Aerodynamic Shape Optimization using a Full and Adaptive Multilevel Algorithm

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Abstract

We are interested by the general problem consisting of minimizing a functional of a state field solution of a PDE state equation. In Particular in this work, we optimize a 3D wing shape immersed an in inviscid flow to reduce drag. Whence, each evaluation of the cost functional is computationally expensive.

For improving the convergence rate of the optimization algorihm, we propose a *multi-scale algorithm* inspired from the *Full Multi-Grid* method [1], and referred to as the *Full and Adaptive Multi-Level Optimum-Shape Algorithm* (FAMOSA), originally defined in [5].

The proposed method include the following strategies:

- The simplest scheme "one way up " by choosing the parametrization of Bézier type to construct a hierarchy of embedded parametric spaces, via the classical *degree elevation* process [3].
- V-cycle algorithm by using (on the coarse level) " perturbation " unknowns from the latest fine estimate, i.e *deformation* instead of *shapes*.
- Parametrization adaption; from an approximate optimal shape, the parameterization is automatically adapted in order to improve the convergence rate and reach shapes of better fitness.

Numerical experiment will be presented to demonstrate the efficiency of the method. We use the free-form deformation approach for 3D tensorial Bézier paramerization [2], and the Nelder-Mead simplex algorithm for minimizing the cost functional.

1 Shape parameterization

1.1 Properties of Bézier curve

Consider a Bézier parmaterization of degree n given by the usual formula

$$P(t) = \sum_{k=0}^{n} B_n^k(t)\, P_k \tag{1}$$

in which

$$B_n^k(t) = C_n^k\, t^k\, (1-t)^{n-k} \qquad (k = 0, 1, ..., n) \tag{2}$$

are the Bernstein polynomials, which form a basis of polynomials of degree $\leq n$, and $P_k = (x_k, y_k)$ are control points.

A very important property of Bézier parameterizations, and their natural extensions (B-splines, NURBS) is related to the classical *degree-elevation process* [3]. To introduce this process in a precise manner, multiply (1) by $(1-t)+t$ one gets:

$$\begin{aligned} P(t) &= \sum_{k=0}^{n} C_n^k\, t^k\, (1-t)^{n+1-k}\, P_k + \underbrace{\sum_{k=0}^{n} C_n^k\, t^{k+1}\, (1-t)^{n-k}\, P_k}_{\sum_{\ell=1}^{n+1} C_n^{\ell-1}\, t^\ell\, (1-t)^{n+1-\ell}\, P_{\ell-1}} \\ &= \sum_{k=0}^{n+1} C_{n+1}^k\, t^k\, (1-t)^{n+1-k}\, P'_k \\ &= \sum_{k=0}^{n+1} B_{n+1}^k\, P'_k \end{aligned} \tag{3}$$

provided the following definitions are made:

$$\begin{cases} P'_0 = P_0 \\ P'_k = \dfrac{C_n^{k-1}\, P_{k-1} + C_n^k\, P_k}{C_{n+1}^k} = \dfrac{k}{n+1}\, P_{k-1} + (1 - \dfrac{k}{n+1})\, P_k \qquad (1 \leq k \leq n) \\ P'_{n+1} = P_n \end{cases} \tag{4}$$

This proves the following classical result [3]:

Theorem 1

Consider the Bézier curve defined by (1), and associated with the $n+1$ control points $P_k = (x_k, y_k)$ $(k = 0, 1, ..., n)$; the new sequence of $n+2$ points $P'_k = (x'_k, y'_k)$ $(k = 0, 1, ..., n+1)$ defined by $P'_0 = P_0$, $P'_{n+1} = P_n$, and for $1 \leq k \leq n$, by

$$\boxed{P'_k = \frac{k}{n+1}\, P_{k-1} + (1 - \frac{k}{n+1})\, P_k} \tag{5}$$

constitutes an alternate control polygon of the same geometrical curve, here viewed as a Bézier curve of degree $n+1$, and described identically as the parameter t varies.

