Bayesian model updating using incomplete modal data without mode matching

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ABSTRACT

This study investigates a new probabilistic strategy for model updating using incomplete modal data. A hierarchical Bayesian inference is employed to model the updating problem. A Markov chain Monte Carlo technique with adaptive random-work steps is used to draw parameter samples for uncertainty quantification. Mode matching between measured and predicted modal quantities is not required through model reduction. We employ an iterated improved reduced system technique for model reduction. The reduced model retains the dynamic features as close as possible to those of the model before reduction. The proposed algorithm is finally validated by an experimental example.

Keywords: Bayesian model updating, model reduction, Markov chain Monte Carlo, modal data, mode matching, structural health monitoring

1. INTRODUCTION

Model updating conditional on observed data is a key component in structural health monitoring (SHM). Considerable efforts have been made on this topic.[1–8] In general, model updating seeks to determine a set of the most plausible parameters that best describe the structure given the measured system responses. To address the uncertainties associated to model updating, probabilistic approach such as Bayesian inference can be applied.[5] Bayesian model updating makes possible to identify a set of plausible models with probabilistic distributions describing the model uncertainties of a structural system.

Recently, a number of Bayesian model updating approaches have been proposed. For example, Beck and Katafygiotis[9] first presented a statistical framework for Bayesian model updating, which was then extended and applied to update various types of structural models using Markov chain Monte Carlo (MCMC) sampling techniques.[10–12] Nichols *et al.*[13] applied the MCMC to sample the parameter distributions of nonlinear structural systems and extended this approach to damage detection of composites. Beck[14] presented a rigorous framework to quantify modeling uncertainty and perform system identification using probability logic. Green[15] presented a Data Annealing-based MCMC algorithm for probabilistic system identification. Yan *et al.*[16] investigated a reverse jump MCMC method for Bayesian updating of flaw parameters. Sun and Büyüköztürk proposed a MCMC approach with adaptive random-walk steps for probabilistic model updating of buildings.[17]

Bayesian model updating based on modal characteristics has been popular.[18, 19] However, mode matching is typically required for the majority of existing Bayesian updating approaches using modal data. In practice, when incomplete measurements of mode shapes are only available, mode matching is not an easy task. In addition, when some of the measured modes are missing or the mode orders are unclear, mode matching becomes more difficult. Mode switching due to structural damage even makes the case worse.[20] Recently, Bayesian methods without requiring mode matching have been proposed for model updating.[20–22] This is realized through introducing the concept of system mode shapes. In the updating process, the system mode shapes become extra parameters to be updated as well, which might bring convergence difficulty to the algorithm. To address the mode matching problem in model updating, we propose a new strategy for Bayesian model updating using incomplete modal data. This is realized by employing a model reduction technique and MCMC with adaptive random-walk steps.

Here is the organization of this paper. Section 2 presents the probabilistic model updating framework based on hierarchical Bayesian inference using incomplete modal data, in which mode matching is not required. Section 3 describes the sampling technique using MCMC with adaptive random-walk steps. Sections 4 discusses a numerical example to validate the proposed model updating technique. Finally, Section 5 provides the conclusions.

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2. PROBABILISTIC MODEL UPDATING WITHOUT MODE MATCHING

2.1 Hierarchical Bayesian inference for model updating

We consider a linear structure model with n degrees-of-freedoms (DOFs). The mass matrix $\mathbf{M} \in \mathbb{R}^{n \times n}$ is assumed to be known and the stiffness matrix $\mathbf{K} \in \mathbb{R}^{n \times n}$ is parameterized by $\boldsymbol{\theta}$, namely, $\mathbf{K} = \mathbf{K}(\boldsymbol{\theta})$, where $\boldsymbol{\theta} \in \mathbb{R}^{N_\theta \times 1}$ is the model parameters to be updated and N_θ is the number of parameters. In Bayesian model updating, the posterior probability density function (PDF) of the model parameters ($\boldsymbol{\theta}$), given a specified model class, can be obtained based on the Bayes' theorem:[5]

$$p(\boldsymbol{\theta}|\mathcal{D}) = c^{-1} p(\mathcal{D}|\boldsymbol{\theta}) p(\boldsymbol{\theta}) \tag{1}$$

with c being the normalizing factor (the evidence given by data $\mathcal{D}$) which can be written as:

