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THE COGNITIVE AND LINGUISTIC UNDERPINNINGS
OF MATHEMATICAL ABILITIES OF CHILDREN WITH READING DISABILITIES

by

NICOLE LIM

Under the Direction of Rose Sevcik, PhD

ABSTRACT

There is high comorbidity between reading disabilities and mathematical learning difficulties, yet the reasons behind this comorbidity has not been determined. Research, however, have suggested some correlates including linguistic abilities and executive functioning skills that influence mathematical skills. A comprehensive examination of how these factors relate to mathematical ability has not been determined. This study aims to investigate the possible influence of cognitive functioning, verbal skills, and reading skills, on the arithmetic competency of second and third graders with reading disabilities between the ages of 78 and 102 months. The data utilized in this study were from a longitudinal project which evaluated the effectiveness of various reading intervention programs. The first objective of this present study was to explore

how performance on basic and advanced mathematical concepts related to verbal skills and reading skills. The results generally did not illustrate any differences in the way these constructs related to the mathematical concepts. The second objective of the study was to analyze the influence of verbal skills, reading skills, and cognitive functioning skills, on the mathematical ability in children, and to develop a parsimonious model of mathematical ability for children with reading disabilities. Various models were assessed using path analyses. The two-construct model of verbal skills and mathematical skills was determined to be the best model describing the mathematical skills of children with reading disabilities. Supplementary analyses were conducted which clarified the various constructs' relationship to specific mathematical concepts. These analyses provided understanding to the impact of verbal skills, as well as other constructs', influence on specific mathematical concepts. The findings of this study have important educational implications and provide insight on more effective methods for developing the mathematical skills of children with reading disabilities. Finally, these findings foster future research in determining more effective interventions methodologies for children with reading disabilities.

INDEX WORDS: mathematical performance, arithmetic competency, verbal skills, reading skills

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OF MATHEMATICAL ABILITIES OF CHILDREN WITH READING DISABILITIES

by

NICOLE LIM

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

in the College of Arts and Sciences

Georgia State University

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Nicole Cybil Sook Yi Lim
2017

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DEDICATION

To my loving parents. They are the ones who have always been supportive of anything that I have put my heart and mind into. Thank you for everything. Without both of your constant encouragement and unyielding faith, this would not have been possible. I have, and will always appreciate everything you both have done to make this dream a reality.

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1 OVERVIEW

Mathematics is a multifaceted subject matter which involves the study of quantity, space and computations. The understanding of mathematics as a skill is complex because it is not a standalone subject. Language is very much interconnected to mathematical skills; the acquisition of mathematical skills has been akin to reading comprehension. Children must understand mathematical vocabulary to be able to comprehend what they are reading in arithmetic problems (Monroe & Orme, 2002). Just as language acquisition is met with certain problems, various issues can arise which impedes the acquisition of competent mathematical skills.

It is estimated that 5 to 8% of grade school children experience difficulties with mathematics (Geary, 2004; Shalev & Gross-Tsur, 2001). The incidence of mathematical difficulties is alarming because can impact the child in many ways. Children experiencing difficulties with mathematics tend to develop a pessimistic outlook on arithmetic; this often escalates to anxiety in regards to mathematics (Krinzinger, Kaufmann, & Willmes, 2009; Ramirez, Chang, Maloney, Levine, & Beilock, 2016). Math anxiety perpetuates the cycle of difficulty that children experience with mathematics, as children learn to associate mathematical learning as an unenjoyable task (Hembree, 1990). This has the unintended consequence of further avoidance of mathematical learning and practice (Hembree, 1990). The earlier an individual experiences math anxiety, the greater the length of math avoidance (Hembree, 1990). A lack of adequate mathematical instruction could unfortunately lead to lifelong impacts on the child, including having a negative impact on their academic careers, occupational training and opportunities in the future (Gerber, 2012). As such, it is essential that researchers explore the factors that play a role in mathematical competence.

When attempting to discover what gives rise to a certain set of skills, it is often beneficial to diagnose its shortcomings. Therefore, to understand mathematical achievement, it is essential that attention is devoted to determine the factors that hinder mathematical performance.

Underachievement in mathematics has been linked to both internal and external factors such as motivation, family background, and socioeconomic status (Nonoyama-Tarumi, Hughes, & Willms, 2015; Suárez-Álvarez, Fernández-Alonso, & Muñiz, 2014). In addition, researchers also have associated mathematical achievement with cognitive abilities such as executive functioning skills and linguistic skills; these are the primary interests of the current study.

According to Miyake and Friedman's model of executive functions (2000), there are three components of executive functioning: updating, inhibiting and shifting. Updating refers to the monitoring and updating of information in the working memory (Miyake et al., 2000), while inhibition describes the ability to attend to relevant information while inhibiting irrelevant information (Miyake et al., 2000). Finally, shifting refers to the change in attentional focus, or, the individual's ability to task-switch depending on the demands of the task. These skills individually and collectively are needed when engaging in mathematical tasks.

Working memory permits the withholding of necessary information and the manipulation of information (Miyake et al., 2000). In mathematical computation, the individual needs to know what each numerical symbol represents, hold multiple aspects of the question in mind, and then subsequently manipulate the various variables. In order to successfully respond to the question, the individual also needs to think about the strategies involved for computation. Poor working memory skills have been demonstrated to be the source of poor mathematical skills in children. Poor central executive functioning has been identified as the reason behind poor mathematical skills in children with reading disabilities (Klaczewski et al., 2015), while higher executive

functioning abilities have been linked to better arithmetic skills in kindergarten and grade school (Fuhs, Hornburg, & McNeil, 2016).

With regards to how inhibitory control relates to mathematical computation, the individual must disregard the many distractors in the question and their surroundings in order to focus on the information pertinent to answering the questions accurately. Lastly, the attention shifting facet of the Miyake and Friedman model of executive functions (2000) highlights the importance of an individual's ability to switch focus, which is critical in mathematical problems whereby multiple computations are necessary to respond correctly to the question at hand.

At present, the relationship between working memory and Intelligence Quotient (IQ) is inconclusive (Mahdi & Adel, 2010). Some researchers have found an association between working memory and fluid intelligence, that is, the intelligence that is associated with identifying patterns and solving problems (Cattell, 1963; Jaeggi, Buschkuhl, Perrig, & Walter, 2008); yet others did not establish such a relationship between working memory and IQ. The varied results are due to the usage of different types and quantities of test measures as well as the cognitive levels of the individuals tested. This matter is even more complicated when the groups of interest are children with mathematical difficulties since learning disabilities may be identified, in part, based on scores on the IQ measure. Given this, it is essential that researchers devote attention to understand how cognitive differences impact the mathematical performance of children with learning disabilities.

Researchers studying children with mathematical difficulties and children with comorbid mathematical and reading difficulties have cited language-based dysfunctions as the root of the mathematical difficulties experienced by these children (Fletcher, 2005). The association between language competency and mathematical skills has been further substantiated by the

predictive ability of early literacy skills such as knowledge of print and vocabulary on later arithmetic skills (Purpura, Hume, Sims, & Lonigan, 2011). It is a fair assumption that expressive and receptive language difficulties might interfere with the children's ability to acquire mathematical skills. For these children, poor receptive language skills limit their ability to process lectures which could impede their acquisition of mathematical concepts. On the same note, children with poor expressive verbal skills might struggle to participate in discussions or engage in self-speech as a strategy for mathematical computation. As a result, mathematical skills can be affected by poor language skills. This could have lasting effects as lower performance on mathematics in adolescence has been linked to early language impairments (Snowling, Adams, Bishop, & Stothard, 2001).

Language skills and mathematics abilities could be closely related within the test measure itself. Assessments of mathematical concepts and skills might not have discriminant validity between measures of children's mathematical and language skills (Rhodes, Branum-Martin, Morris, Ronski, & Sevcik, 2015). Poor discriminant validity between language and mathematical constructs within a test might place children with difficulties with language at a further disadvantage when assessing their mathematical abilities (Rhodes et al., 2015).

Difficulties with mathematics also have been suggested to be related to difficulties with reading. The high incidence of overlap between reading and mathematical difficulties generates questions about an interconnection between those two difficulties (Badian, 1999). There is some evidence that suggests that the comorbidity of the two dysfunctions are rooted in numerous shared and independent underlying processes, such as working memory and processing speed (Willcutt et al., 2013). Phonological processing abilities and Rapid Automated Naming (RAN), two skills that have been determined to be critical for competent reading skills (Wolf & Bowers,

1999), often have been explored to determine how these skills relate to mathematical abilities. The links between phonological awareness and mathematical skills have been demonstrated to influence early numerical ability such as competency in arithmetic facts operations (Vukovic, Lesaux & Siegel, 2010). Deficits on RAN also has been found to be linked to mathematical difficulties, but it has a closer relation to reading disabilities than to mathematical difficulties, highlighting that these disabilities have shared dysfunctions, yet in varying degrees (Mazzocco & Grimm, 2013). In general, there appears to be some link between the deficient cognitive processes that result in reading difficulties and mathematical difficulties. Further exploration of how these two difficulties relate to one another will benefit children with learning disabilities as reading difficulties might elicit additional obstacles during the acquisition of competent mathematical skills.

Together, these cognitive and linguistic-based complications might prevent children from cultivating competent mathematical skills. Considering that mathematical achievement in elementary school is a strong determinant of later academic success in both reading and mathematics (Duncan et al., 2007) and continued success in life, it is critical that researchers uncover what gives rise to mathematical competence.

1.1 Mathematical Difficulties

Mathematical difficulties have been described as the persistent struggle with the acquisition of arithmetic facts and concepts (Geary, 2006). *The Diagnostic and Statistical Manual of Mental Disorders* (5th ed.; DSM-5; American Psychiatric Association, 2013) specifies that a child must perform below what is expected of their chronological age, intelligence, and instruction, to be diagnosed with a learning disability. As such, unlike other learning disabilities (e.g., dyslexia), the term mathematical difficulty is used in place of the term dyscalculia.

Typically, children who score below the 20th or 25th percentile on a mathematical achievement test are classified as experiencing mathematical difficulties (Geary, Hamson, & Hoard, 2000).

Karagiannakis, Baccaglini-Frank and Papadatos (2014) proposed a comprehensive model of mathematical difficulties which describes various subtypes of mathematical learning difficulties. The model classifies subtypes into deficits with core number, memory, reasoning, and visual-spatial (Karagiannakis, Baccaglini-Frank, & Papadatos, 2014). Each of these categories illustrate specific difficulties encountered by the individual when engaging in mathematical tasks (Karagiannakis, Baccaglini-Frank, & Papadatos, 2014). In general, children who possess difficulties with mathematics often have trouble comprehending fundamental mathematical concepts such as number sense (understanding of numbers), counting, and basic arithmetic. These children typically exhibit slow and tedious experiences with calculations, and are usually inaccurate in their computations.

The struggle experienced with these basic mathematical facts and computations often sets the stage for further obstacles in the acquisition of mathematical skills as these foundational concepts are essential for more advance mathematical computations. Duncan et al. (2007) illustrated the relationship between school readiness including early mathematical skills and social-emotional skills to later academic achievement, and found that early mathematical skills was most predictive of later academic performance in mathematical and reading skills over other academic and behavioral characteristics. Also, in Lehl, Kluczniok and Rossbach's (2016) examination of the relationship between the quality of preschool mathematical instruction and development of mathematical skills in 554 first to third grade children, numerical skills at preschool along with the children's socioeconomic status were found to be predictive of the children's mathematical skills at first grade. Development of mathematical skills from first to

third grade was also found to be positively predicted by the quality of mathematical instruction in preschool when home learning environment is controlled (Lehrl, Kluczniok, & Rossbach, 2016). Duncan et al. (2007 and Lehrl et al.'s (2016) findings highlight the importance of establishing early mathematical skills as these skills are associated with later educational success.

Inadequate performance in mathematics can have continuous impact across the lifespan; not only does early mathematical performance impact academic achievement, but this effect could persist to adulthood. Poor mathematical achievement has been shown to limit career opportunities and be related to lower salaries after being employed (Parsons & Bynner, 2005). As such, given that mathematical skills and achievement can have an impact on an individual's life beyond their academic careers, it is important that researchers strive to understand the mechanisms behind competent mathematical abilities.

Current investigations of the source of mathematical abilities and difficulties have explored the contributions of cognitive functioning on mathematical achievement. In particular, researchers have linked deficits in working memory to the difficulties encountered in mathematics. According to Baddeley's model of working memory, our working memory consists of three components, the central executive, the phonological loop, and the visuospatial sketch pad (Baddeley & Hitch, 1974). The central executive element is responsible for the management and coordination of the two latter subsystems. The phonological loop stores and rehearses spoken or written information, while the visuospatial sketch pad stores and processes information in a spatial form. These three components are important in mathematics because mathematical computation involves both the storing and manipulation of numerical information simultaneously. Problems with working memory can impact the speed of processing numerical

information and arithmetic computation. Specifically, the visuospatial sketchpad of the working memory is responsible for mathematical representations and processing of information; children with difficulties in mathematics have been shown to experience difficulties with the representation and retrieval of the semantic memory during mathematical computation (Baddeley & Hitch, 1974; Geary, 1993).

Further evidence in regards to deficits in working memory and their effect on mathematical competence comes from Geary and colleagues' (2009) investigation of the predictors of mathematical difficulties in 6 to 10-year-old children. Using latent growth trajectory analyses, the authors established four groups of children: children with mathematical difficulties, children with low mathematical achievement, children with moderate mathematical achievement, and children with high mathematical achievement (Geary et al., 2009). Geary and colleagues (2009) demonstrated that children with mathematical difficulties possessed deficits in working memory and IQ, along with poor number sense, while children with low achievement did not possess working memory or IQ deficits, but had moderate understanding of number sense. In addition, they also demonstrated that children with high mathematical achievement possessed strong visual-spatial working memory along with a good understanding of number sense (Geary et al., 2009). These findings highlight the importance of visual-spatial working memory in mathematical performance (Geary et al., 2009).

The speed of processing information in working memory has been shown to play a role in mathematical performance, Ackerman and Dykman (1994) examined adolescents with reading disabilities and adolescents with both reading disabilities and mathematical difficulty and demonstrated that speed of processing, a factor reliant on working memory, was the only main difference between those two groups of adolescents among other tasks such as naming speed,

phonological skills, and memory tasks. The results illustrate the importance of speed of processing in mathematical computations. Further examination of children with learning disabilities was conducted by Fletcher (1985). Fletcher (1985) investigated the retrieval ability of children with only reading, spelling, on mathematical disabilities, and a combination of these disabilities, and found that children who experienced difficulties with mathematics, that is, those with only mathematical difficulties, those with comorbid mathematical and spelling difficulties, and those with comorbid mathematical and reading difficulties had lower storage and retrieval on nonverbal tasks, but did not differ significantly with their counterparts on verbal tasks (Fletcher, 1985).

Other researchers that have explored the causes of mathematical difficulties have studied the relationship between mathematical achievement and language skills. The high comorbidity between mathematical difficulty and language-based disorders has gained the interest of researchers as a possible link between these deficits. The complexity behind discerning the contributing factors of mathematical difficulty comes from the fact that learning mathematics relies upon language.

Language allows sharing of knowledge, as such, how efficiently children acquire mathematical skills is dependent on their ability to comprehend what is taught (Pierce & Fontaine, 2009). To respond accurately to certain mathematical problems, such as complex problem sums, children need to comprehend what is presented before them. Problem sums not only include regular words from the language that the children is exposed to, but they also include the use of language that is specific to mathematics – this is referred to as, “mathematics vocabulary” (Pierce & Fontaine, 2009). Mathematical vocabulary have been shown to be important in determining success in mathematical achievement (Pierce & Fontaine, 2009). Thus,

not only must children have adequate language skills outside of the domain of mathematics, but they must also learn terms specific to mathematics, as specific mathematical vocabulary has been shown to influence performance of numeracy (Purpura & Reid, 2016). As such, the learning of mathematics of children with language difficulty especially can be hindered by the language aspects of mathematics. Their poor language skills could bring about difficulties in comprehending mathematical terminology.

The impact of language deficits on mathematical performance could be further amplified by the fact that mathematical questions have been shown to intersect with language. Rhodes et al. (2015) explored the role of language in mathematics in 2nd to 5th graders with mild intellectual disabilities and found that the KeyMath – Revised Test (Connolly, 1988) a commonly used measure to assess the mathematical ability of children, did not have a high discriminant validity between language constructs and mathematics. This suggests that mathematical assessments could be impacted by children’s linguistic skills, and that researchers and educators of mathematics might not attain a “clean” assessment of children’s mathematical skills that is completely independent from their language skills (Rhodes et al., 2015). In essence, it is difficult to completely delineate the impacts of language from conventional mathematical problems.

The majority of the research investigating the links between mathematics and language have studied children with Specific Language Impairments (SLI); less research has focused on children with reading disabilities alone. Morin and Franks (2010) demonstrated how children with less proficient language skills such as children with risk of learning disabilities and SLI, experience ambiguity in knowledge acquisition because instruction is reliant on the processing of language. These language-based instructional environments not only impacted the children’s mathematical abilities, illustrating how inadequate language skills impacts the learning of

mathematics, but Morin and Franks (2010) also showed that the naming speed ability of these children further hindered their mathematical fluency. Further discussion of how language relates to mathematics will be discussed in the later sections of this document.

Language also has been found to play a role in the mathematical competency of children with reading disabilities. Hanich, Jordan, Kaplan and Dick (2001) found that children with comorbid reading and mathematical difficulties did not differ in their abilities of arithmetic approximation and their understanding of place value when compared with children with only mathematical difficulties. However, children with comorbid reading and mathematical difficulties was poorer in their abilities to calculate arithmetic combinations and in problem solving when compared to their peers with only mathematical difficulties (Hanich et al., 2001). This finding showed that the difference in performance between children with mathematical difficulties-only and children with comorbid reading and mathematical difficulties are those mathematical questions that involve language (Hanich et al., 2001).

The relationship between mathematical difficulties and reading disabilities has been explored by researchers. Both difficulties involve an unexpected struggle with their respective subjects due to reasons that cannot be attributed to inadequate instruction, intelligence, or sensory issues (DSM-5, American Psychiatric Association, 2013). The frequent co-occurrence between reading disabilities and mathematical difficulties highlight possible underlying similarities between these two deficits. Despite the research revolving around these two difficulties, the etiology of the overlap between these deficiencies is not fully understood. The comorbidity between these two difficulties range from 30-70% across studies (Badian, 1999); this wide range of difference does not help with specifying the causes of each difficulties. What is known from the study of mathematical difficulties and reading disabilities, is that, in general,

children with both mathematical and reading difficulties seem to possess a more generalized achievement difficulty than children with a single deficit in either reading or mathematics (Dirks, Spyer, & de Sonneville, 2008).

It is logical to assume that children's acquisition of mathematical skills is related to their reading ability, as learning mathematics, for example, gaining an understanding of what each numeral or an arithmetic term means, necessitates formal instructional methods or reading of textbooks. For that reason, the extent to which children grasp mathematical knowledge is reliant upon their reading skills. Willcutt and colleagues (2013) analyzed the social and academic functioning of children with only reading difficulties (RD), children with only mathematical difficulties (MD), children with comorbid reading and mathematical difficulties (RD + MD), and a control group of children with neither difficulty. The authors found that children who experienced any difficulties with reading or math, that is, children from the RD, MD and RD + MD groups were more impaired in all aspects of social and academic functioning than the control group (Willcutt et al., 2013). Willcutt and colleagues (2013) also found that the impairment was most severe for children with difficulties in both areas (RD + MD). These analyses also showed that children with difficulties in both reading and mathematics were linked to shared deficits in working memory, processing speed, and verbal comprehension (Willcutt et al., 2013). They also found that children with only reading difficulties was linked to problems with phoneme awareness and naming speed alone, while children with mathematical difficulties alone had problems with set shifting (Willcutt et al., 2013). These findings illustrated how children with reading disabilities and mathematical difficulties have shared underlying neuropsychological shortcomings (Willcutt et al., 2013).

One of the key characteristics of reading disabilities is difficulty with processing phonemes. The difficulties experienced when processing phonemes could slow down the speed at which problem sums are solved. Processing of phonemes could therefore impact mathematical computation, as more cognitive resources are devoted to processing the question rather than solving the problem presented. Finally, children with reading difficulties have been shown to have deficits in the processing of symbolic language. On the same note, children with mathematical difficulties have been shown to experience difficulties in recognizing numerals and mathematics symbols, this once again, could impact the speed of processing and thus hinder the solving of mathematical problems. Considering the impacts of mathematical and reading difficulties on school achievement (Hakkarainen, Holopainen, & Savolainen, 2013), it is critical to illuminate the elements of these difficulties.

1.2 The Role of Cognitive Skills in Mathematical Computation and Learning

Cognitive skills can largely be described as our ability to think, learn, remember, and organize information in our minds. Mathematical computations and learning are complex processes reliant on the use of general and specific cognitive abilities (Passolunghi, Cargnelutti, & Pastore, 2014). It is estimated that between 5 to 8% of children experience difficulties with the acquisition of mathematical concepts due to certain memory or cognitive deficits (Geary, 2004).

The cognitive processes involved in mathematical calculations are reliant upon our executive functioning skills; links between early mathematical skills and executive functioning have been found (Blair & Razza, 2007; Bull, Espy, & Wiebe, 2008; Moll, Snowling, Göbel, & Hulme, 2015). Clark, Pritchard and Woodward (2010) demonstrated the relationship between children's early executive functioning abilities at age 4, and the children's mathematical performance a year after grade school, at age 6, and found that achievement on set shifting,

inhibitory control, and general executive behavior at age 4, accounted for a significant amount of variance in the children's mathematical achievement at school age. One belief is that executive functioning skills allow children to get accustomed to their learning environment by supporting their concentration on academic tasks, while counteracting irrelevant information (Blair & Diamond, 2008). This could positively impact their capacity to learn by engaging in academically-focused behaviors such as following of directions and controlling of attention, while preventing non-academically-focused behaviors such as disruptive emotions in the classrooms (Blair & Diamond, 2008). Executive functioning skills thus play an important role in school readiness for mathematical achievement (Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014).