This remarkable property, illustrated by FIG. 1 is the *building-block* of our construction embedded parameterized-shape search spaces.

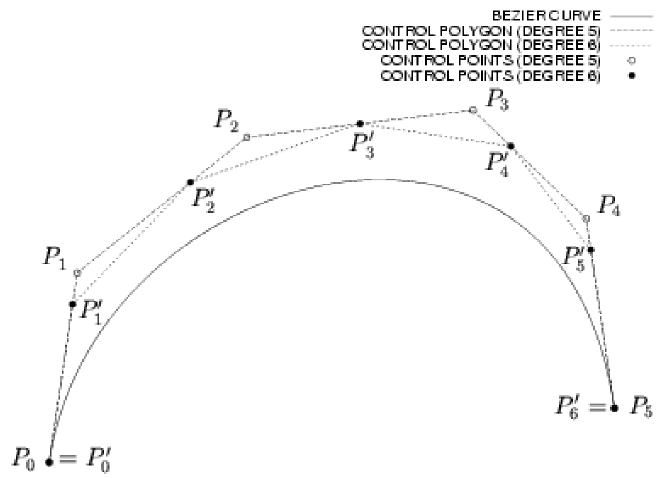

Figure 1: A Bézier curve is initially constructed from the degree-5 parameterization associated with the control polygon connecting the points $\{P_k\}$ $(k = 0, 1, ..., 5)$; an alternate control polygon is constructed thereafter by connecting the new points $\{P'_k\}$ $(k = 0, 1, ..., 6)$ obteained by convex combinations of the former; this new polygon results in a degree-6 Bézier parameterization of the same curve described identically as the parameter t varies.

1.2 Free-Form Deformation approach

One of the inconvenients of The classical Bézier representations is that they describe only smooth objects. An other alternative, which comes from Computer Graphics [2], consists to represent the deformation and not the shape itself. The free form deformation deforms the lattice that was built around the object, the object inside the lattice is deformed by using the tensorial Bézier formula for each node x of the computattional domain:

$$x(s, t, u) = x^0 + \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} B_l^i(s)\, B_m^j(t)\, B_n^k(u)\, \delta P_{ijk} \tag{6}$$

x^0 denotes the position of the node x in the original configuration.
This method has the following advantages:

- it inherits the differentiability and degree-elevation properties of Bézier curves
- can parameterize complexe configuration (e.g. a complete aircraft fuselage)
- the update of the volumic mesh is ncluded in the procedure

2 Hierachical algorithms in shape optimization

The simplest scheme is "one-way up". Its analog in the context of solving a PDE using several mesh discretizations would be: *nested iteration* also referred to as *cascadic multigrid*.

The optimization problem is solved first using a low-degree shape parameterization. This initial step is carried at moderate computational cost, because with a small number of design parameters, the process is not stiff. The converged solution is interpolated onto the support of a higher-degree parameterization. This interpolation, by virtue of our construction, is *exact. This precaution is demonstrated in a forthcoming subsection to be essential to the success of the multiscale strategy.* The optimizer is then restarted, with more design parameters, but an excellent initial guess. At convergence, another degree elevation can be made for an even finer optimal shape description.

Referring momentarily to the analogous context of hierarchical PDE solving, it is very well known today that the key to "$O(N)$ algorithms" is the V-cycle, in which the iteration is initiated, contrary to basic intuition, at the *upper* (fine) level, and *not the coarse*. This concept can be generalized to optimum-shape design, by using (on the coarse level) "perturbation" unknowns from the latest fine estimate, i.e. *deformations* instead of *shapes*. This is depicted skematically in Fig. 2. Again, in this sketch, the upward transfers are exact if the supports of the parameterizations of different degrees are constructed to be embedded.