$$c = p(\mathcal{D}) = \int_\Theta p(\mathcal{D}|\boldsymbol{\theta}) p(\boldsymbol{\theta}) \mathrm{d}\boldsymbol{\theta} \tag{2}$$

where Θ denotes the domain of integration; $p(\boldsymbol{\theta})$ is the prior PDF of $\boldsymbol{\theta}$; and $p(\mathcal{D}|\boldsymbol{\theta})$ is the likelihood function which gives a measure of the agreement between the measured and the predicted data; $p(\boldsymbol{\theta}|\mathcal{D})$ denotes the posterior PDF of $\boldsymbol{\theta}$ conditional on the measured data $\mathcal{D}$ consisting of the extracted modal data from measured system responses, namely,

$$\mathcal{D} = \bigcup_{i=1}^{N_s} \mathcal{D}_i = \bigcup_{i=1}^{N_s} \left\{ \tilde{\omega}_{i,1}, \tilde{\omega}_{i,2}, \ldots, \tilde{\omega}_{i,N_m}, \tilde{\boldsymbol{\phi}}_{i,1}, \tilde{\boldsymbol{\phi}}_{i,2}, \ldots, \tilde{\boldsymbol{\phi}}_{i,N_m} \right\} \tag{3}$$

where $\tilde{\omega}_{i,j}$ and $\tilde{\boldsymbol{\phi}}_{i,j}$ are the ith measured frequency and mode shapes of the ith data set; N_m is the total number of observed modes; N_s is the number of measured data sets used for model updating. The problem of Bayesian model updating can be stated as follows: given the specified model class, the measured data $\mathcal{D}$, and the parameter prior PDF $p(\boldsymbol{\theta})$, one's objective is to determine the posterior PDF $p(\boldsymbol{\theta}|\mathcal{D})$.

2.1.1 Likelihood function

The likelihood function can be formulated based on the prediction error $\boldsymbol{\epsilon}_{i,j} \in \mathbb{R}^{N_o \times 1}$ (e.g., N_o denotes the number of observed DOFs) which represents the discrepancy between the measured and the predicted modal data, where the subscripts i and j denote the $i$the data set and the jth mode, respectively, with $i = 1, 2, \ldots, N_s$ and $j = 1, 2, \ldots, N_m$. Herein $\boldsymbol{\epsilon}_{i,j}$ can be expressed as the eigenvalue equation error below

$$\boldsymbol{\epsilon}_{i,j} = \left[\mathbf{K}_R(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \mathbf{M}_R \right] \tilde{\boldsymbol{\phi}}_{i,j} \tag{4}$$

where $\mathbf{M}_R$ and $\mathbf{K}_R$ are the reduced mass and stiffness matrices defined according to the measured DOFs, which can be obtained using an iterated improved reduced system (IIRS) technique presented in Section 2.2. In this study, we model $\boldsymbol{\epsilon}_{i,j}$ as a discrete zero-mean Gaussian process,[23] namely, $\boldsymbol{\epsilon}_{i,j} \sim \mathbb{N}(\mathbf{0}, \Sigma_\varepsilon) = \mathbb{N}(\mathbf{0}, \sigma_j^2 \mathbf{I})$ where $\mathbf{I} \in \mathbb{R}^{N_o \times N_o}$ is an identity matrix; σ_j^2 denotes the variance of the prediction error of the jth mode, which is an additional unknown variable. Therefore, the likelihood function can be expressed as follows

$$p(\mathcal{D}|\boldsymbol{\theta}) = \frac{1}{\left[(2\pi)^{N_m} \prod_{j=1}^{N_m} \sigma_j^2 \right]^{N_o N/2}} \exp \left[-\sum_{i=1}^{N_s} \sum_{j=1}^{N_m} \frac{1}{2\sigma_j^2} \left\| \left[\mathbf{K}_R(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \mathbf{M}_R \right] \tilde{\boldsymbol{\phi}}_{i,j} \right\|_2^2 \right] \tag{5}$$

where $\| \cdot \|_2$ denotes the ℓ_2 (Euclidean) norm of a vector.

