#### GLOBAL SYSTEMICALLY IMPORTANT FINANCIAL INSTITUTIONS: A STRUCTURAL VAR APPROACH

CHANGHAO ZHANG (B.Sc. (Hons), NUS)

### A THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY DEPARTMENT OF FINANCE NATIONAL UNIVERSITY OF SINGAPORE 2017

Supervisor: Professor Duan Jin-Chuan

Examiners: Associate Professor Wang Tong Dr Denis Tkachenko Assistant Professor Christian Brownlees, Universitat Pompeu Fabra Barcelona

#### DECLARATION

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

houghton

Changhao Zhang 18 April 2017

#### ACKNOWLEDGEMENTS

The author sincerely thanks Jin-Chuan Duan for valuable guidance, and Robert Kimmel, Denis Tkachenko, Tong Wang, Jussi Keppo, Johan Sulaeman, Raja Velu, and Yoshio Nozawa for comments. The author also thanks Zhifeng Wang, Bai Hua and Li Pei of Risk Management Institute, National University of Singapore, for data assistance.

#### SUMMARY

Systemic importance of a financial institution is measured as the additional tail loss induced into the system when the financial institution falls into distress due to its own structural shocks. The use of a structural approach is a step towards addressing a key concern in systemic risk literature, "Is the firm impacting the market, or is the market impacting the firm?"

The identification exploits "too-big-to-fail" restrictions which are implicitly imposed when a dynamic factor model is assumed, and the data reveals "too-interconnected-to-fail", thereby incorporating the two key considerations of systemic importance. Over 21,000 firms listed globally are modelled jointly as a system.

Even though we use only public data, the model's output relates to actual bailout events, and also reflects interactions of firms linked to the same supply chain. In addition, we show how Basel's list of global systemically important banks can be interpreted in our framework.

Keywords: systemic importance, systemic risk, simultaneity, factor model, structural identification, Basel, big data.

### 1 Introduction

The 2008-2011 Global Financial Crisis has inspired researchers to conceptualise and measure systemic risk and systemic importance. Measurement of systemic importance serves the macroprudential objective of identifying financial institutions which may impact the global financial system and the wider economy. Indeed, the area of systemic risk is a growing literature with a variety of ideas being proposed. In this paper, we are interested in measuring systemic importance in a conceptually sound fashion on a global scale, in particular for financial institutions.

To establish the notion of systemic importance, the policy definition from the Basel Committee on Banking Supervision (BCBS) is adopted, that "The Committee is of the view that global systemic importance should be measured in terms of the impact that a bank's failure can have on the global financial system and wider economy..."<sup>1</sup>. In other words, the key question in simple words is, "How does deterioration of a financial institution impact the global community?"

There are two key strengths in the approach laid out in this paper. First, we exploit structural analysis to attribute systemic importance to individual firms. In other words, we measure the impact which a firm would cause to the system, after accounting for the confounding effect of the system's impact on the firm, subject to the limitations of the system we consider. The key systemic risk measures which we reference, SRISK and ∆CoVaR, in our opinion, rely on attribution mechanisms which are too narrow and inherently difficult to interpret. SRISK conditions the system on a drop in the stock market, and assesses the capital shortfall which a firm would experience due to its market beta. The difference in our approach is that we condition on the firm's own structural deterioration, instead of a single market stress test scenario, and this is closer to the Basel definition above. Therefore, the origin of risk is directly attributed to the firm instead of subjecting the said firm to an external situation and then making the additional argument that the firm must raise its capital shortfall from the market. In the case of  $\Delta \text{CoVaR}$ , a firm's systemic risk contribution is

<sup>1</sup>Extracted from: Basel Committee on Banking Supervision, Global Systemically Important Banks: Updated Assessment Methodology and the Higher Loss Absorbency Requirement, July 2013

defined as the difference between the financial system's value-at-risk conditional on the firm being under distress, and the financial system's value-at-risk conditional on the median state of the firm. The authors note that ∆CoVaR does not distinguish whether the contribution is causal or due to a common factor, and provide the example of being "systemic as part of the herd". In other words, a firm's ∆CoVaR might not reflect its own contribution to systemic risk or its own capacity to cause a crisis. A tiny firm could have large ∆CoVaR simply by being part of a "herd" of firms affected by a common external factor such as the liquidity environment. As per our understanding, identifying and drawing conclusions from such situations requires additional non-trivial analysis. The papers relevant to SRISK include Acharya et al (2010), Brownlees and Engle (2012), and Acharya et al  $(2012)$ , while  $\Delta$ CoVaR is discussed in Adrian and Brunnermeier (2011).

Second, we use only public data. Specifically, the global scale of over 21,000 listed firms is modelled jointly as a single system without relying on private inter-firm data. The literature on systemic risk can be broadly divided into those employing network structures with private inter-firm assets and liabilities, and those using public data but relying more heavily on correlation analysis. The advantage of the former approach is that the risk transmission channels are clearly defined and is well-suited for small to medium scale analysis of network phenomena. It does, however, require a reasonable extent of modelling and private inter-firm data which is challenging if not impossible to put together in a consistent manner on a global scale. Clearly, its applicability to global systemic risk rankings, which is our focus here, is limited. Examples of studies in this area include Nier et al (2007), Canedo and Jaramillo (2009), Anand et al (2013), and Duan and Zhang (2013). On the other hand, the latter approach which uses public data often relies on correlation analysis and meets with the question of causality – does the institution's failure cause the crisis, or is the crisis causing the institution's failure? Our approach, while relying only on public data, attempts to impose some structure by incorporating "too-big-to-fail" restrictions and letting the data reveal "too-interconnected-to-fail", which are the two key considerations of systemic importance. To demonstrate robustness, we relate the model's output to actual bailout events for the subset of US banks with readily available data. In addition, we study two cases of supply chains driven by large US firms and find that our model exhibits the expected dependencies.

