (6.1.8) we can write

$$\begin{aligned}
-\frac{\mathcal{L}_{{\rm kin},b}}{\sqrt{-g}} &\supset \frac{1}{2\tau_1^2}\partial_\mu\hat{b}_1\partial^\mu\hat{b}_1 + \frac{1}{2}\tau_1\partial_\mu\hat{b}_2\partial^\mu\hat{b}_2 \approx \\
&\approx \frac{1}{2}\left(\frac{1}{\langle\tau_1\rangle^2} - 2\frac{\delta\tau_1}{\langle\tau_1\rangle^3}\right)\partial_\mu\hat{b}_1\partial^\mu\hat{b}_1 + \frac{1}{2}\Big(\langle\tau_1\rangle + \delta\tau_1\Big)\partial_\mu\hat{b}_2\partial^\mu\hat{b}_2 = \\
&= \frac{1}{2\langle\tau_1\rangle^2}\left(1 - 2\frac{\delta\tau_1}{\langle\tau_1\rangle}\right)\partial_\mu\hat{b}_1\partial^\mu\hat{b}_1 + \frac{1}{2}\langle\tau_1\rangle\left(1 + \frac{\delta\tau_1}{\langle\tau_1\rangle}\right)\partial_\mu\hat{b}_2\partial^\mu\hat{b}_2 = \\
&= \frac{1}{2\langle\tau_1\rangle^2}\left(1 - \frac{4}{\sqrt{3}}\frac{\widehat{\varphi}}{M_{\rm Pl}}\right)\partial_\mu\hat{b}_1\partial^\mu\hat{b}_1 + \frac{1}{2}\langle\tau_1\rangle\left(1 + \frac{2}{\sqrt{3}}\frac{\widehat{\varphi}}{M_{\rm Pl}}\right)\partial_\mu\hat{b}_2\partial^\mu\hat{b}_2
\end{aligned}$$

By setting

$$a_1 = \frac{1}{\langle\tau_1\rangle}\hat{b}_1 \qquad a_2 = \sqrt{\langle\tau_1\rangle}\,\hat{b}_2 \tag{6.1.10}$$

we obtain the Lagrangian of the kinetic coupling between the field $\widehat{\varphi}$ and the normalized axions a_1 and a_2:

$$\mathcal{L}_{\rm kin\ coup} = -\frac{2}{\sqrt{3}}\frac{\widehat{\varphi}}{M_{\rm Pl}}\partial_\mu a_1\partial^\mu a_1 + \frac{1}{\sqrt{3}}\frac{\widehat{\varphi}}{M_{\rm Pl}}\partial_\mu a_2\partial^\mu a_2 \tag{6.1.11}$$

Since we have that the axions a_1 and a_2 are almost massless [24], [25] i.e.

$$\Box a_1 \approx 0 \qquad\qquad \Box a_2 \approx 0$$

we can write

$$\begin{aligned}
\widehat{\varphi}\partial_\mu a_1 \partial^\mu a_1 &\doteq -\partial_\mu \widehat{\varphi} a_1 \partial^\mu a_1 \doteq \Box\widehat{\varphi} a_1 a_1 + \partial_\mu \widehat{\varphi}\partial^\mu a_1 a_1 = \\
&= -m_\varphi^2 \widehat{\varphi} a_1 a_1 + \partial_\mu \widehat{\varphi}\partial^\mu a_1 a_1 \doteq \\
&\doteq -m_\varphi^2 \widehat{\varphi} a_1 a_1 - \widehat{\varphi}\partial_\mu a_1 \partial^\mu a_1 \\
\implies \widehat{\varphi}\partial_\mu a_1 \partial^\mu a_1 &\doteq -\frac{1}{2} m_\varphi^2 \widehat{\varphi} a_1 a_1
\end{aligned}$$

where $\doteq$ denotes that we have integrated by parts and neglected boundary terms. Moreover, we have employed the equation of motion of $\widehat{\varphi}$ al tree level. So, we can finally write

$$\boxed{\mathcal{L}_{\widehat{\varphi}\to aa} = \frac{1}{\sqrt{3}} \frac{m_\varphi^2}{M_{\text{Pl}}} \widehat{\varphi} a_1 a_1 - \frac{1}{2\sqrt{3}} \frac{m_\varphi^2}{M_{\text{Pl}}} \widehat{\varphi} a_2 a_2} \tag{6.1.12}$$

6.2 Suppressed decays into visible sector fields

We now systematically analyse the possible decays of the inflaton. The articles [24], [23] and [26] point out that in fibre inflation the following decay channels are actually *suppressed*:

1. MATTER SCALARS (SQUARKS, SLEPTONS),
   Our model belongs to the so-called class of *non-sequestered* string models where, as we remarked in the previous chapter, the mass of gravitino fixes the scale of the soft terms. Usually, in the context of the cMSSM (constrained MSSM), the mass of the matter scalars are set all equal to the mass of the gravitino around the energy scale of Grand Unification $1 \cdot 10^{16}$GeV [11]. In our model, φ decays at energy $m_\varphi \approx 2 \cdot 10^{13}$GeV, so we can safely neglect corrections due to renormalization running (see [11]) and we can still consider

   $$m_{\text{matter scalars}} \approx m_{3/2} \sim \frac{M_{\text{Pl}}}{\mathcal{V}} \approx 10^{14}\text{GeV} \gg m_\varphi$$

   Thus, the decay into scalar matter particles is kinematically forbidden.

2. matter fermions (quarks, leptons),
   Given $c_{\widehat{\varphi}\to ff}$ the decay fraction of the inflaton into matter fermions, it turns out that
   $$c_{\widehat{\varphi}\to ff} \sim \left(\frac{m_f}{m_\varphi}\right)^2 \ll 1$$
   This is a model independent feature called *chirality suppression*

3. gauginos.
   Ref. [27] shows that the gaugino masses have the same magnitude as the gravitino mass, so the inflaton decay in these particles is kinematically forbidden.

Therefore, we are left with the following ***unsuppressed decay channels***:

1. ***gauge bosons***

2. ***Higgs bosons***

The magnitude of the couplings between the inflaton and the particles of the visible sector depends crucially on the position of the Standard Model (SM) in the Calabi-Yau three-fold. In our model it is localised on a stack of $D7$ branes which wrap the four-cycle associated with the fibre modulus τ_1. Fig. 6.1 gives a pictorial view of the Calabi-Yau volume in our model: it can be schematically seen as a parallelepiped whose lateral faces are the base modulus τ_2 and the fibre modulus τ_1. The blow-up modulus τ_3 can be viewed as as a "hole" in the volume in the parallelepiped.

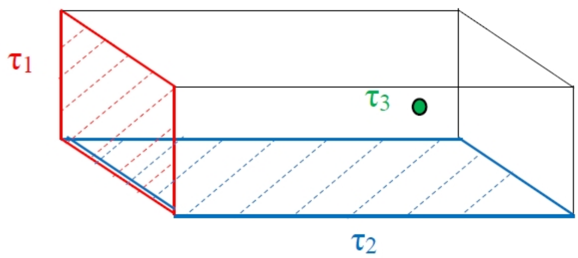

Figure 6.1: Pictorial view of the Calabi-Yau volume. The Standard Model lies on $D7$ branes wrapping the four-cycle related to the fibre modulus τ_1.

Without entering into technical details, we mention that a Dp brane is a $(p+1)$ dimensional subspace of spacetime where "D" stands for "Dirichlet". The endpoints of open strings lie on D-branes which give Dirichlet boundary conditions on the motion of the open string endpoints [16]. Type IIB string theory is compatible only with Dp branes with odd p [16].

For the gauge bosons the couplings with moduli arise from the real part of the gauge kinetic function f_{ab}, which depends on the volume of the four-cycle wrapped by the D-branes containing the Standard Model. In the case of the Higgs boson, the correct normalization and the couplings with the moduli can be derived from the Kähler potential (4.4.3).

6.3 Moduli couplings with the Higgs bosons

6.3.1 Giudice-Masiero term

We now focus on the possible interactions of the inflaton with the Higgs sector. Following ref. [21] and ref. [27] we assume a diagonal Kähler matter metric, so the Kähler potentials for the Higgs sector reads

$$\mathcal{K}_{\text{matter}} = \tilde{K}_{H_u} H_u H_u^* + \tilde{K}_{H_d} H_d H_d^* + Z\,(H_u H_d + \text{ h.c.}) \tag{6.3.1}$$

The functions Z, $\tilde{K}_{\mathrm{H_u}}$, $\tilde{K}_{\mathrm{H_d}}$ are in general unknown and hard to compute since they are not holomorphic. Nonetheless, it is possible to infer the moduli dependence of these functions using some scaling arguments regarding the physical Yukawa couplings (4.4.5). As shown in [27], in our model we have two components for the Kähler matter metric for each Higgs boson

$$\tilde{K}_{\parallel} = \frac{k_i}{\tau_2} \qquad \tilde{K}_{\perp} = k_i \qquad \text{with } \ i = H_u, H_d$$

This is due to the fact that the two Higgs doublet H_u and H_d come from string modes which can be located inside the $D7$ branes or orthogonal to them [27]. $k_{\mathrm{H_u}}$, $k_{\mathrm{H_d}}$ are real functions on the complex structure moduli and the axion dilaton. Assuming that U_α and S have been already stabilised at their minima, we can safely treat $k_{\mathrm{H_u}}$, $k_{\mathrm{H_d}}$ simply as constants. Let be

$$\tilde{K}_{\mathrm{H_u}} = k_{\mathrm{H_u}} \tau_2^{-\lambda_{H_u}} \qquad \tilde{K}_{\mathrm{H_d}} = k_{\mathrm{H_d}} \tau_2^{-\lambda_{H_u}} \tag{6.3.2}$$

The function Z scale in the following way

$$Z = z\sqrt{\tilde{K}_{\mathrm{H_u}} \tilde{K}_{\mathrm{H_d}}} = \frac{z}{\tau_2{}^{\overline{\lambda}}} \sqrt{k_{\mathrm{H_u}} k_{\mathrm{H_d}}} \qquad \text{with } \ \overline{\lambda} = \frac{\lambda_{H_u} + \lambda_{H_d}}{2}, \quad z \in \mathbb{R} \tag{6.3.3}$$

where z is a real parameter. There are two possible cases: $\overline{\lambda} = 1$ and $\overline{\lambda} = 1/2$.