2.1.2 Prior distributions

Let us assume that the system parameters θ_k ($k = 1, 2, \ldots, N_\theta$) follow the Laplace prior distribution given by[24]

$$p(\boldsymbol{\theta}|\lambda) = \prod_{k=1}^{N_\theta} p(\theta_k|\lambda) = \left(\frac{\lambda}{2} \right)^{N_\theta} \exp \left(-\lambda \| \boldsymbol{\theta} - \bar{\boldsymbol{\theta}} \|_1 \right) \tag{6}$$

where λ is the parameter of the Laplace distribution ($\lambda > 0$) called the regularization parameter (note that λ becomes another unknown parameter in the Bayesian updating process); $\bar{\boldsymbol{\theta}} \in \mathbb{R}^{N_\theta \times 1}$ is the mean for the prior distribution; $\| \cdot \|_1$ denotes the ℓ_1 (Taxicab) norm.

Since σ_j^2 ($j = 1, \ldots, N_m$) and λ are always positive, their prior distributions can be modeled by an inverse Gamma and a Gamma distribution, respectively: $p(\sigma_j^2) \sim \mathcal{G}^{inv}(\alpha, \beta)$ and $p(\lambda) \sim \mathcal{G}(a, b)$, where α, β, a, b are positive constant hyperparameters, defined as[23]

$$p(\boldsymbol{\sigma}^2 | \alpha, \beta) = \prod_{j=1}^{N_m} p(\sigma_j^2) = \prod_{j=1}^{N_m} \frac{\beta^\alpha}{\Gamma(\alpha)} \sigma_j^{-2(\alpha+1)} \mathrm{e}^{-\beta/\sigma_j^2} \tag{7}$$

$$p(\lambda | a, b) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} \exp\left(-b\lambda\right) \tag{8}$$

where $\mathcal{G}(\cdot, \cdot)$ and $\mathcal{G}^{inv}(\cdot, \cdot)$ denote the Gamma and the inverse Gamma distribution function, respectively, and $\Gamma(\cdot)$ is the Gamma function. In addition, the hyperparameters are fixed to be small (e.g., $\alpha = a = 1 \times 10^{-3}$ and $\beta = b = 1 \times 10^{-6}$), leading to a non-informative process.

2.1.3 Final form of the posterior distribution

Following the hierarchical Bayesian modeling framework,[5] we obtain the augmented posterior PDF for unknown parameters $\{\boldsymbol{\theta}, \boldsymbol{\sigma}^2, \lambda\}$ as follows

$$p(\boldsymbol{\theta}, \boldsymbol{\sigma}^2, \lambda | \mathcal{D}) \propto p(\mathcal{D} | \boldsymbol{\theta}, \boldsymbol{\sigma}^2) p(\boldsymbol{\theta} | \lambda) p(\boldsymbol{\sigma}^2 | \alpha, \beta) p(\lambda | a, b) \tag{9}$$

The substitution of Equations (5), (6), (7) and (8) into Equation (9) leads to the final form of the posterior PDF of the unknown parameters:

$$p(\boldsymbol{\theta}, \boldsymbol{\sigma}^2, \lambda | \mathcal{D}) \propto \frac{\lambda^{N_2}}{\left[\prod_{j=1}^{N_m} \sigma_j^2\right]^{N_1}} \exp\left\{ -\sum_{j}^{N_m} \frac{1}{2\sigma_j^2} \left[\sum_{i}^{N_s} \left\| \left[\mathbf{K}_R(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \mathbf{M}_R\right] \tilde{\boldsymbol{\phi}}_{i,j} \right\|_2^2 + 2\beta \right] - \lambda \left(\|\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}\|_1 + b \right) \right\} \tag{10}$$

where $N_1 = N_o N_s / 2 + \alpha + 1$ and $N_2 = N_\theta + a - 1$. The total number of unknown parameters in Equation (10) is $N_\theta + N_m + 1$. The conditional posterior PDFs of $p(\boldsymbol{\theta}, \boldsymbol{\sigma}^2, \lambda | \mathcal{D})$ in Equation (10) can be sampled using the MCMC with adaptive random-walk steps described in Section 3. Noteworthy, the analytical solution of the conditional distributions for σ_j^2 and λ are written as

$$\{\sigma_j^2 | \boldsymbol{\theta}, \mathcal{D}, \alpha, \beta\} \sim \mathcal{G}^{inv} \left(\alpha + \frac{N_o N_s}{2}, \beta + \frac{1}{2} \sum_{i}^{N_s} \left\| \left[\mathbf{K}_R(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \mathbf{M}_R\right] \tilde{\boldsymbol{\phi}}_{i,j} \right\|_2^2 \right) \tag{11}$$

and

$$\{\lambda | \boldsymbol{\theta}, a, b\} \sim \mathcal{G} \left(N_\theta + a, \|\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}\|_1 + b \right) \tag{12}$$

Following Equations (11) and (12), σ_j^2 and λ can be easily sampled given $\boldsymbol{\theta}$.