N_2 design iterations in $\mathcal{B}_2 : \Gamma_1$ $\searrow$ N_1 design iterations in $\mathcal{B}_1 : \Gamma_1' = \Gamma_1 \oplus \delta\Gamma_1'$ $\nearrow$ N_2 design iterations in $\mathcal{B}_2 : \Gamma_1''$

Figure 2: Skematic of two-level optimization V-cycle ($\mathcal{B}_1$ and $\mathcal{B}_2$ are the search spaces associated with the coarse and fine Bézier parameterizations respectively); Γ_1, Γ_1', and Γ_1'', are shapes; $\delta\Gamma_1'$ is a free-form deformation potentially expressed *identically* in the fine ($\mathcal{B}_2$), or coarse ($\mathcal{B}_1$) parameterization provided embedded supports are used.

Finally, combining both of the "one-way up" method and V-cycle, a new method, *FMOSA, Full and Multilevel Optimum Shape Algorithm* can be defined formally analogously to the *Full Multigrid Method (FMG)*.

3 Numerical experiments of piaggio-wing shape optimization

3.1 Testcase description

- Mach Number: 0.83
- Angle of attack: 2°
- Cost function:

  The cost function reflects a drag minimization subject to the constraint of a constant lift coefficient. This constraint is weakly enforced by penalization:

$$J = \frac{C_D}{C_{D_0}} + 10^4 \cdot max\left(0, 0.999 - \frac{C_L}{C_{L_0}}\right),$$

The calculations are made using a planform provided by Piaggio and depicted on Fig. 3.

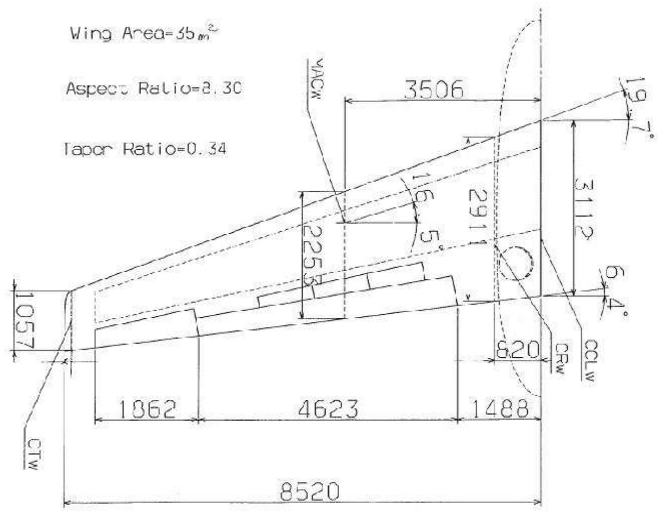

Figure 3: Planform provided by Piaggio.

Our flow solver is based on unstructered grids. The initial cross sections are homothetic to the classical NACA0012 airfoil, but this has little inference on the final results.

This mesh is deformed by the optimization process only in a region about the wing defined as the "bounding-box". This box is depicted on Fig. 4.

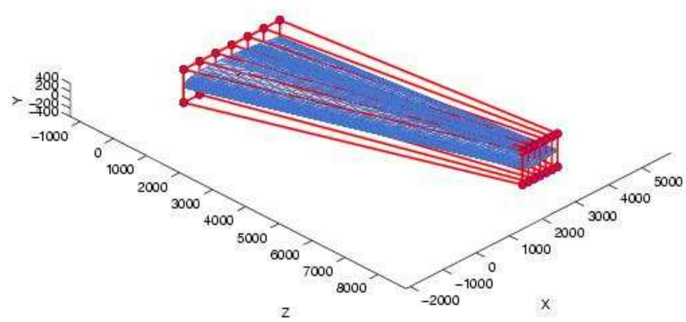

Figure 4: Bounding box in which the 3D mesh is subject to the free-form deformation.

An iteration has been constructed based on the Nelder-Mead Simplex method.

Three methods corresponding to different ways of handling the geometrical parameterization are compared:

- Basic method: single parameterization throughout the whole convergence process;
- Progressive degree elevation: here only 2 levels (coarse and fine) have been considered for simplicity;
- FMOSA: using the same 2 levels of parameterization in strategy including a V-cycle.

