

## Role of Magnetic Reconnection in Magnetohydrodynamic Turbulence

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The current understanding of magnetohydrodynamic (MHD) turbulence envisions turbulent eddies which are anisotropic in all three directions. In the plane perpendicular to the local mean magnetic field, this implies that such eddies become current-sheetlike structures at small scales. We analyze the role of magnetic reconnection in these structures and conclude that reconnection becomes important at a scale  $\lambda \sim LS_L^{-4/7}$ , where  $S_L$  is the outer-scale ( $L$ ) Lundquist number and  $\lambda$  is the smallest of the field-perpendicular eddy dimensions. This scale is larger than the scale set by the resistive diffusion of eddies, therefore implying a fundamentally different route to energy dissipation than that predicted by the Kolmogorov-like phenomenology. In particular, our analysis predicts the existence of the subinertial, reconnection interval of MHD turbulence, with the estimated scaling of the Fourier energy spectrum  $E(k_\perp) \propto k_\perp^{-5/2}$ , where  $k_\perp$  is the wave number perpendicular to the local mean magnetic field. The same calculation is also performed for high (perpendicular) magnetic Prandtl number plasmas ( $Pm$ ), where the reconnection scale is found to be  $\lambda/L \sim S_L^{-4/7} Pm^{-2/7}$ .

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**Introduction.**—Turbulence is a defining feature of magnetized plasmas in space and astrophysical environments, which are almost invariably characterized by very large Reynolds numbers. The solar wind [1], the interstellar medium [2,3], and accretion disks [4,5] are prominent examples of plasmas dominated by turbulence, where its detailed understanding is almost certainly key to addressing long-standing puzzles such as electron-ion energy partition, cosmic ray acceleration, magnetic dynamo action, and momentum transport.

Weak collisionality implies that kinetic plasma physics is required to fully describe turbulence in many such environments [6]. However, turbulent motions at scales ranging from the system size to the ion kinetic scales, an interval which spans many orders of magnitude, should be accurately described by magnetohydrodynamics (MHD).

The current theoretical understanding of MHD turbulence largely rests on the ideas that were put forth by Kolmogorov and others to describe turbulence in neutral fluids (the K41 theory of turbulence [7]), and then adapted to magnetized plasmas by Iroshnikov and Kraichnan [8,9] and, later, Goldreich and Sridhar (GS95) [10]. Very briefly, one considers energy injection at some large scale  $L$  (the forcing, or outer, scale), which then cascades to smaller scales through the inertial range where, by definition, dissipation is negligible and throughout which, therefore, energy is conserved. At the bottom of the cascade is the dissipation range, where the gradients in the flow are sufficiently large for the dissipation to be efficient.

Turbulence in magnetized plasmas fundamentally differs from that in neutral fluids due to the intrinsic anisotropy

introduced by the magnetic field. GS95 suggests this leads to turbulent eddies which are longer in the direction aligned with the local field than in the direction perpendicular to it. The relationship between the field-parallel and perpendicular dimensions is set by critical balance:  $V_{A,0}/\ell \sim v_\lambda/\lambda$ , where  $V_{A,0}$  is the Alfvén velocity based on the background magnetic field  $B_0$ ,  $\ell$  and  $\lambda$  are, respectively, the field-aligned and field-perpendicular dimensions of the eddy, and  $v_\lambda$  is the velocity perturbation at that scale.

More recently, it was argued [11] that the GS95 picture of turbulence needs to be amended to allow for angular alignment of the velocity and magnetic field perturbations at scale  $\lambda$ . As a result, eddies are also anisotropic in the plane perpendicular to the local magnetic field, being thus characterized by three scales:  $\ell$  along the field, and  $\lambda$  and  $\xi$  perpendicular to the field. Although the precise structure of MHD turbulence remains an open research question, observational and numerical evidence in support of 3D anisotropic eddies has since been reported [12–14].

A particularly interesting feature of 3D anisotropic eddies is that they can be thought of as current sheets of thickness  $\lambda$  and length  $\xi$  in the field-perpendicular plane (with  $\xi \gg \lambda$ ). Following the standard Kolmogorov-like arguments, one would then conclude that the inertial interval ends when the scale  $\lambda$  becomes comparable to the dissipation scale. Below this scale the energy is strongly dissipated in the current sheets. Currently available numerical simulations indicate that a considerable fraction of small-scale current sheets look like sites of magnetic reconnection [15–20]. We note that such a dissipation channel is not a feature of the GS95 model, which predicts filamentlike eddies at small scales.