2.2 Model reduction

Let us write the generalized eigenvalue problem of a n-DOF linear system containing the first m modes, with the partitioned mass and stiffness matrices and mode shapes governed by the master and slave DOFs, as follows[25]

$$\begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{ms}^T & \mathbf{K}_{ss} \end{bmatrix} \left\{ \begin{array}{c} \boldsymbol{\Phi}_{mm} \\ \boldsymbol{\Phi}_{sm} \end{array} \right\} = \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{ms}^T & \mathbf{M}_{ss} \end{bmatrix} \left\{ \begin{array}{c} \boldsymbol{\Phi}_{mm} \\ \boldsymbol{\Phi}_{sm} \end{array} \right\} \boldsymbol{\Omega}_{mm} \tag{13}$$

where $\mathbf{M}$ and $\mathbf{K} \equiv \mathbf{K}(\boldsymbol{\theta})$ are the mass and stiffness matrices, respectively; $\boldsymbol{\Phi} \in \mathbb{R}^{n \times m}$ is the mass-normalized mode shape matrix; $\boldsymbol{\Omega} \in \mathbb{R}^{m \times m}$ is the diagonal eigenvalue matrix consisting of the eigenvalues ω_i ($i = 1, 2, ..., m$); m and s denote the number of master and slave DOFs, respectively, satisfying $m + s = n$. Let us denote

$\boldsymbol{\Phi}_{sm} = \mathbf{t}\boldsymbol{\Phi}_{mm}$, where $\mathbf{t} \in \mathbb{R}^{s \times m}$ is a transformation matrix, and substitute it into the second set of Equation (13) to obtain

$$\mathbf{t} = -\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T + \mathbf{K}_{ss}^{-1}\left[\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{t}\right]\boldsymbol{\Phi}_{mm}\boldsymbol{\Omega}_{mm}\boldsymbol{\Phi}_{mm}^{-1} \tag{14}$$

The substitution of $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_{mm} \quad \boldsymbol{\Phi}_{sm}]^T = \mathbf{T}\boldsymbol{\Phi}_{mm}$ into Equation (13) pre-multiplied by $\mathbf{T}^T$ yields

$$\mathbf{M}_R^{-1}\mathbf{K}_R = \boldsymbol{\Phi}_{mm}\boldsymbol{\Omega}_{mm}\boldsymbol{\Phi}_{mm}^{-1} \tag{15}$$

where $\mathbf{T} = [\mathbf{I} \quad \mathbf{t}]^T$ and $\mathbf{I} \in \mathbb{R}^{m \times m}$; $\mathbf{M}_R$ and $\mathbf{K}_R$ are the mass and stiffness matrices of the reduced order model, namely,

$$\mathbf{M}_R = \mathbf{T}^T\mathbf{M}\mathbf{T} \quad \text{and} \quad \mathbf{K}_R = \mathbf{T}^T\mathbf{K}\mathbf{T} \tag{16}$$

The substitution of Equation (15) into (14) yields

$$\mathbf{t} = -\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T + \mathbf{K}_{ss}^{-1}\left[\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{t}\right]\mathbf{M}_R^{-1}\mathbf{K}_R \tag{17}$$

It is noted that Equation (17) forms an implicit function with $\mathbf{t}$ as the unknown parameter which can be solved through an iterative process. Friswell $et\ al.$[26] proposed an IIRS technique to solve for Equations (16) and (17) iteratively to obtain the reduced mass and stiffness matrices $\mathbf{K}_R(\boldsymbol{\theta})$.

3. MARKOV CHAIN MONTE CARLO SAMPLING

The Markov chain Monte Carlo (MCMC) has been proven to be successful for quantifying the uncertainties of the model parameters. The idea is to create stationary chains of samples to approximate the parameter distributions. We sample the model parameters $\boldsymbol{\theta}$, the perdition error variance $\boldsymbol{\sigma}^2$ and the prior regularization parameter λ sequentially using Gibbss sampling.[23] Note $\boldsymbol{\sigma}^2$ and λ can be directly sampled using Equations (11) and (12).