1. $\overline{\lambda} = 1$

   The Kähler matter metric for H_u and H_d is respectively

$$\tilde{K}_{\mathrm{H_u}} = \frac{k_{\mathrm{H_u}}}{\tau_2} \qquad \tilde{K}_{\mathrm{H_d}} = \frac{k_{\mathrm{H_d}}}{\tau_2}$$

   The corresponding Giudice-Masiero term is

$$Z = \frac{z}{\tau_2}\sqrt{k_{\mathrm{H_u}} k_{\mathrm{H_d}}} \approx \frac{{\tau_1}^{1/2}}{\mathcal{V}} z \sqrt{k_{\mathrm{H_u}} k_{\mathrm{H_d}}}$$

   If we expand near the vev and then use eq. (6.1.8) we obtain

$$Z \approx \frac{\langle \tau_1 \rangle^{1/2}}{\langle \mathcal{V} \rangle}\left(1 + \frac{\widehat{\varphi}}{\sqrt{3} M_{\mathrm{Pl}}}\right) z \sqrt{k_{\mathrm{H_u}} k_{\mathrm{H_d}}} \tag{6.3.4}$$

2. $\overline{\lambda} = 1/2$

   In this case there are two possible choices of the Kähler matter metric for the H_u and H_d fields:

$$\tilde{K}_{H_u} = \frac{k_{\mathrm{H_u}}}{\tau_2} \qquad \tilde{K}_{k_{\mathrm{H_d}}} = k_{\mathrm{H_d}} \qquad \text{or} \qquad \tilde{K}_{H_u} = k_{\mathrm{H_u}} \qquad \tilde{K}_{H_d} = \frac{k_{\mathrm{H_d}}}{\tau_2}$$

   Both choices lead to the Giudice-Masiero term

$$Z = \frac{z}{{\tau_2}^{1/2}}\sqrt{k_{\mathrm{H_u}} k_{\mathrm{H_d}}} \approx \frac{{\tau_1}^{1/4}}{\mathcal{V}^{1/2}} z \sqrt{k_{\mathrm{H_u}} k_{\mathrm{H_d}}}$$

   If we expand near the vev and then use eq. (6.1.8) we obtain

$$Z \approx \frac{\langle \tau_1 \rangle^{1/4}}{\langle \mathcal{V} \rangle^{1/2}}\left(1 + \frac{\widehat{\varphi}}{2\sqrt{3} M_{\mathrm{Pl}}}\right) z \sqrt{k_{\mathrm{H_u}} k_{\mathrm{H_d}}} \tag{6.3.5}$$

6.3.2 Normalization of Higgs fields

We now report the canonical normalization for the Higgs field for the two Kähler matter metrics.

1. $\overline{\lambda} = 1$

$$\tilde{K}_{H_u} H_u H_u^* + \tilde{K}_{H_d} H_d H_d^* = \frac{\alpha \langle \tau_1 \rangle^{1/2}}{\langle \mathcal{V} \rangle}\left(1 + \frac{1}{\sqrt{3}} \frac{\widehat{\varphi}}{M_{\mathrm{Pl}}}\right)\left(k_{\mathrm{H_u}} H_u H_u^* + k_{\mathrm{H_d}} H_d H_d^*\right)$$

   The canonically normalized Higgs fields are

$$\widehat{H}_{\mathrm{u}} = \sqrt{2}\frac{\langle \tau_1 \rangle^{1/4}}{\langle \mathcal{V} \rangle^{1/2}} \sqrt{k_{\mathrm{H_u}}} H_u \qquad \widehat{H}_{\mathrm{d}} = \sqrt{2}\frac{\langle \tau_1 \rangle^{1/4}}{\langle \mathcal{V} \rangle^{1/2}} \sqrt{k_{\mathrm{H_d}}} H_d \tag{6.3.6}$$

2. $\overline{\lambda} = 1/2$

   We choose for concreteness $\tilde{K}_{H_u} = k_{\mathrm{H_u}}$ and $\tilde{K}_{H_d} = k_{\mathrm{H_d}}/\tau_2$, therefore

$$\tilde{K}_{H_u} H_u H_u^* + \tilde{K}_{H_d} H_d H_d^* = k_{\mathrm{H_u}} H_u H_u^* + \frac{\langle\tau_1\rangle^{1/2}}{\langle\mathcal{V}\rangle}\left(1 + \frac{1}{2\sqrt{3}}\frac{\widehat{\varphi}}{M_{\mathrm{Pl}}}\right) k_{\mathrm{H_d}} H_d H_d^*$$

   The canonically normalized Higgs fields are

$$\widehat{H}_{\mathrm{u}} = \sqrt{2}\sqrt{k_{\mathrm{H_u}}} H_u \qquad \widehat{H}_{\mathrm{d}} = \sqrt{2}\frac{\langle\tau_1\rangle^{1/4}}{\langle\mathcal{V}\rangle^{1/2}}\sqrt{k_{\mathrm{H_d}}} H_d \tag{6.3.7}$$

6.3.3 Interaction Lagrangian

It can be shown that inflaton decays into the Higgs bosons of the type $\widehat{\varphi} \to \widehat{H}_{\mathrm{u}}\widehat{H}_{\mathrm{u}}$ and $\widehat{\varphi} \to \widehat{H}_{\mathrm{d}}\widehat{H}_{\mathrm{d}}$ are suppressed due to the equations of motion at tree level [26]. So, the only unsuppressed decays come from the Giudice-Masiero term. Thus, the interaction Lagrangian is given by

$$\mathcal{L}_{\mathrm{cubic}} \supset -\frac{1}{2}\frac{m_\varphi^2 \widehat{\varphi}}{\sqrt{3} M_{\mathrm{Pl}}} z\left(\widehat{H}_{\mathrm{u}}\widehat{H}_{\mathrm{d}} + \mathrm{h.c.}\right) \qquad \overline{\lambda} = 1$$

$$\mathcal{L}_{\mathrm{cubic}} \supset -\frac{1}{2}\frac{m_\varphi^2 \widehat{\varphi}}{2\sqrt{3} M_{\mathrm{Pl}}} z\left(\widehat{H}_{\mathrm{u}}\widehat{H}_{\mathrm{d}} + \mathrm{h.c.}\right) \qquad \overline{\lambda} = \frac{1}{2}$$

We summarize the previous expression by writing

$$\boxed{\mathcal{L}_{\mathrm{cubic}} \supset -\Lambda_{\mathrm{G.M.}} m_\varphi^2 \widehat{\varphi}\left(\widehat{H}_{\mathrm{u}}\widehat{H}_{\mathrm{d}} + \mathrm{h.c.}\right)} \tag{6.3.8}$$

where

$$\Lambda_{\mathrm{G.M.}} = \begin{cases} \dfrac{1}{2\sqrt{3}}\dfrac{z}{M_{\mathrm{Pl}}} & \text{for } \overline{\lambda} = 1 \\[2ex] \dfrac{1}{4\sqrt{3}}\dfrac{z}{M_{\mathrm{Pl}}} & \text{for } \overline{\lambda} = \dfrac{1}{2} \end{cases}$$

From now on we follow the notations of ref. [26]. We set

$$\widehat{H}_{\mathrm{u}}^{+} = \frac{1}{\sqrt{2}}(\mathrm{Re}\,\widehat{H}_{\mathrm{u}}^{+} + i\mathrm{Im}\,\widehat{H}_{\mathrm{u}}^{+}) \qquad \widehat{H}_{\mathrm{d}}^{-} = \frac{1}{\sqrt{2}}(\mathrm{Re}\,\widehat{H}_{\mathrm{d}}^{-} + i\mathrm{Im}\,\widehat{H}_{\mathrm{d}}^{-})$$

$$\widehat{H}_{\mathrm{u}}^{0} = \frac{1}{\sqrt{2}}(\mathrm{Re}\,\widehat{H}_{\mathrm{u}}^{0} + i\mathrm{Im}\,\widehat{H}_{\mathrm{u}}^{0}) \qquad \widehat{H}_{\mathrm{d}}^{0} = \frac{1}{\sqrt{2}}(\mathrm{Re}\,\widehat{H}_{\mathrm{d}}^{0} + i\mathrm{Im}\,\widehat{H}_{\mathrm{d}}^{0})$$

$$h_1 = \mathrm{Re}\,\widehat{H}_u^+ \qquad h_2 = \mathrm{Re}\,\widehat{H}_d^- \qquad h_3 = \mathrm{Re}\,\widehat{H}_d^0 \qquad h_4 = \mathrm{Re}\,\widehat{H}_u^0$$
$$h_5 = \mathrm{Im}\,\widehat{H}_d^0 \qquad h_6 = \mathrm{Im}\,\widehat{H}_u^0 \qquad h_7 = \mathrm{Im}\,\widehat{H}_u^+ \qquad h_8 = \mathrm{Im}\,\widehat{H}_d^-$$

hence we can write

$$\mathcal{L}_{\mathrm{cubic}} \supset -\Lambda_{\mathrm{G.M.}} m_\varphi^2 \widehat{\varphi} \sum_{i=1}^{4} (-1)^{i+1} h_{2i-1} h_{2i} = \frac{1}{2}(\widehat{H}_u \widehat{H}_d + \mathrm{h.c.}) =$$
$$= (\widehat{H}_u^+, \widehat{H}_u^0) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \widehat{H}_d^0 \\ \widehat{H}_d^- \end{pmatrix} + (\widehat{H}_u^{+*}, \widehat{H}_u^{0*}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \widehat{H}_d^{0*} \\ \widehat{H}_d^{-*} \end{pmatrix} =$$
$$= \widehat{H}_u^+ \widehat{H}_d^- - \widehat{H}_u^0 \widehat{H}_d^0 + \widehat{H}_u^{+*} \widehat{H}_d^{-*} - \widehat{H}_u^{0*} \widehat{H}_d^{0*} = \widehat{H}_u^+ \widehat{H}_d^- - \widehat{H}_d^0 \widehat{H}_u^0 + \mathrm{h.c.}$$