In this Letter we propose that at sufficiently large magnetic Reynolds numbers, the route to energy dissipation in MHD turbulence is fundamentally different from that envisioned in the Kolmogorov-like theory. This happens since the anisotropic, current-sheetlike eddies become the sites of magnetic reconnection *before* the formal Kolmogorov dissipation scale is reached. This Letter presents the first analytical attempt to quantify this phenomenon and to characterize the role of reconnection in MHD turbulence.

*Background.*—The 3D anisotropic eddies that we envision are depicted in Fig. 2 of Ref. [11]. We will characterize them by the smallest of their field-perpendicular dimensions,  $\lambda$ ; other quantities of interest to us here are related to  $\lambda$  as follows [11]:

$$\xi \sim L(\lambda/L)^{3/4}, \quad (1)$$

$$\ell \sim L(\lambda/L)^{1/2}, \quad (2)$$

$$b_\lambda \sim B_0(\lambda/L)^{1/4}, \quad (3)$$

$$v_\lambda \sim V_0(\lambda/L)^{1/4}, \quad (4)$$

$$\tau \sim \ell/V_{A,0} \sim \lambda^{1/2}L^{1/2}/V_{A,0}, \quad (5)$$

$$V_{A,\lambda} \sim V_{A,0}(\lambda/L)^{1/4}, \quad (6)$$

where  $b_\lambda$  and  $v_\lambda$  are the magnetic field and velocity perturbations at scale  $\lambda$ ,  $\tau$  the eddy turnover time,  $V_0$  is the velocity at the outer scale, and the other quantities have already been introduced [21]. The scalings (1)–(6) imply the Fourier energy spectrum of MHD turbulence  $E(k_\perp) \propto k_\perp^{-3/2}$  [22–31]. The turbulence is governed by the shear-Alfvén modes, it is strongly nonlinear and essentially three dimensional [30,32].

We will, for simplicity, consider the case where the turbulence is critically balanced at the outer scale such that the outer-scale Lundquist number,  $S_L \equiv LV_{A,0}/\eta$  is comparable to the outer-scale magnetic Reynolds number,  $R_m \equiv LV_0/\eta$ . We also introduce the Lundquist number associated with scale  $\lambda$ ,  $S_\lambda \equiv \lambda V_{A,\lambda}/\eta$  [33]. A lower bound on the dissipation scale can be obtained from these scalings by equating  $\tau$  with the eddy resistive diffusion time,  $\lambda^2/\eta$ ; this can be thought of as the Kolmogorov inner scale for a turbulent cascade defined by (1)–(6). It yields

$$\lambda/L \sim S_L^{-2/3} \sim R_m^{-2/3}. \quad (7)$$

*Magnetic reconnection.*—Let us begin by observing that the aspect ratio of an eddy in the perpendicular direction is

$$\xi/\lambda \sim (L/\lambda)^{1/4}; \quad (8)$$

i.e., it increases as  $\lambda \rightarrow 0$ . So, in the perpendicular plane, eddies become ever more elongated current sheets as  $\lambda$  gets smaller. This is qualitatively different from the GS picture,

where both field-perpendicular dimensions are the same, and so the eddy tends to a point in the perpendicular plane as  $\lambda \rightarrow 0$ .

It is therefore natural to ask at what scale (i.e., aspect ratio) does reconnection of these current sheets (eddies) become an important effect, if ever. If it does, it should leave a well-defined signature in both the magnetic and kinetic energy spectra. It may not, however, correspond to the energy dissipation scale, since reconnection, in addition to dissipating magnetic energy, also accelerates flows.

*Sweet-Parker reconnection of eddies.*—The simplest estimate that can be done for eddy reconnection stems from the Sweet-Parker model [34,35], according to which the scale  $\lambda$  at which an eddy would reconnect is given by

$$\lambda/\xi \sim S_\xi^{-1/2}, \quad (9)$$

where  $S_\xi = \xi V_{A,\lambda}/\eta$  is the Lundquist number pertaining to a current sheet of length  $\xi$ , at scale  $\lambda$ , defined with the Alfvén velocity based on the perturbed magnetic field at that scale, Eq. (6). Using (1)–(6) above, one finds that Eqs. (9) and (7) are equivalent statements [19].