Executive functioning skills not only support mathematical learning by promoting positive classroom-learning behaviors, but they have also been linked directly to acquisition of mathematical skills. Fuhs and colleagues (2016) investigated the relationship between executive functioning skills and children's mathematical performance from kindergarten to second grade (Fuhs et al, 2016). The authors demonstrated that higher executive functioning skills was associated with higher number sets identification in kindergarten. Children with higher executive functioning skills performed better than their counterparts at the speed and accuracy of identifying number sets (Fuhs et al., 2016). Their findings also exemplified the predictive ability of executive functioning on growth in mathematical skills from kindergarten to second grade (Fuhs et al., 2016). The authors speculated that children with better executive functioning skills were able to remember larger number sets which allowed them to identify sets as wholes (rather than as individual parts); this in turn, allowed the children to be less distracted by the individual numbers as compared to children with lower executive functioning skills sets (Fuhs et al., 2016).

The ability to focus on wholes rather than smaller parts, has been suggested to be related to children's ability to acquire more advanced mathematical concepts in the early elementary grades (Fuhs et al., 2016). Their study illustrated how executive functioning skills might impact mathematical skills acquisition. The effect of executive functioning skills on mathematical abilities is further demonstrated by Espy and colleagues' (2004) examination of developing mathematical skills in preschool children. Their results indicated that both working memory and inhibitory control have predicted arithmetic competency even after controlling for the children's age, their vocabulary size and their mothers' educational attainment levels (Espy et al., 2004). Other studies also have highlighted the influence of executive functioning skills on the development of mathematical skills. Moll and colleagues (2015) analyzed the language and executive functioning skills of children at risk for and not at risk for dyslexia, and found that early language and executive functioning skills accounted for variations in preschool verbal number skills, which in turn, predicted arithmetic skills in elementary school (Moll et al., 2015), while Passolunghi, Lanfranchi, Altoè, and Sollazzo (2015) showed that in an examination of 100 kindergarten children, processing speed and working memory had a direct relationship to early mathematics skills.

The links between executive functioning skills and early arithmetic abilities also have been demonstrated to have an influence in elementary school. Cantin and colleagues (2016) investigated the influence of the executive functioning on reading, mathematics, and theory of mind performance in 93 7 to 10-year-old elementary school children (Cantin, Gnaedinger, Gallaway, Hesson-McInnis, & Hund, 2016). They found that mental flexibility, a form of executive functioning, accounted for the difference in reading comprehension and mathematical performance. Further evidence of executive functioning on mathematical performance is shown

in Jerman, Reynolds and Swanson's (2012) investigation of children with reading disabilities. They examined how development of various cognitive and executive functioning abilities predicts development in reading and mathematical performance (Jerman, et al., 2012). Swanson and Jerman (2007) also found that children without any learning disabilities had higher working memory growth within a three year period as compared with children with reading disabilities and children with comorbid reading and mathematical difficulties. Altogether, these findings underscore the importance of executive functioning in mathematical skills.

At this point, it is essential to note the difference between the number of items that can be held in working memory, and the resolution or precision of those representation. Xu and Chun (2006) have shown that the relationship between working memory capacity and standard measures of fluid intelligence is mediated by the number of representations that can be held in the working memory at a single point in time, rather than by the precision of those representations. Their work highlights the relationship between working memory and fluid intelligence. As mentioned earlier, the relationship between IQ scores and measurement of fluid intelligence has been undetermined due to the employment of varied types and quantities of test measures and participants' cognitive level. Yet some researchers are able to link working memory, which is associated with fluid intelligence, to IQ scores (Passolunghi et al., 2014). Evidence of an association between fluid intelligence and mathematical performance also has been shown (Foster, Anthony, Clements, & Sarama, 2015).

Finally, it is noteworthy that research investigating the relationship between working memory and mathematical performance also has revealed a relationship between mathematical difficulties and reading disabilities. When working memory deficits exists, children have been shown to have an increased risk of experiencing mathematics difficulties and reading disabilities

in first grade (Morgan, Li, Farkas, Cook, & Pun, 2016). Also, when the inhibitory control aspect of the executive functioning skills are deficient in children with reading disabilities, their growth in both reading and mathematical skills are hindered; these studies highlight how our executive system underlies performance on reading and math measures (Jerman et al., 2012). Given the large role that cognitive skills play in mathematical performance, and how reading difficulties could amplify their impact, it is beneficial to understand the relationship between executive functioning skills and mathematics.

1.3 The intersection of Mathematics and Language

Language is thought to permeate and impact much of our thoughts; therefore, it is unsurprising that mathematics, a cognitively engaging subject, has been consistently linked to be influenced by language. In mathematics, language is used to understand the concepts of numbers and symbols, used for making mathematical connections, and to aid in our verbalization of the steps involved in solving mathematical sums. As such, more proficient language skills could support the development of arithmetic skills, and research seems to support this notion.

The relationship between early arithmetic skills and language has been indicated to be relatively strong in children without any learning disabilities. Purpura et al. (2011) investigated the relationship between early literacy skills and early mathematical skills of 3 to 5-year-old children and found that knowledge of print and vocabulary was predictive of the children's mathematical ability a year later. However, in this particular study, Purpura and colleagues (2011) did not find an association between phonological awareness and mathematical ability a year after assessment.

Some research has investigated the influence of cognitive and linguistic functioning on mathematical abilities, in particular, Purpura and Ganley (2014) assessed the relationship

between language skills, working memory, and 10 specific domains of mathematical skills such as verbal counting, number comparison, set comparison, and story problems, in 199 preschool and kindergarten children (Purpura & Ganley, 2014). They found that language skills were related to all the domains of mathematics, while working memory was related to some mathematical skills (Purpura & Ganley, 2014). The authors also found that children's later mathematical ability was built upon foundational mathematical skills acquired earlier. Their findings demonstrate the importance of establishing early mathematical skills, and how working memory and language skills influence the growth of mathematical skills (Purpura & Ganley, 2014).

Even though there is evidence that supports the relationship between mathematical and language skills, some researchers have suggested that the relationship between language and arithmetic skills is not entirely convincing because the research that explores the association between language and mathematical skills does not delineate if mathematical language is the factor that accounts for most of the relationship between the two variables. Mathematical language is defined as mathematics-content-specific vocabulary which are necessary for acquiring and applying mathematical knowledge and skills (Harmon, Hedrick, & Wood, 2005; Powell & Driver, 2015), e.g. words like more, less, and spatial language, e.g. above. Research that has explored the relationship between mathematics and language have utilized more general language measures without specifying how domain-specific measures of mathematical language might account for that relationship. Purpura and Reid (2016) argued that when mathematical language and general language skills are analyzed separately to predict arithmetic skills, only mathematical language was found to be a significant predictor of numeracy performance. Yet, it is difficult to discount the relationship between language and mathematics purely based on this

finding, because other research that has explored both general language measures and mathematics-specific language measures also have established that language, in general, is related to mathematical performance. In Toll and Van Luit's (2014) exploration of how oral language skills relate to early arithmetic abilities in kindergarten children, they found a significant mutual relationship between general language skills and early numeracy skills using latent growth modeling. They also were able to show that specific mathematics language mediated the relationship between general language skills and numeracy performance (Toll & Van Luit, 2014). Their findings demonstrate the importance of general and specific language skills in mathematical abilities.

Other studies that have explored mathematical language specifically, also have underscored the relationship between mathematics and language. Purpura and Logan (2015) assessed how performance on various academic and cognitive assessments relates to early mathematical performance in preschool children. Their results indicated that mathematical language influenced children's mathematical skills acquisition at any level of development, while the understanding of an approximate number system's influence on mathematical skills acquisition was dependent on children's level of development. This finding reveals that language does have a strong influence on mathematical skills development, and that influence even exceeds the influence of early arithmetic abilities on later mathematical performance. Their study also showed that cognitive measures did not influence mathematical abilities (Purpura & Logan, 2015).

It is essential to note that not only can language and mathematics be related in their underlying skill sets, but these domains can be further related within the test measure itself. Rhodes et al. (2015) found that the KeyMath-Revised Test (Connolly, 1998), an assessment of

essential mathematical concepts and skills, did not demonstrate discriminant validity with measures of children's language skills, that is, some measurements of mathematical skills might be assessing the language construct within their test. As such, the confluence of language and mathematics within the test measure itself makes the delineation of language influence on mathematical achievement even more difficult. Their study also highlights that children with language difficulties might have an additional disadvantage when their mathematical abilities are assessed.

Due to the close association between language and mathematics, children with problems with language may have corresponding issues with mathematics. Vukovic (2012) investigated the development of mathematical skills of 203 children, followed from kindergarten to the third grade. Using latent growth modeling, the researchers assessed how mathematical difficulties with and without reading disabilities were related to measures of working memory, short-term memory, cognitive processing speed, early numerical skills, and phonological processing, (independent of reading, and found that phonological processing skills influenced the growth in mathematics from kindergarten to third grade (Vukovic, 2012).

Other studies also have established a similar association between language and mathematics in children with learning disabilities. Van Daal and colleagues (2013) examined various language and cognitive skills such as executive functioning and fluency's relationship to mathematical and reading skills in 13- and 14-year-old students. The students were grouped according to their reading and mathematical abilities: students with reading disability (RD), students with mathematical difficulties (MD), students with both reading and mathematical difficulties (RD + MD), students with difficulties in reading, mathematics, and listening comprehension (RD + MD + LC), and students with typical achievement (TA). Van Daal and

colleagues (2013) found that students with difficulties in reading, that is, students in the RD, RD + MD and RD + MD + LC groups experienced difficulties with phonological processing and rapid automatic naming (van Daal, van der Leij, & Adèr, 2013). They also found that for children with both reading and mathematical difficulties (RD + MD and RD + MD + LC groups) experienced additional issues with executive functioning and digit span (van Daal et al., 2013). Their findings highlight further deficiencies experienced by children with comorbid difficulties.

This finding of children with mathematical difficulties having multiple deficits other than language is shown by Cirino and colleagues' (2015) assessment of the various cognitive and mathematical skills of second-grade children with learning disabilities. Cirino, Fuchs, Elias, Powell and Schumacher (2015) compared the performance of children with reading difficulty only (RD), children with mathematical difficulty only (MD), children with both reading and mathematical difficulties (RD + MD), and children without any difficulties (TA), on various cognitive and mathematical assessments such as working memory, language, numerical competency, problem solving, etcetera. They found that children without any learning disabilities (TA) performed better than their RD, MD, and RD + MD peers in areas of working memory, language, numeracy, computation and problem solving, demonstrating that the children with reading, or, mathematical, or, both difficulties were comparatively less proficient in processing speed and language (Cirino et al., 2015). The authors also found that children with only mathematical difficulties (MD) outperformed children with both reading and mathematical difficulties (RD + MD) (Cirino et al., 2015). This study highlights how children with problems in mathematics have less proficient language skills than their peers without such difficulties (Cirino et al., 2015).

Further links between language skills and mathematics are demonstrated by the replication of similar findings in children with intellectual disabilities. Rhodes (2012) examined the influence of linguistic complexity on the predictive ability of mathematical performance in 144 second and third graders with mild intellectual disabilities. Rhodes (2012) found that children's language skills had a significant influence on mathematical achievement, but this relationship was not stable across time. Rhodes's (2012) finding also showed that children with better language skills performed better on mathematical tasks that included linguistic content than children with less proficient language skills, showing that mathematical performance is dependent not only on the children's language skills, but also dependent on the linguistic content of the tests (Rhodes, 2012). Finally, Foster and colleagues' (2015) also examined the relationship between language and mathematical skills for children with intellectual disabilities and showed that both phonological awareness skills and naming speed was predictive of mathematical problem solving, with phonological awareness evidencing a stronger predictive ability of mathematical skills than naming speed (Foster, Sevcik, Ronski, & Morris, 2015).

Together, these findings illustrate what appears to be a strong relationship between language skills and mathematics; because both domains are important in school achievement. Further exploration of how specific language components relate to mathematical skills is needed.

1.4 The Relationship between Reading Ability, Reading Disability, and Mathematical Difficulty

Reading is a complex process that is reliant on numerous perceptual and cognitive functions. The Simple View of Reading conceptualizes reading comprehension as a function of decoding and language comprehension (Hoover & Gough, 1990). Decoding involves translating printed words into sounds of spoken words, and then retrieving semantic information at the word

level, while language comprehension involves using semantic information at the word level to deduce interpretations (Hoover & Gough, 1990). Both of these processes are important steps in reading comprehension; if either of the components are deficient, children will experience reading difficulties. To become efficient decoders, children must acquire the letter-sound correspondences of their language; this skill is reliant on the children's phonological awareness abilities.

Reading disability is generally described as an unexpected difficulty with reading despite having average or above average intelligence and adequate exposure to education (Fletcher et al., 1994). Traditionally, the Intelligence-Quotient (IQ) and reading achievement discrepancy was used to diagnose reading disability. With this definition, children's IQ had to be significantly higher than their reading scores to be diagnosed as having a reading disability. Since then, other ways in which reading disabilities is recognized in children has emerged (Fuchs & Fuchs, 2006). Contingent upon the definition employed within a particular study, it is estimated that 5 to 18% of the general population evidence reading disabilities (Shaywitz & Shaywitz, 2005).

Consistent with the decoding element of the Simple View of Reading (Hoover & Gough, 1990), reading researchers are generally in consensus that reading disability is linked to core deficits in areas of phonological awareness and phonological processing (Morris et al., 1998). Phonological awareness is described as the ability to identify and manipulate sounds of a language, while phonological processing involves segmenting words into speech sounds, and then knowing the pronunciation of words from the combination of sounds (Stahl & Murray, 1994). Having deficits in the phonological processes hinders children's ability to acquire word recognition skills, which hampers fluent reading. It has been suggested that the effort devoted to the tedious process of decoding results in less efficient capacity to interpret what is read (Perfetti,

1987). Fuchs and colleagues (2012) were able to show how phonological processing skills impacts reading. In their investigation of the cognitive predictors for reading disability, Fuchs and colleagues (2012) have shown that children's phonological processing skills, oral language comprehension abilities, and nonverbal reasoning skills at first grade were predictive of whether the children had acquired the status of reading disability at fifth grade (Fuchs et al., 2012).

The significance of phonological processing skills in reading is further exemplified by Vellutino and Scanlon's (1987) examination of the relationship between phonological coding, phonemic segmentation and reading disability. They found that phonemic segmentation skills and alphabetic mapping predicted children's ability to identify words (Vellutino & Scanlon, 1987). Their findings indicated how phonological coding difficulties was a critical element in reading disabilities (Vellutino & Scanlon, 1987). Ryder and colleagues (2008) also showed how the development of phonemic skills is related to improvements in reading skills in an examination of how explicit instruction in phonemic awareness and phonemic-based decoding skills impacts children's reading skills. They examined 24 6 and 7-year-old children with reading difficulties that were assigned to an instructional group with phonemic-based instruction and to a control group (Ryder, Tunmer, & Greaney, 2008). After 56 lessons in phonemic awareness and alphabetic coding skills, the results showed that the group that received phonemic instruction performed better than the control group in not only phonemic awareness and decoding, but also on context free word recognition assessed by Burt Word Reading Test, and reading comprehension, assessed by Neale Accuracy Subtest (Ryder et al., 2008). Their findings highlight how phonological processing abilities is related to performance on word reading and comprehension. Ryder and colleagues (2008) also showed the long-term benefits of phonological awareness instruction on word reading; based on their two-year follow-up examination, they

found that the mean reading age of the children in the instructional group was 9 months ahead of the control group in Burt Word Reading test and 14 months ahead on the Neale Accuracy Subtest (Ryder et al., 2008). Based on their findings, the authors suggested that without adequate literacy-related skills such as phonological awareness, children rely on less efficient word identification strategies such as contextual-guessing and partial word-level cues which negatively impacts their reading abilities (Ryder et al., 2008). Their findings demonstrate the importance of phonological skills in reading performance.

While the importance of phonological processing in reading skills has been recognized, other researchers have cited deficits in naming speed (determined by rapid automatized naming [RAN]), to be a critical factor in reading skills. RAN tasks assess the speed at which individuals name objects, colors, or symbols (Wolff, 2014). Wolf and Bowers (1999) introduced the double-deficit hypothesis which suggests that difficulties experienced with reading could stem from independent deficits in either phonological awareness or fluency (RAN), or a co-occurrence of problems in both areas (Wolf & Bowers, 1999). Based on the double-deficit hypothesis, difficulty with reading could arise from three different conditions: 1) children could have problems with decoding yet have intact fluency, 2) have unimpaired ability to decode, but possess issues with fluency, or 3) issues with both decoding and fluency. The third condition - the children with a “double-deficit”, will result in poorer reading skills than those with deficit in only one area, or, a “single-deficit” (Wolf & Bowers, 1999). Bowers and Wolf (1993) examined the speed and accuracy of children naming symbols and letters, and found that poor readers are slower and less accurate in naming symbols and letters than good readers. They proposed that speed and precise timing mechanisms are necessary for words to have phonemic representation. Their finding that naming speed and phonological-awareness skills contribute uniquely to

reading is illustrated as children with a “single-deficit” have less severe reading difficulties than those with a “double-deficit”; and children with intact skills in both areas were able to read fluently (Bowers & Wolf, 1993). Even though Bowers and Wolf (1993) illustrated a unique relationship between phonological processing skills and naming speed, it is essential to note that there is a strong interrelationship between decoding skills and rapid automatized naming skills; as presented in Compton’s (2003) model of the development of decoding skills and rapid automatized naming skills in first grade children. Compton (2003) found a bidirectional relationship between decoding skills and RAN numbers, that is, performance on the rapid automatized naming tasks was found to support the development of decoding skills, and decoding skills were found to support the growth of rapid automatized naming skills.

Other studies also have demonstrated the significance of phonological processing and naming speed in reading. Catts and colleagues (2002) examined how speed of processing, RAN skills and phonological awareness impacted reading achievement in 279 third grade children (Catts, Gillispie, Leonard, Kail, & Miller, 2002). They found that poor readers were slower on the RAN object task as compared to good readers, which indicates that poor readers have a general deficit in speed of processing which impacts their reading abilities. The authors also illustrated the contributions of IQ, phonological awareness, and naming speed in determining reading achievement. Further illustrations of phonological processing and naming speed’s influence on reading ability were shown by de Groot et al.’s (2015) classification of children as poor and good readers. The authors were interested in determining the predictability of reading group classification based upon performance in phonological awareness and RAN (de Groot, Van den Bos, Minnaert, & van der Meulen, 2015). The authors showed that utilizing both phonological awareness ability and RAN skills in combination produced the best prediction of

group membership, and this was particularly evident for poor readers (de Groot, Van den Bos, Minnaert, & van der Meulen, 2015). They also found that the predictive ability of both variables varied depending on the severity of the reading dysfunction, with deficient phonological awareness skills being the mark of reading disability, while RAN performance was a critical predictor for above-average or excellent reading proficiency (de Groot, Van den Bos, Minnaert, & van der Meulen, 2015).

Other researchers also have proposed that orthographic processing is critical in determining reading ability. Orthographic processing involves being able to recognize a letter or a word, and knowing what it sounds like. Ehri's four phases of word reading (Ehri, 2005) describes how children acquire orthographical knowledge via four consecutive stages that encourage the connection of written alphabets and words to their pronunciations in memory. Ehri's four phases of word reading include: the pre-alphabetic phase, the partial-alphabetic phase, full-alphabetic phase and the consolidated-alphabetic phase. In the first phase, children memorize the visual features of words. In the second phase, just as the name suggests, children recognize some letters of the alphabet and use the context to decipher words. At the third phase, children have already acquired the grapheme and phoneme associations in words, and can store some words in memory. At this stage, children are able to engage in sight word reading. Sight word reading permits faster recognition of words in print without having to rely on decoding. At the final stage, larger grapheme-phoneme "sets" are formed for words that the children are frequently exposed to; this allows for even faster processing of words via print. Altogether, these processes can be largely described as orthographic mapping – a map from printed words to their sound and meaning. Efficiency of orthographic processing is reliant upon the amount of print

exposure, that is, increased exposure to printed words encourages the development of orthographic mapping (Apel, 2011).

The significance of orthographic processing in reading ability has been demonstrated in several studies. In a study of early reading skills, Cunningham and Stanovich (1997) found that exposure to print was demonstrated to account for differences in performance on reading comprehension from first grade to 11th grade even after the effects of cognitive ability was accounted for. Orthographic processing also has been shown to aid in vocabulary learning and lessen reliance on phonological processing (Rosenthal & Ehri, 2008). However, it is important to note that despite evidence showing the influence of orthographic processing on reading, Apel (2011) has suggested that orthographic processing is still dependent upon phonology, because even after recognizing a word, the retrieval of linguistic information from memory is reliant upon phonological processing.

Children could possess difficulty with phonological processes, rapid automatized naming or orthographical processing which will present themselves as roadblocks for the acquisition of proficient reading skills. Given that reading is a vehicle by which children acquire knowledge, difficulty experienced with reading could result in difficulties in other subject areas. This gives rise to the question at hand: how do reading skills relate to mathematical skills?

There are evidence showing that reading skills are essential in solving mathematical sums. Korhonen, Linnanmäki, and Aunio (2012) examined the relationship between various language measures such as word comprehension, reading comprehension and spelling to mathematical performance in ninth grade children and found that reading performance accounted for 52% of variance in mathematical performance, showing that reading skills was a strong predictor of mathematical performance. Their findings also illustrated that reading skills that

focuses on understanding of texts are important in solving mathematical tasks (Korhonen et al., 2012).

Also, the association between reading skills and mathematical skills can be exemplified by the relationship between reading and mathematical learning disabilities. At a glance, the incidence of reading and mathematical difficulties differ; with 5 to 18% of children with reading disabilities, and only 5 to 8% of children experiencing difficulties with mathematics (Geary, 2004; Shalev & Gross-Tsur, 2001). Yet, when individuals have either disorder, the likelihood of having both disorders increases to 30-70% (Badian, 1999). Despite the high comorbidity between these two disorders, the etiology of the comorbidity is not fully understood. Part of the reason for the uncertainty revolving around the etiology could stem from the different focus taken by reading and mathematical researchers (Fletcher, 2005). Reading researchers investigating the comorbidity of reading and mathematical difficulties typically compare children with reading disabilities only, and children with both reading and mathematical difficulties (Fletcher, 2005). While mathematical researchers typically compare children with mathematical difficulties only, and children with comorbid reading and mathematical difficulties when investigating the causes of comorbidity (Fletcher, 2005). This difference in focus and sample gives rise to different interpretation of results, with reading researchers generally presenting the comorbid issue as stemming from deficits in reading, while mathematical researchers understanding the issue of comorbidity as a language-based problem. Yet, a comparison of both difficulties as a whole, reveal both parallels and dissimilarities between both difficulties.