Since in the cMSSM electroweak symmetry breaking takes place at energies of order of the gravitino mass while the inflaton decays at energies of order $m_\varphi \ll m_{3/2}$ [26], we need to switch from the basis of gauge eigenstates to the basis of mass eigenstates. In the latter basis we have the neutral fields $\hat{A}^0$, $\hat{h}^0$, $\widehat{H}^0$, $\widehat{G}^0$ and the charge fields $\widehat{G}^\pm$, $\widehat{H}^\pm$ with $\widehat{G}^- = \widehat{G}^{+*}$and $\widehat{H}^- = \widehat{H}^{+*}$. The gauge eigenstates are related to the mass eigenstates by the following relations

$$\begin{pmatrix} \widehat{H}_d^0 \\ \widehat{H}_u^0 \end{pmatrix} = \begin{pmatrix} v_d \\ v_u \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \widehat{H}^0 \\ \hat{h}^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta_0 & -\cos\beta_0 \\ \cos\beta_0 & \sin\beta_0 \end{pmatrix} \begin{pmatrix} \hat{A}^0 \\ \widehat{G}^0 \end{pmatrix}$$
$$\begin{pmatrix} \widehat{H}_u^+ \\ \widehat{H}_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta_\pm & \cos\beta_\pm \\ -\cos\beta_\pm & \sin\beta_\pm \end{pmatrix} \begin{pmatrix} \widehat{G}^+ \\ \widehat{H}^+ \end{pmatrix}$$

so we have

$$h_1 = \sin\beta_\pm \mathrm{Re}\,\widehat{G}^+ + \cos\beta_\pm \mathrm{Re}\,\widehat{H}^+ \qquad h_2 = -\cos\beta_\pm \mathrm{Re}\,\widehat{G}^+ + \sin\beta_\pm \mathrm{Re}\,\widehat{H}^+$$
$$h_3 = \sqrt{2} v_d + (\cos\alpha\,\widehat{H}^0 + \sin\alpha\,\hat{h}^0) \qquad h_4 = \sqrt{2} v_u + (-\sin\alpha\,\widehat{H}^0 + \cos\alpha\,\hat{h}^0)$$
$$h_5 = -\cos\beta_0\,\widehat{G}^0 + \sin\beta_0\,\hat{A}^0 \qquad h_6 = \sin\beta_0\,\widehat{G}^0 + \cos\beta_0\,\hat{A}^0$$
$$h_7 = \sin\beta_\pm \mathrm{Im}\,\widehat{G}^+ + \cos\beta_\pm \mathrm{Im}\,\widehat{H}^+ \qquad h_8 = \cos\beta_\pm \mathrm{Im}\,\widehat{G}^+ - \sin\beta_\pm \mathrm{Im}\,\widehat{H}^+$$

Note that

$$\widehat{H}_d^{-*} = \mathrm{Re}\,\widehat{H}_d^- + i\mathrm{Im}\,\widehat{H}_d^-$$
$$= -\cos\beta_\pm(\mathrm{Re}\,\widehat{G}^+ + i\mathrm{Im}\,\widehat{G}^+) + \sin\beta_\pm(\mathrm{Re}\,\widehat{H}^+ + i\mathrm{Im}\,\widehat{H}^+)$$
$$\Longrightarrow \mathrm{Im}\,\widehat{H}_d^- = \cos\beta_\pm \mathrm{Im}\,\widehat{G}^+ - \sin\beta_\pm \mathrm{Im}\,\widehat{H}^+$$

Provided that v_u and v_d minimize the tree level potential, one finds that

$$\beta_0 = \beta_\pm = \beta \qquad m^2_{\widehat{G}^0} = m^2_{\widehat{G}^\pm} = 0 \qquad m^2_{\widehat{H}^\pm} = m^2_{\hat{A}^0} + m^2_{\mathrm{W}}$$

$$m^2_{\hat{h}^0,\widehat{H}^0} = \frac{1}{2}\left(m^2_{\hat{A}^0} + m^2_{\mathrm{Z}} \mp \sqrt{(m^2_{\hat{A}^0} - m^2_{\mathrm{Z}})^2 + 4m^2_{Z} m^2_{\hat{A}^0}\sin^2(2\beta)}\right)$$

$$m^2_{\hat{A}^0} = \frac{2b}{\sin 2\beta} = 2|\hat{\mu}|^2 + m^2_{\widehat{H}_{\mathrm{u}}} + m^2_{\widehat{H}_{\mathrm{d}}} \quad \text{with } b = B\hat{\mu}$$

The β angle is defined by $\tan\beta = v_{\mathrm{u}}/v_{\mathrm{v}}$ with $\beta \in \left]0, \frac{\pi}{2}\right[$ since we consider positive vev by definition. The mixing angle α is determined, at the tree level, by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\left(\frac{m^2_{\widehat{H}^0} + m^2_{\hat{h}^0}}{m^2_{\widehat{H}^0} - m^2_{\hat{h}^0}}\right) \qquad \frac{\tan 2\alpha}{\tan 2\beta} = \left(\frac{m^2_{\hat{A}^0} + m^2_{\mathrm{Z}}}{m^2_{\hat{A}^0} - m^2_{\mathrm{Z}}}\right)$$

and is usually chosen to be negative, i.e $\alpha \in]-\frac{\pi}{2}, 0[$. Finally, the Lagrangian of the couplings for the Giudice - Masiero term is given by

$$\begin{aligned}\mathcal{L}_{\mathrm{cubic}} \supset \Lambda_{\mathrm{G.M.}} m^2_{\varphi}\widehat{\varphi}\bigg\{ & \frac{1}{2}\sin(2\alpha)\left[(\hat{h}^0)^2 - (\widehat{H}^0)^2\right] + \cos(2\alpha)\left[\hat{h}^0\widehat{H}^0\right] + \\ & + v_{\mathrm{u}}v_{\mathrm{d}} + \sqrt{2}\left[v_{\mathrm{u}}(\cos(\alpha)\widehat{H}^0 + \sin(\alpha)\hat{h}^0) + v_{\mathrm{d}}(-\sin(\alpha)\widehat{H}^0 + \cos(\alpha)\hat{h}^0)\right] \\ & + \frac{1}{2}\sin(2\beta)\left[|\widehat{G}^+|^2 - |\widehat{H}^+|^2\right] + \cos(2\beta)\left[\mathrm{Re}\,\widehat{H}^+\mathrm{Re}\,\widehat{G}^+ + \mathrm{Im}\,\widehat{H}^+\mathrm{Im}\,\widehat{G}^+\right] \\ & + \frac{1}{2}\sin(2\beta)\left[(\widehat{G}^0)^2 - (\hat{A}^0)^2\right] + \cos(2\beta)\left[\hat{A}^0\widehat{G}^0\right]\bigg\}\end{aligned} \tag{6.3.9}$$

6.4 Dominant decays into visible and hidden sector fields

Decay rates into the Higgs sector

The $\widehat{\varphi}$ field may decay only into the $\hat{h}^0, \widehat{G}^0, \widehat{G}^\pm$ fields, since all other decays are kinematically forbidden [26]. So, the couplings of our interest are

$$\mathcal{L}_{\mathrm{cubic}} \supset \frac{\Lambda_{\mathrm{G.M.}}}{2} m^2_{\varphi}\widehat{\varphi}\left\{\sin(2\beta)|\widehat{G}^+|^2 + \sin 2(\beta)(\widehat{G}^0)^2 + \sin(2\alpha)(\hat{h}^0)^2\right\} \tag{6.4.1}$$

with

$$\mathcal{L}_{\widehat{\varphi}\widehat{G}^+\widehat{G}^-} = \frac{\Lambda_{\text{G.M.}}}{2} m_\varphi^2 \sin(2\beta)\,\widehat{\varphi}\widehat{G}^+\widehat{G}^- \tag{6.4.2}$$

$$\mathcal{L}_{\widehat{\varphi}\widehat{G}^0\widehat{G}^0} = \frac{\Lambda_{\text{G.M.}}}{2} m_\varphi^2 \sin(2\beta)\,\widehat{\varphi}\widehat{G}^0\widehat{G}^0 \tag{6.4.3}$$

$$\mathcal{L}_{\widehat{\varphi}\hat{h}^0\hat{h}^0} = \frac{\Lambda_{\text{G.M.}}}{2} m_\varphi^2 (\sin 2\alpha)\,\widehat{\varphi}\hat{h}^0\hat{h}^0 \tag{6.4.4}$$

