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Keywords

engineering education, mathematics education, science education, STEM education, STEM integration, mechanistic reasoning, mathematical description, psychometrics

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Abstract

Because reasoning about mechanism is critical to disciplined inquiry in science, technology, engineering, and mathematics (STEM) domains, this study focuses on ways to support the development of this form of reasoning. This study attends to how mechanistic reasoning is constituted through mathematical description. This study draws upon Smith's (2007) characterization of mathematical description of scientific phenomena as "bootstrapping," where negotiating the relationship between target phenomena and represented relations is fundamental to learning. In addition, the development of mathematical representation presents a viable pathway towards STEM integration. In this study, participants responded to an assessment of mechanistic reasoning while cognitive interviews were conducted to characterize their reasoning about mechanism and mathematical description of the systems of levers represented in the items. Participant item responses were modeled using item response theory and participant talk and gesture were coded according to developed analytic frameworks. Participants were elementary, middle, and high school students as well as college undergraduates, and adults without college education. The results suggest a relationship between participants' tendencies to describe these systems mathematically and their mechanistic reasoning ability. Moreover, there are specific elements of mechanistic reasoning that are more highly associated with mathematical description. In addition, there is a relationship between a participant's propensity to both mathematically describe and mechanistically trace mechanical systems.

Keywords: engineering education, mathematics education, science education, STEM education, STEM integration, mechanistic reasoning, mathematical description, psychometrics

STEM Integration and Mathematical Description

One challenge facing science, technology, engineering, and mathematics (STEM) education is integration, advancing student conceptual development within and across STEM domains (National Research Council [NRC], 2007). The National Academies Press (NAE) (Honey, Pearson, & Schweingruber, 2014) has argued that integrated STEM education should bring together concepts from more than one discipline (e.g., mathematics and science, or science, technology, and engineering); it may connect a concept from one domain to a practice of another, such as applying properties of geometric shapes (mathematics) to engineering design (Weinberg, 2012); or it may combine two practices, such as scientific inquiry (which include modeling, argument, etc.) and engineering design (in which data from a science experiment can be applied).

The NAE (2014) has also argued that STEM education is a potential vehicle for concept development in mathematics through the mathematical description of natural and designed systems in science and engineering. However, in spite of the potential for mathematical learning through STEM integration, studies have shown that most STEM curricula (designed to

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target mathematics content) do not reach this potential (Nathan, Tran, Phelps, & Provost, 2008; Prevost, Nathan, Stein, Tran, & Phelps, 2009; Welty, Katehi, & Pearson, 2008). These studies indicate that the vast majority of such curricula address few mathematics content standards (e.g., National Council of Teachers of Mathematics, 2000) and those standards that are addressed are done so in a shallow, disconnected manner. Moreover, such treatment of mathematics ignores the fundamental role of mathematical description in the other STEM fields (i.e., other than mathematics). The lack of integration in STEM programs, between mathematics and other STEM fields suggests that these programs are either: (1) not targeting content that can readily be described mathematically or (2) failing to mathematically describe content appropriately.

Honey et al. (2014) and the NRC (2007) have argued that mechanistic reasoning is an effective entry point into learning across STEM disciplines. Russ, Scherr, Hammer, and Mikeska (2008) explain that mechanistic reasoning "involves more than noting which causes are associated with which effects; it concerns the process underlying the association" (p. 8). Reasoning about causal mechanism (i.e., mechanistic reasoning) supports the capacity to both diagnose and trace through the components of mechanical systems. In addition, a predisposition for seeking out mechanism within these systems is valuable for disciplined inquiry in STEM fields. For instance, focusing on mechanism is central to the development of capacities to engage in scientific explanation and argumentation (Bolger, Kobiela, Weinberg, & Lehrer, 2012; Russ et al., 2008).

This study proposes that the mathematical description of systems in other STEM disciplines is critical to conceptual change in both those STEM fields and mathematics. This study attends to what is difficult about mechanistic reasoning as well as how mathematical description can support as well as constitute this form of reasoning. For instance, in spite of the vast literature about early children's competencies for mechanistic reasoning in laboratory settings (e.g., Shultz, 1982), previous studies (Bolger et al., 2012; Kobiela, Bolger, Weinberg, Rouse, & Lehrer, 2011; Lehrer & Schauble, 1998; Metz, 1991) indicate that even for simple systems, such as gears and levers, constructing coherent mechanistic explanations is not trivial.

Mathematical Description and Conceptual Change

This study highlights that the difficulty with mechanistic reasoning emerges from the contexts in which participants are asked to employ conceptual resources and not from the absence or presence of said resources. For instance, being able to perceive and diagnose the motion of a lever in one context does not imply the propensity to immediately perceive and diagnose all other levers' motions in all contexts where they appear. Disciplining one's perception to "see" in such a way as to diagnose lever motion across lever types and arrangements no doubt takes time and experience within a legitimate situated context (Lave & Wenger, 1991; Stevens & Hall, 1998) (e.g., K–12 STEM education). As one's experience grows, these systems can become more generally useful for diagnosing and explaining many simple and complex systems. For example, two levers and a screw are the constituent parts of a pair of scissors; bicycles and eggbeaters are other common examples of compound machines. Developing an understanding of the mechanisms within these systems of levers can aid children in understanding the mechanisms of many more machines, including both simple and compound systems. Moreover, a predisposition for seeking out mechanism is valuable for inquiry across the STEM disciplines.

Although mechanistic reasoning is essential for disciplined inquiry in STEM fields, its development is not trivial. diSessa (1993) notes that learning calls for the complex coordination of knowledge. According to his "knowledge-in-pieces" framework (1988), naïve knowledge structures consist of multiple conceptual elements that are spontaneously connected and activated according to the relevance of their contexts. From this perspective, conceptual change involves a process of bottom-up accumulation and coordination of knowledge components. Such conceptual coordination entails changes in student thinking, mediated by new, symbolic, representational resources (Hall & Greeno, 2008). For instance, the coordination of these new representational resources with pre-existing conceptions of the workings of physical systems provides individuals with the capacity to make novel mechanistic inferences.

When individuals learn content in science and engineering, representing relations from other domains can be fundamental to developing new understandings. Smith (2007) describes this process of representing as "bootstrapping," where negotiating the relationship between the content and represented relations is critical to developing new understandings of content. Lehrer and Schauble (2006) supported the development of scientific concepts, known to be challenging to students in the elementary grades, through the utilization of mathematical description. For example, students "invented" Cartesian coordinates in order to more effectively describe the growth of Wisconsin Fast Plants™. They note, "Inscriptional [representational] development transformed the conceptual terrain, so that students began to pose new questions about the plants" (p. 376). Such a use of mathematical description of content in other STEM fields is central to the process of reasoning about that content (Lehrer, Strom, & Comfrey, 2002; Smith, 2007). For instance, Lehrer et al. (2002), also working with Wisconsin Fast Plants™, leveraged third grade students' perceptions of everyday experiences to help understand multiple forms of mathematical similarity. For instance, here students used volume measurement to describe and compare the plant canopies.

When students learn content in science and engineering, representing relations from mathematics involves the use of

Table 1.
The analogues between the geometry of circles and levered systems.

Mathematical description	Definition	Physical system	Definition
Center	A point inside the circle. All points on the circle are equidistant (same distance) from the center point	Fixed pivot (fulcrum)	The fulcrum (fixed pivot) is fixed to the board and all points on the lever remain equidistant
Radius	The radius is the distance from the center to any point on the circle. It is half the diameter	Link (lever)	Every point on the lever is a constant distance from the fixed pivot
Circumference	The circumference is the distance around the circle	Distance	As the lever rotates about the fixed pivot, it traces a circular path. This circular path is the circumference

what Hall and Greeno (2008) term *representational resources*. Representational resources, which include the mathematical description of scientific phenomena, are central to the development of reasoning about mechanism in science and engineering. For example, attending to both the physical system and its analogue mathematical description is necessary to understand and explain the mechanism of a system of levers. For instance, this explanation indicates how representational relations buttress descriptions of the physical system of levers. For example, in Weinberg (2012), Kayla (a sixth grader) noted, "the curved distance travelled by the output (*circular path*) for a given input, for this lever, depends on the relationship between the distance of a point on the lever and the fulcrum (*radius*)."

This study focuses on content in science and engineering education (i.e., systems of levers) that lends itself to mathematical description through its analogue to the geometry of circles (Table 1); this system also supports reasoning about causal mechanism because of its "simplicity" (i.e., all machine parts and mechanisms are visible and inspectable). In addition, this work investigates how individuals bring mathematical representations to bear to mediate their reasoning.

Mechanistic Reasoning and Mathematical Description

Kobiela et al. (2011) have shown that when students engage in the mathematical description of systems of levers it makes their mechanisms more salient and comprehensible. Their study characterizes the relationship between mathematical description and mechanistic reasoning about systems of levers within engineering design. In addition, it investigates how mathematical description mediates the mechanistic tracing of these systems from input to output.

Bolger et al. (2012) showed that by mathematically describing systems of levers, participants supported both mathematical and mechanistic reasoning. In Weinberg (2012), when working with systems of levers, 76% ($n = 56$) of those participants who could causally trace through systems of levers, from input to output, on at least one item made a reference to the mathematics of circles (a representational analogue of the system of levers) when responding to an assessment and a cognitive interview. For example, participants used the following terms (in the interview) to explain the machine motion: "circle," "center of the circle," "radius," "circumference," "axis of rotation." In addition, those participants who spontaneously referenced mathematics to explain machine motion, on at least one item, had higher mechanistic reasoning ability scores on the assessment ($M = 0.43$ logits; $p < 0.0001$) than those who did not reference mathematics in their explanations ($M = -1.38$ logits). These findings suggest that mathematical description supports the identification and causal coordination of mechanistic elements.