In Willcutt and colleagues' (2013) comparison of academic and social functioning of children with a mixture of learning disabilities, i.e., children with only reading difficulties (RD), children with only mathematical difficulties (MD), children with comorbid reading and

mathematical difficulties (RD + MD), and a control group of children with neither difficulties, they found that impairments in reading and mathematics were linked to both shared and disparate difficulties in various functioning (Willcutt et al., 2013). Deficits in reading and mathematics were linked to issues with working memory, processing speed, and verbal comprehension, while reading difficulties were uniquely related to problems in phonemic awareness and naming speed, and difficulties with mathematics were uniquely linked to deficits in set shifting (Willcutt et al., 2013). Despite Willcutt and colleagues (2013) illustrating some underlying differences in reading and mathematical difficulties, there is some evidence that suggests a genetic explanation for the comorbidity between reading and mathematical difficulties (Knopik, Alarcón & DeFries, 1997), which prompts further examination of the comorbidity of the two disorders.

In general, as stated earlier, the foundation of reading disabilities comes from an impairment in phonological processing, yet, there is a subgroup of children without deficits in phonological processing, but still possess difficulties with reading. As illustrated in the Simple View of Reading, language comprehension, i.e., the ability to acquire meaning from words, is the other necessary component (besides decoding) for reading comprehension (Hoover & Gough, 1990). It appears that this subgroup of children with reading difficulties without a phonological deficit have a reading disorder named the Specific Reading Comprehension Deficit (SRCD; Bailey, Hoefft, Aboud, & Cutting, 2016). Children with the SRCD have intact phonological processing abilities, and thus are able to sound out the words presented on a page, yet are unable to interpret the meaning of the written information. Vukovic et al. (2010) examined the mathematical skills of third graders with reading disabilities of the phonological-deficit nature and those of the SRCD nature. They found that children with the phonological-based deficit experienced more difficulty with arithmetic fact fluency and operations (Vukovic et al., 2010)

showing that impairments in phonological processing accounts for some of the problems experienced in arithmetic functioning. Since phonological-deficit based reading disabilities is the more common subtype of reading disabilities, it is plausible that deficits in phonological processing accounts for some of the issues represented in mathematical difficulties. Furthermore, there is evidence that shows that phonological processing abilities is important in mathematical functioning. Vukovic (2012) examined the progression of mathematical dysfunction using latent growth modeling and demonstrated that early numerical skills and phonological processing influenced the growth of mathematical skills from kindergarten to third grade, regardless of whether the children had reading difficulties or not. The influence of phonological processing on the development of mathematical skills is further exemplified by Foster and colleagues' examination of the developing mathematical skills of kindergarten children (Foster et al., 2015). The authors investigated how various cognitive processes relate to mathematical performance and found that fluid intelligence and phonological awareness skills were associated with performance on numeracy and applied problems (Foster et al, 2015). Their findings showed the impact of phonological processing on mathematical performance. The importance of phonological awareness on mathematical ability also has been demonstrated; phonological awareness skills have been shown to mediate the relationship between verbal working memory and early numerical skills, or more specifically, the ability to learn the number word sequence (Michalczyk, Krajewski, Preßler, & Hasselhorn, 2013).

Most studies that investigate the overlap between reading and mathematical difficulties do not differentiate reading disabilities into the phonological-based and SRCD subgroups. However, it is easy to see how children with the SRCD subtype might encounter issues with mathematical word problems purely based on the difficulties experienced with the interpretation

of word passages. For children with the phonological-based reading difficulties, difficulties with arithmetic word problems comes from difficulties experienced with decoding, which might hinder the fluency at which the word problem is understood. Problems with reading fluency come from underlying issues with naming speed; and as mentioned previously, naming speed is a critical element in reading. As such, rapid automatized naming (RAN) has been speculated as a possible indicator for mathematical difficulties. Mazzocco and Grimm (2013) studied how response time on RAN tasks contrasted between children with reading disability and children with mathematical difficulties, as well as children without difficulties in reading and mathematics. They found that children with either reading or mathematical difficulties were slower on the RAN tasks as compared to children without any difficulties in kindergarten (Mazzocco & Grimm, 2013). They also found that even though deficits on RAN number and letter performances were associated with both reading and mathematical difficulties, performance on RAN tasks was more closely related to the children with reading difficulties than children with mathematical difficulties (Mazzocco & Grimm, 2013). Peng and colleagues (2016) were also able to show that, in addition to decoding ability, numerical competence and processing speed was predictive of calculation skills at the first grade, with the latter two further showing predictive ability of calculation at the third grade (Peng et al., 2016). These studies highlight the possible role that RAN performance has on mathematical abilities and difficulties.

Although there is evidence that suggests that naming speed and fluency play a role in mathematical difficulties, there also is evidence that indicate otherwise. In an investigation of how RAN performance relate to reading and mathematical ability, Georgiou and colleagues (2013) examined the relationship of speed of processing, response inhibition, working memory and phonological awareness performance (what they determined to be the various components of

the RAN tasks) to reading and mathematical ability in 72 children from kindergarten to the end of the first grade (Georgiou, Tziraki, Manolitsis, & Argyro, 2013). Pause time, i.e., length of time between naming one symbol to the next, was found to be the important element in both the relationship between RAN tasks and reading performance, and RAN tasks and mathematical performance (Georgiou et al., 2013). Pause time shared most of its predictive ability of reading and mathematical performance with speed of processing and working memory. Their findings highlight that none of the RAN components is uniquely linked to mathematical performance (Georgiou et al., 2013).

Researchers also have explored how word order might impact performance on arithmetic word problems. Peake and colleagues (2015) examined how syntactic awareness, that is, the understanding of word order impacts performance on word problems (Peake, Jiménez, Rodríguez, Bisschop & Villarroel, 2015). Peake et al. (2015) found that children with either reading disabilities or mathematical difficulty, and children with both difficulties were less efficient at problem solving as compared to their typically developing peers, but syntactic awareness only mediated the effect for children with reading difficulties and not children with mathematical difficulties (Peake et al., 2015).

Despite these findings that indicate how certain underlying factors that predict reading do not relate to mathematical abilities in the same way, Wise and colleagues (2008) analyses of children with reading disabilities suggest that depending on the criterion in which mathematical difficulty is defined, the relationship between certain language variables and mathematical skills could be impacted (Wise et al., 2008). Wise and colleagues (2008) examined 114 second and third grade children with reading disabilities who were at risk for, and without risk for mathematical difficulties (MD), and demonstrated that using either a 15th or a 25th percentile-

cutoff point on the KeyMath-Revised Test (Connolly, 1988) to define mathematical difficulties resulted in differences in whether language variables were predictive of mathematical skills. Specifically, they showed that when the 15th percentile cutoff for mathematical difficulty was employed, RAN performance was predictive of the performance on the measurement subscale for the children with reading disabilities (Wise et al, 2008). Yet, when the 25th percentile cutoff score for mathematical difficulty was used, neither phonological awareness skills, nor RAN skills were found to significantly predict mathematical performance. Wise et al.'s (2008) study highlights how the relationship between different variables could vary depending on how it is defined, and it cautions researchers to consider this factor prior to disassociating the relationship between certain variables.

Finally, it is important to note that the reliance on cutoff points for the categorical classification of reading and mathematical difficulties might be arbitrary because reading and mathematical difficulties are based on a continuum of severity, rather than clear-cut criteria; this could impact the way in which various variables relate to one another, and once again, is important for researchers to note this possibility. Branum-Martin, Fletcher and Stuebing (2013) exemplified this in their simulations of cognitive and achievement data of children without any categorical constructs and found that the patterns produced are a product of cutoff points and the correlational structure of the data.

2 RESEARCH PURPOSE AND SIGNIFICANCE

Altogether, the literature reviewed within this paper suggests that a relationship between cognitive, verbal, reading, and mathematical abilities exists; yet, there are several reasons why further clarification of these associations is needed. Firstly, the interrelationship between these variables have only recently started gaining the interests of researchers. As such, the findings are

relatively few. To the author's knowledge, the research that have examined the relationship between cognitive skills, verbal abilities, and reading skills have been able to demonstrate in separate studies that mathematical performance are linked to executive functioning skills, phonological awareness skills, fluency skills (e.g. Fuhs, et al., 2016; Mazzocco & Grimm, 2013; Vukovic et al., 2010). However, it appears that a full exploration of how these variables relate to specific mathematical skills and concepts is not fully understood. To the best of the author's knowledge, an all-inclusive consideration of these variables within a single study has not been done.

Moreover, research looking at mathematical skills has examined arithmetic ability in terms of early skills such as counting, number identification, or later skills such as problem solving, or, the research had looked at mathematical achievement in school, or performance on IQ tests; it is to the author's knowledge that a comprehensive look at how cognitive, verbal and reading skills relate to performance on different concepts of mathematics have not been examined. It is pertinent that attention is given towards understanding how cognitive, verbal skills, and reading skills relate to different mathematical concepts, as children with different subtypes of mathematical difficulties may experience difficulty in some areas of mathematics, but may be competent in other mathematical concepts. Present research that explored the relationship between these variables and mathematical skills have yet to demonstrate the distinctive relationship between these skills and specific mathematical skills and concepts. Only with further clarification of the possible factors that contribute to the different mathematical concepts can a path towards more efficient mathematical instruction be fostered.

Another reason why it is essential to explore this relationship further is because some of the present research that have studied the relationship between these variables and mathematical

skills have utilized the arithmetic component of the IQ test as a measure of mathematical skills, which might have little relevance to mathematical achievement tests in school. To remedy this issue, this present study aims to elucidate the relationship between cognitive, verbal abilities, and reading skills and performance on specific mathematical concepts in the KeyMath-Revised Test (Connolly, 1988). The KeyMath-Revised Test (Connolly, 1988) assesses specific mathematical concepts that are relevant to the curricula from kindergarten through the ninth-grade. Examining the Addition, Geometry, Measurement, Numeration, Subtraction, and Time & Money subtests on the KeyMath-Revised Test (Connolly, 1988) will provide a better understanding of how these variables impact specific mathematical competencies that will have more applicability to mathematical academic performance.

Also, further exploration of how verbal skills and reading skills relate to mathematical abilities is critical because even though attention has been given to consider the possible role of verbal skills and reading skills in mathematical abilities, most of these studies have examined these constructs separately. That is, when researchers explored the relationship between reading and mathematics, verbal skills are neglected; and when the association between verbal skills and mathematical skills is analyzed, the relationship between mathematical skills and reading skills is disregarded. Failing to consider verbal skills and reading skills as separate constructs within an analysis may present an inaccurate understanding of how these skills relate to mathematical performance; and this is especially the case for children with reading disabilities whose verbal skills and reading skills might not be parallel to one another. Thus, questions remain as to how each of these skills independently and collectively interact with mathematical skills, as such, this study will determine how these constructs relate to mathematical skills.

Additionally, further analyses will be valuable to the literature because the studies that have examined the relationship between cognitive abilities and mathematical skills have mostly solely focused on the contributions of executive functioning on mathematical performance; as such, these studies typically employ neuropsychological tests that measure specific underlying cognitive functions such as working memory or inhibitory control. Even though, the information gained is useful, what is lacking in the literature is an understanding of how general cognitive ability relates to mathematical skills. General cognitive ability has significance in the individuals' ability to learn and engage in problem-solving tasks among other things, as such it is critical to learn how general cognitive ability relates to mathematical performance. As such, to counter this gap in the literature, this study will consider the relationship between general cognitive ability (based on IQ assessment) and mathematical performance.

Finally, considering the percentage of children affected by comorbid reading and mathematical difficulties, sufficient information in regards to the underlying factors that give rise to mathematical difficulty in children with reading disabilities has yet to be obtained. This study thus aims to close the gap in what is lacking in the literature by investigating the cognitive, verbal, and reading components that have suggested to be linked to mathematical skills and in effect develop a parsimonious model of mathematical skills for children with reading disabilities. With the development of a simple model exemplifying how various skills and abilities relate to mathematical skills, more effective means towards mathematical learning could be established.

2.1 Project Aims

In this current study, the mathematical ability of second and third graders with reading disabilities are examined to address the possible factors that might influence their arithmetic competency. One of the aims of this study is to explore how different mathematical skills, as

presented by the subcomponents of the KeyMath-Revised Test (Connolly, 1988), relate to verbal abilities and reading skills. Another aim of the study is to consider the role of verbal skills, reading skills and cognitive skills in mathematical ability, and to determine the main factors that characterize the mathematical ability of children with reading disabilities. With these aims, the following questions are addressed:

Question 1: What is the relationship between children’s performance on different concepts of mathematics and their verbal and reading skills? It is hypothesized that the mathematical concepts that are more reliant on language skills for their reasoning and application will be more closely related to the reading and verbal skills, while the mathematical skills that have a stronger basis in “pure” quantitative application will be less related to language skills. Specifically, it is anticipated that stronger associations will be found between the children’s scores on the Geometry, Measurement and Time & Money subcomponents of KeyMath – Revised Test (Connolly, 1988) and their verbal and reading skills, as compared to the relationship between Numeration, Addition, Subtraction and their verbal and reading skills. That is, it is predicted that there will be a stronger relationship between “advanced” mathematical skills and verbal skills and reading skills, as compared to the relationship between “basic” mathematical skills and verbal skills and reading skills.

Question 2: How can the mathematical ability of children with reading disabilities be characterized by their verbal skills, reading skills and cognitive functions? By addressing this question, the authors will gain understanding of how verbal skills, reading skills, and cognitive skills relate to mathematical skills. It is expected that taking into consideration all these factors will provide the best estimation of the children’s mathematical ability.

3 METHODS

3.1 Study Design

This current study utilizes data from a larger study which aimed to evaluate the efficiency of different reading intervention programs for second and third graders (Morris et al., 2012). Potential participants were introduced to the study after their teachers identified them as having trouble with reading. To qualify for the study, the children had to meet the criteria of having a reading disability based on the study's screening battery. The *Kaufman Brief Intelligence Test* (K-BIT; Kaufman & Kaufman, 1990) composite standard score was used to determine intellectual ability, while reading ability was determined by any of these calculations: 1) the average of the standard scores on the *Woodcock Reading Mastery Test – Revised* (WRMT-R; Woodcock, 1987) Passage Comprehension, WRMT-R Word Identification, WRMT-R Word Attack, and WRAT-3 Reading subtest; 2) the WRMT-R Basic Skills Cluster score; and/or 3) the WRMT-R Total Short Scale score. These different options were used to increase the homogeneity of the sample's reading profiles.

The children were randomly assigned to groups of four, and to one of four different intervention conditions. The conditions included either 1) a combination of PHAB and CSS, or 2) Math and CSS, 3) PHAB and WIST (PHAST), or 4) PHAB and RAVE-O (brief description listed below, for further information see Morris et al., 2012 for details). The children participated in 70 hours of intervention during the school year, along with four assessments of their abilities: once at the start of the program (0 hours), one in the middle of the program (35 hours), one at the end of the program (70 hours) and a final evaluation a year after the intervention.

3.1.1 Condition 1 – PHAB + CSS

PHAB (Phonological Analysis and Blending/Direct Instruction component; Engelmann & Bruner, 1988). The PHAB component of the intervention concentrated on developing phonological analysis and blending skills in children via training in letter-sound correspondences.

Classroom Survival Skills component (CSS; Archer & Gleason, 1991). Consisted of classroom etiquette, life skills, and organizational strategies with an emphasis on academic problem solving and self-help techniques. Parts of the CSS component were based off the Skills for School Success program (Archer & Gleason, 1991).

3.1.2 Condition 2 – MATH + CSS

MATH (The Mathematics Program component). The MATH portion of the intervention taught the participants basic math concepts, number facts, computational skills, and problem-solving strategies through direct instruction and metacognitive techniques.

3.1.3 Condition 3 – PHAB + WIST (PHAST)

WIST (Word Identification Strategy Training component; Lovett et al., 1994). In the WIST component of the program, the children were taught word identification strategies via four techniques: 1) via analogy, 2) looking for part of the word that is familiar, 3) trying to pronounce the vowels, and 4) removing prefixes and suffixes in a multisyllabic word.

PHAST program (Phonological and Strategy Training Program; PHAB + WIST; Lovett, Lacerenza, & Borden, 2000). The PHAST used a combination of techniques from the PHAB and WIST program that promotes the children's phonological, orthographical, and morphological skills.

3.1.4 Condition 4 – PHAB + RAVE-O

RAVE-O program (Retrieval, Automaticity, Vocabulary, Engagement with language, and Orthography; Wolf, Miller, & Donnelly, 2000). The RAVE-O program includes training in decoding via phonological processes but also includes orthography, semantics, syntax, and morphology for fluent comprehension.

3.2 Participants

The original study recruited 279 second and third graders between the ages of 78 months (6 years; 6 months) and 102 months (8 years; 6 months). Participants were enrolled from three metropolitan cities, Atlanta, Boston and Toronto. The inclusion criteria for participation included having English as a first language, normal hearing, vision and neurological functioning and attaining a composite score of 70 or above on the *Kaufman Brief Intelligence Test* (K-BIT; Kaufman & Kaufman, 1990) and a standard score of equal to or less than 85 on the Woodcock Reading Mastery Test-R (WRMT-R; Woodcock, 1987). Exclusion criteria included children who repeated a grade. This exclusion criterion was implemented to prevent the possible influence of past experiences on the results.

Of the 279 children employed for the original study, data of 130 participants were excluded from this present study as these participants did not complete the KeyMath – Revised Test (Connolly, 1988), which is an essential measure of this present study. The remaining 149 participants included in this present study were from the MATH + CSS and PHAB + CSS intervention groups. These participants had completed the KeyMath – Revised Test (Connolly, 1988) and have completed all of the measures required for the purposes of the study.

3.3 Setting

The instructional interventions took place during the school year and at the schools in which the children were enrolled. The venue at which the interventions and assessments were carried out were typically in unoccupied offices or classrooms. The interventions took place in groups of four while the test assessments were administered individually to each child.

Experienced and trained teachers carried out the interventions.

3.4 Assessment Measures

In the original study, nationally normed and standardized assessments were carried out pre-intervention (time 0), after 35 hours of intervention (time 35), after 70 hours of intervention (time 70), and 1-year after the intervention was completed (1-year follow-up). For the purposes of this current study, only measures at time 0 will be utilized for analyses.

3.4.1 *Mathematical Ability*

KeyMath – Revised – Test (Connolly, 1998) was used to determine the children's arithmetic ability for specific mathematical skills. *KeyMath – Revised Test* (Connolly, 1988) is a test that is administered individually by a trained examiner and it is meant for participants from 4 years; 6 months of age to 21 years; 11 months of age. *KeyMath – Revised Test* (Connolly, 1988) was nationally normed based on the United States (U.S.) Census reports. The sample was stratified by geographic region, grade, sex, socioeconomic level, race, and parents' level of educational achievement. The examiner's manual did not report information about intellectual or learning disabilities. There were a total of 258 questions on the test that belong to three different themes: basic concepts, operations and application. Each of the sections includes several subsections. Numeration, Rational Numbers and Geometry are under the section of basic concepts; addition, subtraction, multiplication, division, and mental computation are within the

Operations section; and measurement, time/money, estimation, interpretation of data, and problem solving are under the Applications section of the KeyMath – Revised Test (Connolly, 1988). For the purposes of this study, only addition, geometry, measurement, numeration, subtraction and time & money sections are included in the data analyses.

The KeyMath – Revised Test (Connolly, 1988) administration is carried out by the examiner by asking the participant to respond to questions orally, written computation is only needed on some of the subtests of the operations area (e.g., Addition, Subtraction, and Multiplication). The test is carried out from the first item on the test and discontinued if the participant answers three consecutive questions incorrectly.

3.4.2 Reading and verbal skills

The following measures were used to assess the children's reading and verbal proficiency. Based on the analyses, some of the measures from both reading skills and verbal skills are combined.

3.4.3 Reading skills

3.4.3.1 Comprehensive Test of Reading Related Phonological Processes (CTRRPP; Torgesen & Wagner, 1996), i.e., Comprehensive Test of Phonological Processing (CTOPP; Wagner, Torgesen, & Rashotte, 1999)

CTRRPP was the prepublication research version and forerunner of the published CTOPP (1999). The CTOPP is a test meant to assess individuals from the ages of 5 to 25 years of age. The test was norm-referenced based on 1,656 individuals between the ages 6 to 24. The manual indicated that the sample was representative of the U.S. school population including children with disabilities (Wagner et al., 1999). There are a total of 13 subtests in CTRRPP (1996), but for the purposes of this paper, only two of the subtests of the CTRRPP (1996) will be

used: blending and elision. These tests are individually administered. The blending section assesses the participant's ability to manipulate phonemes, or, to put sounds together to form words (e.g. "pop-corn" would require the correct response of "popcorn"). The elision section examines the participant's phoneme deletion ability, or the participant's ability to remove sounds of spoken words (e.g., "Say cat." "Now say cat without saying /k/." A correct response would be, "at."). These subtests are carried out from the first item on the test and discontinued if the participant missed three consecutive items.

3.4.3.2 *Woodcock Reading Mastery Test – Revised (WRMT-R; Woodcock, 1987)*

The WRMT (1987) is an individually administered test that assesses the reading skills of children and adults from ages of 5-75. The WRMT-R was based on a norm-referenced sample of 6,089 which was comparative to the U.S. population based on 1980 U.S. census information. The sample included children with learning disabilities, and the total score on the WRMT-R was found reliable for children from both the group with learning disabilities and those without. For the purposes of this study, three subtests of the WRMT-R will be used: word identification, word attack and passage comprehension. The word identification section evaluates the participants' noncontextual word reading skills; a word is shown to the participant and the participant has to read and pronounce the word presented without any contextual cues. Word attack measures the participant's ability to decode non-words, i.e., the participant has to attempt to pronounce nonsense words. The passage comprehension subscale assesses the participant's competency in reading a short passage of typically two to three sentences long, and the participant's also has to determine a missing key word within a passage. The WRMT also provides the Basic Skills Cluster score which is a composite score of the word identification and word attack sections,

while the passage comprehension subscale is one out of two (the other being word comprehension) subscales within the Reading Comprehension Cluster.

