# ADAPTIVE FUZZY CONTROL DESIGN FOR THE MOLTEN STEEL LEVEL IN A STRIP CASTING PROCESS

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This paper studies the adaptive fuzzy control problem of the molten steel level for a class of twin roll strip casting systems. Based on fuzzy logic systems (FLSs) and the mean value theorem, a novel adaptive tracking controller with parameter updated laws is effectively designed. It is proved that all the closed-loop signals are uniformly bounded and the system tracking errors can asymptotically converge to zero by using the Lyapunov stability analysis. Simulation results of semi-experimental system dynamic model and parameters are provided to demonstrate the validity of the proposed adaptive fuzzy design approach.

Key words: strip, casting process, molten steel level control, fuzzy logic systems (FLSs), dynamic model

### INTRODUCTION

Due to the rapid development of machine industry, the demand of thin steel strips combining two processes of continuous casting and hot rolling into a single production has received widely attention in recent years. Correspondingly, it can bring in many advantages including lower investment cost, energy saving, less space requirements and so on. For the continuous casting technique, a lot of successful control methods [1-3] have been investigated. In [4], adaptive neural network control scheme was proposed for the molten steel level control of strip casting processes based slide-mode technique. By using the perturbation method, a decoupling control algorithm in [5] was designed to obtain a uniform sheet thickness and keep a constant roll separating force in the strip casting process.

As is well known, the model of twin roll strip casting system is nonlinear, and its controller design process is very difficult. Meanwhile, a few of robust fuzzy tracking control strategies [6,7] are developed for a class of nonlinear systems. In [8], the asymptotic tracking control problem was considered for a class of nonlinear systems with actuator nonlinearities by introducing approximate adaptive laws with positive integrable time-varying functions. On the basis of this, the authors in [9] further studied the actuator saturation constrain problem of adaptive fuzzy tracking control is considered for a class of uncertain nonaffine nonlinear timedelayed systems. Especially, inspired by [9], by utilizing fuzzy approximation technique and the implicit function theorem, a novel fuzzy tracking controller with adaptation laws is designed for the molten steel level of twin roll strip casting process in this paper. Moreover, it is shown that the state signal of the molten steel level can asymptotically converge to the desired reference signal via the Lyapunov stability analysis.

# SYSTEM MODEL OF THE STRIP CASTING PROCESS

## Molten metal level equation

Here, similar to (1)-(4) in [4], the mathematical model for the molten steel leveling dynamics can be described as follows

$$\frac{dy}{dt} = F^{-1}(x_g, y)L^{-1}(ku - Lx_gv - Ly\frac{dx_g}{dt})$$
(1)

Where *y* is the height of molten metal, *L* is the length of the roll cylinders, *u* is the electric servomotor control, k(t) is the corresponding control gain,  $\nu$  is the roll surface tangential velocity, and  $F(x_g, y) = x_g + 2R - \sqrt{R^2 - y^2}$  with *R* being the roll radius.

Then, by introducing the coordinate transformations  $x_1 = y$ ,  $x_2 = \frac{dy}{dt}$  and applying Newton's Second Law, it follows from (1) that the following SISO nonaffine non-linear system can be obtained

$$\begin{aligned} x_1 &= x_2 \\ \dot{x}_2 &= \tau u + f(x, x_g, \dot{x}_g, \ddot{x}_g, u) \end{aligned}$$
 (2)

Where  $x = [x_1, x_2]^T$  stands for the state variable,  $f(x, x_g, \dot{x}_g, \ddot{x}_g, u) = \frac{d^2 y}{dt^2}$  is the unknown and smooth nonaffine nonlinear function, and  $\tau > 0$  is the proper control gain parameter.

The objective of this paper is to design an adaptive fuzzy controller *u* such that all the closed-loop error signals are uniformly ultimately bounded, and the system state *x* can asymptotically track a reference trajectory  $x_d = [y_d, \dot{y}_d]^T$  with  $y_d$  being a desired signal. Then, to

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achieve this control objective, the necessary assumptions are required for the nonaffine nonlinear system (2).

Assumption 1.([2]) The desired reference trajectory  $y_d$  is known and smooth, and its first and second order derivatives are also continuous. That is, there exist unknown positive constants  $\overline{d}$ ,  $\overline{d}$  and  $\overline{d}$  such that  $|y_d(t)| \le \overline{d}$ ,  $|\dot{y}_d(t)| \le d$ , and  $|\ddot{y}_d| \le d$ , respectively.