To establish the chain for the model parameters $\boldsymbol{\theta}$, we herein apply the Metropolis-Hastings (M-H) algorithm.[27,28] Let us assume that random samples are generated from a target distribution denoted with $\pi(\boldsymbol{\theta})$ (e.g., $p(\boldsymbol{\theta}|\boldsymbol{\sigma}^2, \lambda, \mathcal{D})$ in this study). The M-H algorithm generates a sequence of samples $\boldsymbol{\theta}^{(p)}$ from the target distribution through a rejection sampling procedure. At a generic pth iteration, a candidate solution $\boldsymbol{\theta}^*$ is generated based on the current value $\boldsymbol{\theta}^{(p-1)}$, which can be sampled from a chosen proposal or a transition distribution function $g\left(\boldsymbol{\theta}^* \mid \boldsymbol{\theta}^{(p-1)}\right)$. A Bernoulli trial is then performed with a success probability defined as

$$\gamma = \min\left\{\frac{\pi(\boldsymbol{\theta}^*)g\left(\boldsymbol{\theta}^{(p-1)} \mid \boldsymbol{\theta}^*\right)}{\pi\left(\boldsymbol{\theta}^{(p-1)}\right)g\left(\boldsymbol{\theta}^* \mid \boldsymbol{\theta}^{(p-1)}\right)}, \quad 1\right\} \tag{18}$$

Note that if the result of the trial is successful (e.g., $r_0 \leq \gamma$, where r_0 is a uniform random number sampled from $[0, 1]$), $\boldsymbol{\theta}^{(p)}$ is replaced by $\boldsymbol{\theta}^*$; otherwise (e.g., $r_0 > \gamma$), $\boldsymbol{\theta}^{(p)}$ is kept as $\boldsymbol{\theta}^{(p-1)}$. The acceptance-rejection process is repeated many times until the chain becomes stationary. The samples in the non-stationary part of the chain are called "burn-in" samples and the rest stationary samples are called "retained" samples. In this paper, we apply a uniform distribution to describe the transition proposal, e.g.,

$$\left\{\theta_k^* \mid \theta_k^{(p-1)}\right\} \sim U\left(\theta_k^{(p-1)} - \frac{L_k^{(p-1)}}{2}, \theta_k^{(p-1)} + \frac{L_k^{(p-1)}}{2}\right) \tag{19}$$

where U denotes a uniform distribution; θ_k^* and $\theta_k^{(p-1)}$ represent the kth ($k = 1, 2, \ldots, N_\theta$) parameter in θ^* and $\theta^{(p-1)}$; $L_k^{(p-1)}$ is the interval length (random-work step) of the uniform distribution for θ_k. Here, $L_k^{(p-1)}$ changes adaptively along with the iterations.[13] For example, $L_k^{(p)} = \kappa L_k^{(p-1)}$, where κ is the adaptivity coefficient. In the burn-in period, if the sampling trial is successful, κ is adopted 1.01; otherwise (sample is rejected), κ is chosen 0.99. It is noteworthy that in the retained period, κ is selected to be 1. The initial value $L_k^{(0)}$ is set to be $0.05\left(\theta_k^{max} - \theta_k^{min}\right)$. The use of an adaptive random-walk step leads to a more efficient sampling process, meanwhile, keeping the tuning capability of the algorithm.

4. NUMERICAL EXAMPLE: A NINE-STORY SHEAR-TYPE BUILDING

In order to test the performance of the proposed algorithm for probabilistic model updating, a nine-storey shear-type building with synthetic measurements is studied here. The building has uniformly distributed mass and stiffness parameters, e.g., $\bar{m} = 150$ metric tons and $\bar{k} = 200$ MN/m. The first four frequencies are 0.960, 2.853, 4.669 and 6.357 Hz.

We first generate the synthetic ambient response time histories sampled at 100 Hz by applying ambient ground motions to the building. The damping ratio of each mode is chosen to be 3% in the simulation. Accelerometers are placed at the 1st, 3rd, 5th, 7th and 9th floors. To test the effect of measurement noise on parameter updating, white noise (20%) has been considered in the synthetic measurements. Eight data sets are synthesized and used for modal identification (e.g., $N_s = 8$). The frequency domain decomposition (FDD) is applied to extract the modal characteristics (e.g., frequencies and mode shapes). The identified first four modes are used for model updating. The identified frequencies are 0.957 (0.89%), 2.845 (0.96%), 4.660 (0.87%) and 6.338 (0.92%) Hz, where the percentages in the parentheses denote the coefficient of variation.