3.4.4 Verbal skills

3.4.4.1 Sound Symbol Identification (SSI; Lovett, et al., 1994)

The SSI is an individually administered test that consists of four subtests. The subtests include Letter Sound Identification, Sound Combination Identification, Onset identification, and Rime identification. These subtests examine the children's ability to say the sounds of the letters or combination of letters that are presented to them by an administrator. A composite score based on the sum of all the subtests was used for analyses in this study.

3.4.4.2 Kaufman Brief Intelligence Test (Verbal IQ) (K-BIT IQ; Kaufman & Kaufman, 1990)

The KBIT is an individually-administered test that is meant to assess the verbal and nonverbal intelligence of children and adults from the ages of 4 years; 0 months to 90 years; 11 months. The K-BIT was standardized based on the 1990 U.S. census representative sample of 2,022 individuals and was based on gender, geographical region, socioeconomic status, and race and ethnic group. The KBIT includes verbal and nonverbal scales that do not require reading or spelling. The KBIT consists of three subtests: verbal knowledge, riddles and matrices. The KBIT-Verbal IQ was used to assess the children's verbal skills in this study, and the Verbal IQ score includes the verbal knowledge and riddles subtests. Verbal knowledge examines the individual's receptive vocabulary and general knowledge, and riddles assesses the individual's comprehension, reasoning and vocabulary knowledge. The KBIT also provides a composite score for IQ.

3.4.4.3 Wechsler Intelligence Scale for Children (Verbal) (WISC-IV; Wechsler, 2004)

The WISC-IV is a paper and pencil, individually administered measure of intelligence intended for children from the age of 6 years to 16 years and 11 months. The WISC-IV standardization was based on a sample of 2,200 individuals, representative of the 1988 U.S. Census on gender, socioeconomic status, race and ethnicity, and geographic region. The sample also included special groups such as children that were intellectually gifted, children with intellectual and learning disabilities. The WISC-IV assesses the cognitive ability of its participants via evaluation of their verbal comprehension, perceptual reasoning and working memory. The WISC-IV consists of 13 subtests with 6 of the subtests assessing Verbal IQ; these subtests include the information, digit span, vocabulary, arithmetic, comprehension, and similarities subtests, and the remaining 7 subtests assesses the participants' performance IQ which is used in this study to assess the children's cognitive functioning skills.

3.4.5 Cognitive Functioning

3.4.5.1 Kaufman Brief Intelligence Test (Non-Verbal IQ) (K-BIT IQ; Kaufman & Kaufman, 1990)

The KBIT non-verbal intelligence measure is a subtest within the larger KBIT test which was described above. The non-verbal measure includes the matrices subtest of the KBIT, and it measures the individual's understanding of relationship and visual analogies. The KBIT non-verbal IQ measures fluid reasoning ability and is used as an assessment of cognitive functioning in this study.

3.4.5.2 *The Wechsler Intelligence Scale for Children (Performance IQ) (WISC-IV; Wechsler, 2004)*

The WISC-IV Performance IQ is a subscale within the larger WISC-IV as described above. The WISC-IV Performance IQ includes picture completion, picture arrangement, block design, object assembly, coding, mazes, and symbol search.

4 RESULTS

4.1 Data Screening and Descriptive Statistics

Histograms, scatterplots, and $q-q$ plots were created to screen for outliers and to determine the distribution of data. Skewness analyses indicated that all of the variables had a normal distribution, i.e., a skewness value of ± 1 , except for the Word Attack subset of the WRMT-R (Woodcock, 1987) which was positively skewed at 1.65. Some researchers have indicated a more stringent value of ± 1 for skewness as the acceptable range for normality (e.g., Bulmer, 1979), while other researchers have stated that skewness values between ± 2 represent normality of distribution (Field, 2000 & 2009; Trochim & Donnelly, 2006).

Kurtosis analyses showed that all of the variables met the criteria for normality, i.e., ± 2 (George & Mallery, 2010), with values ranging from -1.21 to .92 for all of the variables, with the exception of Word Attack and Sound Combination. Word Attack had a kurtosis value of 2.02, which was deemed as an acceptable value for normality. Sound Combination had a kurtosis value of 2.45, which violates the criteria for normality. However, Sound Combination was not analyzed independently; rather, a composite score was attained by combining the scores from the other subscales of the Sound Symbol Identification Test (Lovett, 1994): Letter Sound, Onset, and Rime. These scores together produced a composite score which had a kurtosis of -.93, which was deemed acceptable.

To determine if this data set would benefit from an adjustment, a constant of 1 was added prior to an application of a logarithmic transformation for all of the variables. With this adjustment, the skewness of the Word Attack subset of the WRMT-R (Woodcock, 1987) improved, from the original value of 1.65 to a value of .64, which met the more stringent criteria for normality. However, the other variables were negatively impacted by this adjustment. As such, the raw data were utilized for analyses.

Participants' demographics are presented in Table 1 while descriptive statistics (i.e., mean, standard deviation, range, skewness and kurtosis) of all of the variables utilized in the present study are presented in Table 2. The data utilized in this study are from Time 0, the pre-intervention phase.

Table 1: Demographics of Participants

	Mean (SD)	Minimum	Maximum
Child's Age in Months	92.15 (6.43)	80.82	107.53
Fathers' Education in Years	12.4 (2.34)	6	17
Mothers' Education in Years	12.47 (2.33)	2	19

Table 2: Descriptive Statistics for subscales in KeyMath-R, SSI, KBIT, WISC, CTRRPP, and WRMT at Time 0

	Mean (SD)	Range	Skewness (SE)	Kurtosis (SE)
KeyMath-R				
Numeration	8.51 (3.07)	3 - 19	.69 (.20)	.45 (.40)
Geometry	8.48 (3.31)	0 - 15	-.28 (.20)	-.51 (.40)
Measurement	7.43 (2.76)	1 - 12	-.32 (.20)	-.66 (.40)
Addition	6.23 (2.39)	1 - 13	.40 (.20)	.14 (.40)
Subtraction	3.52 (2.02)	0 - 9	.53 (.20)	-.51 (.40)
Time/Money	4.08 (2.59)	0 - 12	.68 (.20)	.59 (.40)
SSI				
Letter Sound	20.94 (8.79)	0 - 35	-.70 (.20)	-.39 (.40)
Onset	4.78 (5.07)	0 - 15	.55 (.20)	-1.21 (.40)
Rime	3.89 (.37)	0 - 19	1.14 (.20)	.73 (.40)
Sound				
Combination	4.84 (.34)	0 - 24	1.18 (.20)	2.45 (.40)
Composite	34.37 (19.14)	0 - 74	.12 (.20)	-.93 (.40)
K-BIT				
Verbal	28.65 (5.57)	19 - 43	.20 (.20)	-.79 (.40)

WISC	Non-Verbal	19.06 (3.71)	11 - 32	.76 (.20)	.92 (.40)
	Verbal IQ	89.07 (11.90)	62 - 124	.40 (.21)	.05 (.41)
	Performance IQ	91.84 (14.63)	62 - 125	.02 (.21)	-1.02 (.41)
CTRRPP	Blending Words	8.92 (4.69)	0 - 22	.02 (.20)	-.56 (.40)
	Elision	7.88 (3.72)	0 - 21	.66 (.20)	.50 (.40)
WRMT	Word ID	17.55 (12.19)	0 - 45	.30 (.20)	-1.02 (.40)
	Word Attack	1.96 (.23)	0 - 11	1.65 (.20)	2.02 (.40)
	Passage				
	Comprehension	8.15 (6.42)	0 - 27	.67 (.20)	-.48 (.40)

Note. KeyMath-R (KeyMath-Revised Test, Connolly, 1998), SSI (Sound-Symbol Identification, Lovett et al., 1994), K-BIT (Kaufman Brief Intelligence Test, Kaufman & Kaufman, 1990), WISC (Wechsler Intelligence Scale for Children, Wechsler, 2004), CTRRPP (Comprehensive Test of Reading Related Phonological Processes, Torgesen & Wagner, 1996), WRMT-R (Woodcock Reading Mastery Test-Revised, Woodcock, 1987).

4.2 Examining the relationship between mathematical performance and verbal skills, reading skills, and cognitive functioning

To attain a better understanding of how children's performance on different concepts of mathematics relate to their verbal skills, reading skills, and cognitive functioning, a correlation matrix based on bivariate Pearson's correlations was created. The Addition, Geometry, Measurement, Numeration, Subtraction, and Time & Money subscales of the KeyMath – Revised Test (Connolly, 1998) were used to represent the various mathematical concepts that are pertinent in grade school. The Sound-Symbol Identification (Lovett et al., 1994) test, the Verbal subset of the K-BIT (Kaufman & Kaufman, 1990), and the Verbal IQ subscale of the WISC (Wechsler, 2004) were used to determine children's verbal skills. The Blending and Elision subtests of the CTRRPP (Torgesen & Wagner, 1996), and the Word Identification, Word Attack, and Passage Comprehension subtests of the WRMT-R (Woodcock, 1987) were used to represent the children's reading skills. The Non-Verbal section of the K-BIT (Kaufman & Kaufman, 1990), and the Performance IQ subscale of the WISC (Wechsler, 2004) represented the

children's cognitive functioning. Table 3 displays the correlations of all of the variables utilized in this study.

Table 3: Zero-order correlations

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.
1. KeyMath-R Numeration	1.0															
2. KeyMath-R Geometry	.52**	1.0														
3. KeyMath-R Addition	.61**	.45**	1.0													
4. KeyMath-R Subtraction	.65**	.53**	.63**	1.0												
5. KeyMath-R Measurement	.61**	.44**	.50**	.52**	1.0											
6. KeyMath-R Time/Money	.70**	.47**	.63**	.61**	.54**	1.0										
7. SSI Composite	.56**	.35**	.55**	.55**	.48**	.47**	1.0									
8. K-BIT Verbal	.48**	.39**	.36**	.40**	.48**	.42**	.55**	1.0								
9. K-BIT Non-Verbal	.50**	.33**	.39**	.50**	.45**	.41**	.45**	.43**	1.0							
10. WISC Verbal IQ	.48**	.39**	.34**	.44**	.56**	.40**	.44**	.54**	.46**	1.0						
11. WISC Performance IQ	.34**	.23*	.29**	.35**	.54**	.25*	.26*	.27**	.37**	.37**	1.0					
12. CTRRPP - Blending	.35**	.32**	.29**	.30**	.36**	.22**	.67**	.60**	.27**	.45**	.31**	1.0				
13. CTRRPP - Elision	.47**	.30**	.41**	.43**	.37**	.35**	.58**	.44**	.44**	.33**	.21*	.52**	1.0			
14. WRMT - Word ID	.60**	.40**	.59**	.60**	.37**	.51**	.68**	.46**	.43**	.28**	.05	.41**	.50**	1.0		
15. WRMT - Word Attack	.40**	.24*	.40**	.44**	.31**	.36**	.71**	.43**	.41**	.37**	.15	.48**	.52**	.63**	1.0	
16. WRMT - Passage Comprehension	.59**	.40**	.58**	.58**	.41**	.56**	.63**	.46**	.49**	.28**	.09	.37**	.49**	.87**	.60**	1.0

Note: Two-tailed significance, * $p < .05$, ** $p < .001$. KeyMath-R (KeyMath-Revised Test, Connolly, 1998), SSI (Sound-Symbol Identification, Lovett et al., 1994), K-BIT (Kaufman Brief Intelligence Test, Kaufman & Kaufman, 1990), WISC (Wechsler Intelligence Scale for Children, Wechsler, 2004), CTRRPP (Comprehensive Test of Reading Related Phonological Processes, Torgesen & Wagner, 1996), WRMT-R (Woodcock Reading Mastery Test-Revised, Woodcock, 1987).

4.2.1 Overall relationship between all variables

In general, low to high strong positive correlations were found between all of the measures, $r = .24$ to $r = .87$, $p < .001$ to $p < .05$, except for the correlations between Performance IQ (WISC; Wechsler, 2004) and Word Identification, Word Attack and Passage Comprehension (WRMT-R; Woodcock, 1987), $r = .05$, $r = .15$, and $r = .09$, $p = n.s.$ (Cohen, 1988). Also, the correlation matrix indicated that all of the relationships among the variables within and between constructs were of an expected direction, i.e., higher performance on Numeration was correlated with higher performance on the Time & Money section of the KeyMath – Revised Test (Connolly, 1998), as well as better performance in mathematics was related to better verbal skills. From these results, it shows that in general, higher verbal skills, reading skills, and cognitive functioning abilities are associated with higher performance on mathematical concepts.

4.2.2 Performance across different mathematical concepts

The correlation matrix indicates a moderate to high association between the various mathematical concepts, $r = .44$ to $r = .70$, $p < .001$ (Cohen, 1988). This suggests that mathematical performance across concepts are related to one another, e.g., children with better performance on Numeration tend to perform better on Addition, $r = .61$, $p < .001$, and, children with better performance on Geometry tend to perform better on Time & Money, $r = .47$, $p < .001$.

4.3 The relationship between the various mathematical concepts and verbal skills and reading skills

Research Question 1: What is the relationship between children's performance on different concepts of mathematics and their verbal and reading skills?

The original hypothesis was that stronger associations would be found between the Geometry, Measurement, and Time & Money subcomponents of KeyMath – Revised Test (Connolly, 1988) and children’s verbal and reading skills, as compared with the relationship between Numeration, Addition, Subtraction and reading and verbal skills. The hypothesis was based on the belief that the former mathematical concepts are deemed to be more “advanced” and thus necessitate stronger verbal and reading skills for their understanding and application as compared to the latter, which are comparatively more “basic”.

4.3.1 Assessing the two-construct model of “basic” and “advanced” mathematical skills

To determine if “basic” and “advanced” mathematical concepts related differently to verbal skills and reading skills, path analyses were first conducted to determine if these mathematical concepts can be classified according to their levels of difficulty, i.e., “basic” and “advanced”. Path analysis is a statistical methodology which enables researchers to determine the direction of relationships among a set of variables, and to determine the fit of constructs. Path analysis also allows the researcher to investigate the unique relationship between each of the variables, and provide an indication of the weight of influence of the variables employed. Path analysis was conducted using the Mplus Version 5.21 (Muthén & Muthén, 2007).

The results of this model and subsequent models were determined to have acceptable or good fit based on the standards provided by the literature, i.e., (1) an insignificant chi-square (χ^2) value (Wheaton, Muthen, Alwin, & Summers, 1977), (2) a RMSEA (The Root Mean Square Error of Approximation) value between 0.05 to 0.10 is considered acceptable, with values close to 0.07 indicating good fit (Browne & Cudeck, 1993). (3) A CFI (Comparative Fit Index) value greater than 0.90 is deemed an acceptable fit, while a CFI value of .95 and above is deemed as a good fit model (Kline, 2005). (4) SRMR (Standardized Root Mean Square Residual) values of

.08 and lower represents acceptable fit, with values of less than .05 representing a good fit model (Byrne, 1998).

Numeration, Addition, and Subtraction (KeyMath – Revised Test; Connolly, 1988) were employed as indicators of “basic” mathematical skills, and Geometry, Measurement, and Time & Money were used as indicators of “advanced” mathematical skills. The two-construct model of “basic” and “advanced” mathematical skills was then assessed. The results of the model fit were as followed: $\chi^2(8) = 6.80, p = n.s., RMSEA = 0, CFI = 1.00, SRMR = .02$. This model only met the standards of a good fit based on its chi-square value (Wheaton, et al., 1977). The correlation coefficient between “basic” mathematical skills and “advanced” mathematical skills was 1.03, suggesting that these constructs should be assessed as a single construct of mathematical skills. This finding also suggests that verbal skills and reading skills would not relate differently to “basic” and “advanced” mathematical skills; subsequent analyses explored how the variables of interest related to mathematical skills as a single construct. Figure 1 displays the graphic representation of the model.

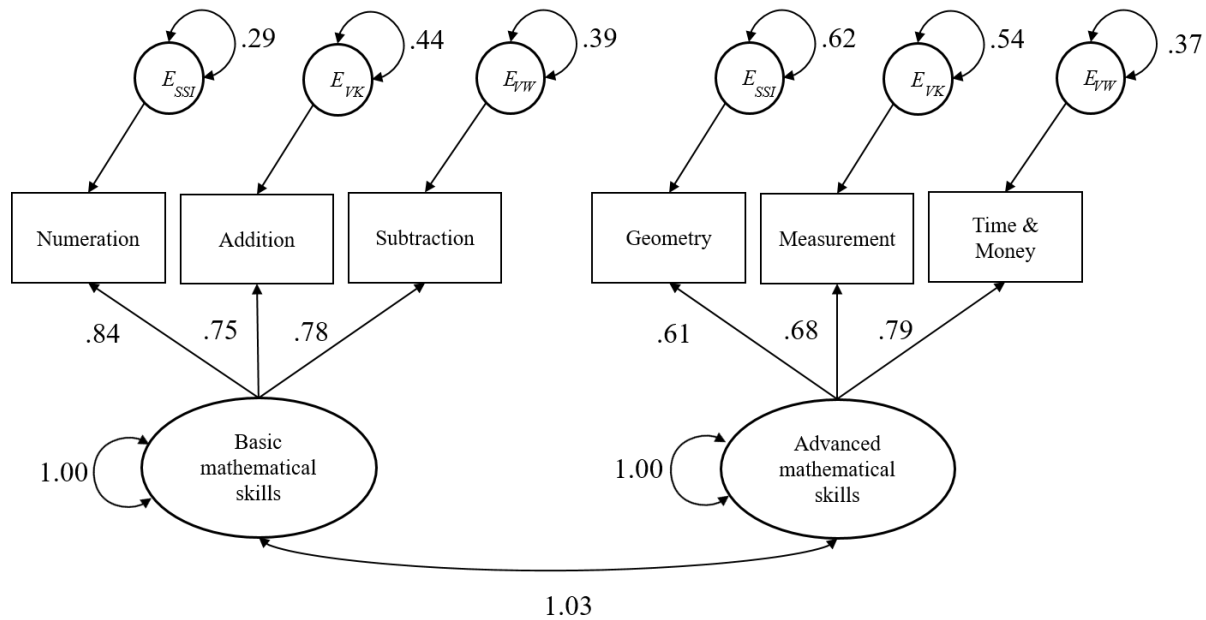


Figure 1: A two-construct model of “basic” and “advanced” mathematical skills. Note. Numeration, Addition, Subtraction, Geometry, Measurement and Time & Money represents the subscales from the KeyMath – R (KeyMath-Revised Test; Connolly, 1998).

4.3.2 Assessing mathematical ability as a single construct

Mathematical skills were then assessed as a single construct with Numeration, Addition, Subtraction, Geometry, Measurement, and Time & Money as indicators. The graphic representation of this model is displayed in Figure 2. The results indicated that mathematical skills as a single construct is valid, $\chi^2(9) = 7.58, p = .058, RMSEA = 0, CFI = 1.00, SRMR = .02$.

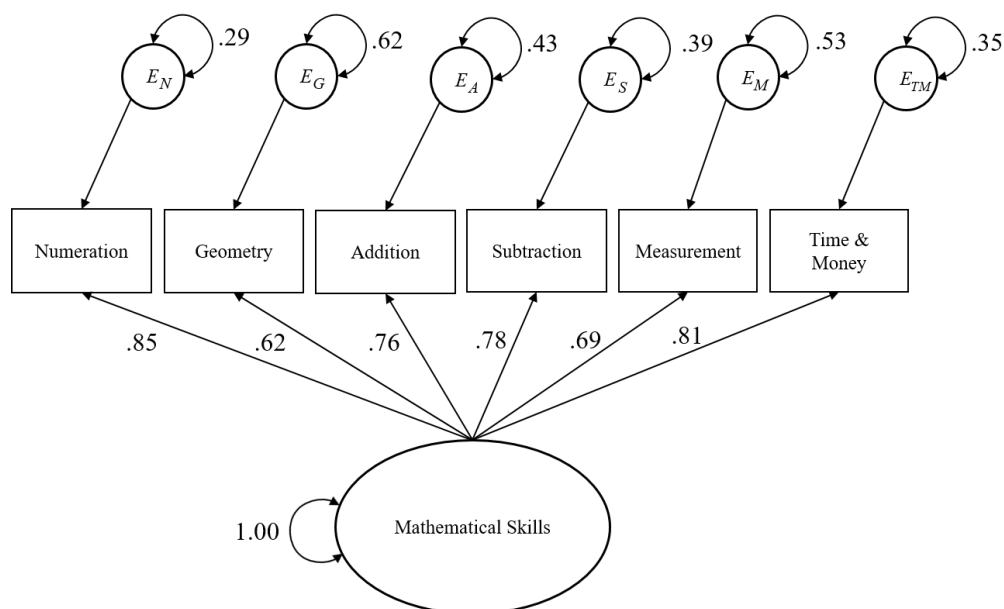


Figure 2: Mathematical skills as a single construct.

Note. Numeration, Addition, Subtraction, Geometry, Measurement and Time & Money represents the subscales from the KeyMath – R (KeyMath-Revised Test; Connolly, 1998).

4.4 The relationship between mathematical skills, verbal skills, reading skills, and cognitive functioning

Research Question 2: *Can the mathematical ability of children with reading disabilities be characterized by their verbal skills, reading skills and cognitive functioning?*

The second objective of this study was to attain an understanding of the influence of verbal skills, reading skills, and cognitive functioning on mathematical skills, and to determine the extent in which these constructs predict mathematical performance. Essentially, these analyses aimed to develop a model of mathematical abilities for children with reading disabilities. To investigate how the hypothesized contributing constructs interacted with one other, path analyses were conducted with the variables of interest.

The following variables were employed as indicators of constructs for the subsequent goodness of fit tests. Specifically, Sound-Symbol Identification (Lovett et al., 1994), K-BIT – Verbal (Kaufman & Kaufman, 1990), and WISC – Verbal (Wechsler, 2004) were employed as

indicators for verbal skills. The Blending and Elision subsets of the CTRRPP (Torgesen & Wagner, 1996), and the Word Identification, Word Attack, and Passage Comprehension of the WRMT-R (Woodcock, 1987) were utilized as indicators within the construct of reading skills. When the construct that was assessed was a combination of both verbal skills and reading skills, the indicators from both verbal and reading skills were used. The Non-Verbal section of the K-BIT (Kaufman & Kaufman, 1990) and the Performance IQ subscale of the WISC (Wechsler, 2004) were used as indicators for cognitive functioning.