The decays rates are

$$\Gamma_{\widehat{\varphi}\to\widehat{G}^+\widehat{G}^-} = \frac{1}{16\pi m_\varphi}\frac{\Lambda^2_{\text{G.M.}}}{4} m_\varphi^4 \sin^2(2\beta) \tag{6.4.5}$$

$$\Gamma_{\widehat{\varphi}\to\widehat{G}^0\widehat{G}^0} = \frac{1}{8\pi m_\varphi}\frac{\Lambda^2_{\text{G.M.}}}{4} m_\varphi^4 \sin^2(2\beta) \tag{6.4.6}$$

$$\Gamma_{\widehat{\varphi}\to\hat{h}^0\hat{h}^0} = \frac{1}{8\pi m_\varphi}\frac{\Lambda^2_{\text{G.M.}}}{4} m_\varphi^4 \sin^2(2\alpha) \tag{6.4.7}$$

where

$$\Lambda_{\text{G.M.}} = C_j \frac{z}{2\sqrt{3}}\frac{1}{M_{\text{Pl}}} \qquad \text{with } C_j = \begin{cases} 1 & \text{for } \overline{\lambda} = 1 \\ \frac{1}{2} & \text{for } \overline{\lambda} = 1/2 \end{cases}$$

thus

$$\Gamma_{\text{Higgs}} = \frac{\Lambda^2_{\text{G.M.}}}{64\pi} m_\varphi^3 \Big[3\sin^2(2\beta) + 2\sin^2(2\alpha)\Big] = C_j^2 \frac{z^2}{16}\frac{1}{48\pi}\frac{m_\varphi^3}{M_{\text{Pl}}^2}\Big[3\sin^2(2\beta) + 2\sin^2(2\alpha)\Big]$$

Finally, we rewrite the previous expressions as follows

$$\boxed{\Gamma_{\text{Higgs}} = C_j^2 \left(\frac{z}{4}\right)^2 \Gamma_0 \Big[3\sin^2(2\beta) + 2\sin^2(2\alpha)\Big]} \qquad \text{with } \Gamma_0 = \frac{1}{48\pi}\frac{m_\varphi^3}{M_{\text{Pl}}^2} \tag{6.4.8}$$

Decays rates into visible and hidden sector fields

We find the following decay rates for the canonically normalized modulus φ:

$$\text{Decays into DR:} \begin{cases} \Gamma_{\widehat{\varphi}\to a_1 a_1} = \frac{1}{24\pi}\frac{m_\varphi^3}{M_{\text{Pl}}^2} \\ \Gamma_{\widehat{\varphi}\to a_2 a_2} = \frac{1}{96\pi}\frac{m_\varphi^3}{M_{\text{Pl}}^2} \end{cases} \tag{6.4.9}$$

$$\Longrightarrow \Gamma_{\widehat{\varphi}\to \mathrm{DR}} = \Gamma_{\widehat{\varphi}\to a_1 a_1} + \Gamma_{\widehat{\varphi}\to a_2 a_2} = \frac{5}{96\pi}\frac{m_\varphi^3}{M_{\mathrm{Pl}}} = \frac{5}{2}\Gamma_0 \tag{6.4.10}$$

$$\text{Decays into SM:} \begin{cases} \Gamma_{\widehat{\varphi}\to A_1 A_1} = \dfrac{N_g}{48\pi}\dfrac{m_\varphi^3}{M_{\mathrm{Pl}}} \\ \Gamma_{\widehat{\varphi}\to \mathrm{Higgs}} = C_j^2 \dfrac{z^2}{16}\Gamma_0\Big[3\sin^2(2\beta) + 2\sin^2(2\alpha)\Big] \end{cases} \tag{6.4.11}$$

$$\Longrightarrow \Gamma_{\widehat{\varphi}\to \mathrm{SM}} = \Gamma_{\widehat{\varphi}\to A_1 A_1} + \Gamma_{\widehat{\varphi}\to \mathrm{Higgs}} =$$
$$= \left[N_g + C_j^2 \frac{z^2}{16}\Big(3\sin^2(2\beta) + 2\sin^2(2\alpha)\Big)\right]\Gamma_0 \tag{6.4.12}$$

where N_g is the number of gauge bosons. Throughout the next calculation we have set $N_g = 12$.

6.5 Reheating temperature

After the end of inflation, φ starts oscillating around its minimum when $H \sim m_\varphi$, at frequency $k_0 = m_\varphi$ (if we ignore effects associated with particle creation). The oscillation amplitude will fall off as $[a(t)]^{-3/2}$ and the energy of the field decreases in same way as the density of non relativistic particles of mass m_φ [28]. So, during reheating the equation of state is $p = 0$, implying

$$a(t) \sim t^{3/2} \qquad H = \frac{2}{3t} \qquad \varphi \sim a^{-3/2} \sim t^{-1}$$

In the slow roll approximation, we can treat φ as the amplitude of a homogeneous field.

Now, let us consider the quantum corrections to the equation of motion of this field, oscillating at a frequency $k_0 = m_\varphi \gg H(t)$:

$$\ddot{\varphi} + 3H(t)\dot{\varphi} + \Big[m_\varphi^2 + \Pi(k_0)\Big]\varphi = 0$$

Here $\Pi(k_0)$ is the polarization operator for the field φ at a four- momentum $k = (k_0, 0, 0, 0)$. When k_0 is greater than the threshold of particles pair production, $\Pi(k_0)$ acquires an imaginary part $\mathrm{Im}\,\Pi(k_0)$. For $m_\varphi^2 \gg \mathrm{Im}\,\Pi(k_0)$,

$m_\varphi^2 \gg H^2$ and neglecting the time-dependence of H, we obtain a solution which describes the damped oscillations of the field near the minimum [28]

$$\varphi = \varphi_0 \exp(i m_\varphi t) \cdot \exp\left[-\frac{1}{2}\left(3H + \frac{\operatorname{Im}\Pi(m_\varphi)}{m_\varphi}\right) t\right]$$

From the unitary relations it follows that

$$\operatorname{Im}\Pi(m_\varphi) = m_\varphi \Gamma_{\rm tot}$$

where $\Gamma_{\rm tot}$ is the total decay width of the inflaton. Hence, when $\Gamma_{\rm tot} \gg 3H$, the energy density of the field decreases exponentially in a time less than the typical expansion time of the universe $\Delta t \approx H^{-1}$:

$$\rho_\varphi = \frac{m_\varphi^2 \varphi^2}{2} \approx \rho_0 \, e^{-\frac{\Gamma_{\rm tot}}{2} t}$$

If the coupling constant with the other fields are small, then initially

$$\Gamma_{\rm tot} < 3H(t) = 2/t$$

In this case, the energy density of φ simply decreases due to the expansion of the Universe. The fraction of total energy converted in particle production is small since the particles produced during the oscillations are diluted and the Universe cannot essentially reach thermal equilibrium. This remains true until

$$\Gamma_{\rm tot} \approx \frac{3}{2} H(t^*)$$

At the time t^* the contribution of the newly created particles becomes significant and after t^* practically all the energy of the field φ is transformed into particles production. In slow roll approximation it is possible to write

$$H^2 = \frac{1}{3}\frac{V(\varphi)}{M_{\rm Pl}^2} = \frac{1}{3}\frac{\rho}{M_{\rm Pl}^2} \implies$$

$$\implies \rho = 3H^2 M_{\rm Pl}^2 = \frac{4}{3}\Gamma_{\rm tot}^2 M_{\rm Pl}^2$$

If the reheating process occurs rapidly enough, virtually all the energy from the oscillating field will be transformed into thermal energy, and the matter acquires a temperature $T_{\rm rh}$ where

$$\rho = \frac{\pi g_*(T_{\rm rh})}{30} T_{\rm rh}^4 \approx \frac{4}{3}\Gamma_{\rm tot}^2 M_{\rm Pl}^2 \tag{6.5.1}$$

Reheating temperature in fibre inflation

In fibre inflation, the total decay rate Γ_ϕ of the inflaton has two main contributions, which comes from the decay into visible and hidden dof:

$$\Gamma_{\chi_1} = \Gamma_{\rm vis} + \Gamma_{\rm hid} = (c_{\rm vis} + c_{\rm hid})\Gamma_0 \qquad \text{with } \Gamma_0 = \frac{1}{48\pi}\frac{m_\varphi^3}{M_{\rm Pl}^2} \tag{6.5.2}$$

The hidden sector dof is constituted by axions, which never thermalise during thermal history of the universe, so the reheating temperature is given by (we use the same notation of ref. [26])

$$T_{\rm rh} = \left(\frac{30\rho_{\rm vis}}{\pi^2 g_*(T_{\rm rh})}\right)^{1/4}$$

where

$$\rho_{\rm vis} = \frac{c_{\rm vis}}{c_{\rm tot}} 3H^2 M_{\rm Pl}^2 \qquad \text{with } c_{\rm tot} = c_{\rm vis} + c_{\rm hid}$$