3.2 Aerodynamic coefficients

The aerodynamic coefficients obtained after 500 iterations of each method are compiled in Table 1. The lift coefficient is approximately maintained or slightly increased by the shape optimization process. Significant reductions in the drag coefficient are observed.

Because method C produces a much better-converged solution, as demonstrated subsequently, the drag reduction is far superior (5 % more reduction).

	Method	CL	CD	Cost
Reference		0.319201	0.026353	1.
Test A	Single Param. (9-1-1)	0.322667	0.014522	0.551042044
Test B	Degre Elev (3-1-1,9-1-1)	0.319363	0.013736	0.521216929
Test C	FMOSA (3-1-1,9-1-1)	0.319854	0.013252	0.502875191

Table 1: Aerodynamic coefficients compared to reference values as given by three methods after 500 iterations by the simplex methods

3.3 Convergence history plots

Fig. 5 depicts the convergence history of the basic method (single parameterization) for three different parameterizations. With a coarse parameterization, a very fast convergence is observed, but the converged value of the cost functional is not very small (poor accuracy). Increasing the number of geometrical parameters results in an improved accuracy at the cost of the numerically more costly computation of a greater number of iterations.

Fig. 6 depicts the convergence for the three methods under consideration. The method based on progressive degree elevation is significantly faster (more than a factor of 2). FMOSA is still more efficient. In fact, 500 iterations is insufficient to achieve full convergence in the other cases.

3.4 Flows

Fig. 7 depicts the Mach number on the wing upper surface in the original configuration, and Fig. 8 as a result of shape optimization by each method. The supersonic region has been reduced in extent, and the shock in strength.

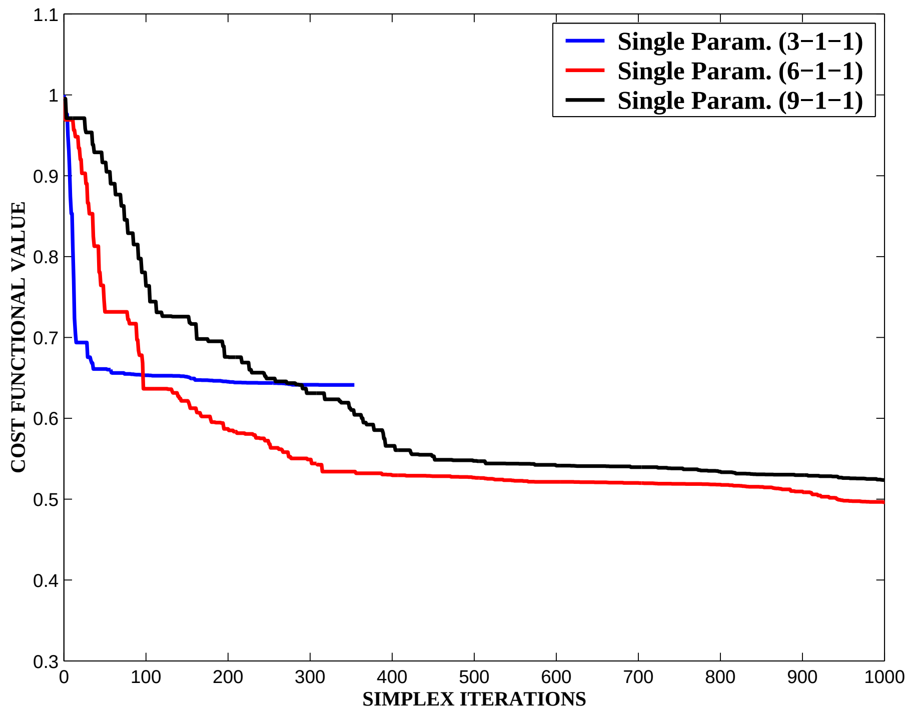

Figure 5: Basic method (single-parameterization): convergence history plot for three different parameterizations (blue: coarse; red: medium; black: fine).