4.4.1 Testing models for mathematical ability

The constructs included in the proposed models were selected based on evidence in the literature that shows a relationship to mathematical skills (e.g., Korhonen et al., 2012; Passolunghi et al, 2014; Purpura & Ganley, 2014). Several proposed models were tested: from simpler models with two constructs, to more complex models involving three to four constructs. Two construct models included models with mathematical skills and verbal skills; mathematical skills and reading skills; mathematical skills and verbal and reading skills combined; and mathematical skills and cognitive functioning. The models with three and four constructs examined the relationship between mathematical skills and a combination of the earlier mentioned factors, ensuring that all variations for combinations between the constructs were tested.

The modification indices output of the Mplus Version 5.21 (Muthén & Muthén, 2007) provides information on how the model could be adjusted for the best possible fit. With the model modifications, several models emerged as acceptable models for children's mathematical skills. These models are displayed in the appendices (Appendix A-G). Based on the principle of parsimony and the comparison of the AIC (Akaike Information Criterion) values among the

models with the best fit, the two-construct model of mathematical skills and verbal skills was determined to be the best model of mathematical skills. Other models that were assessed but did not result in a good fit as compared to the other models, are discussed briefly.

4.4.2 Assessing model fit for mathematical skills and verbal skills

Two models with verbal skills predicting mathematical skills were examined. One model was assessed prior to model modification, and the other was assessed post model modification, as suggested by the model indices. These models included the same constructs and variables, with only the paths between the variables differing. These models are displayed in Appendices A and B, with Appendix A representing pre-model modification, and Appendix B representing the model following model modification.

Prior to model modification, the results suggested that verbal skills met some of the criteria for a good fit, $\chi^2(26) = 44.01, p = .02, RMSEA = 0.07, CFI = 0.97, SRMR = .04$. Specifically, the model met the standards of a good fit based on its RMSEA, CFI and SRMR values without model modification (displayed in Appendix A).

As suggested by the model indices, a path representing a relationship between Measurement (KeyMath- R; Connolly, 1998) and Verbal IQ (WISC; Woodcock, 1987) was added. This resulted in an even better model for mathematical skills, $\chi^2(25) = 32.85, p = .13, RMSEA = 0.05, CFI = 0.99, SRMR = .04$. This model met the criteria for a good fit for all of the tests. That is, the model's chi-square value was insignificant, had a RMSEA value close to 0.07, a CFI value above 0.95 and a SRMR value below .05. This model had a regression coefficient of .83, suggesting that improvements in verbal skills are linked to an increase in mathematical performance. The model also indicates that the residual variance for mathematical skills is 0.30,

suggesting that 30% of variance in mathematical performance is unaccounted for by verbal skills. This model is displayed in Figure 3 and in Appendix B.

4.4.3 Assessing model fit for mathematical skills and reading skills

Three different models with reading skills predicting mathematical skills were examined. The first model was assessed prior to model modification. The latter two models were assessed post model modification, with slight modifications, as suggested by the model indices. The difference between these models are the absence and addition of paths indicating the relationship between variables within reading skills (details of each model are discussed below). The models are displayed in Appendices C, D, and E accordingly.

The results of the model with mathematical skills and reading skills without model modification indicated that it met some of the standards of a good model, $\chi^2(43) = 85.43, p = .00$, RMSEA = 0.08, CFI = 0.96, SRMR = .06. This model met the requirements for RMSEA, CFI, and SRMR, and is displayed in Appendix C. Based on the suggestions from the model indices, two different models were developed with slight variations between each model.

The first model assessed post-model modification included the addition of two paths indicating a relationship between the Elision and Blending subsets of the CTRRPP (Torgesen & Wagner, 1996) and relationship between Word Identification and Passage Comprehension subtests of the WRMT-R (Woodcock, 1987) (displayed in Appendix D). The two-path modification was deemed appropriate as the paths between the variables that were added, were from the same test, e.g., Elision and Blending subtest are both from the CTRRPP (Torgesen & Wagner, 1996). The fit of the adjusted model is as follows: $\chi^2(41) = 57.52, p = .05$, RMSEA = .05, CFI = .98, SRMR = .04, which signifies an acceptable fit for χ^2 , and a good fit based on its RMSEA, CFI and SRMR values. The regression coefficient of this model is .79, suggesting that

improvements in reading skills are linked to increase mathematical performance. The residual variance for mathematical skills in this model is 0.38, suggesting that verbal skills (.30 both before and after model modification) explained a higher variance in mathematical skills as compared to reading skills.

The second model assessed post-model modification included the addition of three paths to the unmodified model of reading skills predicting mathematical skills. As done in the earlier model modification (displayed in Appendix D), two paths between the Elision and Blending (CTRRPP; Torgesen & Wagner, 1996) and between Word Identification and Passage Comprehension (WRMT; Woodcock, 1987) were added. In addition, this model also includes a new path between Blending and Word Attack (WRMT; Woodcock, 1987) as suggested by the model indices. This model is displayed in Appendix E. The fit of this model is as follows: $\chi^2(40) = 52.71, p = .09, RMSEA = .05, CFI = .99, SRMR = .04$, which signifies a good fit for χ^2 , RMSEA, CFI and SRMR. The residual variance for mathematical skills in this model is 0.37, showing that the addition of the new path did not improve the model substantially.

4.4.4 Assessing other two construct models

Another 4 models (two pre- and two post-modification), consisting of two constructs 1) mathematical skills and cognitive functioning, 2) mathematical skills and a combination of verbal skills and reading skills were assessed. These models did not meet as many goodness of fit standards as compared to the two construct models of mathematical skills and reading skills, and mathematical skills and verbal skills.

4.4.5 Assessing model fit for mathematical skills and cognitive functioning

In the model whereby cognitive functioning predicted mathematical skills, the model fit after the addition of a path linking Measurement (KeyMath – R; Connolly, 1998) and

Performance IQ (WISC; Woodcock, 1987), as suggested by the modification indices, was $\chi^2 (18) = 15.98, p = .59, RMSEA = .00, CFI = 1.00, SRMR = .03$. The results of the model met the criteria of an acceptable fit for chi-square, and a good fit for CFI and SRMR (Byrne, 1998; Kline, 2005; Wheaton, Muthen, Alwin, & Summers, 1977).

4.4.6 Assessing model fit for mathematical skills, and verbal skills and reading skills combined

In the model, whereby verbal skills and reading skills together predicted mathematical skills, modification indices suggested the addition of many paths for the improvement of model fit. With the addition of three paths indicating a relationship between Sound-Symbol Identification (Lovett et al., 1994) and Blending (CTRRPP; Torgesen & Wagner, 1996); K-BIT – Verbal (Kaufman & Kaufman, 1990) and Blending (CTRRPP; Torgesen & Wagner, 1996); and Word Identification and Passage Comprehension (both from the WRMT-R; Woodcock, 1987), the results of the goodness of fit was: $\chi^2 (72) = 149.36, p = .00, RMSEA = .09, CFI = .94, SRMR = .07$. The results of the model met the criteria of an acceptable fit for RMSEA, CFI, and SRMR (Browne & Cudeck, 1993; Byrne, 1998; Kline, 2005).

4.4.7 Comparing all models with two constructs

Based on the standards of a model fit, as well as the principle of parsimony, the model involving mathematical skills and verbal skills was deemed to be the best model for mathematical skills for children with reading disabilities. As a single construct, verbal skills consist of fewer variables as compared to reading skills. Furthermore, only a single path between Measurement (KeyMath – R; Connolly, 1998) and Verbal IQ (WISC; Woodcock, 1987) was added to attain an improvement in the model as compared to the post-modification model with mathematical skills and reading skills. In addition, as discussed above, verbal skills were more

highly related to mathematical performance as compared to reading skills (.83 for verbal skills as compared to .79 for reading skills). Goodness of fit statistics of all of the models tested are displayed in Appendix H.

4.4.8 Assessing model fit for mathematical skills, cognitive functioning, and verbal skills

Models with three constructs consisting of a series of combination and omissions of the constructs verbal skills, reading skills, verbal skills and reading skills combined, and cognitive functioning and mathematical skills were analyzed next. The model with mathematical skills, cognitive functioning and verbal skills produced the best model fit post-adjustments based on the model indices. These models are displayed in Appendices F and G, with these two models only having variations in the direction of paths.

Prior to model modification, a model indicating a mutual relationship between verbal skills and cognitive functioning, both predicting mathematical skills independently, produced a model fit of $\chi^2(41) = 99.04, p < .01, RMSEA = .10, CFI = .93, SRMR = .06$. This signifies a mediocre fit for RMSEA, CFI and SRMR. This model is displayed in Appendix F. Model indices suggested the addition of two paths indicating the relationship between Performance IQ (WISC; Woodcock, 1987) and Measurement (KeyMath – R; Connolly, 1998) and Performance IQ and Verbal both from the WISC (Woodcock, 1987) for improvement of model fit. Post model modification, the model fit was as follows: $\chi^2(39) = 54.66, p = .05, RMSEA = .05, CFI = .98, SRMR = .04$, which signifies an acceptable fit for χ^2 , and good fit based on its RMSEA, CFI and SRMR values. This model is displayed in Appendix G.

Prior to model modification, the regression coefficient of verbal skills was .47, and .39 for cognitive functioning. Post-modification, the regression coefficient was .45 for both constructs. In the model post-modification, only 26% of the variance in mathematical

performance was left unexplained. This model also showed that verbal skills is highly related to cognitive functioning, $r = .84$. Appendix H displays the results of the model fit of all of the models tested predicting mathematical skills as a single construct.

4.4.9 Assessing the fit of other three construct models

As mentioned above, other three construct models were assessed. In total, the fit of 6 other models (three pre- and three post-modification) were assessed; they were 1) verbal skills and reading skills predicting mathematical skills, 2) reading skills and cognitive functioning predicting mathematical skills and 3) cognitive functioning, and verbal skills and reading skills combined predicting mathematical skills. These models did not meet as many goodness of fit criteria as compared to the earlier mentioned model of cognitive functioning and verbal skills predicting mathematical skills.

4.4.10 Assessing model fit for mathematical skills, verbal skills, and reading skills

The results of the model fit of verbal skills and reading skills predicting mathematical skills, with the addition of paths linking Word Identification and Passage Comprehension (WRMT-R; Woodcock, 1987); and Sound-Symbol Identification (Lovett et al., 1994) and Word Attack (WRMT; Woodcock, 1987), was $\chi^2 (72) = 171.36, p = .00, RMSEA = .10, CFI = .92, SRMR = .07$. The results of the model fit met the criteria of an acceptable fit for RMSEA, CFI, and SRMR (Browne & Cudeck, 1993; Byrne, 1998; Kline, 2005).

4.4.11 Assessing model fit for mathematical skills, reading skills, and cognitive functioning

The model fit of reading skills and cognitive functioning predicting mathematical skills, with the addition of 4 paths including (1) mathematical skills to the Non-Verbal subscale (KBIT; Kaufman & Kaufman, 1990), (2) Performance IQ (WISC, Wechsler, 2004) and Measurement (KeyMath-Revised Test, Connolly, 1998), (3) Word Identification and Passage Comprehension

(both from the WRMT-R; Woodcock, 1987), and (4) Elision and Blending (both from the CTRRPP, Torgesen & Wagner, 1996), was $\chi^2(72) = 102.45$, $p = .59$, RMSEA = .07, CFI = .96, SRMR = .0. This model met the criteria for chi-square, and a good fit for RMSEA and CFI (Browne & Cudeck, 1993; Kline, 2005; Wheaton, et al., 1977).

4.4.12 Assessing model fit for mathematical skills, verbal skills and reading skills combined, and cognitive functioning

The model fit of cognitive functioning and verbal and reading skills combined predicting mathematical skills, with the addition of three paths including (1) cognitive functioning to Verbal IQ (WISC, Wechsler, 2004), (2) Performance IQ and Verbal IQ (both from the WISC, Wechsler, 2004), (3) and Word Identification and Passage Comprehension (both from the WRMT-R; Woodcock, 1987), was $\chi^2(98) = 224.47$, $p = .00$, RMSEA = .09, CFI = .91, SRMR = .07. This model met the criteria of an acceptable fit for RMSEA, CFI and SRMR (Browne & Cudeck, 1993; Byrne, 1998; Kline, 2005).

4.4.13 Assessing the fit of model with mathematical skills, verbal skills, reading skills, and cognitive functioning

A model consisting of three constructs: verbal skills, reading skills, and cognitive functioning predicting mathematical skills was examined. Model indices suggested that the addition of paths linking Word Identification and Passage Comprehension (WRMT-R; Woodcock, 1987); Performance IQ and Verbal IQ (both from the WISC, Wechsler, 2004); and having some variables indicated as being double loaded to the constructs would improve model fit. The Blending subset of the CTRRPP (Torgesen & Wagner, 1996) was modified to be double loaded onto verbal skills and reading skills, and Verbal IQ (WISC, Wechsler, 2004) was modified to be double loaded to verbal skills and cognitive functioning skills. The results of the

model fit post-modification was $\chi^2(95) = 220.21, p = .00, RMSEA = .09, CFI = .91, SRMR = .07$. This model met the acceptable fit for RMSEA, CFI and SRMR (Browne & Cudeck, 1993; Byrne, 1998; Kline, 2005).

4.4.14 Best Model for Mathematical Skills

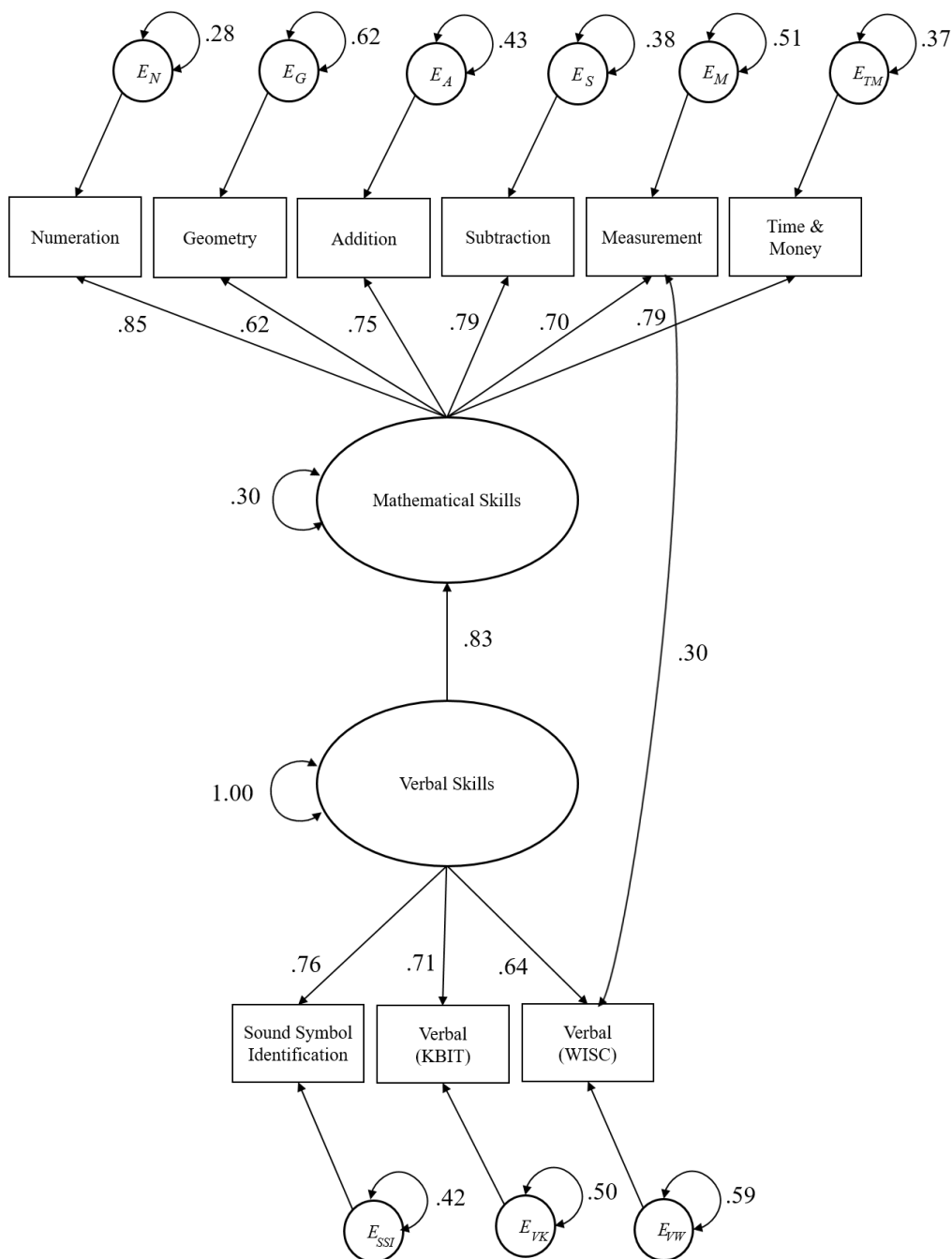


Figure 3: A modified path model indicating the relationship between mathematical skills and verbal skills based on standardized estimates, with a path added linking Measurement (KeyMath) and Verbal (WISC).

Note. Also displayed in Appendix D. Numeration, Geometry, Addition, Subtraction, Measurement and Time and Money represents the subscales from the KeyMath – R (KeyMath-Revised Test; Connolly, 1998); Sound Symbol Identification (Lovett et al., 1994); Verbal (KBIT) is from the Kaufman Brief Intelligence Test (Kaufman & Kaufman, 1990); Verbal (WISC) is from the Wechsler Intelligence Scale for Children (Wechsler, 2004). Curved, double-headed arrows represent the covariance between two variables.

Table 4: Estimates for the best model for mathematical skills

Parameter	Relation/Variable	Estimate	S.E.	Ratio	<i>p</i> -value	Std
Regression	MS by Numeration	1.00	0.00	999.00	999.00	0.85
	MS by Geometry	0.73	0.09	8.10	<0.01	0.62
	MS by Addition	0.89	0.08	10.55	<0.01	0.75
	MS by Subtraction	0.93	0.08	11.19	<0.01	0.79
	MS by Measurement	0.82	0.09	9.50	<0.01	0.70
	MS by Time and Money	0.94	0.08	11.38	<0.01	0.79
	VS by SSI	1.00	0.00	999.00	<0.01	0.76
	VS by Verbal (K-BIT)	0.93	0.12	7.83	<0.01	0.71
	VS by Verbal (WISC)	0.84	0.12	7.17	<0.01	0.64
	MS with VS	0.54	0.09	6.10	<0.01	0.83
	Measurement with Verbal (WISC)	0.16	0.05	3.12	<0.01	0.83
	Variances	MS	0.71	0.11	6.24	<0.01
VS		0.58	0.12	4.97	<0.01	1.00
Numeration		0.28	0.04	6.31	<0.01	0.28
Geometry		0.61	0.08	8.08	<0.01	0.62
Addition		0.43	0.06	7.45	<0.01	0.43
Subtraction		0.38	0.05	7.18	<0.01	0.38
Measurement		0.51	0.07	7.78	<0.01	0.51
Time and Money		0.37	0.05	7.09	<0.01	0.37
SSI		0.42	0.07	5.93	<0.01	0.42
Verbal KBIT		0.49	0.07	6.68	<0.01	0.50
Verbal WISC		0.58	0.08	7.30	<0.01	0.59

Note. SE = Standard Error. Std = Standardized Value. MS = Mathematical skills. Numeration, Geometry, Addition, Subtraction, Measurement and Time and Money represents the subscales from the KeyMath – R (KeyMath-Revised Test; Connolly, 1998); VS = Verbal skills. SSI = Sound Symbol Identification; Verbal (K-BIT) = Verbal subscale from Kaufman Brief Intelligence Test; Verbal (WISC) = Verbal subscale from Wechsler Intelligence Scale for Children.

Table 5: Goodness of fit statistics

Model	χ^2	df	p	CFI	RMSEA	SRMR
Final	32.85	25	0.13	0.99	0.05	0.04

Note. CFI = Comparative Fit Index; RMSEA = Root Measure Square Error of Approximation; Standardized Root Mean Square Residual.

Based on the principle of parsimony and the AIC value, the modified model of verbal skills predicting mathematical skills was deemed to be the best model to describe the mathematical skills of children with reading disabilities. The graphic representation of the model is displayed in Figure 3 as well as in Appendix B. Table 4 indicates the model estimates, while Table 5 displays the goodness-of-fit indices for the best model.

In this model, the observed variables for mathematical skills were Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money, and their standardized estimates were .85, .62, .74, .79, .72, and 0.79, respectively ($p < .01$ for all variables). These estimates indicate that these variables are good measures for mathematical skills. In this model, the observed variables for verbal skills were Sound Symbol Identification, Verbal (KBIT) and Verbal IQ (WISC), and their estimates were .76, .71, and .64, respectively ($p < .01$ for all variables), suggesting that these variables are acceptable measures for verbal skills.

The results of this model suggest that verbal skills are highly related to mathematical skills, and improvements in verbal skills are accompanied with an improvement in mathematical skills as well. With 30.4% of the variance left unexplained by verbal skills, it suggests that other factors could account for the residual variance in mathematical performance

4.5 Supplementary Analyses

Further analyses were conducted beyond the analyses that were proposed. These analyses were conducted because they provided further details as to how verbal skills, reading skills, and cognitive functioning related to specific mathematical concepts.

4.5.1 Supplementary analyses: How does children's performance on different concepts of mathematics relate to their verbal skills, reading skills, and cognitive functioning?

The aim of research question one was to compare the differential relationships between verbal skills and reading skills and “basic” and “advanced” mathematical concepts. To explore this question, “basic” and “advanced” mathematical skills were assessed as two separate constructs. The results indicated that mathematical concepts should not be differentiated into “basic” and “advanced” mathematical skills, but rather, the mathematical concepts belong to a single construct of mathematical skills. As such, the subsequent models assessed provided information on how verbal skills, reading skills, and cognitive functioning related to mathematical skills as a single construct. These analyses provided understanding as to how these constructs related to overall mathematical skills, yet, these analyses did not provide an understanding of how verbal skills, reading skills, and cognitive functioning related to specific mathematical concepts.