Using the relation $9H^2 \approx 4\Gamma_\varphi^2$, we find

$$\begin{aligned}\rho_{\rm vis} &= \frac{4}{3}\frac{c_{\rm vis}}{c_{\rm tot}}\Gamma_{\chi_1}^2 M_{\rm Pl}^2 = \frac{4}{3}\frac{c_{\rm vis}}{c_{\rm tot}}(c_{\rm vis}+c_{\rm hid})^2\Gamma_0^2 M_{\rm Pl}^2 = \\ &= \frac{4}{3}c_{\rm vis}(c_{\rm vis}+c_{\rm hid})\left(\frac{1}{48\pi}\right)^2\frac{m_\varphi^6}{M_{\rm Pl}^2} = \frac{4}{3}c_{\rm vis}\,c_{\rm tot}\left(\frac{1}{48\pi}\right)^2\frac{m_\varphi^6}{M_{\rm Pl}^2}\end{aligned}$$

Thus, the reheating temperature can be rewritten as

$$\begin{aligned}T_{\rm rh} &= \left(\frac{30\rho_{\rm vis}}{\pi^2 g_*(T_{\rm rh})}\right)^{1/4} = \left[\frac{30}{\pi^2}\frac{1}{g_*(T_{\rm rh})}c_{\rm vis}\,c_{\rm tot}\frac{4}{3}\left(\frac{1}{48\pi}\right)^2\right]^{1/4} \\ &= \frac{1}{\pi}\left(\frac{5}{288}\frac{c_{\rm vis}\,c_{\rm tot}}{g_*(T_{\rm rh})}\right)^{1/4} m_\varphi\sqrt{\frac{m_\varphi}{M_{\rm Pl}}}\end{aligned} \tag{6.5.3}$$

This reheating temperature has to be larger than about 1 MeV in order to preserve the successful BBN predictions. Using the relation (5.1.23) we can recast the reheating temperature as

$$\begin{aligned}T_{\rm rh} &= \frac{1}{\pi}\left(\frac{5}{288}\frac{c_{\rm vis}\,c_{\rm tot}}{g_*(T_{\rm rh})}\right)^{1/4} m_\varphi\sqrt{\frac{m_\varphi}{M_{\rm Pl}}} = \\ &= \frac{1}{\pi}\left(\frac{5}{288}\frac{c_{\rm vis}\,c_{\rm tot}}{g_*(T_{\rm rh})}\right)^{1/4}\left(\frac{1}{256 g_s^2}\right)^{1/4}\left(\frac{g_s}{8\pi}C_{\rm tuning}\right)^{3/4}\frac{M_{\rm Pl}}{\langle\mathcal{V}\rangle^{5/2}}\end{aligned}$$

In our model, the total decay of the φ field in the visible sector is

$$\Gamma_{\rm vis} = \Gamma_{\widehat{\varphi}\to A_1 A_1} + \Gamma_{\widehat{\varphi}\to \rm Higgs} = \\ = \left[12 + C_j^2\left(\frac{z}{4}\right)^2\left(3\sin^2 2\beta + 2\sin^2 2\alpha\right)\right]\Gamma_0 = c_{\rm vis}\Gamma_0$$

whereas the total decay in the hidden sector is

$$\Gamma_{\rm hid} = \Gamma_{\widehat{\varphi}\to \rm DR} = \frac{5}{2}\Gamma_0 = c_{\rm hid}\Gamma_0 \tag{6.5.4}$$

We define

$$f(\alpha,\beta) := \left[3\sin^2(2\beta) + 2\sin^2(2\alpha)\right] \tag{6.5.5}$$

with $f(\alpha,\beta) \in [0,5]$ in principle. So, we have

$$c_{\rm vis} = 12 + C_j^2\frac{z^2}{16}f(\alpha,\beta) \qquad c_{\rm hid} = \frac{5}{2} \tag{6.5.6}$$

thus

$$c_{\rm vis}\,c_{\rm tot} = c_{\rm vis}(c_{\rm vis} + c_{\rm hid}) = c_{\rm vis}^2 + \frac{5}{2}c_{\rm vis}$$

In our computations we have used the following expression for the reheating temperature

$$\boxed{T_{\rm rh} = \frac{1}{4\pi}\left(\frac{5}{288}\frac{c_{\rm vis}^2 + \frac{5}{2}c_{\rm vis}}{g_*(T_{\rm rh})}\right)^{1/4}\left(\frac{g_s^{1/3}}{8\pi}\mathcal{C}_{\rm tuning}\right)^{3/4}\frac{M_{\rm Pl}}{\langle\mathcal{V}\rangle^{5/2}}} \tag{6.5.7}$$

6.5.1 Predictions for the reheating temperature

As to make some numerical predictions for $T_{\rm rh}$, we consider the following parameters

$$\alpha = \beta = \pi/4 \qquad z = 1 \qquad C_j = 1/2 \tag{6.5.8}$$

For these values the constant $c_{\rm vis}$ reads

$$c_{\rm vis} = 12 + \left(\frac{1}{2}\right)^2\frac{1}{16}f\left(\frac{\pi}{4},\frac{\pi}{4}\right) \simeq 12.1$$

Since we expect that $z \approx \mathcal{O}(1)$, the constant $c_{\rm vis}$ does not affect the order of magnitude of the reheating temperature, that instead is basically fixed by the inflaton mass. Using the benchmark values of table 5.3 and eq. (6.5.8), our model predict the reheating temperatures of table 6.1 , which are in agreement with [18]. We have set $g_*(T_{\rm rh} = 106.75)$ since the reheating temperature is always higher than 30 GeV.

6.5.2 Predictions for n_s and r

In the previous chapter we have calculated the spectral index n_s and the tensor-to-scalar-ratio r by taking into account a range of plausible numbers of e-foldings for different values of the parameter R. The reheating temperature combined with $M_{\rm inf}$ naturally select a number of e-folding N_e thanks to eq. (5.2.4):

$$N_e \simeq 62 + \ln\left(\frac{M_{\rm inf}}{10^{16}}\right) - \frac{1-3w}{3(1+w)} \ln\left(\frac{M_{\rm inf}}{T_{\rm rh}}\right)$$

Thus, *fibre inflation predictions for n_s and r turn out to depend essentially only on the parameter R*. Table 6.1 shows N_e related to the reheating temperatures listed in the same table and the benchmarks values of table 5.3. In our calculations we have set $\omega = 0$ i.e. we assumed that the reheating period is *matter dominated*. It is clear now that the fibre inflation model can predict

	$T_{\rm rh}$ (GeV)	N_e	n_s	r
$R = 2.3 \cdot 10^{-6}$	$1.0 \cdot 10^{10}$	57.2	0.970	$5.70 \cdot 10^{-3}$
$R = 2.7 \cdot 10^{-5}$	$1.5 \cdot 10^{10}$	57.4	0.999	$1.02 \cdot 10^{-2}$

Table 6.1: Reheating temperatures and related number of e-foldings, n_s and r. The benchmark values for $M_{\rm inf}$ reheating temperatures of table 5.3. We have set $\omega = 0$.

observable primordial gravity waves nowadays i.e. $r \simeq 0.01$ only if the parameter R is in range of $\mathcal{O}(2 \cdot 10^{-5})$. As we have seen in the previous chapter, n_s and r are not independent at all, so $r \simeq 0.01$ implies necessarily $n_s \simeq 1.000$.

6.6 Dark radiation predictions

Our final task is to calculate $\Delta N_{\rm eff}$ in order to verify if our model can actually accommodate a $n_s \simeq 1.000$. In fact, from fig. 2.1 we deduce that $n_s \simeq 1.000$ requires at least $\Delta N_{\rm eff} \simeq 0.5$. If the decay of the inflaton into axions is too much suppressed, the experimental constraints do not allow $n_s \simeq 1.000$ for our model, so a prediction of $r \simeq 0.01$ is forbidden in fibre inflation.
From now on we consider $\Delta N_{\rm eff} = 0.5$ as lower bound for dark radiation.

6.6.1 Extra neutrino species

We have seen previously that ΔN_{eff} is given by (2.2.27)

$$\Delta N_{\text{eff}} = \frac{43}{7}\frac{B_a}{1-B_a}\left[\frac{10.75}{g_*(T_{\text{rh}})}\right]^{1/3}$$

where B_a is the branching ratio for the φ decays into axions. In our case we have

$$\Delta N_{\text{eff}} = \frac{43}{7}\frac{c_{\text{hid}}}{c_{\text{vis}}}\left[\frac{10.75}{g_*(T_{\text{rh}})}\right]^{1/3} = \frac{43}{7}\frac{5/2}{12 + C_j^2\frac{z^2}{16}f(\alpha,\beta)}\left[\frac{10.75}{106.75}\right]^{1/3} \tag{6.6.1}$$

where the function $f(\alpha,\beta)$ is defined by eq. (6.5.5)

6.6.2 Parameter space constraints from dark radiation

We are now able to give some predictions for ΔN_{eff}, which turns out to be a function of the Giudice - Masiero term z and of the angles α and β. Moreover, it depends on the theoretical D-brane setup of the model through the constant C_j that we have seen it can take two possible values:

$$C_{1/2} = \frac{1}{2} \qquad\qquad C_1 = 1 \tag{6.6.2}$$

The search for realistic α and β angles is beyond the aim of this thesis, so we simplify our analysis by considering the function $f(\alpha,\beta)$ as an independent variable which takes values in $[0,5]$. Fig. 6.2 and fig. 6.3 show ΔN_{eff} versus $f(\alpha,\beta)$ for different values of z in the two setup $C_j = 1/2$ and $C_j = 1$.