Research Questions

This study characterizes the ways that participants spontaneously deploy mathematical description to buttress mechanistic reasoning. The following research questions are addressed: (1) in what ways do participants mathematically describe simple systems of levers and (2) how does this mathematical description mediate their mechanistic reasoning about these systems?

Method

Participants

The participant groups that comprise the sample are shown in Table 2. The elementary, middle, and high school students come from public and private schools in the southeastern United States. The university undergraduates come from three universities, two in the southeastern and one in the mid-western United States. Of the two universities in the southeastern United States, one is a highly ranked private university and the other is a large lower-ranked public university. The university in the mid-western United States is a highly ranked private liberal arts college.

The public elementary, middle, and high schools belong to Centennial Public School District (a pseudonym). The percent of children attending these three schools qualifying for free or reduced lunch ranges between 60 and

Table 2.
Participants.

Participants	Number
Elementary school students	28 (female = 17)
Middle school students	25 (female = 16)
High school students	20 (female = 4)
University undergraduates (non-science majors)	16 (female = 13)
University undergraduates (engineering majors)	13 (female = 5)
Adults (without college education)	10 (female = 8)
Total	112 (female = 63)

90% from year to year. The adults without college degrees ($n = 10$) are 10% Caucasian and 90% African-American.

The participants in elementary, middle, and high school, according to their teachers, represent a wide spectrum of academic achievement. The undergraduates, both engineering majors and non-science majors, represent students along a continuum of academic success (i.e., from less to more highly rated universities). It was hypothesized that the engineering majors would perform well on this assessment because of the benefit of their academic training. The adults without college degrees are likely from different populations than the other adults in the study (i.e., college undergraduates) and their experiences are also likely different. It was conjectured that these populations would draw upon diverse conceptual resources as a result of their disparate histories and cultures. Thus, individuals in the study represent various ethnic backgrounds and life experiences. These participants were chosen because of the hypothesis that representational resources would aid in mechanistic reasoning in diverse ways across all different cultures and histories.

Procedure

The data in this study were collected from a cognitive interview that was conducted while participants worked on an assessment of mechanistic reasoning (Weinberg, 2012). The results of the assessment were also analyzed.

Cognitive interview

Each participant responded to a cognitive interview while working on each assessment item. The total interview (i.e., assessment administration) was completed during one session and lasted an average of 37.5 minutes (ranging from 17 minutes to 78 minutes). Interviews were recorded using one camera, with a table microphone. The camera was positioned at the side of the participant, about one half foot away from the table, angled down to capture what the participant was looking at as well as gestures made over the paper. Interview sessions were digitally rendered for further analysis.

Assessment

The assessment contained 21 paper-and-pencil items, which were presented to participants across seven forms, to maximize the number of items to which participants could respond. Each participant was given one form containing 15 items. There were three common items (or link items) that appeared on all forms so that scores and item difficulties across the forms could be compared. Link items are central to the process of test equating in item response theory (IRT) (Kolen & Brennan, 2013). The mean number of respondents for each item was 53.5, with a range from 38 to 112 (link items); the median number of respondents per item was 44.

Elementary and middle school students completed ten items per form, while high school students, undergraduates, and non-college educated adults completed fifteen items per form. Five items were indicated on each form that elementary and middle school students were to skip.

Short-answer items that required respondents to draw predicted motion were used in this assessment. All items represented simple systems of levers. The assessment features two types of items, System Tracing (Appendix A) and Machine Prediction (Appendix B). System Tracing items require the participant to predict the motion of all levers in the system, while Machine Prediction items only require the participant to predict the motion of the output lever(s). Participants drew predicted motion of the levers under indicated directed inputs. In addition, the different systems can be broken down into four categories according to: (1) number of levers (e.g., fourteen systems were composed of three or more levers, while seven systems were composed of two or fewer), (2) the arrangement of levers (e.g., seven systems were constructed with one or more intermediate link(s) between the input and output, while fourteen were constructed with no intermediate links, (3) lever type (e.g., five systems were composed of class 1 levers, while five systems were composed of class 3 levers), and (4) the presence of specialized and unfamiliar levers (e.g., two systems contained an intermediate link that was a bent crank, while the remaining nineteen systems did not contain unfamiliar levers).

Related direction and rotation could be assessed on 21 and 19 items (respectively); lever arms, constraint via the fixed pivot, and tracing could be assessed on 11 items. The mechanistic elements that can be scored for each item are given in Table 3.

Conduct of the Interview

While participants responded to each assessment item, they were asked to: (1) read the problem aloud and (2) think aloud as they responded to each item. When the participant completed the item, the participant was asked to explain again, if necessary, the rationale for the observed item response with interviewer probes. Finally, participants were asked to report any words that they found confusing, as well as whether there was any confusion about what the item was asking. The interview was conducted in this

Table 3.
Item Wright Map results: mean item difficulty estimates, standard errors, and mechanistic elements assessed.

Item	Mechanistic elements assessed
Hands Fixed Pivot-Opposite	RD, R
Machine Prediction-A2	RD, R
Sequential Tracing-D1	RD, R, LA, CFP, T
Sequential Tracing-E2	RD, R, LA, CFP, T
Hands Fixed Pivot-Same	RD, R
Machine Prediction-A1	RD, R
Machine Prediction-A3	RD, R
Machine Prediction-A3'	RD, R
Machine Prediction-B2	RD, R
Machine Prediction-B2'	RD, R
Machine Prediction-D1	RD, R
Machine Prediction-D1'	RD, R
Sequential Tracing-A1	RD, R, LA, CFP, T
Sequential Tracing-A3	RD, R, LA, CFP, T
Sequential Tracing-A3'	RD, R, LA, CFP, T
Sequential Tracing-B1	RD, R, LA, CFP, T
Sequential Tracing-B1'	RD, R, LA, CFP, T
Sequential Tracing-B2	RD, R, LA, CFP, T
Sequential Tracing- D1'	RD, R, LA, CFP, T
Sequential Tracing-E1	RD, R, LA, CFP, T
Sequential Tracing-CMT	RD, R, LA, CFP, T

Note. Estimate is constrained. RD = related direction, R = rotation, LA = lever arms, CFP = constraint via the fixed pivot, and T = tracing.

order to: (1) determine spontaneous thinking throughout participant interaction with each item (i.e., think-aloud) and (2) assess mechanistic reasoning that was present, but possibly not elicited during the think-aloud (i.e., retrospective explanations) with interviewer probes.

Analysis

The assessment

The assessment instrument used in this study characterized how participants deployed different mechanistic elements (i.e., naïve resources for making sense of machine motion). These elements were (a) *related direction* (attention to the coordinated direction of the input and output of a linkage), (b) *rotation* (attention to the rotary motion of the levers), (c) *lever arms* (attention to the coordinated opposite motion of the two lever arms), and (d) *constraint via the fixed pivot* (attention to the causal relation between the pivot being fixed to the board and the resultant motion). These mechanistic elements were developed in a previous study (Bolger et al., 2012). In addition, the assessment characterizes participant proclivity to construct causal schemes, coordinating these elements from input to output (i.e., *tracing*). Each item was scored according to its exemplar (Appendix A, Appendix B). An exemplar is a scoring guide designed for each assessment item. These scoring guides are ordered from the least to most difficult mechanistic elements for participants to endorse. A demonstration of how one item was scored, according to its exemplar, is presented (Appendix A, Appendix B). Exemplars contain three scoring categories: (1) the missing code (i.e., scores for missing responses), (2) the no linking code (i.e., scores for responses that do not link to the construct map), and (3) construct linking codes (i.e., scores for responses that link to the construct map).

Missing

The "missing" code was assessed when participant responses were not present. However, in this study participants responded to all items.

No linkage

The "no link" code was assessed when participants gave responses that provided evidence that they did not understand the nature of the task. This is seen in responses like "I don't know."

Construct linking codes

The construct linking codes include participant responses that (a) do not reason about causal mechanism, (b) reason about the four mechanistic elements (i.e., *related direction*, *rotation*, *lever arms*, *constraint via the fixed pivot*), and (c) causally coordinate all mechanistic elements from input to output (i.e., *tracing*). Each construct linking code is displayed below.

No mechanistic elements are shown

These participant responses are not mechanistic. These may indicate participant reasoning about individual system components, machine structure, or idiosyncratic rules about machine motion. This is seen in the participant response scored at level 0.

Related direction

These participant responses indicate identification of the coordinated motion of input and output. This is seen in the participant response scored at level 1.

Rotation

These participant responses indicate identification of the rotary paths of the systems' levers. This is seen in the participant response scored at level 2.

Lever arms

These participant responses indicate identification of the coordinated opposite direction of the lever's arms. This is seen in the participant response scored at level 3.

Constraint via the fixed pivot

These participant responses indicate identification of the coordinated motion around the fixed pivot. This is seen in the participant response scored at level 4.

Tracing

These participant responses indicate (a) identification of all mechanistic elements and (b) the sequential coordination of these elements from input to output. This is seen in the participant response scored at level 5.