It is believed that analyzing how our constructs of interest relate to specific mathematical concepts will provide information that is beneficial as children might perform well on certain mathematical concepts and not others. Classifying mathematical skills as a single construct could result in the loss of some potentially important information. Hence, supplementary analyses were conducted to provide further information on the relationship between these constructs and specific mathematical concepts. These models assessed verbal skills, reading skills, or a combination of both verbal and reading skills variables, and cognitive functioning's relationship to Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money. A total of 13 models were assessed to provide insight into the best model that describes the performance on specific mathematical concepts. The model with verbal skills predicting specific mathematical

concepts, and the model with reading skills predicting specific mathematical concepts resulted in the best model fit, as such, they are described in further detail.

4.5.2 Assessing model fit for verbal skills predicting specific mathematical concepts

In the model where verbal skills predicted Numeration, Geometry, Addition, Subtraction, Measurement and Time & Money, the results of the goodness of fit were $\chi^2 (12) = 27, p < .001$, RMSEA = 0.09, CFI = 0.98, SRMR = .03. This model met the standards of a good fit based on its RMSEA and CFI value. Model indices indicated that no further model modification was needed. Table 6 displays the model estimates and Figure 4 presents a graphic display of the model.

As presented in Figure 4, the indicators are demonstrated to be an acceptable measure for verbal skills as a construct. The results of the analysis further demonstrated that in general, verbal skills explained a significant amount of variance in the various mathematical concepts. Specifically, verbal skills explained 51% of the variance in Numeration, 28% of the variance in Geometry, 35% of the variance in Addition, 43% of the variance in Subtraction, 50% of the variance in Measurement, and 37% of the variance in Time & Money.

Based on the results, verbal skills are demonstrated to have the strongest relationship with Numeration, then Measurement, followed by Time & Money, and Subtraction. The weakest relationship is with Geometry and Addition.

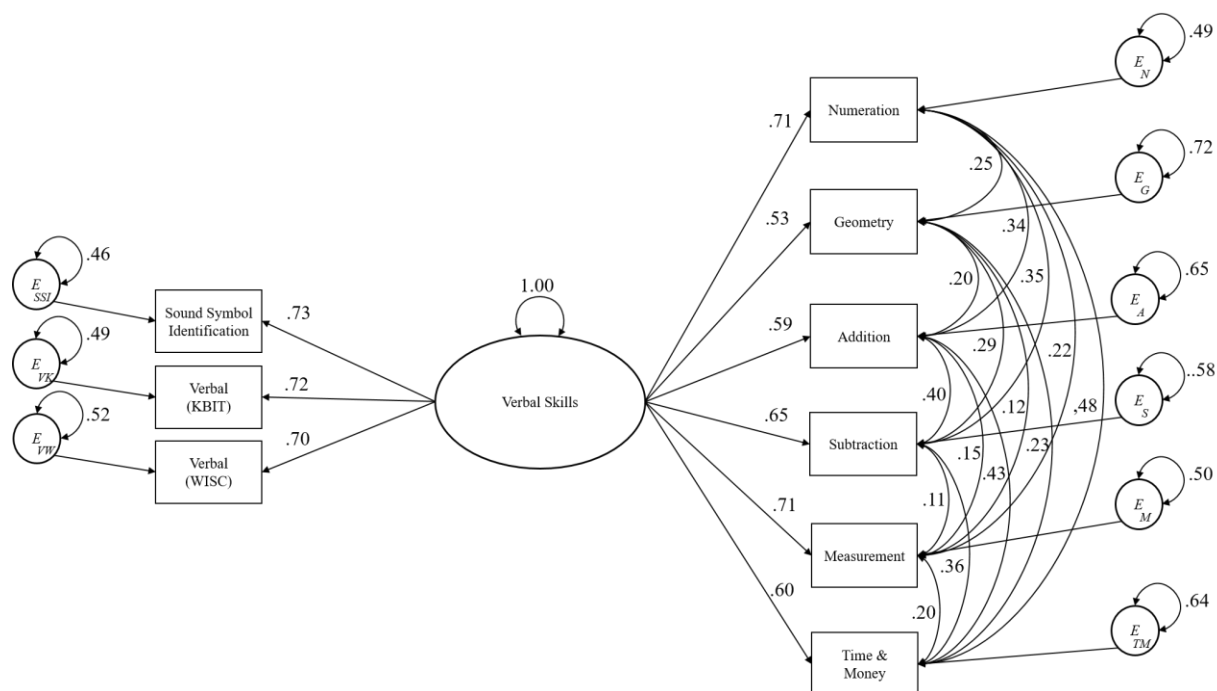


Figure 4: A path model of verbal skills predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money independently.

Note. Numeration, Geometry, Addition, Subtraction, Measurement and Time & Money represents the subscales from the KeyMath – R (KeyMath-Revised Test; Connolly, 1998); Sound Symbol Identification (Lovett et al., 1994); Verbal (KBIT) is from the Kaufman Brief Intelligence Test (Kaufman & Kaufman, 1990); Verbal (WISC) is from the Wechsler Intelligence Scale for Children (Wechsler, 2004).

Table 6: Estimates for alternate model with verbal skills

Parameter	Relation/Variable	Estimate	S.E.	Ratio	p-value	Std
Regression	VS by SSI	1.02	0.13	7.82	<0.01	0.73
	VS by Verbal (K-BIT)	0.98	0.13	7.82	<0.01	0.72
	VS by Verbal (WISC)	0.95	0.13	7.61	<0.01	0.70
	Num on VS	0.97	0.13	7.48	<0.01	0.71
	Geo on VS	0.72	0.13	5.55	<0.01	0.53
	Add on VS	0.81	0.13	6.23	<0.01	0.59
	Sub on VS	0.89	0.13	6.86	<0.01	0.65
	Mea on VS	0.96	0.13	7.41	<0.01	0.71
	TM on VS	0.83	0.13	6.37	<0.01	0.60
	Num with Geo		0.15	0.06	2.27	<0.05
Num with Add		0.19	0.06	2.94	<0.01	0.34
Num with Sub		0.19	0.06	2.94	<0.01	0.35
Num with Mea		0.11	0.06	1.81	0.07	0.22
Num with TM		0.27	0.07	4.01	<0.01	0.48
Geo with Add		0.14	0.07	2.06	<0.05	0.20

	Geo with Sub	0.19	0.07	2.78	<0.01	0.29
	Geo with Mea	0.07	0.06	1.11	0.27	0.12
	Geo with TM	0.15	0.07	2.25	<0.05	0.23
	Add with Sub	0.24	0.07	3.60	<0.01	0.40
	Add with Mea	0.08	0.06	1.35	0.18	0.15
	Add with TM	0.27	0.07	3.89	<0.01	0.43
	Sub with Mea	0.06	0.06	1.00	0.32	0.11
	Sub with TM	0.22	0.07	3.23	<0.01	0.36
	Mea with TM	0.11	0.06	1.83	0.07	0.20
Variiances	VS	0.53	0.11	4.77	<0.01	1.00
Standardized Variiances	Num	0.51	0.08	6.55	<0.01	
	Geo	0.28	0.08	3.64	<0.01	
	Add	0.35	0.08	4.44	<0.01	
	Sub	0.43	0.08	5.38	<0.01	
	Mea	0.50	0.08	6.41	<0.01	
	TM	0.37	0.08	4.63	<0.01	
	SSI	0.54	0.07	7.43	<0.01	
	Verbal (K-BIT)	0.51	0.07	7.05	<0.01	
	Verbal (WISC)	0.48	0.07	6.59	<0.01	

Note. SE = Standard Error. Std = Standardized Value. VS = Verbal skills; SSI = Sound Symbol Identification (Lovett et al., 1994); Verbal (K-BIT) = Verbal subscale from Kaufman Brief Intelligence Test (Kaufman & Kaufman, 1990); Verbal (WISC) = Verbal subscale from Wechsler Intelligence Scale for Children (Wechsler, 2004). Numeration (Num), Geometry (Geo), Addition (Add), Subtraction (Sub), Measurement (Mea) and Time & Money (TM) represents the subscales from the KeyMath – R (KeyMath-Revised Test; Connolly, 1998).

4.5.3 Assessing model fit for reading skills predicting specific mathematical concepts

In the model where reading skills predicted Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money, the results of the model fit prior to model modification were $\chi^2(29) = 68.85, p = .00, RMSEA = 0.10, CFI = 0.96, SRMR = .06$. This model only met the standards of a mediocre fit based on its RMSEA and SRMR value, and a good fit based on its CFI value without model modification. Based on model indices, three paths representing a relationship between (1) Blending and Elision (both from the CTRRPP; Torgesen & Wagner, 1996), (2) Blending (CTRRPP; Torgesen & Wagner, 1996) and Word Attack

(WRMT-R; Woodcock, 1987), and (3) Word Identification and Passage Comprehension (from the WRMT-R; Woodcock, 1987) were added. This resulted in an improved model, $\chi^2 (26) = 36.49, p = n.s., RMSEA = 0.05, CFI = 0.99, SRMR = .04$. The modified model met the criteria for a mediocre fit for RMSEA and SRMR, and a good fit based on its chi-square and CFI. Table 7 indicates the model estimates and Figure 5 is a graphic representation of the model.

As presented in Figure 5, the indicators are demonstrated to measure the construct of reading skills well. The results also showed that reading skills accounted for a significant amount of variance in the mathematical concepts. Reading skills accounted for 46% of the variance in Numeration, 20% of the variance in Geometry, 43% of the variance in Addition, 45% of the variance in Subtraction, 21% of the variance in Measurement, and 35% of the variance in Time & Money.

The results also showed that reading skills has a stronger relationship with Numeration, Subtraction and Addition as compared to its relationship with Time & Money, Measurement and Geometry.

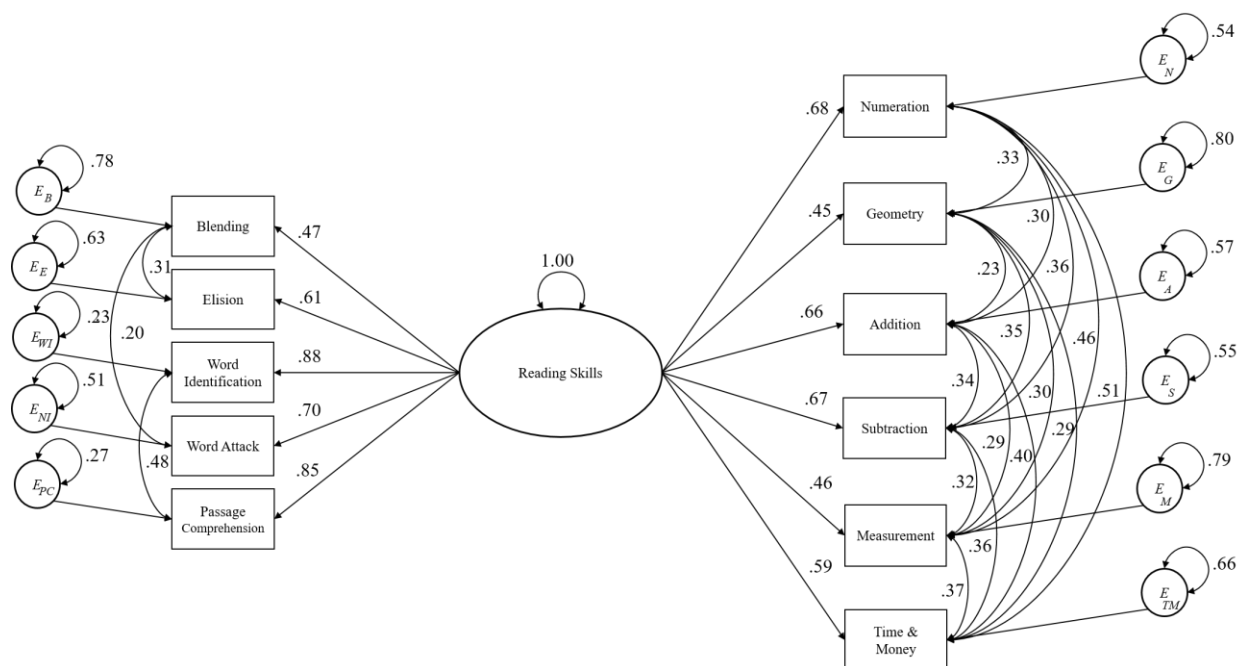


Figure 5: A modified path model of reading skills predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money independently.

Note. Numeration, Geometry, Addition, Subtraction, Measurement and Time & Money represents the subscales from the KeyMath – R (KeyMath-Revised Test; Connolly, 1998); Blending and Elision are subtests of the CTRRPP (Torgesen & Wagner, 1996), Word Identification, Word Attack, and Passage Comprehension are subtests from the WRMT-R (Woodcock, 1987).

Table 7: Estimates for alternate model for reading skills

Parameter	Relation/Variable	Estimate	S.E.	Ratio	<i>p</i> -value	Std
Regression	RS by Blending	0.76	0.13	5.84	<0.01	0.47
	RS by Elision	1.32	0.23	5.84	<0.01	0.61
	RS by Word ID	1.89	0.34	5.56	<0.01	0.88
	RS by Word Attack	1.51	0.26	5.76	<0.01	0.70
	RS by Pass Comp	1.83	0.33	5.50	<0.01	0.85
	Num on RS	1.45	0.28	5.15	<0.01	0.68
	Geo on RS	0.96	0.23	4.08	<0.01	0.45
	Add on RS	1.41	0.28	5.08	<0.01	0.66
	Sub on RS	1.45	0.28	5.15	<0.01	0.67
	Mea on RS	0.99	0.24	4.18	<0.01	0.46
	TM on RS	1.26	0.26	4.81	<0.01	0.59
	Correlation	Num with Geo	0.22	0.06	3.41	<0.01
Num with Add		0.17	0.06	2.88	<0.01	0.30
Num with Sub		0.19	0.06	3.34	<0.01	0.36
Num with Mea		0.30	0.07	4.48	<0.01	0.46
Correlation	Num with TM	0.30	0.06	4.72	<0.01	0.51
	Geo with Add	0.16	0.06	2.49	<0.05	0.23

	Geo with Sub	0.23	0.06	3.56	<0.01	0.35
	Geo with Mea	0.23	0.07	3.25	<0.01	0.30
	Geo with TM	0.21	0.07	3.09	<0.05	0.29
	Add with Sub	0.19	0.06	3.21	<0.01	0.34
	Add with Mea	0.20	0.06	3.07	<0.01	0.29
	Add with TM	0.24	0.06	3.90	<0.01	0.40
	Sub with Mea	0.21	0.06	3.28	<0.01	0.32
	Sub with TM	0.21	0.06	3.50	<0.01	0.36
	Mea with TM	0.27	0.07	3.91	<0.01	0.37
	Word ID with Pass Comp	0.12	0.06	2.15	<0.05	0.48
	Blending with Elision	0.21	0.06	3.41	<0.01	0.31
	Word Attack with Blending	0.12	0.08	2.23	<0.05	0.20
Variances	RS	0.22	0.08	2.82	<0.01	1.00
Standardized Variances	Num	0.46	0.07	6.35	<0.01	
	Geo	0.20	0.07	3.05	<0.01	
	Add	0.43	0.07	5.95	<0.01	
	Sub	0.45	0.07	6.32	<0.01	
	Mea	0.21	0.07	3.22	<0.01	
	TM	0.35	0.07	4.77	<0.01	
	Blending	0.22	0.07	3.32	<0.01	
	Elision	0.38	0.07	5.29	<0.01	
	Word ID	0.78	0.06	12.25	<0.01	
	Word Attack	0.49	0.07	7.12	<0.01	
	Pass Comp	0.73	0.07	10.91	<0.01	

Note. SE = Standard Error. Std = Standardized Value. VS = Verbal skills; SSI = Sound Symbol Identification (Lovett et al., 1994); Verbal (K-BIT) = Verbal subscale from Kaufman Brief Intelligence Test (Kaufman & Kaufman, 1990); Verbal (WISC) = Verbal subscale from Wechsler Intelligence Scale for Children (Wechsler, 2004). RS = Reading skills; Blending and Elision are subtests from CTRRPP (Comprehensive Test of Reading Related Phonological Processes; Torgesen & Wagner, 1996); Word Identification (Word ID), Word Attack, and Passage Comprehension (Pass Comp) are subtests from the WRMT (Woodcock Reading Mastery Test-Revised; Woodcock, 1987); Numeration (Num), Geometry (Geo), Addition (Add), Subtraction (Sub), Measurement (Mea) and Time & Money (TM) represents the subscales from the KeyMath – R (KeyMath-Revised Test; Connolly, 1998).

4.5.4 Assessing model fit for verbal skills and reading skills predicting specific mathematical concepts

The model fit for verbal skills and reading skills predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money was assessed. This model did not meet as many goodness of fit standards as compared to verbal skills and reading skills separately predicting the mathematical concepts. The goodness of fit for the model where verbal skills and reading skills together predicted mathematical concepts were $\chi^2(55) = 223.94, p = <.01$, RMSEA = .14, CFI = .87, SRMR = .07. This model only met the criteria for a good fit for SRMR (Byrne, 1998). The correlation coefficient between verbal skills and reading skills was 1.02, suggesting that these constructs should be tested as a single construct with verbal and reading skills combined.

4.5.5 Assessing model fit for verbal skills and reading skills combined predicting specific mathematical concepts

The model fit for verbal skills and reading skills combined predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money was assessed next. The goodness of fit for the model where verbal skills and reading skills combined predicted mathematical concepts prior to model modification was: $\chi^2(62) = 254.25, p = <.01$, RMSEA = .14, CFI = .85, SRMR = .08. This model only met the criterion for a good fit based on its SRMR value (Byrne, 1998). Based on the model indices, 4 paths indicating a relationship between (1) Word Identification and Passage Comprehension (both from the WRMT-R; Woodcock, 1987), (2) Passage Comprehension (WRMT-R; Woodcock, 1987) and Blending (CTRRPP; Torgesen & Wagner, 1996), (3) Sound-Symbol Identification (Lovett et al., 1994) and Blending (CTRRPP; Torgesen & Wagner, 1996), and (4) K-BIT – Verbal (Kaufman & Kaufman, 1990) and Blending

(CTRRPP; Torgesen & Wagner, 1996) were added. The results of the goodness of fit post-model modification was: $\chi^2 (58) = 136.21, p = <.01, RMSEA = .10, CFI = .94, SRMR = .07$. This model met the criteria for a mediocre fit for RMSEA and CFI, and a good fit for SRMR (Browne & Cudeck, 1993; Byrne, 1998; Kline, 2005). These results indicate that this model did not meet as many goodness of fit standards as compared to verbal skills and reading skills independently predicting the mathematical concepts.

4.5.6 Assessing the model fit for verbal skills and cognitive functioning predicting specific mathematical concepts

The model fit for verbal skills and cognitive functioning predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money was assessed. The goodness of fit for the model prior to model modification was: $\chi^2 (22) = 67.59, p = <.01, RMSEA = .12, CFI = .94, SRMR = .05$. This model met the criterion of a mediocre fit based on its CFI and SRMR value (Byrne, 1998). Based on the model indices, a path indicating a relationship between Performance IQ and Verbal IQ (both from the WISC-IV; Wechsler, 2004) was added. Post-model modification, the goodness of fit was: $\chi^2 (21) = 41.98, p = <.01, RMSEA = .08, CFI = .97, SRMR = .04$. This model met the criteria for a good fit for RMSEA, CFI, and SRMR (Browne & Cudeck, 1993; Byrne, 1998; Kline, 2005).

4.5.7 Assessing the model fit for reading skills and cognitive functioning predicting specific mathematical concepts

The model fit for reading skills and cognitive functioning predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money was assessed. The goodness of fit for the model prior to model modification was: $\chi^2 (43) = 122.80, p = <.01, RMSEA = .11, CFI = .93, SRMR = .08$. This model met the criterion of a mediocre fit based on its CFI and

SRMR value (Byrne, 1998; Kline, 2005). Based on the model indices, three paths indicating a relationship between Word Identification and Passage Comprehension (both from the WRMT-R; Woodcock, 1987); Blending and Elision (both from the CTRRPP; Torgesen & Wagner, 1996); and Word Attack (WRMT-R; Woodcock, 1987) and Blending (CTRRPP; Torgesen & Wagner, 1996) were added. The goodness of fit of the model post-model modification was: $\chi^2 (39) = 82.15$, $p = <.01$, RMSEA = .09, CFI = .96, SRMR = .06. This model met the criteria for a mediocre fit based on its RMSEA and SRMR value, and a good fit based on its CFI value (Browne & Cudeck, 1993; Byrne, 1998; Kline, 2005).

4.5.8 Assessing the model fit for verbal skills and reading skills combined and cognitive functioning predicting specific mathematical concepts

The model fit for verbal skills and reading skills combined and cognitive functioning predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money was assessed. The goodness of fit for the model prior to model modification was: $\chi^2 (82) = 339.36$, $p = <.01$, RMSEA = .15, CFI = .82 SRMR = .09, all of which did not meet the criteria for an acceptable fit. The model indices suggested the addition of four paths to improve the model; they included: (1) Word Identification and Passage Comprehension (both from the WRMT-R; Woodcock, 1987), (2) Performance IQ and Verbal IQ (both from the WISC-IV; Wechsler, 2004), (3) Word Identification (WRMT-R; Woodcock, 1987) and Performance IQ (WISC-IV; Wechsler, 2004), (4) and Sound Symbol Identification (SSI; Lovett, et al., 1994) and Blending (CTRRPP; Torgesen & Wagner). The goodness of fit of the model post-model modification was: $\chi^2 (78) = 198.55$, $p = <.01$, RMSEA = .10, CFI = .92, SRMR = .07, which met the criteria for a mediocre fit based on its RMSEA, CFI and SRMR value (Browne & Cudeck, 1993; Byrne, 1998; Kline, 2005).

4.5.9 Assessing the model fit for verbal skills, reading skills, and cognitive functioning predicting specific mathematical concepts

The model fit for verbal skills, reading skills, and cognitive functioning predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money was assessed. The results of the goodness of fit were $\chi^2(76) = 301.20, p = <.01, RMSEA = .14, CFI = .78$ SRMR = .10. This model did not meet any criteria of an acceptable fit. The correlation coefficient between verbal skills and reading skills was 1.01, which explains the poor fit.