This important observation immediately points to the problem with the Kolmogorov-like transition to the dissipation regime. A robust conclusion of the past decade of reconnection research is that Sweet-Parker current sheets above a certain critical aspect ratio, corresponding to a Lundquist number  $S_c \sim 10^4$ , are violently unstable to the formation of multiple magnetic islands, or plasmoids (see Ref. [36] for a recent review). One straightforward implication of this instability [37–40] is that the Sweet-Parker current sheets cannot be formed in the first place [37,41–43]. We now demonstrate that the MHD turbulent cascade will be affected by this instability *before* it has a chance to form Sweet-Parker current sheets at small scales, thus qualitatively changing the route to energy dissipation in MHD turbulence.

*Dynamic reconnection onset and eddy disruption by the tearing instability.*—According to the above discussion, turbulent eddies may be viewed as a hierarchy of current sheets, whose dynamical (eddy turnover) times are given by Eq. (5). Consider an eddy at some scale  $\lambda$  and ask what would be the rate of a linear tearing instability triggered by its magnetic profile. Obviously, if the rate of the linear instability turns out to be much higher than the eddy turnover rate, the eddy evolution may be considered slow and the linear theory would apply. The dynamics of the eddy at such a scale then would be dominated by reconnection. By the same token, the eddies whose turnover rates are much higher than the reconnection rates will not be affected by reconnection.

The central result of our work is that for large enough Lundquist numbers, the small-scale part of the inertial interval formally predicted by the Alfvénic theory (1)–(6) inevitably falls in the reconnection-dominated domain. Indeed, as the scale  $\lambda$  decreases, the rate of the corresponding tearing instability increases faster than the eddy turnover

rate (5). We can therefore define a critical scale  $\lambda_{\text{cr}}$  at which the two rates become comparable. Below this scale the Alfvénic turbulence must cross over to the new, reconnection dominated regime.

In order to estimate the critical scale  $\lambda_{\text{cr}}$  we note that the tearing instability has two well-known regimes, Furth-Killeen-Rosenbluth (FKR) (small tearing mode instability parameter,  $\Delta'$ ) [44] and Coppi (large  $\Delta'$ ) [45]. The  $N = 1$  mode, related to the tearing perturbation wave number through  $k/2\pi = N/\xi$ , is the most unstable mode until it transitions into the Coppi regime. This happens at the scale that satisfies

$$(\xi/\lambda)S_\lambda^{-1/4} \sim 1, \quad (10)$$

yielding the *transition scale* for the  $N = 1$  mode

$$\lambda_{\text{tr},1}/L \sim S_L^{-4/9}. \quad (11)$$

In other words, if  $\lambda > \lambda_{\text{tr},1}$ , the most unstable mode in the current sheet is an FKR mode; if the opposite is true, it is instead a Coppi mode which is the most unstable.

We define the critical scale for any mode  $N$ ,  $\lambda_{\text{cr},N}$ , as the scale at which the growth rate  $\gamma$  of that mode matches the eddy turn over time at that scale, given by Eq. (5). Strictly speaking, the tearing mode analysis cannot be performed on a background which is evolving on a time scale comparable to the growth time of the instability; however, as we discussed above, the criterion  $\gamma\tau \sim 1$  provides a reasonable estimate for the turbulence scale at which tearing becomes important. For the  $N = 1$  mode while in the FKR regime, the growth rate is  $\gamma_1^{\text{FKR}} \sim \xi^{2/5} V_{A,\lambda}^{2/5} \lambda^{-2} \eta^{3/5}$ . The equation  $\gamma_1^{\text{FKR}} \tau \sim 1$  therefore yields

$$\lambda_{\text{cr},1}/L \sim S_L^{-6/11}. \quad (12)$$

We see that  $\lambda_{\text{cr},1} < \lambda_{\text{tr},1}$ , implying that the modes that will become critical are not FKR modes, but rather Coppi modes. For these modes the largest growth rate is  $\gamma_{\text{max}}^{\text{Coppi}} \sim \tau_{A,\lambda}^{-1} S_\lambda^{-1/2}$ , where  $\tau_{A,\lambda} \equiv \lambda/V_{A,\lambda}$ , corresponding to a mode number  $N_{\text{max}}^{\text{Coppi}} \sim \xi/\lambda S_\lambda^{-1/4}$ . The criticality condition that the tearing mode growth rate becomes comparable to the eddy turnover rate  $\gamma_{\text{max}}^{\text{Coppi}} \tau \sim 1$  now yields

$$\lambda_{\text{cr}}^{\text{Coppi}}/L \sim S_L^{-4/7}, \quad (13)$$

which is the main result of our work.