Assumption 2. ([9]) For a given compact set  $\Omega_0$ , there always exist positive constants  $F_1$  and  $F_2$  such that the following inequality holds

$$0 < F_1 \le \frac{\partial f(x, u)}{\partial u} \le F_2, \forall (x, u) \in \Omega_0$$
(3)

Similar to [7], the optimal parameter vectors  $\theta_0$  of FLS is defined as

$$\theta^* = \arg\min_{\theta \in \Omega_{\theta}} [\sup_{X \in \Omega_X} \left| F(X) - \theta^T \phi(X) \right|]$$
(4)

Where  $\Omega_0$  and  $\Omega_X$  are compact regions for  $\theta$  and X, respectively. Furthermore, the optimal parameter vector  $\theta^*$  is defined as

$$F(X) = \theta^{*T} \phi(X) + \delta^{*}(X), \forall X \in \Omega_X \subset \mathbb{R}^n$$
 (5)

# ADAPTIVE FUZZY TRACKING CONTROLLER DESIGN AND STABILITY ANALYSIS Adaptive tracking controller design

In this subsection, the adaptive fuzzy asymptotic tracking control scheme will be developed for the molten steel leveling dynamics system (2). Consequently, taking the time derivative of the tracking error  $e = x - x_d$  with respect to *t* yields

$$\dot{e} = Ae + B(\tau u + f(x, x_g, \dot{x}_g, \ddot{x}_g, u) - \ddot{y}_d)$$
 Where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
(7)

Thus it follows from (7) that A + BL is a stable matrix by approximately choosing a gain vector *L*. Furthermore, these exists a  $P = P^T > 0$  such that the Lyapunov equation  $(A + BL)^T P + P(A+BL) = -Q$  holds for any a given  $Q = Q^T > 0$ . At the same time, by using the mean value theorem for the nonaffine term f(X, u) with  $X = [x, x_g, \dot{x}_g, \ddot{x}_g]^T$ from Assumption 2, one can obtain that  $f(X, u) = f(X, 0) + \frac{\partial f(X, u_\lambda)}{\partial u} u$  with  $u_\lambda = \lambda u$  and  $0 < \lambda < 1$ .

In addition, by invoking the expression in (5), it is easy to see that

$$f(X,u) = f(X,0) + \frac{\partial f(X,u_{\lambda})}{\partial u}u$$
  
=  $ku + \Gamma(X,u)$  (8)  
=  $ku + \theta^{*T}\xi(X) + \delta^{*}(X), \forall X \in \Omega_{X}$ 

where  $\Gamma(X, u) = (\frac{\partial f(X, u_{\lambda})}{\partial u} - k)u + f(X, 0)$  with *k* being a positive design parameter. The approximation error  $\delta^*(X)$  satisfies  $|\delta^*(X)| \leq \delta^*$  with  $\delta^*$  being an any

small positive constant, and  $\Omega_x$  is a proper compact set. Noting Assumption 1, denote  $M^* = \sup_{t \ge 0} (|\ddot{y}_d(t)| + \delta^*)$ , where  $M^*$  is an unknown upper bound. Then, the adaptive fuzzy controller is given by

$$u = (k+\tau)^{-1} (Le - \hat{\theta}^T \xi(X) - \frac{2M^2 B^T P e}{\left| e^T P B \right| \hat{M} + \rho(t)}$$
(9)

with the corresponding parameter updated laws

$$\hat{\theta}^{=} - \sum \rho \theta + \sum e^{T} P B \xi(X),$$

$$\dot{\hat{M}}^{=} - \gamma \rho M + 2\gamma \left| B^{T} P e \right|$$
(10)

Where  $\hat{M}$  and  $\hat{\theta}$  are the estimates of  $M^*$  and  $\theta^*$ , respectively,  $\sum = \sum^T > 0$  and  $\gamma$  are positive design parameters. In addition, the continuous function  $\rho(t)$  is subject to  $\rho(t) > 0$  and  $\int_0^t \rho(\tau) d\tau \le \overline{\rho} < \infty, \forall t \ge 0$  with  $\overline{\rho}$  being a positive constant.

#### **Stability analysis**

In this subsection, the stability of the resulting closed-loop system is given in the following theorem.