4.5.10 Comparing all models with construct(s) predicting specific mathematical concepts

Based on all of the analyses that assessed the fit of the models with construct(s) predicting specific mathematical concepts, the three models that produced comparatively better fit among all of the models that were assessed included the model with verbal skills predicting the mathematical concepts, the post-modified model of reading skills predicting the mathematical concepts, and the post-modified model of verbal skills and cognitive functioning predicting mathematical concepts. However, once again, based on the principal of parsimony and the models' AIC value, these analyses demonstrated that the model with verbal skills predicting Numeration, Geometry, Addition, Subtraction, Measurement and Time & Money is the best model that describes the mathematical abilities for children with reading disabilities. The graphic representation of verbal skills predicting Numeration, Geometry, Addition, Subtraction, Measurement and Time & Money is displayed in Figure 4 and its model estimates are displayed in Table 6. The goodness of fit statistics of all of the models tested in the supplementary analyses are displayed in Appendix I.

4.5.11 Interpreting the findings from the supplementary analyses

The results of these analyses suggest that improvement in verbal skills and reading skills can contribute to the improvement of mathematical skills across all concepts. Improvements in either skill tend to show larger improvements in certain mathematical concepts such as Numeration, suggesting that Numeration might be more heavily reliant on verbal and reading skills as compared to the other concepts. These analyses also demonstrate that mathematical skills are highly related to verbal and reading skills.

4.6 Overall findings

Overall, the analyses demonstrate a strong relationship between all of the mathematical concepts: Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money. This finding shows that mathematical skills across concepts are highly related to each other. This finding also suggests that children with higher performance on one mathematical concept generally tend to perform better on another mathematical concept.

The aim of the first research question was to determine how verbal skills and reading skills related to “basic” and “advanced” mathematical skills. Models were assessed to determine the validity of “basic” and “advanced” mathematical skills as constructs. The analyses demonstrated that the mathematical concepts should not be differentiated into “basic” and “advanced” mathematical skills, rather these mathematical concepts fall under a single construct of mathematical skills. A follow-up analyses of mathematical skills as a single construct was then assessed. The results of these findings negate the prediction of the first hypotheses wherein a stronger relationship would be found between verbal skills and reading skills and “advanced” mathematical concepts as compared to the relationship between verbal skills and reading skills with “basic” mathematical concepts.

The second objective of this study was to determine the best model of mathematical skills for children with reading disabilities. Various models were assessed using path analyses. Based on the models' goodness of fit, the modified two-construct models of verbal skills and mathematical skills, and reading skills and mathematical skills were determined to be the best models. However, based on the principle of parsimony and the model's AIC value, the model with verbal skills predicting mathematical skills was determined to be the best model for mathematical skills. Given that this model demonstrates that verbal skills accounted for a significant portion of variance in mathematical skills, this study has important implications for the educational practices for children with reading disabilities.

Finally, supplementary analyses were conducted to understand how these constructs related to the specific mathematical concepts Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money, independently, as opposed to all of the mathematical concepts being indicators of a single construct of mathematical skills. These analyses showed that the model of verbal skills predicting the mathematical concepts, the post-modified model of reading skills predicting the mathematical concepts, and the post-modified model of verbal skills and cognitive functioning predicting mathematical concepts were all good models that can predict performance on specific mathematical concepts. However, once again, based on the principal of parsimony and the models' AIC value, these analyses showed that the model with verbal skills predicting Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money is the best model that predicts performance on specific mathematical concepts for children with reading disabilities.

5 DISCUSSION

The aims of this present study were twofold. Firstly, the relationship between “basic” and “advanced” mathematical skills and verbal skills and reading skills were analyzed to provide an understanding of how performance on mathematical skills related to verbal and reading skills in children with reading disabilities. Secondly, this study aimed to develop a model of mathematical skills for children with reading disabilities, with the intention of identifying the skills that are most strongly associated with mathematical skills.

Pearson’s correlation analyses were conducted between specific measures of mathematical concepts: Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money, and the variables from verbal skills, reading skills, and cognitive functioning to provide an overview of how the variables relate to one another. As expected, all of the variables were positively associated with each other. Positive associations between different concepts of mathematics suggest that better performance on one mathematical concept is related to higher performance on another mathematical concept, across all concepts.

The Pearson’s correlational analyses also demonstrate that stronger verbal and reading skills are associated with better mathematical performance across all concepts. Considering the positive relationship between verbal skills, reading skills and performance across all mathematical concepts, these findings can mean that cultivating verbal and reading skills can potentially improve mathematical abilities. These findings are consistent with previous studies that also demonstrate a positive association and the predictive value of verbal and reading skills to mathematics (Korhonen et al., 2012; Purpura & Ganley, 2014; Toll & Van Luit, 2014).

In examining how verbal skills and reading skills related to “basic” and “advanced” mathematical skills, a two-construct model of “basic” and “advanced” mathematical skills was

assessed. The results highlighted that the mathematical concepts examined should not be differentiated into “basic” and “advanced” mathematical skills, instead, these mathematical concepts come from a single construct of mathematical skills. This finding was not in support of the first hypothesis in which it was anticipated that stronger associations would arise between verbal skills and reading skills and the more “advanced” mathematical skills.

Even though these findings differed from what was hypothesized, upon reexamination, it may have been more accurate to classify the mathematical concepts in the manner in which KeyMath – Revised Test (Connolly, 1988) categorized the mathematical concepts. In KeyMath – Revised Test (Connolly, 1988), there are three overarching categories: Basic Concepts, Applications, and Operations. Basic concepts measure the individual’s foundational knowledge and it includes Numeration and Geometry. Applications assess the individual’s ability to use knowledge and computation skills and problem solving, and it includes the Measurement and Time & Money subtests, while Operations assess the individual’s computation skills, and it includes the Addition and Subtraction subtests. It is possible that based on how these mathematical concepts are categorized, there would be a difference in the way they relate to verbal skills and reading skills. It is possible that the mathematical concepts within Basic Concepts would be more highly related to verbal and reading skills, as these skills might be more crucial in developing foundational mathematical knowledge. As for Operations, since the mathematical concepts tested within this category have a heavier emphasis on computations, these mathematical concepts might have a weaker association with verbal and reading skills as compared to the mathematical concepts within the Basic Concepts category. Finally, with both mathematical knowledge and computational skills being relevant to the mathematical concepts

within Applications, it is important to determine how these mathematical concepts will be related to both verbal and reading skills.

5.1 The model of mathematical skills for children with reading disabilities

In order to identify the best plausible model which describes the mathematical skills of children with reading disabilities, several path analyses were conducted. The model indices suggested the addition of paths between variables to improve the model fit. With model re-specifications, several models resulted with acceptable to good fit based on their chi-square, RMSEA, CFI and SRMR values (Browne and Cudeck, 1993; Byrne, 1998; Kline, 2005; Wheaton, et al., 1977). Based upon the principal of parsimony, and the lowest AIC value (lower values indicating better models), this study identified verbal skills as an important factor in influencing the mathematical skills of second graders with reading disabilities. The results of the best model did not support the inclusion of reading skills and cognitive functioning in determining mathematical skills.

After the two-factor model of verbal skills and mathematical skills was tested, the model indices suggested the addition of a path indicating a link between Measurement (KeyMath- R; Connolly, 1998) and Verbal IQ (WISC; Wechsler, 2004) to the model. When the new path was added, there was significant improvement in the goodness of fit. This model suggests that better verbal skills would support better mathematical performance. It is possible that verbal skills could aid a child in understanding when mathematics is taught in class, while at the same time aid in their ability in narrating the steps involved when solving mathematical problems, which explain better mathematical performance. At the same time, this model also shows that improving verbal skills could strengthen mathematical skills. Specifically, the improvement of

verbal skills, including the children's receptive and expressive vocabulary might better the mathematical performance of children.

5.2 Supplementary analyses

Supplementary analyses using path analyses were conducted to provide more in-depth information as to how verbal skills, reading skills, and cognitive functioning relate to the different concepts of mathematics. Even though the model which examined verbal skills' prediction of mathematical skills provided an understanding of how verbal skills were related to overall mathematical skills, this supplementary examination was conducted to determine how verbal skills, reading skills, and cognitive functioning relate to Numeration, Geometry, Addition, Subtraction, Measurement and Time & Money, specifically, as opposed to an overall mathematical ability, in a single model. The results of these analyses showed that the model with verbal skills predicting mathematical concepts, the post-modified model of reading skills predicting mathematical concepts, and the post-modified model of verbal skills and cognitive functioning predicting mathematical concepts were the models that had better goodness of fit compared to all of the models assessed. However, the model with verbal skills predicting Numeration, Geometry, Addition, Subtraction, Measurement and Time & Money was deemed to be the best model based on the principal of parsimony and the model's AIC value. This finding suggests that the mathematical skills of children with reading disabilities could improve across mathematical concepts when their verbal skills are developed.

Based on all of the analyses, a strong relationship between verbal skills and mathematical skills is supported, which encourages educators to expand their focus when striving to improve mathematical performance. This study shows that other abilities such as verbal skills and reading skills could influence mathematical skills, and educators could look to develop verbal skills

when trying to cultivate mathematical skills. However, it is important to note that the strong association between mathematical performance and verbal skills and reading skills could stem from being closely related within the test measure itself due to poor discriminant validity as Rhodes et al. (2015) have illustrated in their investigation of performance on KeyMath-R (Connolly, 1988) in 2nd to 5th graders with mild intellectual disabilities. Rhodes et al.'s (2015) findings suggest that with poor discriminant validity between language and mathematical constructs in KeyMath-R (Connolly, 1988), the test might be assessing the linguistic skills of the children rather providing a “clean” measure of mathematical skills without the influence of verbal skills. This could possibly explain the strong relationship between verbal skills and mathematical abilities found in this study. It is essential that test developers and educators be aware of this possibility to avoid placing an additional obstacle on children with poor verbal skills when their mathematical skills are examined.

In sum, these analyses provide support to what is present in the literature, suggesting the influence of verbal skills on mathematical skills. Even though the best model described in this study does not include reading skills and cognitive functioning, the other models that factored in these variables show that reading skills and cognitive functioning should not be ignored completely when trying to determine children's mathematical skills as these abilities are shown to be related to verbal skills. This model suggests that improvements in verbal skills could provide the most efficient advances in mathematical performance, but does not negate the benefits of improvements in reading skills and cognitive functioning in mathematical performance. As shown in the analyses, reading skills and cognitive functioning could provide some prediction of mathematical skills, but might not be the best indicator for the mathematical skills of children with reading disabilities.

5.3 Limitations

Even though this study contributes to the literature on verbal and reading skills' role in mathematical performance, it is not without some limitations that should be addressed. The main limitation of this study is that the sample comprised of young children in second and third grades. This age limitation might decrease the study's ability to generalize the results to children in later grades. It is possible that children's reliance on verbal skills for understanding mathematical concepts changes across the grades and thus these findings will not accurately describe their mathematical skills.

Secondly, the original study was not developed to examine children with mathematical difficulties. The focus of the larger study was to compare the effectiveness of various reading intervention methodologies. It is possible that some might view this as a drawback, as the children were not recruited based on difficulties with mathematics.

Finally, because mathematical skill was not the interest of the original study, limited assessment examining mathematical skills was conducted. KeyMath-R (Connolly, 1988) is used as the main measure of mathematics in this study, KeyMath-R (Connolly, 1988) was used as a control measure in the original study. As such, limited variation of mathematical concepts was examined in this study, i.e., this study did not examine mathematical concepts beyond what is assessed in KeyMath-R, e.g., Algebra. Even though an understanding of how verbal skills relate to some mathematical concepts is achieved in this study, it did not examine all mathematical concepts that might be of interest.

5.4 Educational implications

This study is the start of the development of a framework in which parents and educators are able to construct plans to nurture their children's developing verbal and mathematical skills.

The finding that improvements in verbal skills as oppose to reading skills or cognitive functioning skills will result in the most improvement in mathematical skills could provide guidance to parents making decisions on the academic areas in which their children should devote the most time and effort.

The results from this study are especially significant for children with speech and language impairments. The findings of this study help teachers understand that difficulties in mathematics could stem from shortcomings in verbal skills rather than a deficit in mathematical computation. This has important implications because rather than devoting attention to the “symptom” of the problem – difficulty in mathematics, attention could be devoted to alleviating verbal difficulties. As such, this current model could possibly encourage educators to work on the literacy and verbal skills of the children which will positively impact the mathematical abilities of children, rather than focusing on working on the mathematical abilities of the children. Merely focusing on the mathematical skills of children demonstrating difficulties with mathematics may not be able to bring about a successful improvement in the mathematical skills of children if only mathematical skills are targeted.

The findings from this study also inform methodology techniques in which instructors should engage. Because verbal skills and mathematical skills are highly related, children with difficulty with mathematics may benefit from instruction that emphasizes verbalization of steps involved in mathematical calculations. One of the strategies used with mathematical learning involve using self-speech as a strategy for mathematical computation (Ostad & Sorensen, 2007). Self-speech can be described as private speech directed to oneself as part of self-guidance, or managing one’s behavior. Children with poor expressive verbal skills might struggle to participate in discussions or engage in self-speech as a strategy for mathematical computation.

As such, teachers should give children opportunities to practice expressing their strategies aloud. Also, rather than relying solely on verbal communication and reading of text to impart mathematical knowledge, a possible alternative is to provide graphical presentations during mathematical instruction. Using alternate techniques to convey mathematical understanding might reduce reliance on verbal skills for comprehension of concept.

5.5 Future Directions

Future research could address some of the limitations of this study. To determine if the predictive ability of verbal skills on mathematical performance is generalizable to ages beyond second and third graders, future research should involve the examination of children in later grades. Examining children of older ages will provide important information on the longitudinal impacts of verbal skills on mathematical performance. Additional research also could involve the analyses of mathematical concepts other than the ones examined in this study: Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money providing further understanding of how verbal skills relates to mathematical performance.

Another possible direction that can be taken includes analyzing the mathematical concepts' relationship with verbal skills and reading skills based on how it was originally categorized in the KeyMath – Revised test (Connolly, 1998). As mentioned earlier, it would be of interest to determine if there would be a difference in the way mathematical concepts within the basic concepts category would relate differently to the mathematical concepts within the operations category, as the former are more heavily reliant on mathematical knowledge as compared to the latter which has a stronger focus on mathematical computations.

Also, additional research could provide an understanding of how changes in verbal and reading skills from children's participation in reading intervention program impact the

mathematical skills of children with reading disabilities. This analysis would provide insight to the impacts of reading interventions beyond reading skills. Finally, research could assess the impact of an intervention focusing directly on development of mathematical skills, as compared an intervention aiming to improve mathematical performance via nurturing verbal skills on overall academic performance for children with speech and language impairments.

5.6 Conclusions

One of the most important implications of this study is the contribution to the growing literature illustrating the links between verbal and reading skills and mathematical skills. From the analyses, verbal skills are demonstrated to have predictive ability of simpler to more complex mathematical concepts including Numeration, Geometry, Addition, Subtraction, Measurement and Time & Money. Verbal skills were shown to provide the best predictive ability of the mathematical performance across concepts for children with reading disabilities; this has important educational implications such as informing the instructional practices of schools and classes developing verbal and mathematical skills.

This study also examined constructs that were previously analyzed separately in investigations striving to understand the factors that contribute to the development of mathematical skills, it provided a holistic examination of the factors that could impact mathematical skills. Taking together all of the findings, although the best model did not include reading skills, reading skills did produce an acceptable model. Thus, it is important not to disregard the positive impact improvements in reading skills would have on mathematical skills. Furthermore, the model that investigated verbal skills and cognitive functioning's relationship to specific mathematical concepts also demonstrated that cognitive functioning does affect mathematical performance. As such, concentrating solely on developing verbal skills would not

be as beneficial to the development of mathematical skills as compared to developing other skills concurrently.

5.7 Concluding remarks

This study has shed light on the linguistic and cognitive correlates of mathematical skills. With the findings demonstrating the weight of influence verbal skills have on mathematical skills, this study has not only provided further evidence of the intersection between verbal skills and mathematical skills, but also have paved the way for future research to explore other factors that could influence mathematical skills. With this study, it is evident that the mathematical performance of children with reading disabilities was affected by their verbal skills. As researchers and educators, we should strive to delineate the extent at which verbal skills affect children's academic achievement beyond the classes that are traditionally language-focused. Only with this understanding can educators best support children's academic needs.

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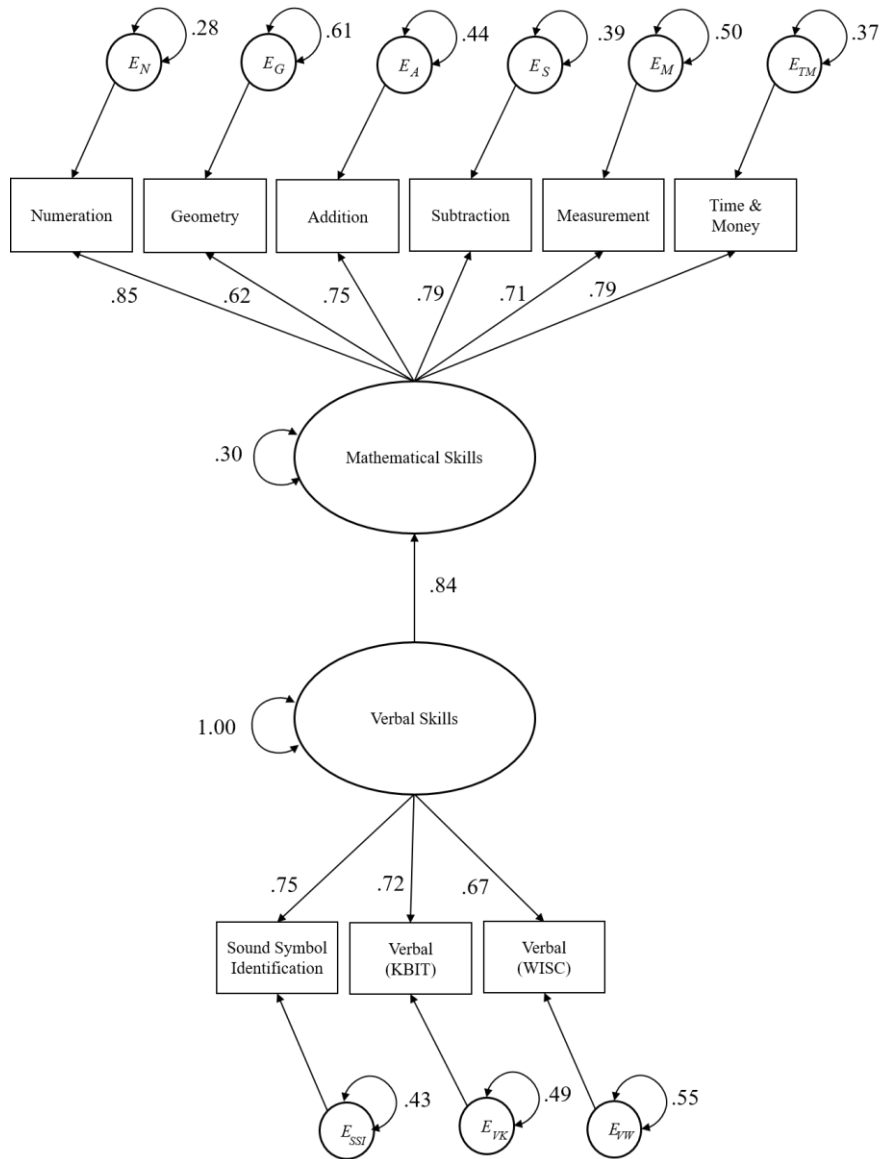
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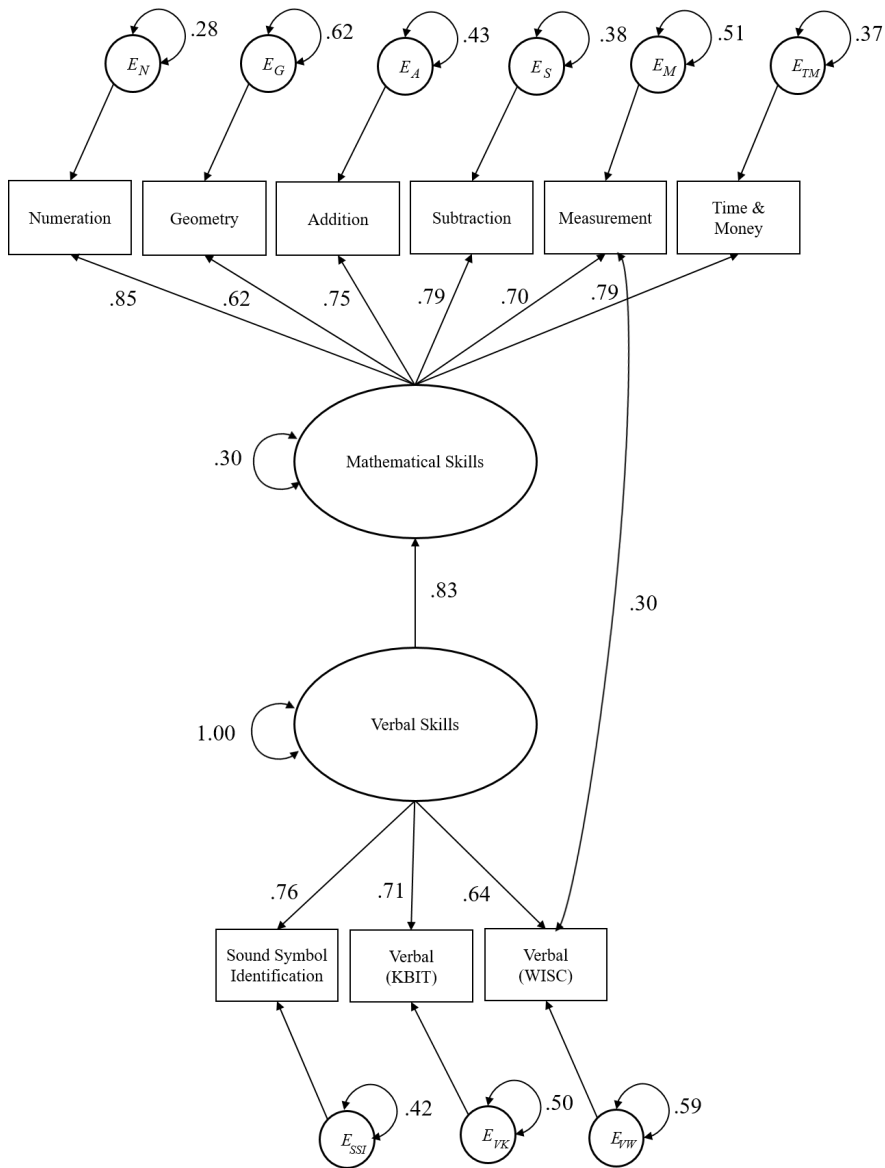
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7 APPENDICES