It is easy to see that Eq. (13) corresponds to a mode number, that is, number of magnetic islands, or plasmoids, that would form inside a sheet of thickness  $\lambda_{\text{cr}}^{\text{Coppi}}$  and length  $\xi_{\text{cr}}^{\text{Coppi}} \equiv \xi(\lambda_{\text{cr}}^{\text{Coppi}}) \sim LS_L^{-3/7}$ , given by

$$N_{\text{max}}^{\text{Coppi}} \sim S_L^{1/14}. \quad (14)$$

Coppi modes, as they become nonlinear, lead to a loss of equilibrium that happens on the Alfvénic time scale at scale  $\lambda$ ,  $\tau_{A,\lambda}$  [46,47]. Therefore, Eq. (13) identifies the scale at

which reconnection becomes dynamically relevant to the turbulence.

Finally, we may compute the width of the inner boundary layer of the tearing instability corresponding to this most unstable mode, which is given by [41]  $\delta_{\text{in}} \sim [\gamma(\xi/NV_{A,\lambda})^2 \lambda^2 \eta]^{1/4}$  evaluated for the scale obtained in Eq. (13):

$$\delta_{\text{in,max}}^{\text{Coppi}}/L \sim S_L^{-9/14}. \quad (15)$$

Whether this scale is larger or smaller than kinetic scales in the plasma at hand [the ion (sound) Larmor radius, or the ion skin depth] decides the adequateness, or lack thereof, of the MHD tearing mode theory in describing the transition to the reconnection-dominated domain of the turbulence spectrum.

*Large magnetic Prandtl number.*—The calculation above can be straightforwardly repeated for cases in which the magnetic Prandtl number,  $Pm \equiv \nu_\perp/\eta$ , is large. We are referring to the perpendicular viscosity, not the parallel one—see the discussion in Sec. II B of Ref. [40]. The perpendicular magnetic Prandtl number that we consider here is  $Pm \sim (m_i/m_e)^{1/2} \beta_i$ , and it can be large in astrophysical plasmas.

The scalings for the linear tearing mode in the small and large  $\Delta'$  regimes at high Prandtl number were derived in [48] and are conveniently summarized in Ref. [40]. Since this calculation is entirely similar to the one in the previous section, we limit ourselves to stating the main results. For the  $N = 1$  mode in the  $Pm \gg 1$  regime, the transition scale is  $\lambda_{\text{tr},1}/L \sim S_L^{-4/9} Pm^{2/9}$ , whereas the critical scale is  $\lambda_{\text{cr},1}/L \sim S_L^{-8/15} Pm^{-2/15}$ . Clearly  $\lambda_{\text{cr},1} \ll \lambda_{\text{tr},1}$  implying, as above, that the modes that will become critical are Coppi modes. The critical scale is now

$$\lambda_{\text{cr}}^{\text{Coppi}}/L \sim S_L^{-4/7} Pm^{-2/7}, \quad (16)$$

corresponding to mode number  $N_{\text{max}}^{\text{Coppi}} = S_L^{1/14} Pm^{2/7}$ . The inner boundary layer now scales as  $\delta_{\text{in,max}}^{\text{Coppi}}/L \sim S_L^{-9/14} Pm^{-1/14}$ .

We see that Eq. (16) yields a smaller scale than its inviscid counterpart, Eq. (13). This makes intuitive sense: viscosity slows down the Coppi modes; as such, the tearing and turbulence timescales can only match at a scale  $\lambda$  smaller (and, therefore, larger current sheet aspect ratio,  $\xi/\lambda$ ) than in the absence of viscosity.