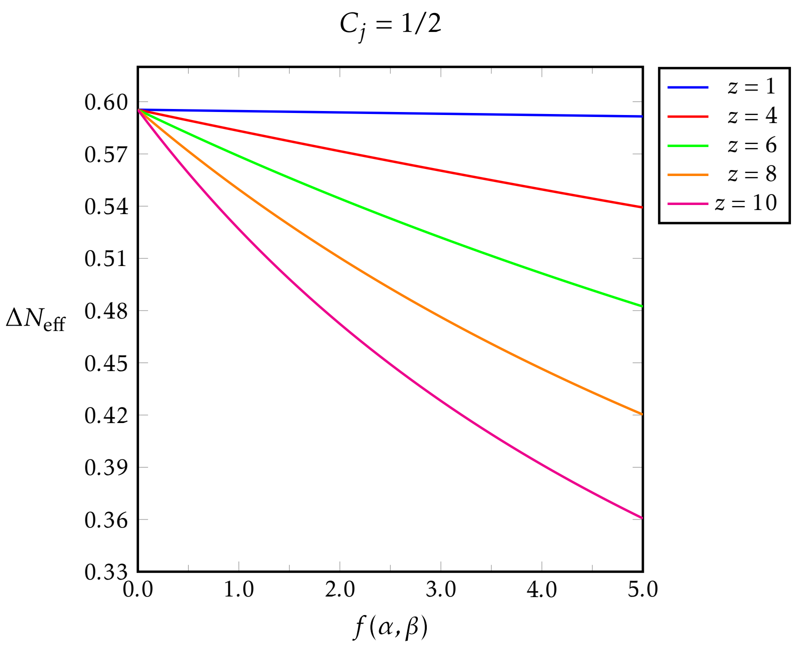

Figure 6.2: ΔN_{eff} vs $f(\alpha, \beta)$ for different values of z. (Case $C_j = 1/2$)

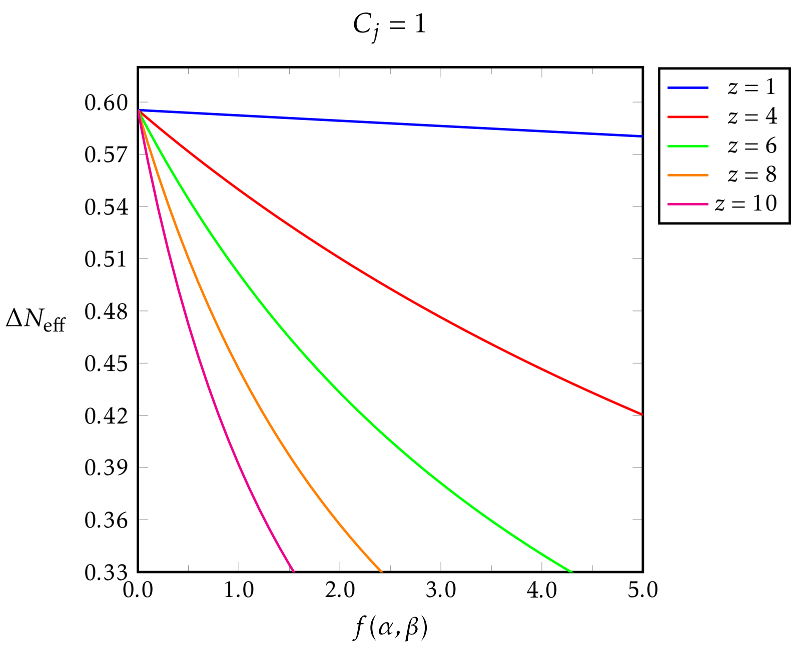

Figure 6.3: ΔN_{eff} vs $f(\alpha, \beta)$ for different values of z. (Case $C_j = 1$)

These graphics show that for all the values of z considered we can have $\Delta N_{\text{eff}} \geq 0.5$ as long as $f(\alpha, \beta)$ is sufficiently small. Notice that for $z = 1$, the extra number of neutrino species is $\Delta N_{\text{eff}} \simeq 0.59$ for all the values of $f(\alpha\beta)$ in both the setup. Moreover, in the limit $z \to 0$ the extra number of neutrino species tends to $\Delta N_{\text{eff}} = 0.595$ without any requirements. This is a very interesting and uncommon feature in string inflation, because a too small Giudice-Masiero term leads to an overproduction of dark radiation in many models unless of the presence of some constraints in the parameters space [24], [23].
For the sake of simplicity let us consider $\alpha = \beta = \pi/4$, so $f(\alpha, \beta) = 5$. From graphic 6.4 we deduce that in order to have $\Delta N_{\text{eff}} \geq 0.5$, the parameter z must be in the following ranges

$$z \leq 3 \quad \text{for } C_j = 1/2 \qquad\qquad z \leq 5.5 \quad \text{for } C_j = 1$$

This in agreement with our expectation that the Giudice-Masiero term $z \approx \mathcal{O}(1)$.

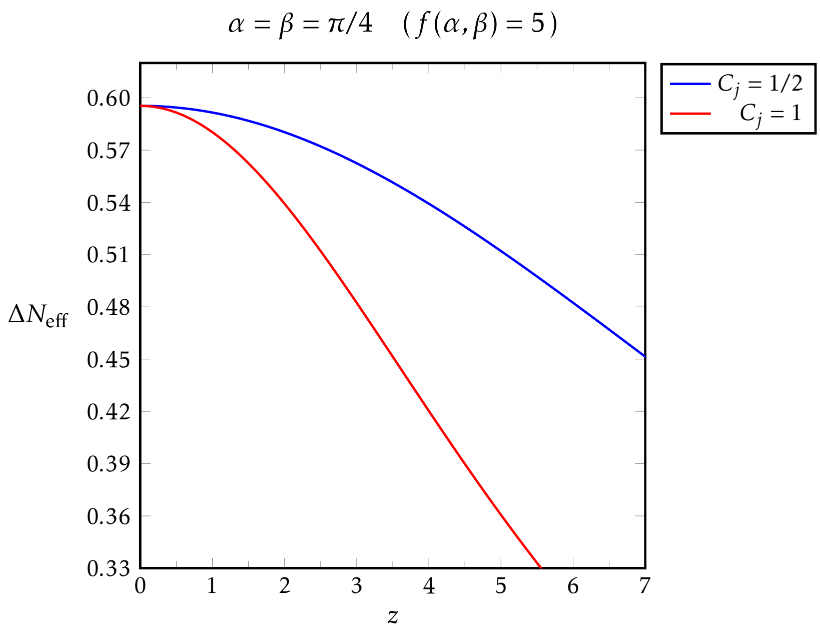

Figure 6.4: ΔN_{eff} vs z for $\alpha = \beta = \pi/4 \quad (f(\alpha, \beta) = 5)$

Summarizing all the previous observations, we can finally state that our model of fibre inflation is capable to predict a spectral index of order $n_s \simeq 1.000$ and so a tensor-to-scalar-ratio $r \simeq 0.01$ since it can easily accommodate $\Delta N_{\text{eff}} \geq 0.5$ for $z \simeq \mathcal{O}(1)$.

6.7 A shortcoming: the Higgs boson mass

Unfortunately, there is a serious shortcoming in our model due to the small Calabi-Yau volume needed to match the COBE normalization: the prediction of fibre inflation for the Higgs boson mass does not fit with its experimental value. In fact, since there is no sequestering, the mass of the soft terms $M_{\rm soft}$ is about the gravitino mass $m_{3/2}$ thus

$$M_{\rm soft} \approx m_{3/2} \sim \frac{M_{\rm P}}{\mathcal{V}} \approx 10^{14} - 10^{15}\,{\rm GeV}$$

Ref. [29] points out that the MSSM with high scale SUSY breaking does not allow for a correct Higgs boson mass. The resulting upper bound is $M_{\rm soft} \lesssim 10^{11}\,{\rm GeV}$. A possible way out is to consider a different D-brane setup for our model: for example, we can in principle set the Standard Model over a blow-up that shrinks to zero, namely sequestering the visible sector of the theory. Nevertheless, as shown in [25], in a sequestered fibre Calabi-Yau there is an overproduction of dark radiation, making this option clearly not viable.
Another possible solution is to consider an alternative method to generate the scalar density perturbations, so that the COBE normalization is satisfied for higher Calabi-Yau volumes than in the vanilla fibre inflation. Ref. [22] and ref. [30] describe this alternatives scenarios: the former article suggests a *curvaton model*, while the latter propose a *modulated reheating scenario*. Both of them require that the modulus driving inflation is not wrapped by a stack of D7 branes containing the visible sector, thus the inflaton coupling with the gauge bosons is drastically reduced respect to our model. As consequence, dark radiation production is strongly enhanced, so the previous models most likely predict an excessive number of neutrino species.

If we want a model of fibre inflation with suitable inflation, dark radiation and the correct Higgs bosons mass at once, we probably have to look for other solutions. A rather drastic and simple way-out is to drop out the MSSM and instead take into account its minimal extension, namely the NMSSM. In fact, ref [31] points out that in the NMSSM $M_{\rm soft} \simeq 10^{14}\,{\rm GeV}$ can lead to the correct Higgs boson mass, making our model still consistent with current experimental constraints. Considering the NMSSM instead of the MSSM does not require any change in our D-brane setup, so there is no needed to modify our calculations of the Kähler matter metric. Moreover, the singlet S in the NMSSM does not introduce new relevant couplings for the inflaton field, thus our decay widths for the inflaton are still valid.

In summary, we may obtain the correct Higgs boson mass if we consider the NMSSM and this possible solution has the virtue of requiring no modifications to our previous decay width calculations, making our fibre inflation predictions for dark radiation, n_s and r unaltered. However, a careful analysis is mandatory in order to verify these preliminary observations.