Participants were scored at the highest level (i.e., most difficult mechanistic element) where they achieved competency. For example, if a participant was assessed at both the levels of *rotation* and *related direction* on an item, they were assessed at the level of *rotation*. To be scored at the level of *tracing*, participants must have indicated a causal coordination of all elements. An outside researcher scored 10% of the total items. The agreement was 85%.

Item response theory modeling

When all the participant responses were scored, IRT analysis was used. IRT modeling determines the ability of each respondent and difficulty of each item through the calculation of a log-odds ratio (i.e., the logit function). Logits are an equal interval level of measurement; as such, the distance between each point on the scale is equal. A probability of 0.5 (50% of participants correctly responding to a dichotomous item) corresponds to a logit of 0. Negative logit values indicate probabilities smaller than 0.5; positive logit values indicate probabilities greater than 0.5. The relationship is symmetrical: logits of −0.2 and 0.2 correspond to probabilities of 0.45 and 0.55, respectively. The absolute distance to 0.5 is identical for both probabilities.

In this study, one-tailed *t*-tests were conducted to compare mean person ability scores.

Cognitive interview

Participant proclivity to employ mechanistic reasoning (Bolger et al., 2012) and mathematical description (Kobiela et al., 2011) was characterized according to analytic frameworks developed in previous studies as participants noticed, described, and explained the motion of these systems of levers. Mechanistic reasoning was coded according to the elements described above: (a) *related direction* (e.g., "When you push the input up, the output goes down"), (b) *rotation* ("The output goes around"), (c) *lever arms* ("When this side [of the lever] goes up, this side goes down"), and (d) *constraint via the fixed pivot* ("Because the brad is stuck to the board, the lever is going to move that way"). In addition, *tracing*, the causal coordination of these mechanistic elements, was determined according to the following criteria: participants (1) referred sequentially, in talk or gesture, to each of the links in linkages and (2) reasoned about all mechanistic elements.

Mathematical description was categorized according to the following: (1) *describing or noticing a mathematical object* and (2) attending to *magnitude relations* (Figure 1).

Describing or noticing a mathematical object included the following subcategories: (a) *circular path* ("this lever goes in a *circle*") (Constance, grade 3), (b) *center of circle* ("this lever moves around this point,") (Kim, undergraduate/engineering major), and (c) *radius of circle* ("the part of the link that will move up is from here [fulcrum] to here [end of lever]) (Roderick, grade 9; underline highlights mathematical description).

Citing *magnitude relations* may include (a) *circumference–radius relations* ("the farther you go out on the link, the more distance it's going to cover when you push it") (Jacqueline, adult), (b) *constant radius* ("as you move the lever around, all points are always going to be the same distance from the fulcrum") (Maddie, 11th grade), and *other relations* ("the

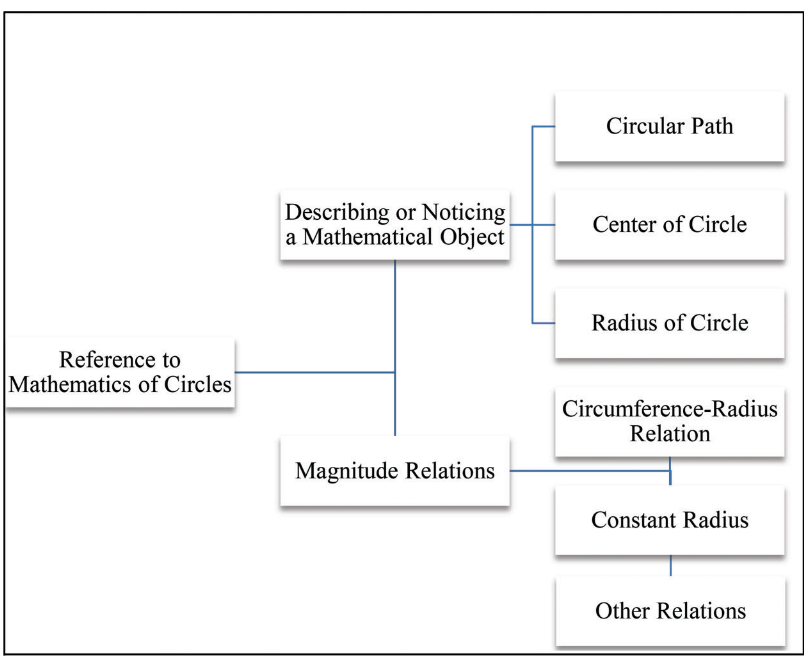

Figure 1. Framework for mathematical description.

speed of the rotation of the lever increases as you move farther from the fixed pivot") (Chris, grade 12; underline highlights mathematical description).

Coding interviews

The coding of the talk and gesture for one high school student, James (pseudonym), is presented below (Figure 2). James begins diagnosing item Sequential Tracing D1 (STD1), indicating the directed motion of the input and linking that to the fixed pivot constraint: "this [input lever]—this is going upward (*related direction, input*) and the point (*fixed pivot, center of circle*) is to the right." Here, the fixed pivot constraint was supported by the mathematical description of the circle's center. Next, James connects the coordinated opposite directed motion of the system's lever arms with resultant motion from the *constraint via the fixed pivot*: "things to the left will go up (*lever arms, left; constraint via the fixed pivot, resulting motion*) because they're attached to that [connected to the input] … I think … this would go up and, um, this one would go down (*lever arms, right*)."

Next, James characterizes the rotary motion of the output by invoking mathematical description of the circular path (Figure 2B): "…but it's like that (indicates motion of outputs; *related direction, output*) … and because the fixed pivot's (*constraint via the fixed pivot, identification*) here, it'll (internal lever) have to move like that (indicates rotary motion; *rotation, circular path*)…"

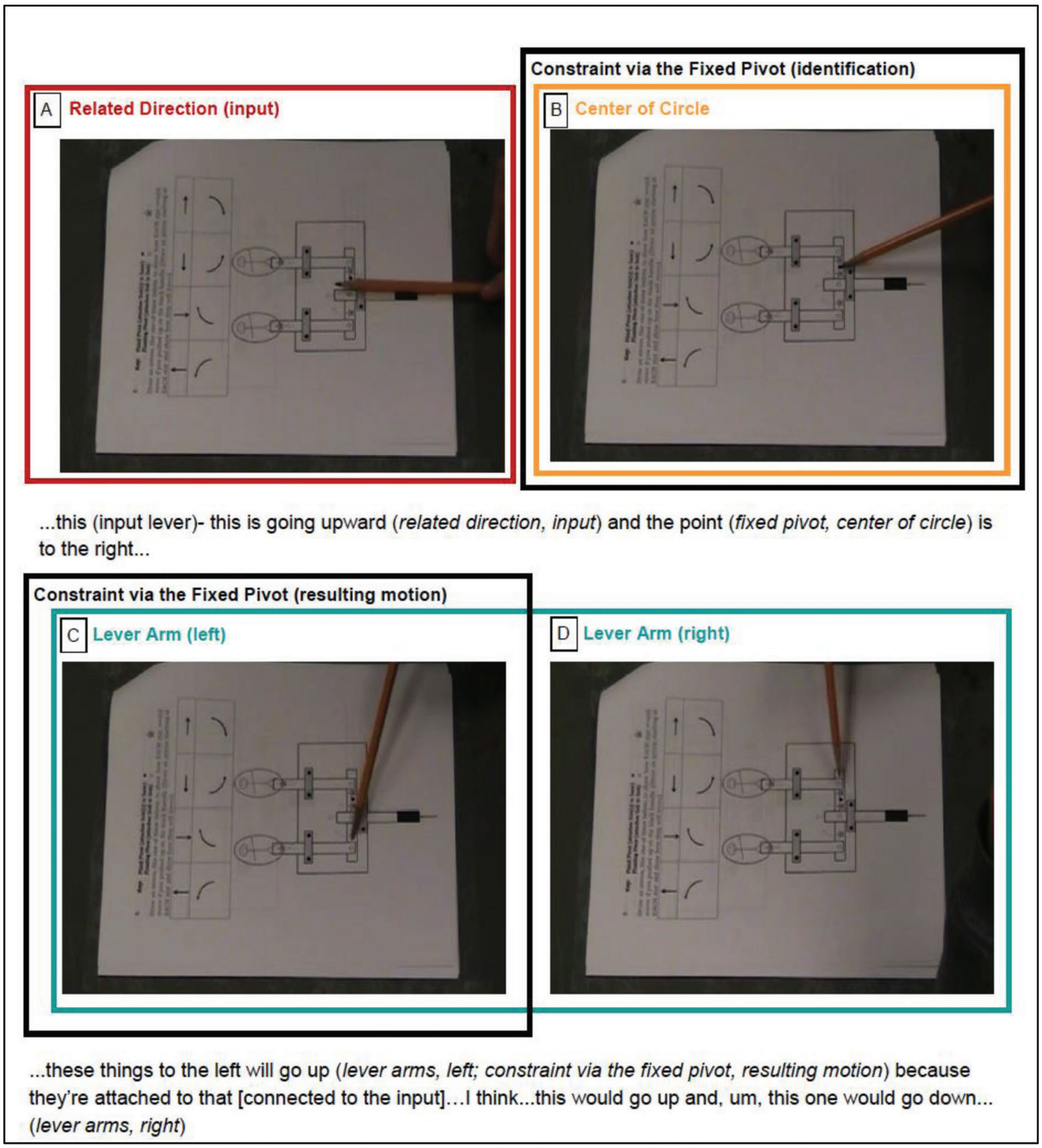

Figure 2. (A) James.