The chain length used in MCMC is $N_{mc} = 6 \times 10^3$ and the burn-in period is $N_b = 2 \times 10^3$. The lower and upper bounds for the parameters are zero and five times the true values. Since the masses are assumed to be known, the updating parameters become $k_1 \sim k_9$, $\sigma_1^2 \sim \sigma_4^2$ and λ (e.g., $N_\theta = 14$). The prior stiffness parameters follow the normal distribution with the mean value of 150 MN/m. The numerical analyses are programmed in MATLAB® (The MathWorks, Inc., MA, USA) on a standard Intel (R) Core (TM) i7-4930K 3.40 GHz PC with 32G RAM.

Figure 1 shows the samples of nine stiffness parameters in the MCMC updating process. It is seen that the algorithm converges quite fast, e.g., the sample chains become stationary after about 300 iterations. The burn-in period of 2000 iterations is sufficient and the retained samples are enough to be used for the representation of the posterior PDFs of the stiffness parameters. Figure 2 shows the quantified posterior PDFs of the ten stiffness parameters by MCMC with small deviations. Note that the posterior PDFs, represented by histograms, are obtained based on the statistics of the samples in the "retained" period. We fit the posterior histograms using the generalized extreme value (GEV) distribution. The identified maximum *a posteriori* (MAP) values are also shown in Figure 2 which are quite close to the true value of 200 MN/m. The 95% confidence intervals are also listed in Figure 2.

Figure 3 shows the depicts the pairwise plots of posterior samples for some typical stiffness parameters. It can be seen that some of the stiffness parameters approximately follow the normal distribution (e.g., pairwise samples are similar to ellipses) which matches the PDFs in Figure 2. Figure 4 shows the posterior PDFs of the prediction error variance σ_1^2, σ_2^2, σ_3^2 and σ_4^2 and the regularization parameter λ. It takes about 4 min CPU time to complete the model updating process in this example. The overall performance of the proposed algorithm for probabilistic model updating is satisfactory.
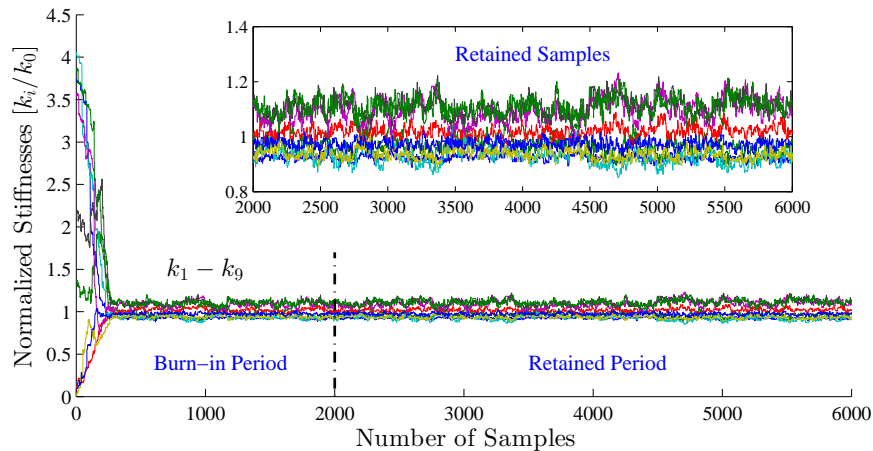