The nonlinear evolution of large  $Pm$  tearing modes is less well understood than the low  $Pm$  case, but we expect a similar loss of equilibrium as in the inviscid case to take place. Thus, as in the inviscid case, Eq. (16) identifies the scale at which the eddies become affected by tearing instability.

*Spectrum of turbulence below the reconnection scale.*—We now address the spectrum below the reconnection scale identified by Eq. (13), or Eq. (16) for  $Pm \gg 1$  plasmas.

A plausible estimate for such a spectrum may be obtained based on a simplified phenomenological picture that approximates turbulent structures as a hierarchy of collapsing plasmoid chains. As the island chain becomes nonlinear, it may undergo  $X$ -point collapse, with new current sheets forming between each two plasmoids. These may themselves be unstable to plasmoid formation, and so on. Assuming one can apply here what is known from the dynamics of large Lundquist number reconnecting systems (see, e.g., Refs. [36,49–53]), the final state would be one where there is a distribution of plasmoid sizes, whose dynamics is dictated by advection out of the current sheet, coalescence, and generation of new plasmoids. This can be viewed as a new subinertial-range interval of turbulence, which may be characterized by its own power spectrum [54–56].

In order to derive the spectrum in this “reconnection interval,” we first note that we expect such plasmoids of many different sizes to be separated from each other by Sweet-Parker current sheets of a length  $L_c$  such that their aspect ratio is marginally stable to plasmoid formation,  $\sim S_c^{1/2}$  [49], where  $S_c = L_c V_{A,\lambda_{cr}^{\text{Coppi}}} / \eta$  is the critical Lundquist number,  $S_c \sim 10^4$ . The rationale is that current sheets longer than  $L_c$ , and, therefore, larger values of the Lundquist number, are unstable to plasmoid formation; if, on the other hand, they are shorter than  $L_c$ , they will be stretched to that length by differential background flows [49] (implying that the total number of plasmoids per current sheet can be estimated as  $N \sim \xi_{cr}^{\text{Coppi}} / L_c \sim S_c^{-1} S_L^{3/7}$ ).

The thickness of these critical current sheets is estimated as

$$\delta_c \sim L_c S_c^{-1/2} \sim \lambda_{cr}^{\text{Coppi}} S_{\lambda_{cr}^{\text{Coppi}}}^{-1} S_c^{1/2}, \quad (17)$$

where  $S_{\lambda_{cr}^{\text{Coppi}}} = \lambda_{cr}^{\text{Coppi}} V_{A,\lambda_{cr}^{\text{Coppi}}} / \eta$ . Using Eq. (13), we thus obtain

$$\delta_c / L \sim S_c^{1/2} S_L^{-6/7}. \quad (18)$$

These critical current sheets are the structures where Ohmic and viscous dissipation is happening [51]. It is reasonable to assume that Eq. (18) sets the dissipation scale, that is, the scale below which the reconnection interval is ultimately terminated by the dissipation. Assuming that the energy spectrum in this interval follows a power law, we write it in the form

$$E(k_{\perp}) \propto k_0^{-3/2} (k_{\perp} / k_0)^{-\alpha}, \quad (19)$$

where  $k_0 \sim 1 / \lambda_{cr}^{\text{Coppi}}$  is the wave number corresponding to the reconnection scale (13), where the reconnection-interval spectrum matches the inertial-interval spectrum of MHD turbulence  $E(k_{\perp}) \propto k_{\perp}^{-3/2}$ . The power-law spectrum (19) extends up to the wave number  $k_* \sim 1 / \delta_c$  corresponding to the dissipation scale (18), after which it is expected to decline fast.

We now calculate the rate of magnetic energy dissipation using the spectrum (19):

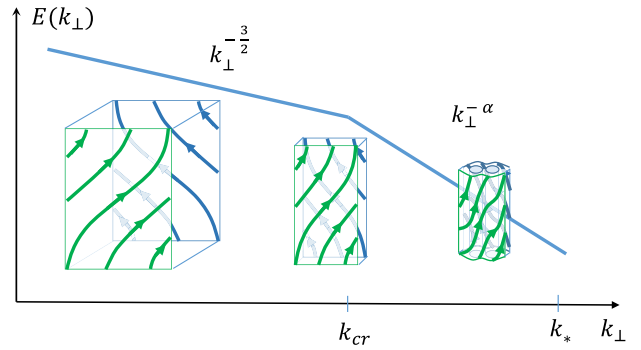


FIG. 1. Sketch of the Fourier energy spectrum (log-log scale) and the shapes of eddies as a function of  $k_{\perp} \sim 1/\lambda$ . The arrows indicate the direction of the magnetic field lines. For  $k_{\perp} < k_{cr} \sim 1/\lambda_{cr}$ , the turbulent eddies become progressively more anisotropic as  $k_{\perp}$  approaches  $k_{cr}$ . For  $k_{\perp} > k_{cr}$ , the tearing instability is an essential part of the turbulence.