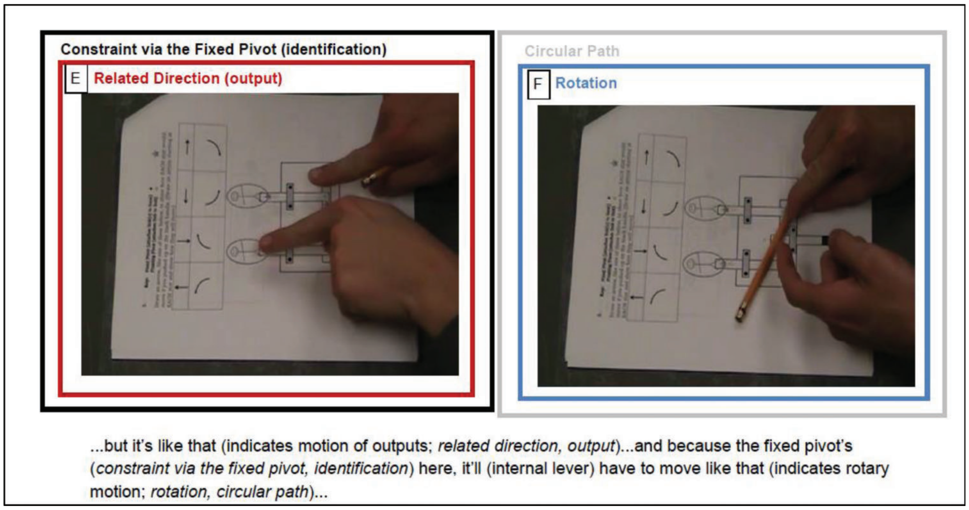

Figure 2. (B) James.

Finally, James provides further evidence that these explanations have been facilitated by the mathematical description of this system (Figure 2C): "these two (stars) look like they will move the same amount because they're … an equal distance away from the fixed point (*constant radius*) and this one's (star) a farther distance. So it'll move, um, further (*circumference-radius relation*)."

A participant's work on one item is defined as a "performance." Participants were coded at the highest level where they achieved competency within each performance for mechanistic reasoning codes. For example, if a participant was coded at both the levels of *constraint via the fixed pivot* and *tracing* within the same instance, they were reported at the level of *tracing*. Participants were double-coded for mathematical description codes. For example, some individuals were coded as citing *circular path* and *circumference–radius relation* to explain machine motion. All performances were coded using NVivo 11.0 software. An outside researcher coded 10% of the total instances. The agreement was 83%.

This qualitative analysis was used to characterize mathematical description and mechanistic reasoning. This characterization was used to triangulate the behaviors and practices engaged in by participants and their assessment IRT person ability scores. In addition, these behaviors and practices were used to understand how participants reasoned when they diagnosed specific mechanistic elements.

Results

In this section, the relationship between mathematical description and mechanistic reasoning is described. In addition, mechanistic elements are explored and clarified. Finally, a participant's propensity to both mathematically describe and trace is characterized.

Mathematical Description Constitutes Mechanistic Reasoning

Mechanistic reasoning, across all participants, was not simple. For instance, 11% ($n = 12$) of participants did not show the propensity to diagnose one mechanistic element on even one item. However, those participants that spontaneously referenced mathematics to explain machine motion, on at least one item, had higher mechanistic reasoning ability scores on the assessment ($M = 0.43$ logits; $p < 0.0001$, one-tailed t-test) than those who did not reference mathematics in their explanations ($M = -1.38$) during the interview. There is also a difference in ability between those who made one reference to mathematics ($M = -0.60$; $p < 0.05$, one-tailed t-test) and those who made no references ($M = -1.38$). This shows the relationship between mathematical description and mechanistic reasoning.

With each additional instance of mathematical description, there was an increase in assessed mechanistic reasoning ability. For instance, of those fifteen participants (13%) who mathematically described the highest proportion of systems, all but one individual was able to *trace* at least one system from input to output. These individuals referenced mathematics on at least five items (*range* = 5–10; $M = 7.93$; $Mdn = 8$).

Mathematical Description Constitutes Specific Mechanistic Elements

Participants had difficulty diagnosing all of the mechanistic elements. For instance, on the assessment the following percentage of participants diagnosed the following mechanistic elements on at least one system of levers: 71% ($n = 80$) of participants diagnosed *related direction*, 30% ($n = 34$) diagnosed *rotation*, 65% ($n = 73$) diagnosed *lever arms*, 44% ($n = 49$) diagnosed *constraint via the fixed* pivot, and 23% ($n = 26$) were assessed as *tracing*. In addition,

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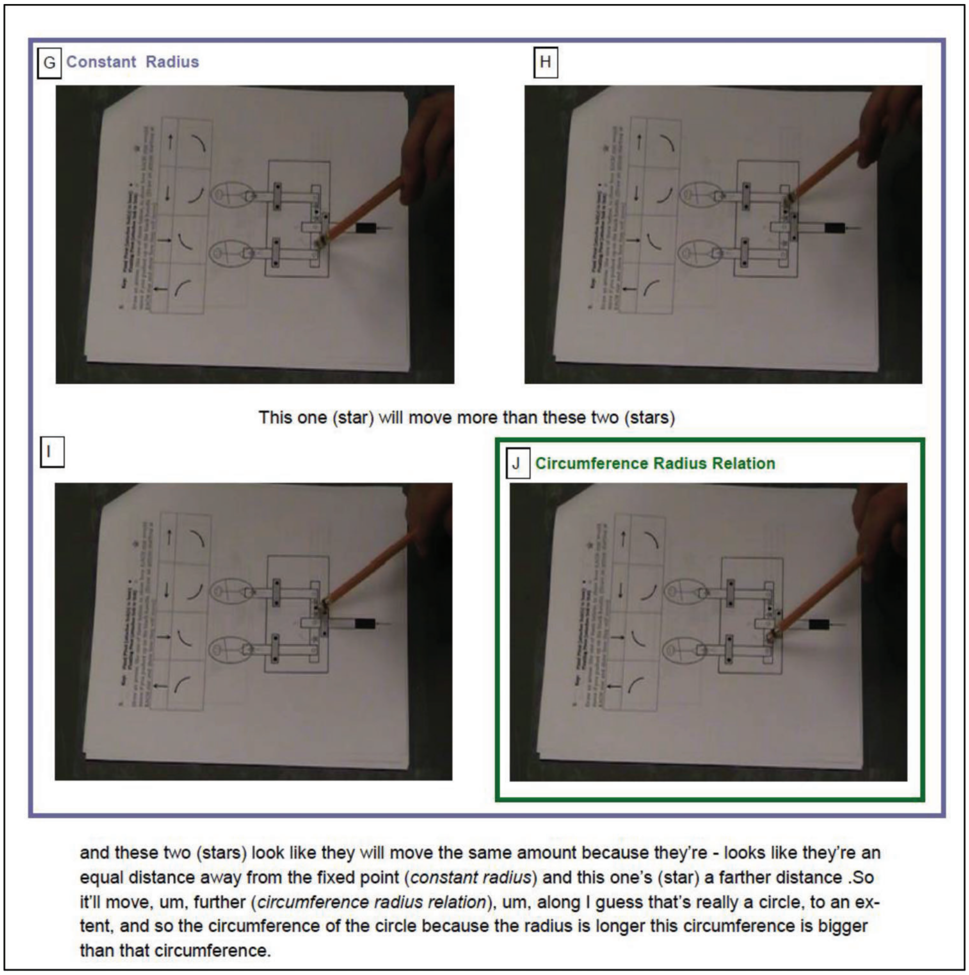


Figure 2. (C) James.

only 1% (n = 1) of the participants deployed all of the mechanistic elements across the assessment items; only 10% (n = 11) of participants diagnosed *related direction*, *rotation*, *lever arms*, and *constraint via the fixed pivot* across the assessment items; only 16% (n = 18) of participants diagnosed *related direction*, *rotation*, and *lever arms* across the assessment items; and only 10% (n = 11) diagnosed *rotation*, *lever arms*, and *constraint via the fixed pivot* across the assessment.

In spite of the difficulty participants had in diagnosing individual mechanistic elements (let alone multiple elements) across the assessment items, there was a relationship between mathematical description and mechanistic reasoning for most mechanistic elements. For instance, participants mathematically described the levered systems more frequently (in more performances) when explaining the following: *rotation* (29%), *constraint via the fixed pivot* (37%), as well as *tracing* (76%) the system from input to output (Table 4).

Participants who were unable to diagnose any mechanistic elements rarely mathematically described the system (3%) (Table 4). Moreover, participants who diagnosed the system exclusively referencing *related direction* only mathematically described 9% of performances (Table 4). In addition, participants who were scored at the level of *lever arms* mathematically described few performances (16%).

Table 4.
Percentage of mathematical description by mechanistic element.

Mechanistic element	Mathematical description
No mechanistic elements (n = 553)	3% (18)
Related direction (n = 219)	9% (20)
Lever arms (n = 114)	16% (18)
Rotation (n = 199)	29% (57)
Constraint via the fixed pivot (n = 109)	37% (40)
Tracing (n = 74)	76% (56)

Further investigation of mathematical description focuses on two mechanistic elements that most frequently corresponded with mathematical description: *rotation* and *constraint via the fixed pivot*. In addition, the causal coordination of all the mechanistic elements (i.e., *tracing*) also shows a significant correspondence with mathematical description.