**Theorem 1.** Consider the molten steel leveling dynamics system (2) satisfying Assumptions 1 and 2. By designing adaptive fuzzy controller (9) and parameter updated laws (10), the tracking error e(t) of the closedloop system can asymptotically converge to zero, that is,

$$\lim_{X \to \infty} e(t) = 0, \forall X \in \Omega_X$$
(11)

Where  $X = [\dot{x}, \ddot{x}_g, \dot{x}_g, \ddot{x}_g]'$  and  $\Omega_x$  is a proper compact set.

**Proof:**Define the Lyapunov function  $V(e, \tilde{\theta}, \tilde{M})$  as

$$V(e,\tilde{\theta},\tilde{M}) = e^{T}Pe + \frac{1}{2}\gamma^{-1}\tilde{M}^{2} + \frac{1}{2}\tilde{\theta}^{T}\Sigma^{-1}\tilde{\theta} \quad (12)$$

Where  $\tilde{M} = \hat{M} - M^*$  and  $\tilde{\theta} = \hat{\theta} - \theta^*$  are the parameter estimation errors.

The derivative of V is

(6)

$$\dot{V} = e\dot{e} + \gamma^{-1}\tilde{M}\dot{\tilde{M}} + \tilde{\theta}^{T} \sum^{-1} \dot{\tilde{\theta}}$$

$$= 2e^{T}P(Ae + B(\tau u + f(X, u) - \ddot{y}_{d})) \qquad (13)$$

$$+ \gamma^{-1}\tilde{M}\dot{\tilde{M}} + \tilde{\theta}^{T} \sum^{-1} \dot{\tilde{\theta}}$$

From (8), substituting (9) into (13) yields

$$\dot{V} = 2e^{T}P(Ae + B((k + \tau)u + \theta^{*T}\xi(X) + \delta^{*}(X))) + \gamma^{-1}\tilde{M}\dot{\tilde{M}} + \tilde{\theta}^{T}\sum^{-1}\dot{\tilde{\theta}}$$

$$\leq -e^{T}Qe - 2e^{T}PB\tilde{\theta}^{T}\xi(X) + 2\left|e^{T}PB\right|M^{*} + \gamma^{-1}\tilde{M}\dot{\tilde{M}} + \tilde{\theta}^{T}\sum^{-1}\dot{\tilde{\theta}}$$
(14)

Invoking the adaptive control laws (10) and triangle inequality, (14) becomes

$$\dot{V} \leq -e^{T}Qe - \frac{2M^{2}\left|e^{T}PB\right|^{2}}{\left|e^{T}PB\right|\hat{M} + \rho(t)} + 2\left|e^{T}PB\right|\hat{M}$$

$$-\rho\tilde{M}\hat{M} - \rho\tilde{\theta}^{T}\hat{\theta} \qquad (15)$$

$$\leq -e^{T}Qe + 2\rho + \frac{1}{2}\rho M^{*2} + \frac{1}{2}\rho \left\|\theta^{*}\right\|^{2}$$

$$\leq -e^{T}Qe + \rho\eta^{*}$$

Where  $\eta^* = \frac{1}{2}M^{*2} + \frac{1}{2}\|\theta^*\|^2 + 2$ . Integrating (15) from 0 to *t* gives

$$V(t) + \int_0^t e^T(\tau) Q e(\tau) d\tau \le V \Big|_{t=0} + \overline{\rho} \eta^*$$
(16)

which implies that  $\int_{0}^{t} e^{T}(\tau) e(\tau) d\tau \leq \{\lambda_{\min}(Q)\}^{-1}(V|_{t=0} + \overline{\rho}\eta^{*}), \text{ Where } \lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of a matrix. Furthermore, it can be concluded that  $e \in L_2$  and  $\lim_{t\to\infty} e(t) = 0$ by using the Barbalat's lemma [10].

This completes the proof.