$$-\frac{dE}{dt} = \eta \int_{k_*}^{k_0} k_{\perp}^2 E(k_{\perp}) dk_{\perp} \propto S_L^{4(-\frac{3}{2}+\alpha) + \frac{6}{7}(3-\alpha) - 1}. \quad (20)$$

In a steady state, the rate of energy dissipation must be equal to the constant rate of energy cascade from the large-scale MHD turbulence, and, hence, independent of the Lundquist number. This defines the scaling of the energy spectrum:  $\alpha = 5/2$ . The energy spectrum and the eddy structure envisioned in our model are represented in Fig. (1).

Finally, note that the validity of Eq. (18) rests on it being smaller than Eq. (13) (or, equivalently,  $\xi_{cr}^{\text{Coppi}} \gg L_c$ ); this yields a criterion for the minimum value of the outer scale Lundquist number that is needed to observe this  $k_{\perp}^{-5/2}$  spectrum:  $S_L \gg S_c^{7/4} \sim 10^7$ .

*Discussion and conclusion.*—The results derived above present a compelling case for revisiting the mechanism of energy dissipation envisioned in existing Kolmogorov-like theoretical models of MHD turbulence. Because of progressively increasing eddy anisotropy at small scales [11,31,57], at a sufficiently small scale the reconnection time becomes comparable to the eddy turnover time. Reconnection, we have argued, disrupts the eddies at that scale, and implies that no such eddies can form at smaller scales. The energy cascade from the large scales, where it is injected, to the smallest scales, where it dissipates, must, therefore, proceed through a new subinertial stage where reconnection, the resulting structures, and associated flows are key players.

As has been noted in the past, the presence of the large-scale magnetic field and the Alfvénic time scale implies that the Kolmogorov first self-similarity hypothesis may not hold for MHD turbulence [58]. In particular, the spectrum of MHD turbulence may depend not only, or not at all, on the Kolmogorov-like dissipation scale. The presented analysis provides physical arguments for the existence of alternative scales (13) and (18) that play a crucial role in MHD turbulence. The dissipation scale (18) decreases faster with the Lundquist number than the

Kolmogorov scale (7). This means, for example, that in order for the numerical simulations of MHD turbulence to be resolved, their discretization scale should decrease faster than the Kolmogorov scale (7) as the Lundquist number increases. This property of MHD turbulence has also been discussed in Ref. [58].

Finally, we point out that the generation of secondary islands in two-dimensional MHD turbulence has been previously numerically detected at Lundquist numbers up to  $\sim 10^4$ – $10^5$  [15–18]. The transition to the new spectral scaling, however, was not observed in these studies. Although the 2D case is qualitatively different from its 3D counterpart [32,59], these numerical results may also be explained by insufficiently large  $S_L$  numbers, or they may, possibly, indicate our incomplete understanding of the reconnection-dominated interval. Indeed, the scaling (18) and the spectrum (19) are phenomenological estimates in that they have not been self-consistently derived from a theory of a reconnection-dominated cascade. Such a theory is not currently available; the definitive conclusion on the scaling and the structure of this interval should, therefore, await further analytic studies and numerical investigations.

We may, however, compare our results with available numerical simulations of reconnection-induced turbulence [55]. Although in these simulations the reconnecting magnetic profile was not generated by a turbulent cascade but rather imposed as an initial condition, they may capture the dynamics of the reconnection-dominated spectral interval. These simulations produce the spectra ranging from  $-2.1$  to  $-2.5$ , which are broadly consistent with our model.

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*Note added.*—In the advanced stages of preparation of this Letter, we became aware that a similar calculation [in particular, the derivation of Eq. (13)] has concurrently and independently been performed by A. Mallet, A. Schekochihin, and B. Chandran [60].

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