The frequency of specific mathematical description codes by specific mechanistic element codes (as well as *tracing*) is shown in Table 5.

Those participants who showed the proclivity to mathematically describe the levered system were all coded at the level of *describing or noticing mathematical objects.* However, those participants who were diagnosed at the level of *rotation*, *constraint via the fixed pivot*, and *tracing* cited *magnitude relations* more than those who did not ($p < 0.0001$; Chi-squared test; Figure 3). This is likely the case because when participants saw circular paths it did not only allow them to see and potentially explain the levers' rotary path (*rotation*). In addition, these participants also likely saw these circles emanating from a "center" (*center of the circle*). This mathematical description cued the relation between the constraint of the fixed pivot (*center of circle*) and the resultant lever motion (*constraint via the fixed pivot*).

Mathematical Description Constitutes System Tracing

Tracing these mechanical systems was more difficult than citing all other mechanistic elements individually (Weinberg, 2012). For example, 77% ($n = 86$) of participants did not show the propensity to trace on even one item. This is because *tracing* requires participants to causally connect all mechanistic elements from input to output. However, in 76% of items where participants traced, they mathematically described the systems (coded as *describing or noticing mathematical objects* or *magnitude relations*). An example of *tracing* while mathematically describing the mechanistic elements, from input to output, on an assessment item is presented in Figure 4. Kelly, an undergraduate engineering major, provided evidence of *tracing* and mathematical description on this paper-and-pencil representation of a system of levers. She first identified the input motion "this star is on this link (indicates input) … when I push this forward the star that's on that link will just go straight up" (*related direction, input*). Kelly then continued to trace the system with the identification of the output lever, indicating the coordinated motion of input and output: "…that this piece (indicates output) … this cross-link will move" (*related direction, output*). She then linked *constraint via the fixed pivot* to the motion of the output lever (identifying the fixed pivot), indicating its rotary path. The identification of these mechanistic elements is constituted and mediated by mathematical description (i.e., *center of circle*): "because … it's constrained over here (*constraint via the fixed pivot, identification*) so it can only rotate around this point (*constraint via the fixed pivot, resulting motion*; *rotation*; *center of circle*)."

Finally, Kelly links the output motion (Figure 4B; *related direction, output*) to the coordinated opposite motion of the two *lever arms*: "these two stars will just move up and to the right as well as this one (*lever arm, left*) whereas this star which is on the opposite side of the fulcrum (*lever arm, right*) will move down and to the left." This reasoning was mediated by representational resources from the

Table 5.
Percentage of specific mathematical description codes by mechanistic element codes.

	Describing or noticing mathematical objects			**Magnitude relations**		
Mechanistic element	Circular path	Center	Radius	Circumference–radius	Constant radius	Other
Rotation ($n = 57$)	91% (52)	46% (26)	0% (0)	12% (7)	2% (1)	2% (1)
Constraint via the fixed pivot ($n = 40$)	80% (32)	40% (16)	3% (1)	33% (13)	0% (0)	15% (6)
Tracing ($n = 56$)	80% (45)	57% (32)	0% (0)	38% (21)	2% (1)	7% (4)

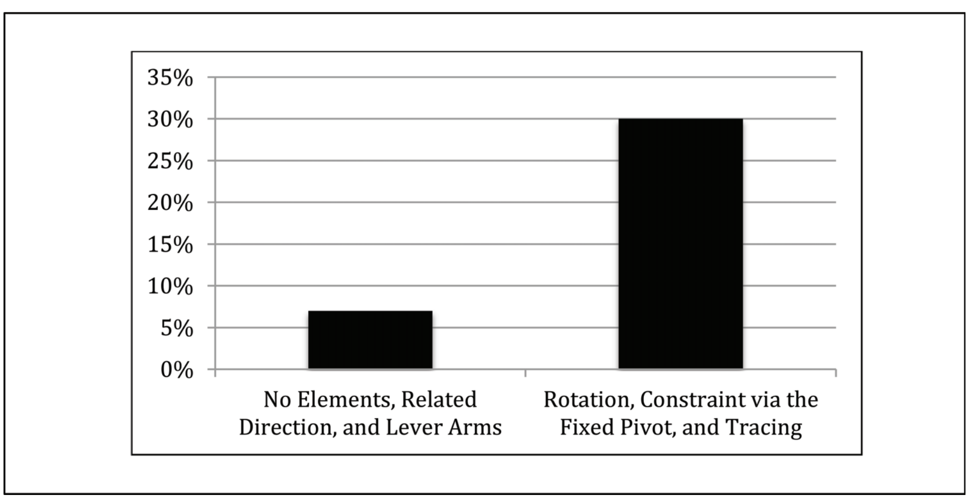


Figure 3. Percentage of relational mathematical description by groups of mechanistic elements.

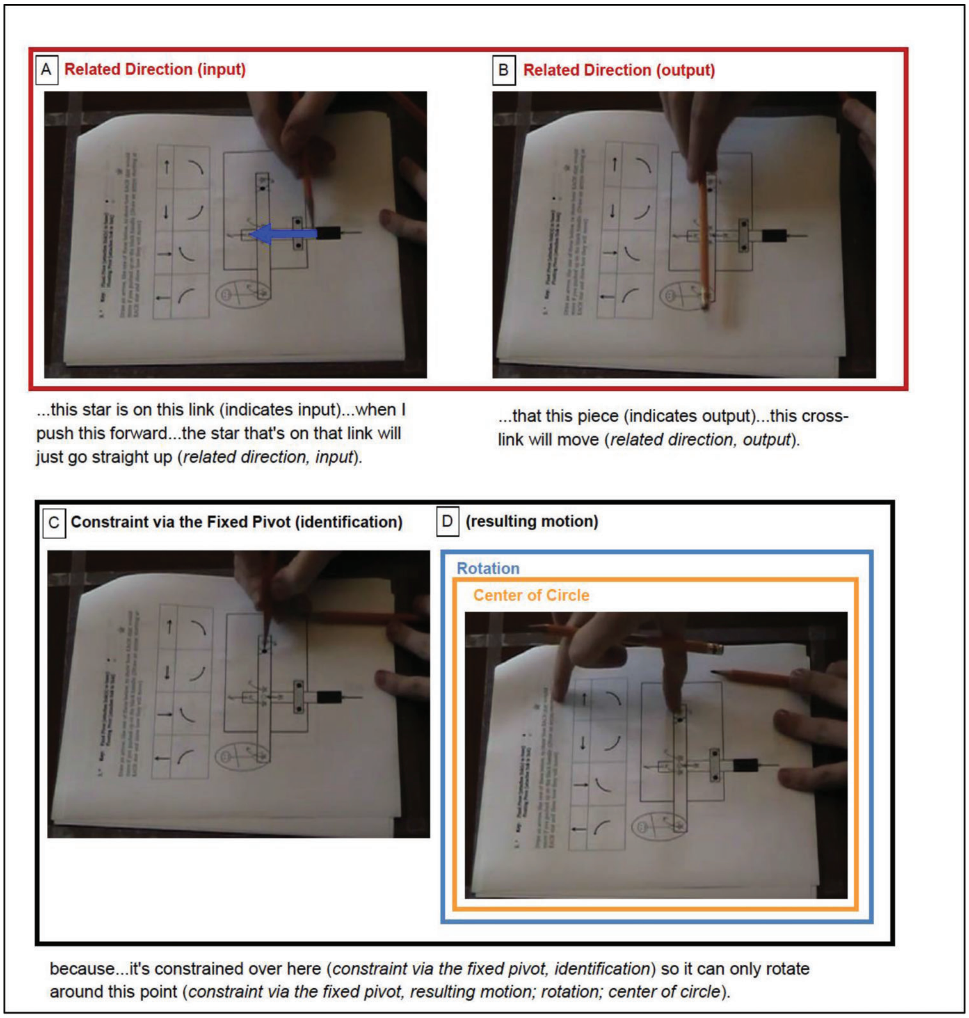


Figure 4. (A) Kelly.

mathematics of circles: "the longer the moment arm from this star to this star, the more distance it's going to travel (*circumference–radius relation*; *radius*). These stars will move. This star will cover more distance and these will be progressively less and less distance the closer you get to the fulcrum of axis." Here, mechanistic reasoning and mathematical description seem to be inseparable. This suggests that the development of representational resources and mechanistic elements are tied.

On items where participants traced, they also mathematically described the system of levers. For instance, when tracing, participants referenced *circular paths* (80%) and the *center of the circle* (57%) in their explanations of machine motion. In addition, they also cited *circumference–radius relations* (38%) as well as other relations (7%). Other relations included the relation between any two mathematical objects (e.g., radius and arc curvature) or quantities (e.g., speed).