Figure 1. Samples of nine stiffness parameters obtained by MCMC using 20% RMS noise measurement.
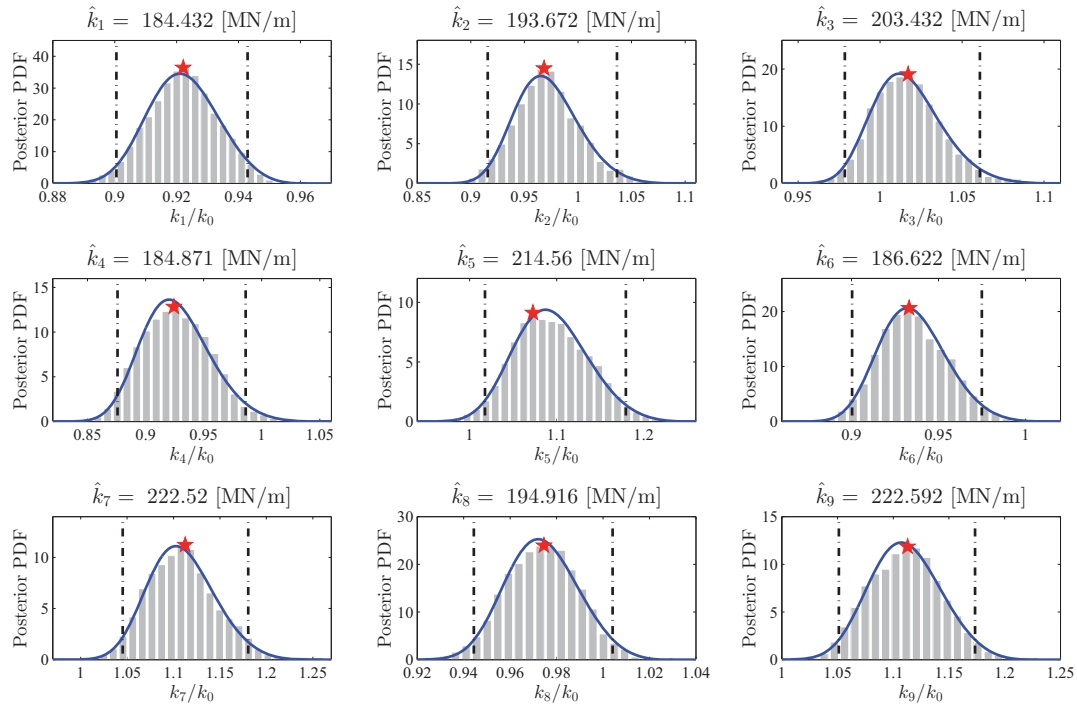
Figure 2. The stiffness parameter posterior PDFs of the 9-storey building identified by MCMC using 20% RMS noise measurement. Note that CI represents the confidence interval. The histograms denote the PDFs obtained from MCMC sampling and the solid blue lines denote the PDFs through curve fitting using the generalized extreme value (GEV) distribution. The red star denotes the maximum *a posteriori* (MAP) estimate.
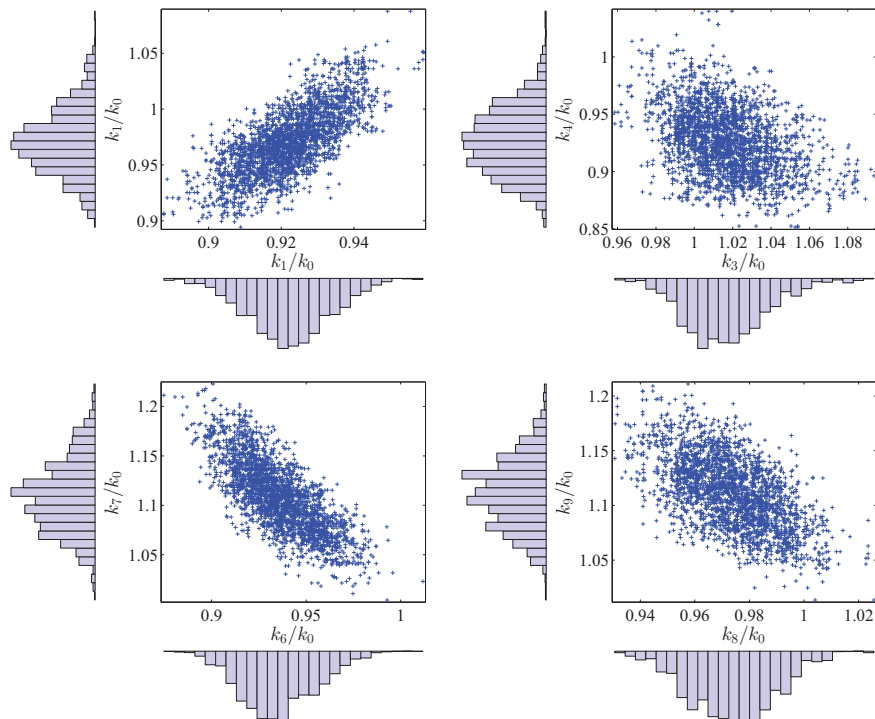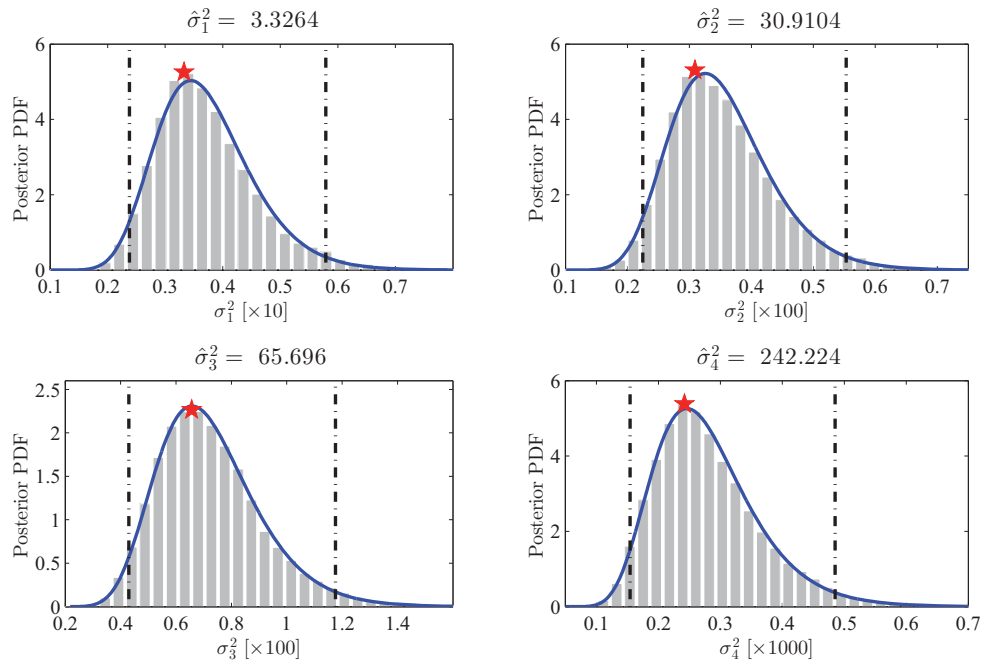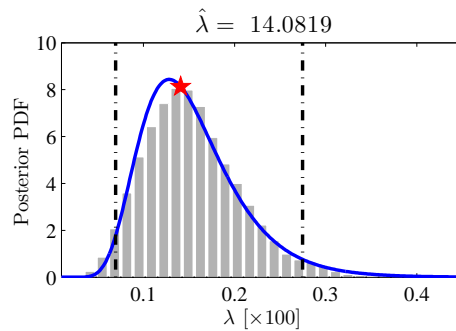