Chapter 7

Conclusions

In the context of string inflation, fibre inflation is a very interesting model since many of its features make it very predictive. From the phenomenological viewpoint, fibre inflation is very appealing since it can predict an axionic dark radiation of order $\Delta N_{\text{eff}} \simeq 0.5-0.6$ together with an observable tensor-to-scalar-ratio $r \simeq 0.010$ and $n_s \simeq 1.000$. Thus, fibre inflation points out the existence of primordial gravity waves combined with a scale invariant power spectrum for the density perturbations, making its predictions very distinctive and easily testable in the next few years.

From the theoretical perspective, this model can easily accommodate dark radiation within experimental constraints even if the coefficient of the Giudice-Masiero term is equal to zero, i.e. the inflaton decay into the Higgs sector is completely suppressed. This nice feature is a natural consequence of the fact that the Standard Model is located on a stack of D7-branes wrapping the fibre τ_1, where the latter has an intersection with base modulus τ_2.

Despite these and other remarkable features, fibre inflation cannot predict the correct Higgs boson mass due to the high scale of supersymmetry breaking [29]. This serious drawback is unfortunately an example of the known tension between large scale string inflation and low energy supersymmetry. A possible bottom up solution could be taking into account the Next to Minimal Supersymmetric Standard Mode, NMSSM for short.

We finally mention that it would be interesting to study possible generalizations of vanilla fibre inflation models which could give more freedom in the construction of a chiral visible sector with D7-branes. For example one could consider a different Calabi-Yau setup leading to the so-called *generalised fibre inflation*. In this context the volume of the Calabi-Yau looks like:

$$\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3} - \tau_s^{3/2}$$

Moduli stabilisation is more involved than vanilla fibre inflation since it requires D-term contributions to fix, for example, τ_2 in terms of τ_1. But after D-term fixing, moduli stabilisation proceeds in the same way as in vanilla fibre inflation models and the resulting single field inflationary potential can predict a tensor-to-scalar ratio in the range of 0.01 [18]. However, an in-depth analysis is needed in order to check if generalised fibre inflation can predict a suitable amount of dark radiation.
In any case, it is certainly worth looking for a solution which gives the correct Higgs boson mass while preserving the nice features of fibre inflation. We are confident that it is possible to find such kind of model.

Appendix A

Canonical normalization

A.1 Moduli kinetic Lagrangian

The kinetic Lagrangian for real part of the moduli is given by (with $M_{\rm Pl} = 1$)

$$
\begin{aligned}
-\frac{\mathcal{L}_{\text{kin},\tau}}{\sqrt{-g}} &= \frac{1}{4}\frac{\delta^2 K^0(\tau_i)}{\delta\tau_i\delta\tau_j}\partial^\mu\tau_i\partial_\mu\tau_j = \\
&= \begin{pmatrix}\partial^\mu\tau_1 & \partial^\mu\tau_2 & \partial^\mu\tau_3\end{pmatrix}\frac{1}{4\tau_2^2}\begin{pmatrix}\frac{\tau_2^2}{\tau_1^2} & \gamma\left(\frac{\tau_3}{\tau_1}\right)^{3/2} & -\frac{3\gamma}{2}\frac{\sqrt{\tau_3}}{\tau_1^{3/2}}\tau_2 \\ \gamma\left(\frac{\tau_3}{\tau_1}\right)^{3/2} & 2 & -3\gamma\sqrt{\frac{\tau_3}{\tau_1}} \\ -\frac{3\gamma}{2}\frac{\sqrt{\tau_3}}{\tau_1^{3/2}}\tau_2 & -3\gamma\sqrt{\frac{\tau_3}{\tau_1}} & \frac{3\alpha\gamma}{2}\frac{\tau_2^2}{\mathcal{V}\sqrt{\tau_3}}\end{pmatrix}\begin{pmatrix}\partial_\mu\tau_1 \\ \partial_\mu\tau_2 \\ \partial_\mu\tau_3\end{pmatrix} = \\
&= \frac{1}{4\tau_1^2}\partial^\mu\tau_1\partial_\mu\tau_1 + \frac{1}{2\tau_2^2}\gamma\left(\frac{\tau_3}{\tau_1}\right)^{3/2}\partial^\mu\tau_2\partial_\mu\tau_1 - \frac{3\gamma}{4\tau_2}\frac{\sqrt{\tau_3}}{\tau_1^{3/2}}\partial^\mu\tau_1\partial_\mu\tau_3 + \\
&\quad + \frac{1}{2\tau_2^2}\partial_\mu\tau_2\partial^\mu\tau_2 - \frac{1}{2\tau_2^2}\frac{3\gamma}{2}\sqrt{\frac{\tau_3}{\tau_1}}\partial^\mu\tau_2\partial_\mu\tau_3 + \frac{3\alpha\gamma}{8}\frac{1}{\mathcal{V}\sqrt{\tau_3}}\partial^\mu\tau_3\partial_\mu\tau_3
\end{aligned}
$$

Bringing out the dependence on the volume $\mathcal{V}$ for each term, we can rewrite the previous expression as

$$-\frac{\mathcal{L}_{\text{kin}}}{\sqrt{-g}} = \frac{1}{4\tau_1^2}\partial^\mu\tau_1\partial_\mu\tau_1 + \frac{\alpha^2\gamma}{2}\frac{\tau_3}{\mathcal{V}^2}\sqrt{\frac{\tau_3}{\tau_1}}\partial^\mu\tau_2\partial_\mu\tau_1 + \\ -\frac{3\alpha\gamma}{4}\frac{\sqrt{\tau_3}}{\mathcal{V}\tau_1}\partial^\mu\tau_1\partial_\mu\tau_3 + \frac{1}{2\tau_2^2}\partial_\mu\tau_2\partial^\mu\tau_2 + \\ -\frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}\tau_2}\partial^\mu\tau_2\partial_\mu\tau_3 + \frac{3\alpha\gamma}{8}\frac{1}{\mathcal{V}\sqrt{\tau_3}}\partial^\mu\tau_3\partial_\mu\tau_3 \tag{A.1.1}$$

We now trade τ_2 for $\mathcal{V}$, in the limit in which τ_1 and τ_2 are much larger than τ_3.

$$\frac{\partial\mathcal{V}}{\partial x^\mu} = \frac{\partial\mathcal{V}}{\partial\tau_1}\frac{\partial\tau_1}{\partial x^\mu} + \frac{\partial\mathcal{V}}{\partial\tau_2}\frac{\partial\tau_2}{\partial x^\mu} \approx \frac{\alpha}{2}\tau_2\tau_1^{-1/2}\frac{\partial\tau_1}{\partial x^\mu} + \alpha\tau_1^{1/2}\frac{\partial\tau_2}{\partial x^\mu}$$

$$\implies \partial_\mu\tau_2 = \frac{1}{\alpha\tau_1^{1/2}}\partial_\mu\mathcal{V} - \frac{1}{2}\frac{\tau_2}{\tau_1}\partial_\mu\tau_1$$

$$\implies \partial_\mu\mathcal{V}\partial^\mu\mathcal{V} = \frac{\alpha^2}{4}\frac{\tau_2^2}{\tau_1}\partial^\mu\tau_1\partial_\mu\tau_1 + \alpha^2\tau_1\partial_\mu\tau_2\partial^\mu\tau_2 + \alpha^2\tau_2\partial^\mu\tau_1\partial_\mu\tau_2 = \\ = \frac{\mathcal{V}^2}{\tau_2^2}\partial^\mu\tau_2\partial_\mu\tau_2 + \frac{1}{4}\frac{\mathcal{V}^2}{\tau_1^2}\partial^\mu\tau_1\partial_\mu\tau_2 + \frac{\alpha\mathcal{V}}{\tau_1^{1/2}}$$

$$\implies \frac{M_{\text{Pl}}^2}{2\mathcal{V}^2}\partial_\mu\mathcal{V}\partial^\mu\mathcal{V} = \\ = \frac{1}{2\tau_2^2}\partial^\mu\tau_2\partial_\mu\tau_2 + \frac{1}{8\tau_1^2}\partial^\mu\tau_1\partial_\mu\tau_1 + \frac{\alpha}{2\mathcal{V}\tau_1^{1/2}}\partial^\mu\tau_1\partial_\mu\tau_2 = \\ = \frac{1}{2\tau_2^2}\partial^\mu\tau_2\partial_\mu\tau_2 + \frac{1}{8\tau_1^2}\partial^\mu\tau_1\partial_\mu\tau_1 + \frac{1}{2\mathcal{V}\tau_1}\partial_\mu\mathcal{V}\partial^\mu\tau_1 - \frac{1}{4\mathcal{V}\tau_1}\frac{\alpha\tau_2}{\tau_1^{3/2}}\partial_\mu\tau_1\partial^\mu\tau_1 \\ = \frac{1}{2\tau_2^2}\partial^\mu\tau_2\partial_\mu\tau_2 + \frac{1}{8\tau_1}\partial^\mu\tau_1\partial_\mu\tau_1 + \frac{1}{2\mathcal{V}\tau_1}\partial_\mu\mathcal{V}\partial^\mu\tau_1 - \frac{1}{4\tau_1^2}\partial_\mu\tau_1\partial^\mu\tau_1$$

$$\implies \boxed{\frac{1}{2\tau_2^2}\partial^\mu\tau_2\partial_\mu\tau_2 = \frac{1}{2\mathcal{V}^2}\partial_\mu\mathcal{V}\partial^\mu\mathcal{V} + \frac{1}{8\tau_1}\partial^\mu\tau_1\partial_\mu\tau_1 - \frac{1}{2\mathcal{V}\tau_1}\partial_\mu\mathcal{V}\partial^\mu\tau_1}$$