Discussion

This study focuses on the development of mechanistic reasoning in science and engineering education and how such reasoning is constituted through mathematical description. Because these forms of reasoning are context specific (diSessa, 1993), this study focused on simple systems of levers. Even though children have conceptual resources for these forms of reasoning under specific conditions, they do not necessarily deploy these forms of reasoning spontaneously in unfamiliar or untutored contexts. This may be the case because these forms of reasoning are heavily dependent on the cues that are provided (and attended to) when participants are considering which components are relevant to system functioning. Many characteristics of simple machines, including aspects of their appearance, embeddedness within other components, and possibly other attributes as well, will likely influence the difficulty with which individuals develop and support mechanistic explanations from a system's input to output. Smith (2007) describes "bootstrapping" as a means to facilitate this context-specific reasoning, whereby individuals negotiate the relationships between systems of levers and their represented relations. For instance, participants that spontaneously referenced mathematics to explain machine motion, on at least one item, had higher mechanistic

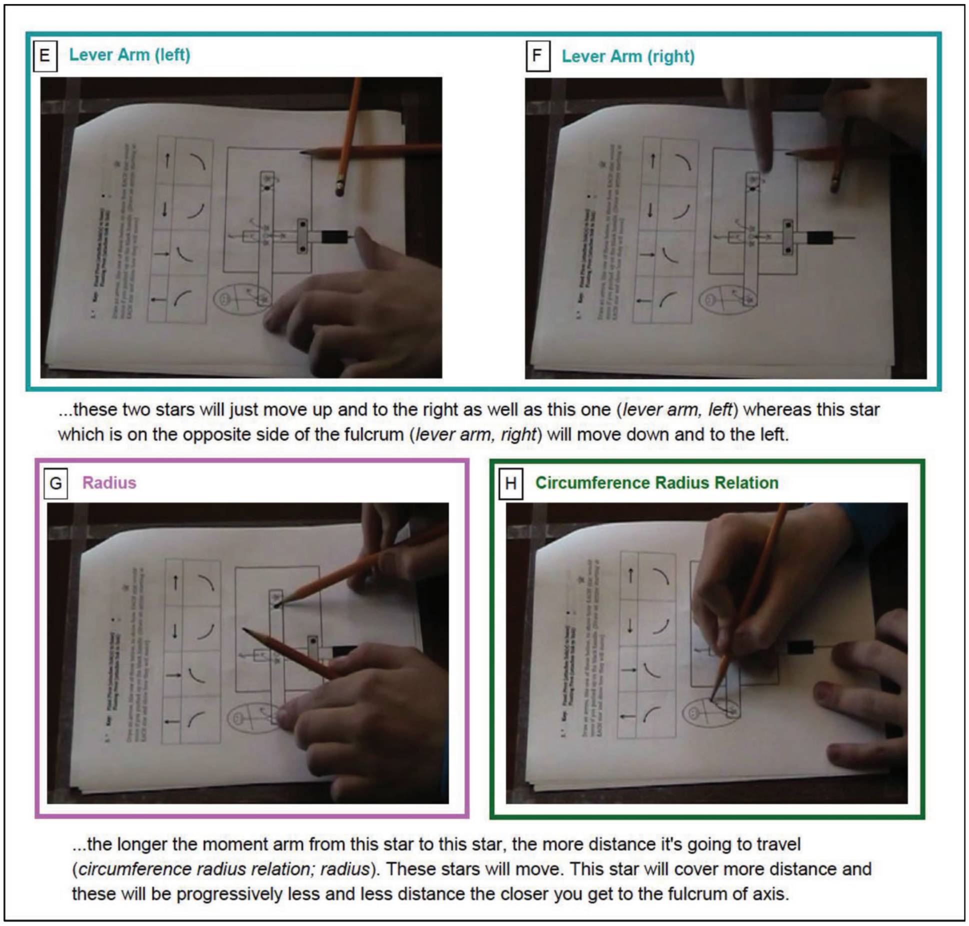


Figure 4. (B) Kelly.

reasoning ability scores on the assessment ($M = 0.43$ logits; $p < 0.0001$, one-tailed t-test) than those who did not reference mathematics in their explanations ($M = -1.38$). Moreover, with each additional reference to mathematics there was an increase in assessed mechanistic reasoning ability. Thus, the capacity to seek out mechanism seems to be constituted through the development of represented resources. This work suggests an available entrée into STEM integration because the development of mechanistic reasoning (i.e., in science and engineering) relies upon representational resources from mathematics.

This study reveals that participants who described a levered system mathematically reasoned more mechanistically than those who did not. In addition, participants who identified relations between mathematical objects and magnitude relations diagnosed the most difficult mechanistic elements: *rotation* and *constraint via the fixed pivot*. For instance, there was a relationship between mathematical description and mechanistic reasoning when explaining the following mechanistic elements: *rotation* (29%), *constraint via the fixed pivot* (37%), as well as *tracing* (76%) the system from input to output (Table 4). These individuals were also able to trace these systems from input to output. This relationship is not surprising because of the correspondence these mechanistic elements have with the geometry of circles. For instance, all points on a lever trace out rotary paths that are equidistant from the fixed pivot, generating circles.

Drawing upon these representational relations seems to be implicated in *tracing* (Table 5). On items where participants traced, they mathematically described the system of levers. When participants traced, they referenced *circular paths* (80%), the *center of the circle* (57%), *circumference–radius relations* (38%), as well as other relations (7%). Describing the relations between mathematical objects and magnitudes is implicit in tracing. For example, if participants reference a circumference–radius relationship within this system, this suggests an understanding that the lever will move in a rotary path (*rotation*), that that path will be generated by the "fixedness" of the fixed pivot (*center of circle*), and suggests a causal link between the fixed pivot constraint and the rotary path (*rotation*, *constraint via the fixed pivot*). Having the proclivity to represent the system mathematically so as to see the relations that undergird the relationships between the mechanisms enables one to causally trace the system from input to output. System tracing is productive when an individual is diagnosing mechanisms in systems where forces are transmitted through visible components, such as those featured in mechanics and engineering.

Mathematical Description is Not Simply a Support

This work takes the perspective that representational resources are not simply peripheral supports (i.e., "scaffolds"); these meditational means are inseparable from the act of reasoning (Wertsch, 1998). The construction and development of representational infrastructures (Hall & Greeno, 2008) (e.g., from mathematics) is requisite within disciplined practice across STEM fields.

This study illustrates the importance of mathematical description in the development of mechanistic reasoning. Previous work on mechanistic reasoning (Bolger et al., 2012; Kobiela et al., 2011; Lehrer & Schauble, 1998; Metz, 1991; Shultz, 1982) and representational description (Hall & Greeno, 2008; Hall, Stevens, & Torralba, 2002; Lynch & Macbeth, 1998) has not considered these constructs together as important co-constituted elements of reasoning towards conceptual change.

diSessa and colleagues (diSessa, 2004; diSessa, Hammer, Sherin, & Kolpakowski, 1991) have been concerned with meta-representational competence, the proclivity to construct, revise, critique, and appropriate representational resources required in making sense of phenomena in the designed or natural world. Work in the history and philosophy of science makes it evident that the invention and development of representations constitutes an integral component of scientific practice (Bazerman, 1988; Brush, 1989; Daston & Galison, 2007; Dear, 2006; Giere, 1988; Gooding, 1990; Goodwin, 1994, 2000; Kaiser, 2000; Knorr-Cetina, 1999; Kuhn, 2012; Latour, 1999; Latour & Woolgar, 1986; Nersessian, 2008; Pickering, 2010). For example, graphing and Cartesian coordinates integrate data presentations from mathematics with functional relations that exist in science, technology, and engineering (e.g., biology, ecology, physics). Thus, it will be important to attend to how students invent and deploy mathematical representations of natural and designed systems. How are these mathematical representations chosen and how are they utilized? For instance, in this study a majority of mathematical description focused on representations that were literal analogues of the system (e.g., *circular paths*). In order for students to begin to develop dispositions towards system tracing, teachers must first support mathematical descriptions that draw relations between the physical system and these literal analogues. However, the further development of these competencies requires that other more suitable mappings be employed (e.g., representational systems, syntactic models, etc.; Lehrer & Schauble, 2006).

The Development of Mathematical Description and Mechanistic Reasoning

The results of this study show that the propensity to mathematically describe mechanical systems and reason mechanistically is not systematically developed in schooling to assist adults (high school aged and older). For instance, in this study 78% (n = 87) of all the participants failed to trace from input to output on all items; in addition, no participant managed to trace on every item. The assessment has characterized mechanistic reasoning and mathematical description about simple levered systems, across age and life experiences within a diverse sample. The ability to trace was not dependent on age, gender, or socio-economic status. When the data were disaggregated, participants with experiences in engineering through their schooling (at least one year) showed significantly greater capacities to mathematically describe and coordinate machine mechanisms, indicating the importance of systematically developing these learning experiences throughout K–12 education. Further data on the impact of engineering training on mathematical description and mechanistic reasoning are extensively detailed in another paper (Weinberg, in process).

Mathematical Description, Mechanistic Reasoning, and STEM Education

When focusing on mathematics learning within STEM integration, the selection of content could more frequently be made based on its capacity to be described mathematically. Systems of levers are particularly good candidates for mathematical description because of their analogue to the mathematics of circles. In addition, it is essential that curriculum designers are mindful of highlighting the potential mappings between physical systems and analogue mathematical representations so they are accessible for educators and students. For instance, Lehrer and Penner (2013) supported K–3 students in the production of mathematical representations and models of local ecosystems in order to support them in reasoning mechanistically about ecosystem composition and function.

In order to support STEM integration using mathematical description, K–12 teachers should be provided with appropriate supports. For instance, teachers must have sufficient knowledge of the target STEM domain, mathematics, as well as strong knowledge of teaching within these domains. The classroom teacher must be able to anticipate what mathematical analogues students will likely make, understand the value in supporting particular analogues over others, and effectively facilitate learning through these analogues.

References

Bazerman, C. (1988). *Shaping written knowledge: The genre and activity of the experimental article in science* (p. 59). Madison,WI: University of Wisconsin Press.