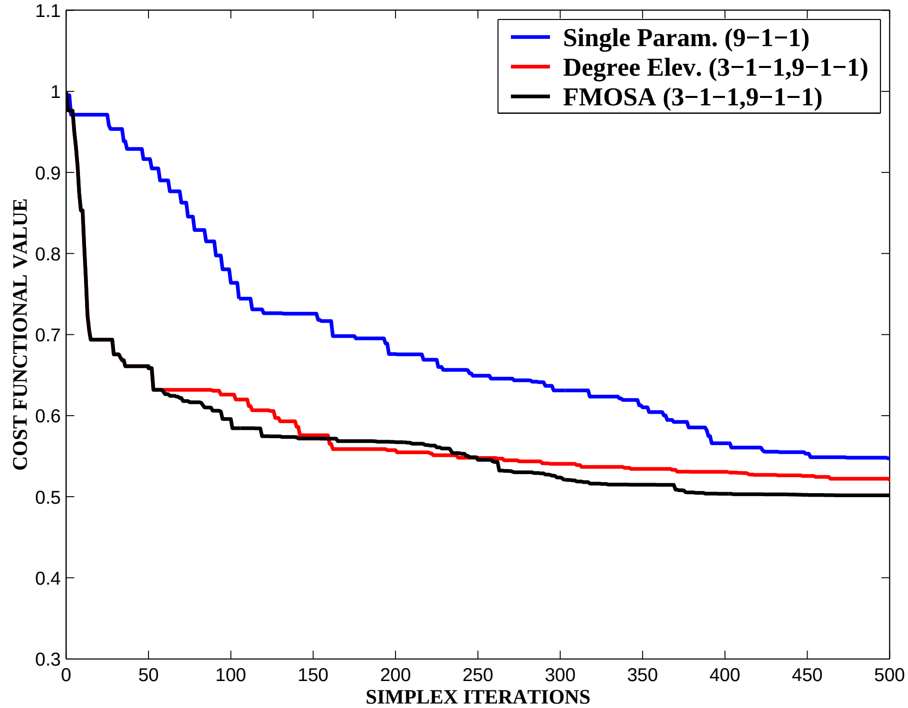

Figure 6: Comparison of three methods in terms of iterative convergence: blue: basic (single-parameterization); red: degree-elevation (2 parameterizations); black: FMOSA (2 parameterizations).