#### SIMULATION STUDIES

In this section, the following numerical simulation cited in [5] is given for SISO nonaffine nonlinear system (2). The corresponding system parameters are selected as R=150 mm, L=200 mm, v=10 mpm. In addition, the desired roll gap is set to be 3 mm, the initial molten steel level is set to be 70 mm, and the reference signal is chosen as sin(t) + cos(0.5 t), respectively. Furthermore, the fuzzy membership functions are chosen as follows  $(V + 1.5)^2$  $(V_{1}, 1)^{2}$ 

$$\mu_{F_{j}^{1}} = \exp[-\frac{(X_{j}+1,5)^{2}}{2}], \mu_{F_{j}^{2}} = \exp[-\frac{(X_{j}+1)^{2}}{2}],$$

$$\mu_{F_{j}^{3}} = \exp[-\frac{(X_{j}+0,5)^{2}}{2}], \mu_{F_{j}^{4}} = \exp[-\frac{X_{j}^{2}}{2}],$$

$$\mu_{F_{j}^{5}} = \exp[-\frac{(X_{j}-0,5)^{2}}{2}], \mu_{F_{j}^{6}} = \exp[-\frac{(X_{j}-1)^{2}}{2}],$$

$$\mu_{F_{j}^{7}} = \exp[-\frac{(X_{j}-1,5)^{2}}{2}], j = 1, 2, \cdots, n.$$
(17)

Define fuzzy basis functions as

$$\phi_i(X) = \frac{\prod_{j=1}^n \mu_{F_j^i}(X_j)}{\sum_{i=1}^7 [\prod_{j=1}^n \mu_{F_j^i}(X_j)]}, i = 1, 2, \cdots, 7.$$
(18)

Where  $X = [X_1, X_2, \dots, X_n]^T$ . The simulation parameters are chosen as  $\tau = 6$ ,  $k = 2, \gamma = 0, 1, \Sigma = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \rho(t) = 5e^{-0, 1t}$ , and the initial values are selected as  $x(0) = [0,07, 0,8]^T$ ,  $\hat{\theta}(0) = [0,1, 0,1]^T$ 

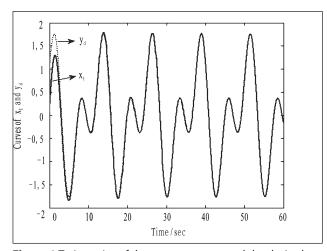
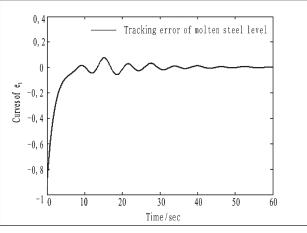
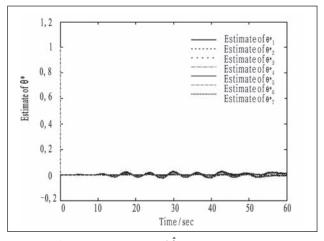


Figure 1 Trajectories of the system state x, and the desired reference signal y<sub>d</sub>

0,1,0,1,0],  $\hat{\mathbf{M}}(0) = 10$ . By choosing  $Q = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} > 0$ , it is to obtain from  $(A - BL)^T P + P(A+BL) = -Q$  that  $\frac{1}{2}$ . The simulation results are obtained in Fig-P =ures 1-5. From Figure 1 and Figure 2, it can be seen that the system state  $x_1$  and the reference signals  $y_d$ . It is easy to see that the state tracking error  $e_1$  of the molten steel level can converge to zero. The boundedness of estimations curves  $\hat{\theta}$  and  $\hat{M}$ , as well as the control signal u are demonstrated in Figures 3-5.



**Figure 2** Trajectories of the state tracking error  $e_1$ 



**Figure 3** The response curves of  $\theta$ 

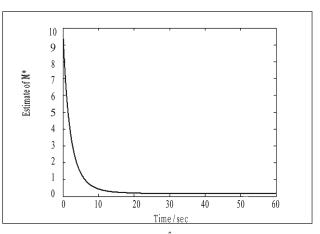


Figure 4 The response curves of M

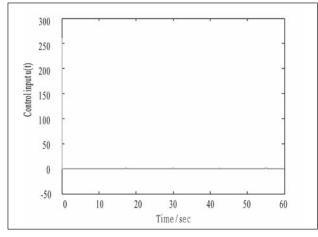


Figure 5 The respond curve of the control signal u

#### CONCLUSIONS

This paper is concerned with the adaptive fuzzy tracking control problem for twin roll strip casting process. Based on fuzzy approximation technique, a novel adaptive fuzzy tracking control scheme is proposed. In particular, the mean value theorem is applied to handle with the nonaffine term, and the parameter updated laws for the parameter regress vector and the unknown upper bound of approximation error is constructed.

Moreover, by using the designed adaptive controller, it is proved that all the closed-loop signals are bounded and the state tracking error of the molten steel level can asymptotically converge to zero via the Lyapunov stability analysis. Also, simulation results show the effectiveness of the proposed adaptive control method.

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