Bolger, M., Kobiela, M., Weinberg, P. J., & Lehrer, R. (2012). Analysis of children's mechanistic reasoning about linkages and levers in the context of engineering design. *Cognition and Instruction*, *30*(2), 170–206. http://doi.org/10.1080/07370008.2012.661815

Brush, S. G. (1989). Prediction and theory evaluation: The case of light bending. *Science*, *246*(4934), 1124–1129. http://doi.org/10.1126/science.246.4934.1124

Daston, L., & Galison, P. (2007). *Objectivity*. New York, NY: Zone Books.

Dear, P. (2006). *The intelligibility of nature: How science makes sense of the world*. Chicago, IL: University of Chicago Press. http://doi.org/10.7208/chicago/9780226139500.001.0001

diSessa, A. A. (1988). Knowledge in pieces. In G. Forman & P. B. Pufall (Eds.), *Constructivism in the computer age* (pp. 49–70). East Sussex, UK: Psychology Press.

diSessa, A. A. (1993). Toward an epistemology of physics. *Cognition and Instruction*, *10* (2&3), 105–225. http://doi.org/10.1080/07370008.1985.9649008

diSessa, A. A. (2004). Metarepresentation: Native competence and targets for instruction. *Cognition and instruction*, *22*(3), 293–331. http://doi.org/10.1207/s1532690xci2203_2

diSessa, A. A., Hammer, D., Sherin, B., & Kolpakowski, T. (1991). Inventing graphing: Meta-representational expertise in children. *Journal of Mathematical Behavior*, *10*(2), 117–160.

Giere, R. (1988). *Explaining science*. Chicago, IL: University of Chicago Press. http://doi.org/10.7208/chicago/9780226292038.001.0001

Gooding, D. W. (1990). *Experiment and the making of meaning* (Vol. 5). New York, NY: Springer Science & Business Media. http://doi.org/10.1007/978-94-009-0707-2

Goodwin, C. (1994). Professional vision. *American Anthropologist*, *96*(3), 606–633. http://doi.org/10.1525/aa.1994.96.3.02a00100

Goodwin, C. (2000). Action and embodiment within situated human interaction. *Journal of pragmatics*, *32*(10), 1489–1522. http://doi.org/10.1016/S0378-2166(99)00096-X

Hall, R., & Greeno, J. G. (2008). Conceptual learning. In T. Good (Ed.), *21st century education: A reference handbook* (pp. 212–221). London, UK: Sage Publications. http://doi.org/10.4135/9781412964012.n23

Hall, R., Stevens, R., & Torralba, T. (2002). Disrupting representational infrastructure in conversations across disciplines. *Mind, Culture, and Activity*, *9*(3), 179–210.

Honey, M., Pearson, G., & Schweingruber, H. (Eds.). (2014). *STEM integration in K–12 education: Status, prospects, and an agenda for research*. Washington, DC: National Academies Press.

Kaiser, D. (2000). Stick-figure realism: Conventions, reification, and the persistence of Feynman diagrams, 1948–1964. *Representations*, 49–86.

Knorr-Cetina, K. D. (1999). *Epistemic cultures: Forms of reason in science*. Cambridge, MA: Harvard University Press.

Kobiela, M., Bolger, M., Weinberg, P., Rouse, R., & Lehrer, R. (2011). Mathematization and embodiment for reasoning about mechanism within an engineering curriculum. Paper presented at the Annual Meeting of the Jean Piaget Society, Berkeley, CA.

Kolen, M. J., & Brennan, R. L. (2013). *Test equating: Methods and practices*. New York, NY: Springer Science & Business Media.

Kuhn, T. S. (2012). *The structure of scientific revolutions*. Chicago, IL: University of Chicago Press. http://doi.org/10.7208/chicago/9780226458144.001.0001

Latour, B. (1999). *Pandora's hope: Essays on the reality of science studies*. Cambridge, MA: Harvard University Press.

Latour, B., & Woolgar, S. (1986). *Laboratory life: The construction of scientific facts*. Princeton, NJ: Princeton University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press. http://doi.org/10.1017/CBO9780511815355

Lehrer, R., & Penner, E. (2013). Supporting the development of models and modeling.Invited paper/workshop presented at STEM Smart: Lessons Learned from Successful Schools, National Science Foundation, Washington, DC.

Lehrer, R., & Schauble, L. (1998). Reasoning about structure and function: Children's conceptions of gears. *Journal of Research in Science Teaching*, *35*(1), 3–25.

Lehrer, R., & Schauble, L. (2006). Cultivating model-based reasoning in science education. In R. Keith Sawyer (Ed.), *The Cambridge handbook of the learning sciences*. London, UK: Cambridge University Press.

Lehrer, R., Strom, D., & Confrey, J. (2002). Grounding metaphors and inscriptional resonance: Children's emerging understanding of mathematical similarity. *Cognition and Instruction*, 20, 345–398. http://doi.org/10.1207/S1532690XCI2003_3

Lynch, M., & Macbeth, D. (1998). Demonstrating physics lessons. In J. Greeno & S. Goldman (Eds.), *Thinking practices in mathematics and science learning* (pp. 269–297). Mahwah, NJ: Lawrence Erlbaum Associates.

Metz, K. E. (1991). Development of explanation: Incremental and fundamental change in children's physics knowledge. *Journal of Research in Science Teaching*, *28*(9), 785–797. http://doi.org/10.1002/tea.3660280906

Nathan, M. J., Tran, N., Phelps, L. A., & Prevost, A. (2008). The structure of high school academic and pre-engineering curricula: Mathematics. *Proceedings of the American Society of Engineering Education Annual Conference and Exposition*. Pittsburgh, PA.

National Academy of Engineering and National Research Council. (2014). *STEM Integration in K-12 Education: Status, Prospects, and an Agenda for Research*. Washington, DC: The National Academies Press.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Research Council (NRC). (2007). *Rising above the gathering storm: Energizing and employing America for a brighter economic future*. Washington, DC: National Academies Press.

Nersessian, N. (2008). *Creating scientific concepts*. Cambridge, MA: MIT Press.

Pickering, A. (2010). *The mangle of practice: Time, agency, and science*. Chicago, IL: University of Chicago Press.

Prevost, A., Nathan, M. J., Tran, N., & Phelps, L. A. (2009). Integration of mathematics in pre-engineering: The search for explicit connections. *Proceedings of the American Society of Engineering Education (ASEE) 2009*, Austin, TX.

Russ, R. S., Scherr, R. E., Hammer, D., & Mikeska, J. (2008). Recognizing mechanistic reasoning in student scientific inquiry: A framework for discourse analysis developed from philosophy of science. *Science Education*, *92*(3), 499–525. http://doi.org/10.1002/sce.20264

Smith, C. L. (2007). Bootstrapping processes in the development of students' commonsense matter theories: Using analogical mappings, thought experiments, and learning to measure to promote conceptual restructuring. *Cognition and Instruction*, *25*(4), 337–398. http://doi.org/10.1080/07370000701632363

Shultz, T. R. (1982). Rules of causal attribution. *Monographs of the Society for Research in Child Development*, *47*, 1–51. http://doi.org/10.2307/1165893

Stevens, R., & Hall, R. (1998). Disciplined perception: Learning to see in technoscience. In M. Lambert & M. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 107–149). Cambridge, UK: Cambridge University Press. http://doi.org/10.1017/CBO9780511571251.007

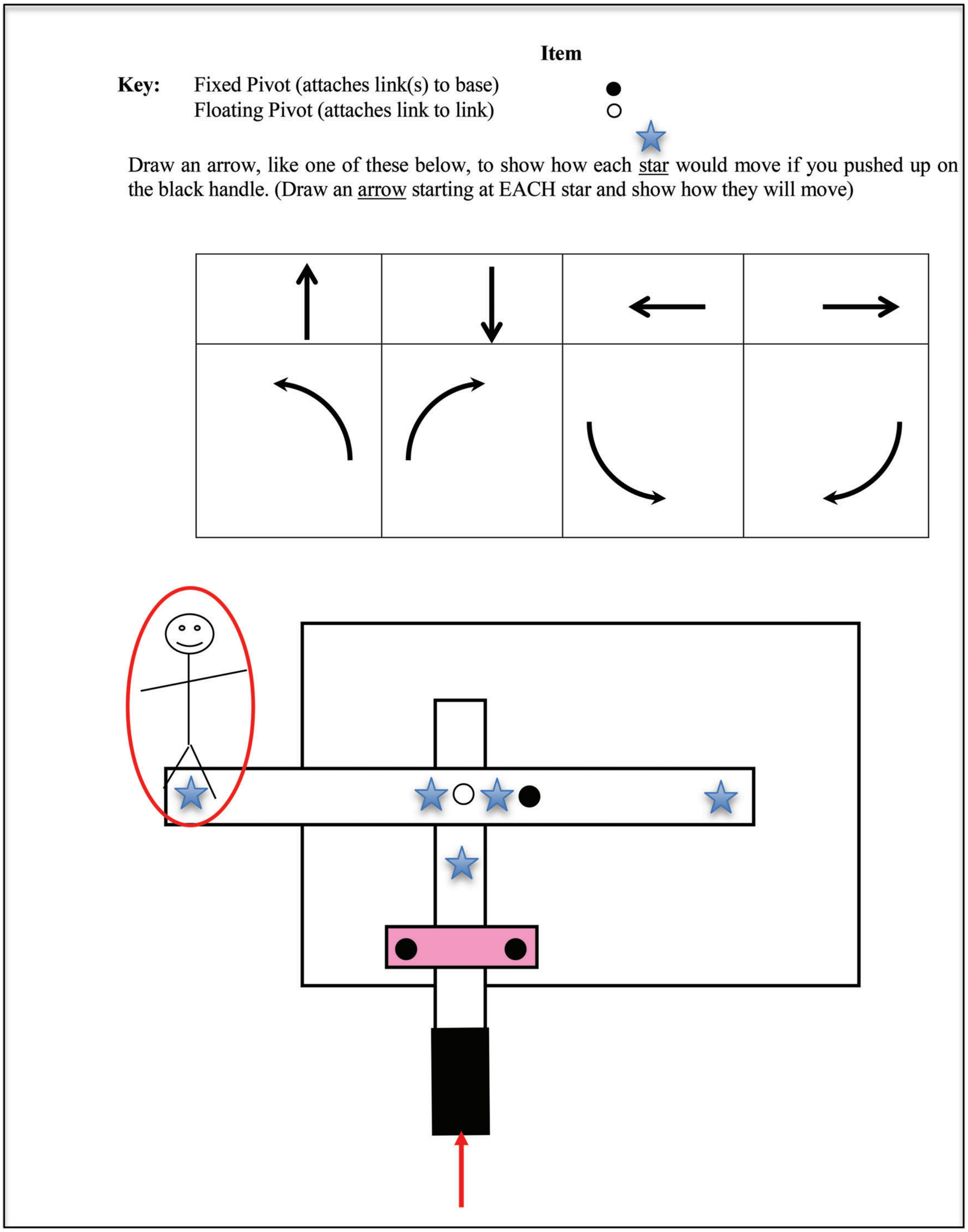
Weinberg, P. J. (2012). *Assessing mechanistic reasoning: Promoting system thinking*. (Doctoral dissertation). Retrieved from Electronic Theses and Dissertations (etd-07112012-004845).