Figure 3. Pairwise plots of posterior samples ("retained" period samples of the MCMC) for some stiffness parameters.

(a) PDF of σ_j^2 $(j = 1, 2, 3, 4)$

(b) PDF of λ

Figure 4. Identified PDFs of the prediction error variance σ_j^2 $(j = 1, 2, 3, 4)$ and the regularization parameter λ. Note that the histograms denote the PDFs obtained from MCMC sampling and the solid blue lines denote the PDFs through curve fitting using the log-normal distribution. The red star denotes the maximum *a posteriori* estimate $\hat{\sigma}_j^2$ and $\hat{\lambda}$.

5. CONCLUSIONS

Model updating is very important in SHM. The identified or updated system parameters can be used to assess the health condition, quantify uncertainties, evaluate the integrity, and estimate the capacity to carry loads and risk of a structure. Traditional model updating seeks to determine a set of the most plausible parameters that best describe the structure given the measured system responses, while Bayesian model updating techniques make possible to identify a set of plausible models with probabilistic distributions and to characterize the modeling uncertainties of a structural system. This study investigates a new probabilistic strategy for model updating using incomplete modal data. A hieratical Bayesian inference is applied to model the updating problem. Mode matching between the measured and the predicted modal quantities is not required in the updating process, which is realized through model reduction. A Markov chain Monte Carlo technique with adaptive random-walk steps is proposed to draw the samples to quantify uncertainties of the model parameters. The proposed algorithm is successfully validated by nine-storey shear-type building example.

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