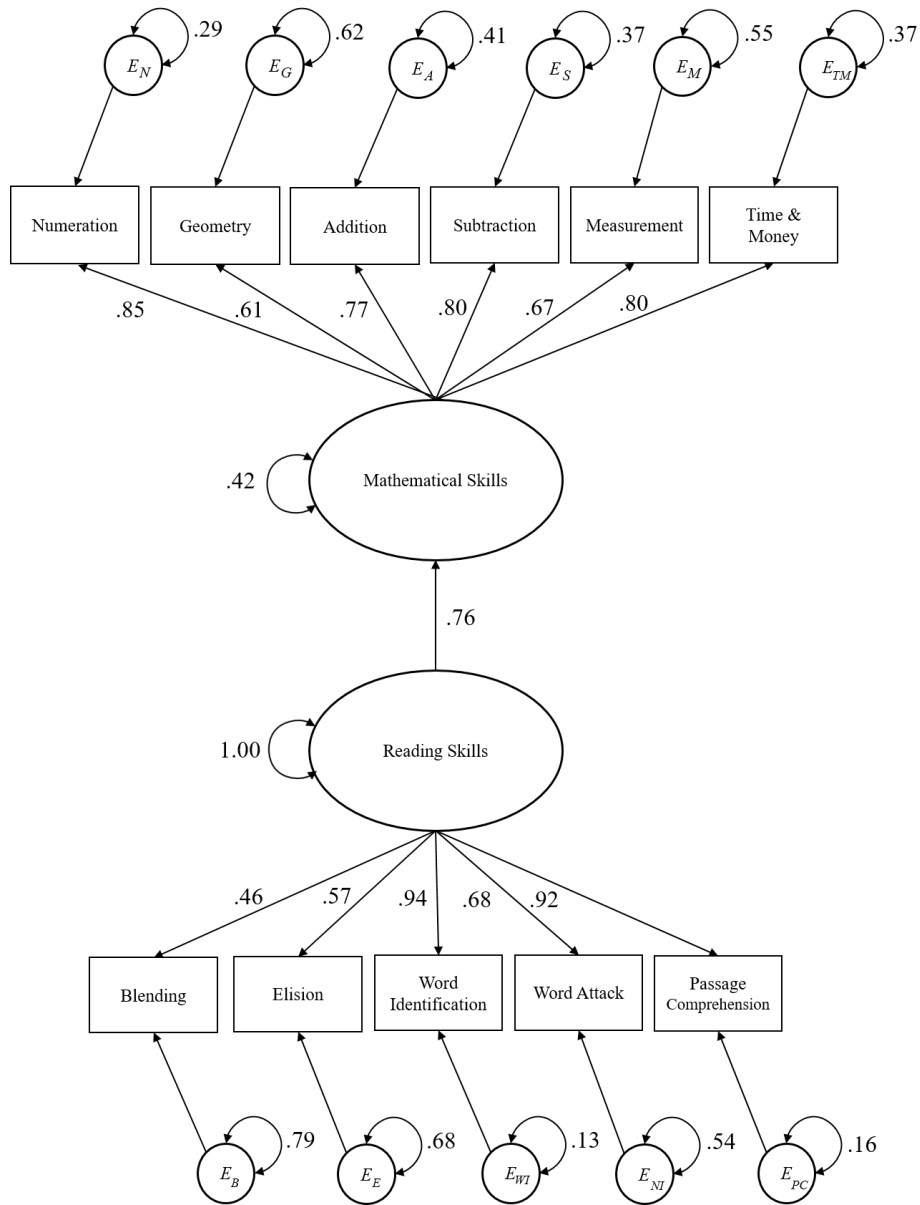
7.1 Appendix A: A path model indicating the relationship between mathematical skills and verbal skills prior to model modification



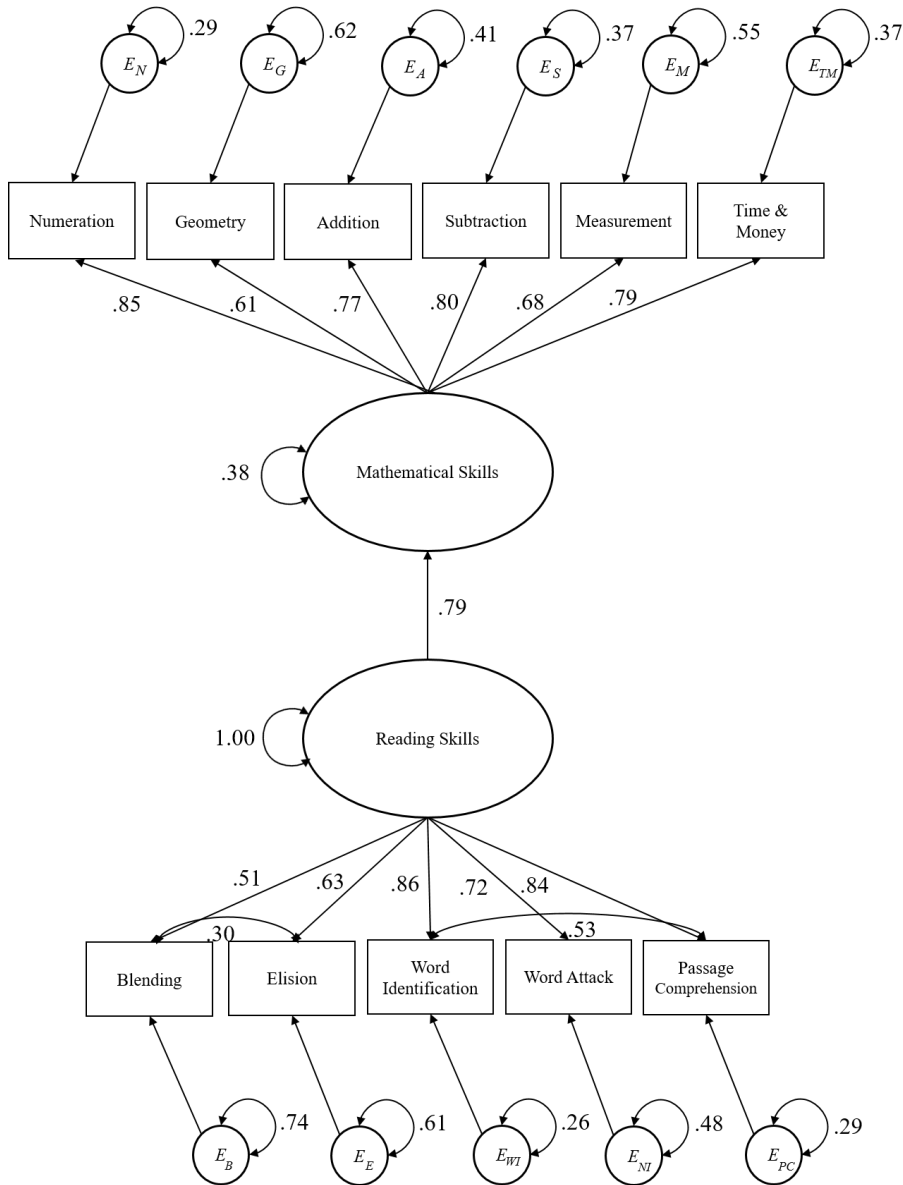
7.2 Appendix B: A modified path model indicating the relationship between mathematical skills and verbal skills based on modification indices, with a path added linking Measurement (KeyMath) and Verbal (WISC)



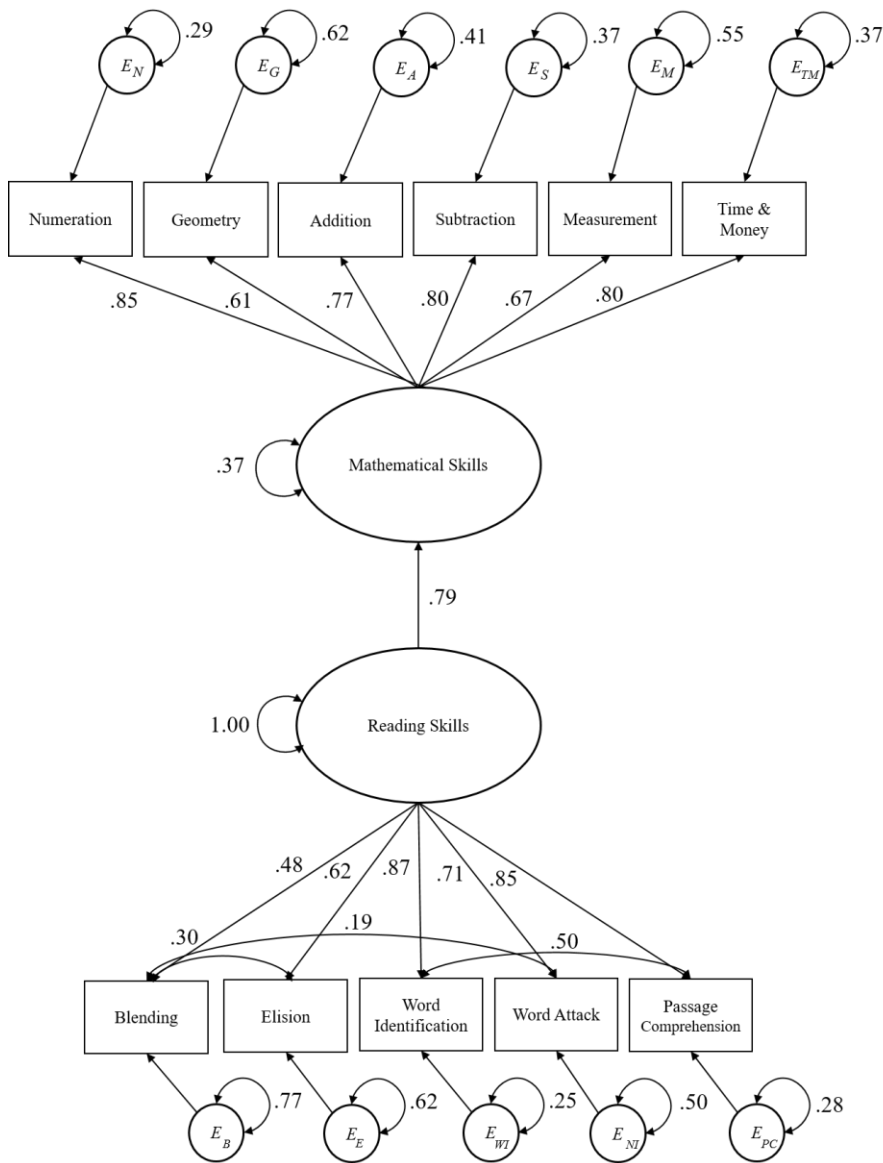
7.3 Appendix C: A path model indicating the relationship between mathematical skills and reading skills prior to model modification



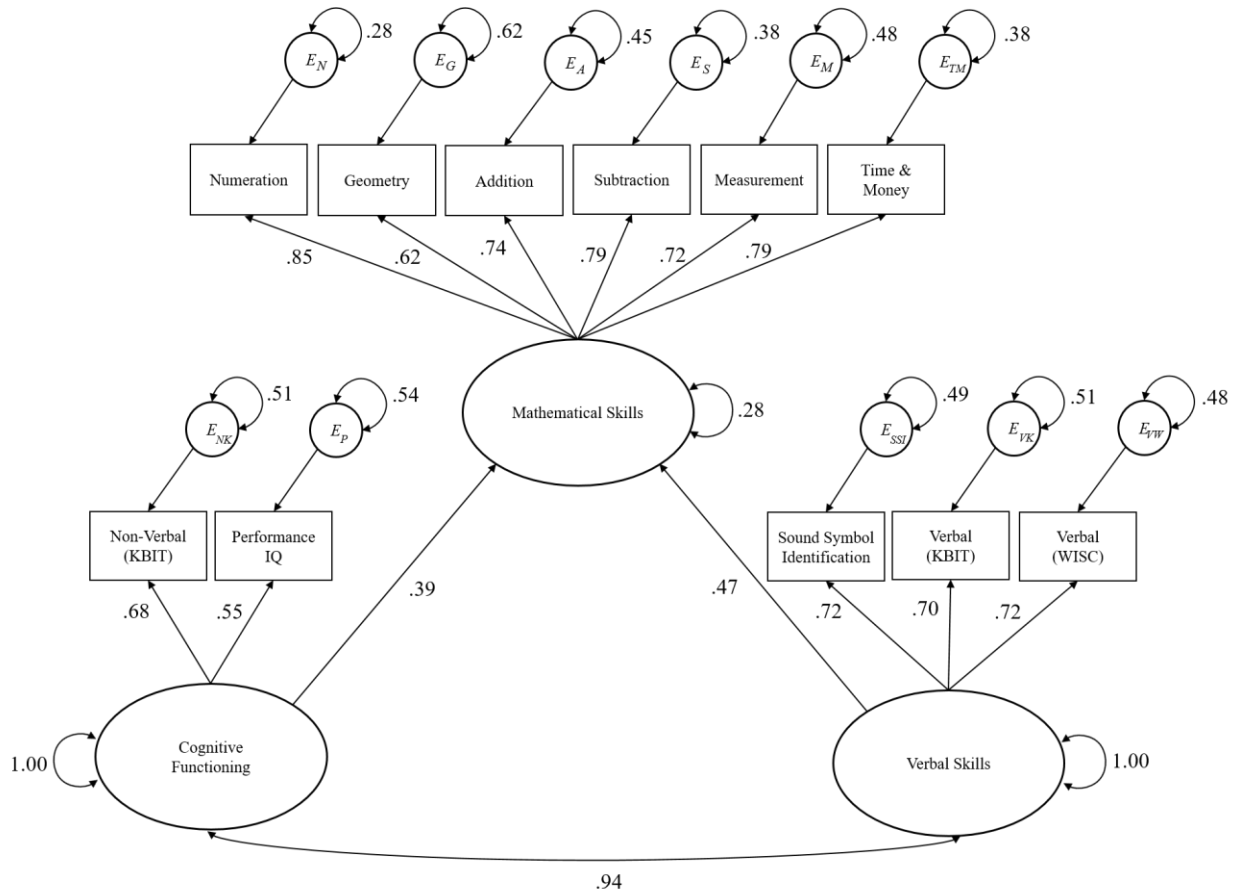
7.4 Appendix D: A modified path model indicating the relationship between mathematical skills and reading skills based on modification indices, with two paths added linking (1) Blending and Elision (CTRRPP) and (2) Word Identification and Passage Comprehension (WRMT)



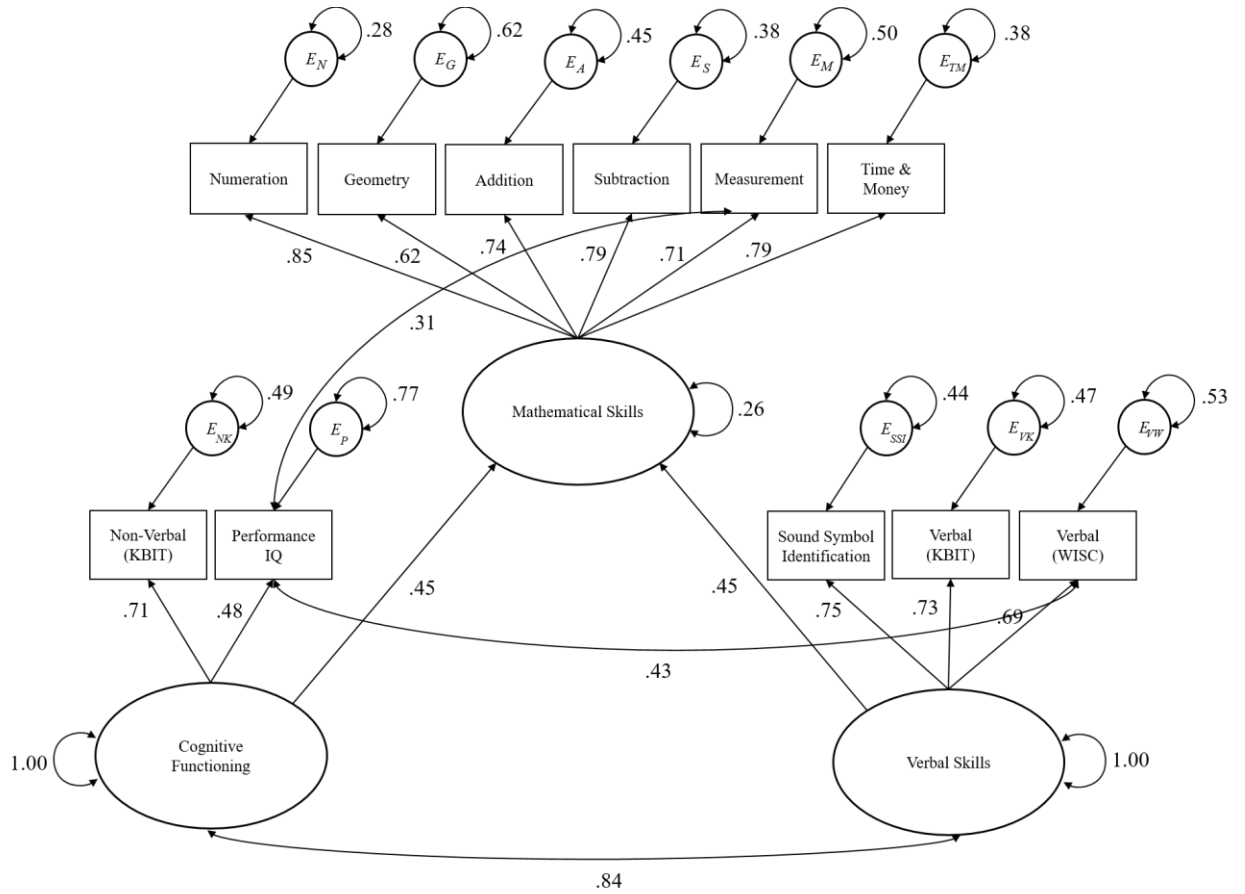
7.5 Appendix E: An alternate modified path model indicating the relationship between mathematical skills and reading skills based on modification indices, with three paths added linking (1) Blending and Elision (CTRRPP), (2) Word Identification and Passage Comprehension (WRMT), and (3) Blending (CTRRPP) and Word Attack (WRMT)



7.6 Appendix F: A path model indicating the relationship between mathematical skills, cognitive functioning, and verbal skills prior to model modification



7.7 Appendix G: A modified path model indicating the relationship between mathematical skills, cognitive functioning, and verbal skills based on medication indices, with added paths indicating a relationship between Performance IQ (WISC) and Measurement (KeyMath) and Performance IQ and Verbal (WISC) and Measurement (KeyMath) and Performance IQ and Verbal (WISC)



7.8 Appendix H: Table listing the results of all the models tested for mathematical skills

Goodness of fit statistics

Model	χ^2	df	<i>p</i>	CFI	RMSEA	SRMR
VS (pre, H)	44.01	26	.02	.97	.07	.04
VS (post, I)	32.85	25	.13	.99	.05	.04
RS (pre, J)	85.43	43	.00	.96	.08	.06
RS (post 2 paths, K)	57.52	41	.05	.98	.05	.04
RS (post 3 paths, L)	52.71	40	.09	.99	.05	.04
CF (post)	15.98	18	<i>n.s.</i>	1.00	.00	.03
VRS (post 3 paths)	149.36	72	<.01	.94	.09	.07
VS, CF (pre, M)	99.04	41	<.01	.93	.10	.06
VS, CF (pre, N)	54.66	39	.05	.98	.05	.04
VS, RS (post 2 paths)	171.36	72	<.01	.92	.10	.07
RS, CF (post 4 paths)	102.45	72	<i>n.s.</i>	.96	.07	.00
CF, VRS (post 3 paths)	224.47	98	<.01	.91	.09	.07
VS, RS, CF (post 3 paths)	220.21	95	<.01	.91	.09	.07

Note. VS = Verbal skills; RS = Reading skills; VRS = Verbal skills and reading skills combined; CF = Cognitive functioning; pre = pre-model-modification; post = post-model-modification; CFI = Comparative Fit Index; RMSEA = Root Measure Square Error of Approximation; Standardized Root Mean Square Residual.

7.9 Appendix I: Table listing the results of all of the model tested for the construct(s) predicting specific mathematical skills: Numeration, Geometry, Addition, Subtraction, Measurement, and Time & Money

Goodness of fit statistics

Model	χ^2	df	<i>p</i>	CFI	RMSEA	SRMR
VS	27.00	12	<.01	.98	.09	.03
RS (pre)	68.85	29	<.01	.96	.10	.06
RS (post)	36.49	26	<i>n.s.</i>	.99	.05	.04
VS, RS	223.94	55	<.01	.87	.14	.07
VRS (pre)	254.25	62	<.01	.85	.14	.08
VRS (post)	136.21	58	<.01	.94	.10	.07
VS, CF (pre)	67.59	22	<.01	.94	.12	.05
VS, CF (post)	41.98	21	<.01	.97	.08	.04
RS, CF (pre)	122.80	43	<.01	.93	.11	.08
RS, CF (post)	82.15	39	<.01	.96	.09	.06
VRS, CF (pre)	339.36	82	<.01	.82	.15	.09
VRS, CF (post)	198.55	78	<.01	.92	.10	.07
VS, RS, CF	301.20	76	<.01	.78	.14	.10

Note. VS = Verbal skills; RS = Reading skills; VRS = Verbal skills and reading skills combined; CF = Cognitive functioning; pre = pre-model-modification; post = post -model-modification; CFI = Comparative Fit Index; RMSEA = Root Measure Square Error of Approximation; Standardized Root Mean Square Residual.