Welty, K., Katehi, L., & Pearson, G. (2008). Analysis of K-12 engineering education curricula in the United States—A preliminary report. *Proceedings of the American Society for Engineering Education Annual Conference and Exposition*. Pittsburgh, PA.

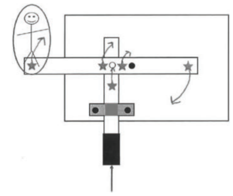
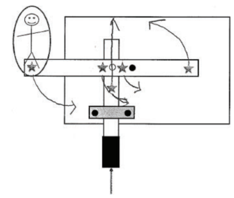
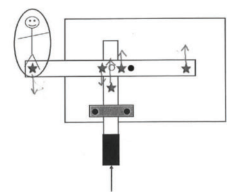
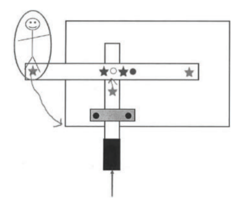
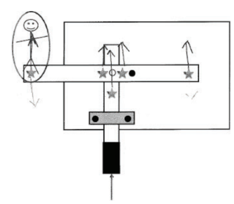
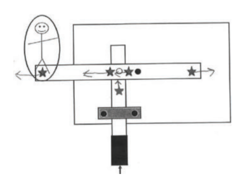
Wertsch, J. V. (1998). *Mind as action*. Oxford, UK: Oxford University Press.

APPENDIX

Appendix A. Item Sequential Tracing A1 (STA1).



Exemplar for Sequential Tracing A1 (STA1).

Level	Mechanistic element	Mechanistic element descriptions	Mechanistic element example
5	Tracing	Participant diagnoses all mechanistic elements (without gaps) from input to output	
4	Constraint via the fixed pivot	Participant draws the opposite motion of the two closest points on opposite sides of the fixed pivot	
3	Lever arms	Participant draws arrows with opposite directions from stars on opposite sides of a lever's arms	
2	Rotation	Participant draws arced paths. However, the location of these paths must reasonably approximate fractions of circles either centered around the fixed or floating pivot[a]	
1	Related direction	Participant draws the coordinated input/output motion	
0	Student diagnoses no mechanistic elements	No mechanistic elements are shown	
NL	No link	It is not clear if the participant understood the nature of the task	"I don't know"
M		Missing response	

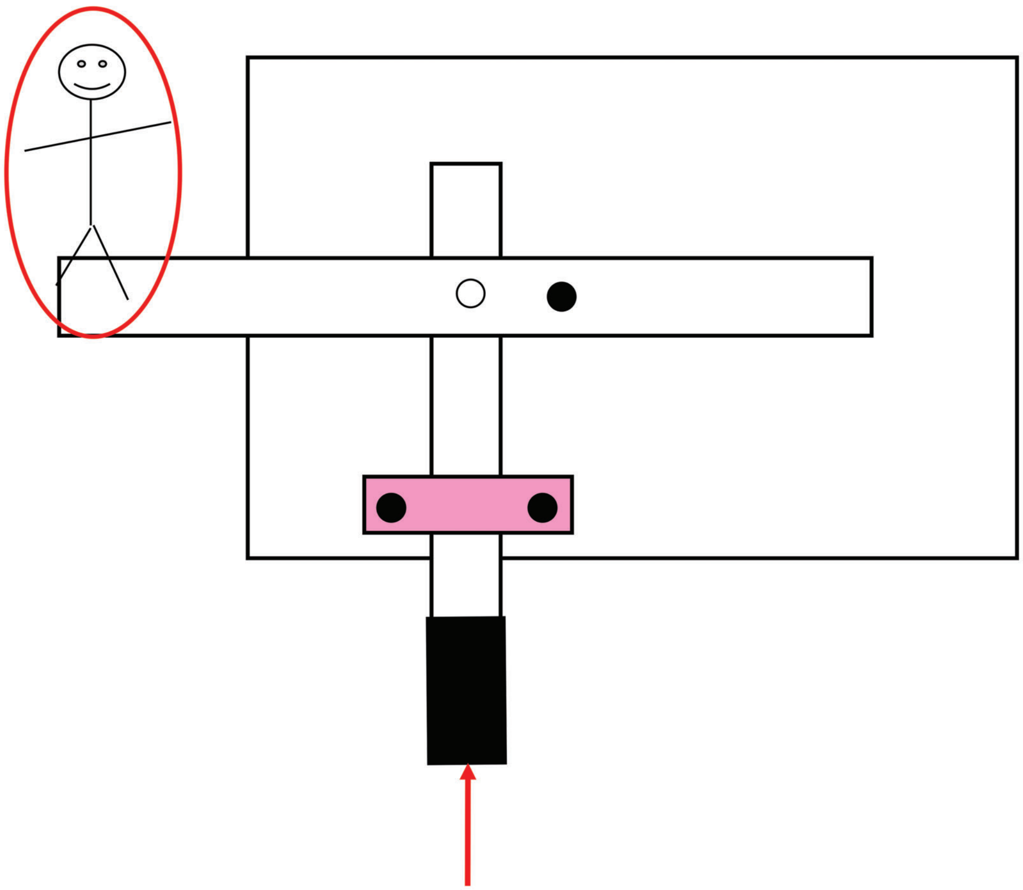
Note. This item assesses a participant's propensity to diagnose the mechanistic elements of *related direction*, *rotation*, *lever arms*, and *constraint via the fixed pivot* as well as *tracing*. No link (NL) indicates an item response does not indicate that the participant understands the nature of the task, while "No elements" provides any evidence of mechanistic reasoning. "Missing" indicates that the item was left completely blank. The "stars" have been placed on the levers to allow participants to indicate lever motion. A "little person" has been included on the output lever to make the system output salient.
[a]Although these paths are centered around the fixed pivot, this element of mechanistic reasoning does not make this distinction.

Appendix B. Machine Prediction A1 (MPA1).

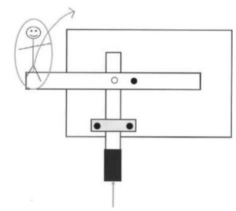
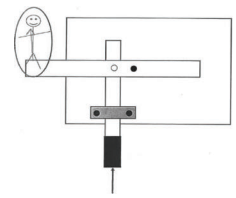
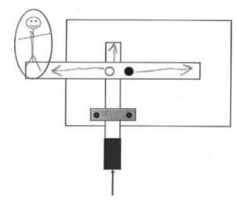
Item

Key: Fixed Pivot (attaches link(s) to base) ●
Floating Pivot (attaches link to link) ○

For the machine below, draw an arrow showing how each little person would move if you PUSHED UP on the black handle (just like the arrow shows). (Draw an <u>arrow</u> starting at each person and show how they will move)



Exemplar for Machine Prediction A1 (MPA1).

Level	Mechanistic element	Mechanistic element descriptions	Mechanistic element example
2	Rotation	Participant draws arced paths. However, the location of these paths must reasonably approximate fractions of circles either centered around the fixed or floating pivot[a]	
1	Linked direction	Participant draws the coordinated input/output motion	
0	No elements	No mechanistic elements are shown	
NL	No link	It is not clear if the participant understood the nature of the task	"I don't know"
M	Missing	Missing response	

Note. This item assesses a participant's propensity to diagnose the mechanistic elements of *related direction* and *rotation*. No link (NL) indicates an item response does not indicate that the participant understands the nature of the task, while "No elements" provides any evidence of mechanistic reasoning. "Missing" indicates that the item was left completely blank. A "little person" has been included on the output lever to make the system output salient.
[a]Although these paths are centered around the fixed pivot, this element of mechanistic reasoning does not make this distinction.

http://dx.doi.org/10.7771/2157-9288.1124