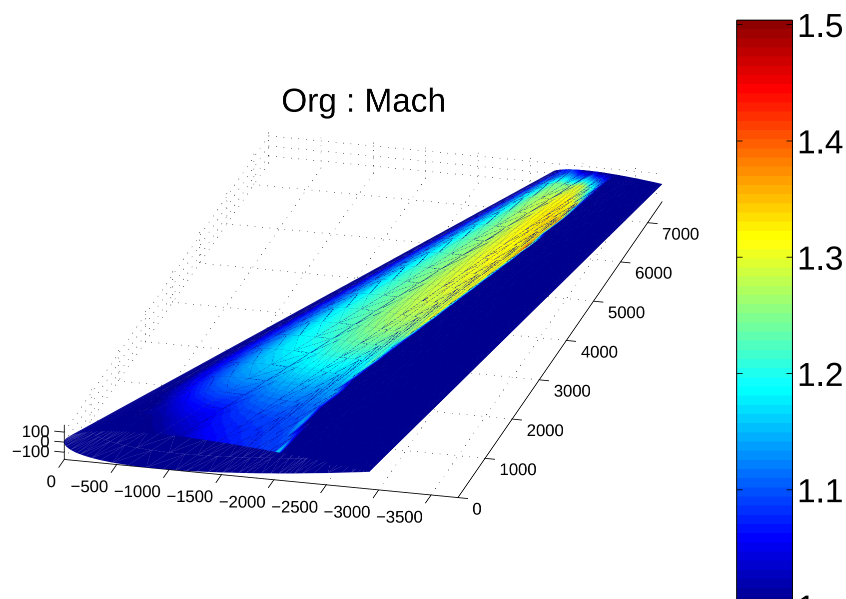

Figure 7: Mach number field in original configuration

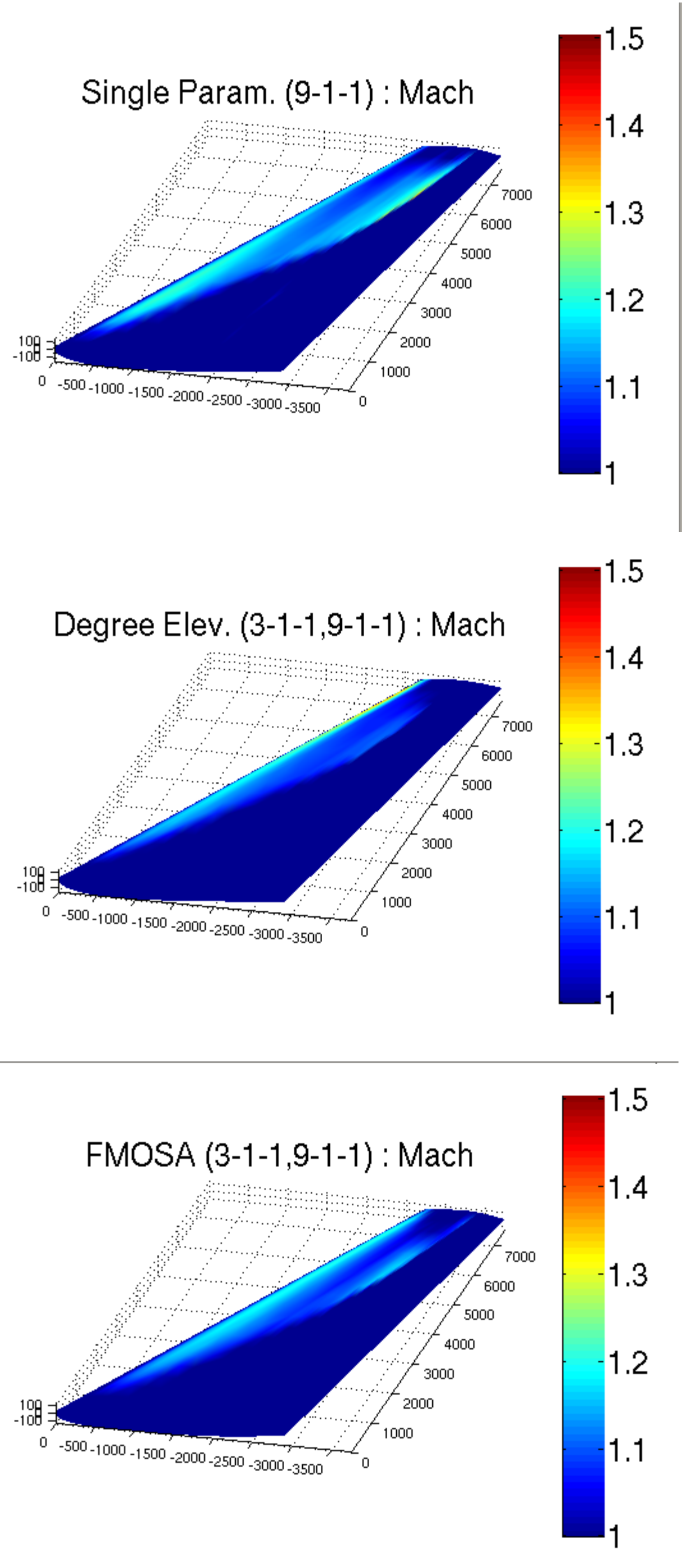

Figure 8: Mach number field after 500 iterations for the three methods under comparison

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