$$\Longrightarrow -\frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}}\left[\frac{1}{2\tau_1}\partial^\mu\tau_1\partial_\mu\tau_3 - \frac{1}{\tau_2}\partial^\mu\tau_3\partial_\mu\tau_2\right] =$$

$$= -\frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}}\left[\frac{1}{2\tau_1}\partial_\mu\tau_1\partial^\mu\tau_3 - \frac{1}{\tau_2}\partial^\mu\tau_3\left(\frac{1}{\alpha\sqrt{\tau_1}}\partial_\mu\mathcal{V} - \frac{1}{2}\frac{\tau_2}{\tau_1}\partial_\mu\tau_1\right)\right] =$$

$$= -\frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}}\left[\frac{1}{2\tau_1}\partial_\mu\tau_1\partial^\mu\tau_3 - \frac{1}{\alpha\sqrt{\tau_1}\tau_2}\partial^\mu\tau_3\partial_\mu\mathcal{V} - \frac{1}{2\tau_1}\partial_\mu\tau_1\partial^\mu\tau_3\right] =$$

$$= -\frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}^2}\partial^\mu\tau_3\partial_\mu\mathcal{V}$$

$$\Longrightarrow \boxed{-\frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}}\left[\frac{1}{2\tau_1}\partial^\mu\tau_1\partial_\mu\tau_3 - \frac{1}{\tau_2}\partial^\mu\tau_3\partial_\mu\tau_2\right] = -\frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}^2}\partial^\mu\tau_3\partial_\mu\mathcal{V}}$$

$$\Longrightarrow \frac{\alpha^2\gamma}{2}\frac{\tau_3^{3/2}}{\mathcal{V}^2\sqrt{\tau_1}}\partial^\mu\tau_1\partial_\mu\tau_2 =$$

$$= \frac{\alpha^2\gamma}{2}\frac{\tau_3^{3/2}}{\mathcal{V}^2\sqrt{\tau_1}}\partial^\mu\tau_1\left[\frac{1}{\alpha\sqrt{\tau_1}}\partial_\mu\mathcal{V} - \frac{1}{2}\frac{\tau_2}{\tau_1}\partial_\mu\tau_1\right] =$$

$$\boxed{= \frac{\alpha\gamma}{2}\frac{1}{\mathcal{V}^2}\frac{\tau_3^{3/2}}{\tau_1}\partial^\mu\tau_1\partial_\mu\mathcal{V} \; - \; \frac{1}{4}\left(\frac{\tau_3}{\tau_1}\right)^{3/2}\frac{\alpha\gamma}{\mathcal{V}\sqrt{\tau_1}}\partial_\mu\tau_1\partial^\mu\tau_1}$$

Putting all the previous expressions together we get

$$-\frac{\mathcal{L}_{\text{kin},\tau}}{\sqrt{-g}} = \frac{3}{8\tau_1^2}\left(1 - \frac{2\alpha\gamma}{3}\frac{\tau_3^{3/2}}{\mathcal{V}}\right)\partial_\mu\tau_1\partial^\mu\tau_1 + \frac{1}{2\mathcal{V}^2}\partial_\mu\mathcal{V}\partial^\mu\mathcal{V} +$$

$$+ \frac{3\alpha\gamma}{8}\frac{1}{\mathcal{V}\sqrt{\tau_3}}\partial_\mu\tau_3\partial^\mu\tau_3 - \frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}^2}\partial_\mu\mathcal{V}\partial^\mu\tau_3 +$$

$$- \frac{1}{2\mathcal{V}\tau_1}\left(1 - \alpha\gamma\frac{\tau_3^{3/2}}{\mathcal{V}}\right)\partial^\mu\tau_1\partial_\mu\mathcal{V}$$

$$\approx \frac{3}{8\tau_1^2}\partial_\mu\tau_1\partial^\mu\tau_1 + \frac{1}{2\mathcal{V}^2}\partial_\mu\mathcal{V}\partial^\mu\mathcal{V} + \frac{3\alpha\gamma}{8}\frac{1}{\mathcal{V}\sqrt{\tau_3}}\partial_\mu\tau_3\partial^\mu\tau_3 +$$

$$- \frac{3\alpha\gamma}{2}\frac{1}{\mathcal{V}^2}\sqrt{\tau_3}\partial_\mu\mathcal{V}\partial^\mu\tau_3 - \frac{1}{2\mathcal{V}\tau_1}\partial^\mu\tau_1\partial_\mu\mathcal{V}$$

where we have neglected subleading terms in $1/\mathcal{V}$. It is convenient to canonically normalize order by order in $1/\mathcal{V}$:

$$\mathcal{L}_{\text{kin},\tau} = \mathcal{L}_{\text{kin},\tau}^{\mathcal{O}(1)} + \mathcal{L}_{\text{kin},\tau}^{\mathcal{O}(\mathcal{V}^{-1})} \tag{A.1.2}$$

The leading terms are

$$-\frac{\mathcal{L}_{\text{kin},\tau}^{\mathcal{O}(1)}}{\sqrt{-g}} = \frac{3}{8\tau_1^2}\partial^\mu\tau_1\partial_\mu\tau_1 + \frac{1}{2\mathcal{V}^2}\partial_\mu\mathcal{V}\partial^\mu\mathcal{V} - \frac{1}{2\tau_1\mathcal{V}}\partial_\mu\tau_1\partial^\mu\mathcal{V} = \tag{A.1.3}$$

$$= \frac{1}{2}\begin{pmatrix} \frac{\partial_\mu\tau_1}{\tau_1} & \frac{\partial_\mu\mathcal{V}}{\mathcal{V}} \end{pmatrix} \cdot \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial_\mu\tau_1}{\tau_1} \\ \frac{\partial_\mu\mathcal{V}}{\mathcal{V}} \end{pmatrix} \tag{A.1.4}$$

while the subleading ones at $\mathcal{O}(\mathcal{V}^{-1})$ are

$$-\frac{\mathcal{L}_{\text{kin}}^{\mathcal{O}(\mathcal{V}^{-1})}}{\sqrt{-g}} = \frac{3\alpha\gamma}{8}\frac{1}{\mathcal{V}\sqrt{\tau_3}}\partial^\mu\tau_3\partial_\mu\tau_3 - \frac{3\alpha\gamma}{2}\frac{\sqrt{\tau_3}}{\mathcal{V}^2}\partial_\mu\tau_3\partial^\mu\mathcal{V} \tag{A.1.5}$$

We can get a canonically normalize Lagrangian by first diagonalizing $\mathcal{L}_{\text{kin},\tau}^{\mathcal{O}(1)}$. This can be done using the following transformations (we reintroduce the Planck mass dependence)

$$\tau_1 = \exp\left(a\frac{\chi_1}{M_{\text{Pl}}} + b\frac{\chi_2}{M_{\text{Pl}}}\right) \tag{A.1.6}$$

$$\mathcal{V} = \exp\left(c\frac{\chi_2}{M_{\text{Pl}}}\right) \tag{A.1.7}$$

$$\begin{pmatrix} \frac{\partial_\mu\tau_1}{\tau_1} \\ \frac{\partial_\mu\mathcal{V}}{\mathcal{V}} \end{pmatrix} = M \cdot \begin{pmatrix} \partial_\mu\chi_1 \\ \partial_\mu\chi_2 \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \cdot \begin{pmatrix} \partial_\mu\chi_1 \\ \partial_\mu\chi_2 \end{pmatrix} \tag{A.1.8}$$

The coefficients a, b and c are obtained from the condition that the matrix M satisfies

$$M^t \cdot \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix} \cdot M = I_2$$

One possible solution of the previous system is

$$a = \frac{2}{\sqrt{3}} \qquad b = \sqrt{\frac{2}{3}} \qquad c = \sqrt{\frac{3}{2}}$$

Now we diagonalize the next-order kinetic term, $\mathcal{L}_{\text{kin}}^{\mathcal{O}(\mathcal{V}^{-1})}$. The first term of (A.1.5) can be canonically normalized by setting

$$\tau_3 = \left(\frac{3\mathcal{V}}{4\alpha\gamma}\right)^{2/3}\left(\frac{\Phi}{M_{\text{Pl}}}\right)^{4/3} \tag{A.1.9}$$

while the second term of (A.1.5) requires a mixing between $\mathcal{V}$ and τ_3. So, by introducing the following subleading corrections

$$\tau_1 = \exp\left(\frac{2}{\sqrt{3}}\frac{\chi_1}{M_{\text{Pl}}} + \sqrt{\frac{2}{3}}\frac{\chi_2}{M_{\text{Pl}}} + \frac{3}{2}\frac{\Phi^2}{M_{\text{Pl}}^2}\right) \tag{A.1.10}$$

$$\mathcal{V} = \exp\left(\sqrt{\frac{3}{2}}\frac{\chi_2}{M_{\text{Pl}}} + \frac{9}{4}\frac{\Phi^2}{M_{\text{Pl}}^2}\right) \tag{A.1.11}$$

we finally get

$$\mathcal{L}_{\text{kin},\tau}^{\mathcal{O}(1)} + \mathcal{L}_{\text{kin},\tau}^{\mathcal{O}(\mathcal{V}^{-1})} = \frac{1}{2}\partial^\mu\chi_1\partial_\mu\chi_1 + \frac{1}{2}\partial^\mu\chi_2\partial_\mu\chi_2 + \frac{1}{2}\partial^\mu\Phi\partial_\mu\Phi \tag{A.1.12}$$

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