Other related papers in the systemic risk literature include Hautsch et al (2014) which considers systemic risk contribution of an individual firm to be the marginal effect of its individual value-atrisk on the value-at-risk of the system, Huang et al (2012) which defines the systemic importance of each bank as *its marginal contribution* to the hypothetical distress insurance premium of the whole banking system, and Billio et al (2012) which measures Granger Causality of returns and aggregates the total outward Granger effects of a financial institution to determine systemic rankings. On the theoretical side, a popular strand stems from Eisenberg and Noe (2001), which explores the existence of a post-liquidation equilibrium in a static setting; that is, to show that an initial shock to a banking network can indeed be resolved through the balance sheets of the banks and there is an end-state after all defaults have taken place. Gourieroux et al (2012) is one example that provides an extension and uses it to explore contagion risk. Chen et al (2016) uses a similar framework but incorporates liquidity contagion as an additional channel. White et al (2015) measures tail dependence using multivariate regression quantiles and is closely related to ∆CoVaR and SRISK. Copula-based approaches for measuring tail dependence include Han et al (2016) and Okhrin et al  $(2015).$ 

More recently, partial correlations have also been used to measure systemic risk, such as CriSIFI by Chan-Lau et al  $(2016)^2$  and CoRisk by Giudici and Parisi  $(2016)$ . The use of partial correlations is attractive because it controls for indirect effects that pass through third parties, appealing to the notion that each partial correlation pair can be attributed to the bilateral interbank relationship. At the bilateral level, however, it is difficult to determine which bank is affecting which, and some form of size is usually incorporated (this is also true for SRISK and ∆CoVaR). Our approach recognises the importance of size upfront and imposes them as restrictions, and control for commonality through and within the factors.

<sup>&</sup>lt;sup>2</sup>Like SRISK, CriSIFI has been operationalised into a live system and rankings are provided online.

The Basel approach used by the Financial Stability Board relies on an indicator approach which uses a variety of asset and liability aggregates, most of which correlate strongly with size. Broadly speaking, this addresses the concern of "too-big-to-fail" without accounting for "too-interconnectedto-fail" directly. We will show how Basel's list can be approximately obtained in our framework by limiting interconnectedness, amplifying the importance of some countries, and downplaying others.

The key technical contribution of this paper is recognising that the assumption of a factor model, commonly used in asset pricing, implicitly provides identifying restrictions. An individual firm moves due to factors as well as its own shock, and at the same time, factors are defined from individual firms. Endogeneity due to simultaneity is embedded within such a system. A typical assumption is to treat factors as exogenous, often because each firm's contribution to the factor is small, for example, the typical CAPM regression. As a result, each firm's own shock will be orthogonal to the factors and often termed an idiosyncratic shock, which is a misnomer as it contradicts the definition of the factors. On the other hand, as we will show, accounting for the simultaneity allows the expression of all factors and all firms in terms of the firms' structural shocks. Systemic importance can then be assessed by conditioning the system on a firm being shocked to a distressed state due to its own structural shock.

As alluded to, "too-big-to-fail" restrictions are automatically imposed by value-weighting each firm during the construction of factors. Precisely, we weight each firm by its total liabilities, since as a precursor, firms with larger liabilities are more likely to impact other firms' credit quality. Although the selection of factors affects the structural identification and may introduce some subjectivity, it turns out that the results are robust to alternative specifications of factor construction. At the baseline, we use hierarchical correlation clustering to group countries together and form 3 factors representing the major global economic regions, while for robustness tests we randomly assign countries to factors.

To our knowledge, the source of identifying restrictions which we use here, i.e. by resolving the simultaneity in a factor model, is new and adds to the list of structural identification strategies which

include timing restrictions, zero restrictions, short and long-run restrictions, sign restrictions, etc. Notably, our motivation differs from the traditional usages of factor models. In pioneering work, Sargent and Sims (1977) proposed a "two-index model" and briefly noted the issue of simultaneity and found that two factors could explain a large variation is US economic data. The setting established by Geweke (1977) is that most economic time series are driven by a small number of unobserved (and exogenous) factors, and the challenge pertained to identification of structural shocks that drove these factors. Here, our setting differs in that the structural shocks are individual firm level shocks, and the factors, being endogenously constructed from individual firms, respond to the collective force of the firms' shocks. Good summaries of the long line of literature on factor models are readily available in Stock and Watson (2005) as well as in Bai and Wang (2016).

As can be seen, our paper stands out in its structural identification strategy, without needing to make stronger assumptions than those commonly used in asset pricing models. Consequently, we can assess systemic importance via inspecting structural causality. Indeed, several of the papers mentioned earlier use factor models but do not recognise the structural implications. Furthermore, factor models are suitable, in fact almost inevitable, for dealing with very large datasets which are necessary to address the question of global systemic importance. A comparison of the key features vis-a-vis SRISK and ∆CoVaR is presented in Table 1.

	<b>SRISK</b>	$\Delta$ CoVaR	This Paper
Conditioning Event	Severe shock of at least $40\%$ to the equity in- $\frac{1}{2}$ dex over a period of 6 months.	Firm in distress vs firm in median state.	Firm being shocked to distress due to its own structural shocks.
Quantity <b>Taken</b>	Capital shortfall of the firm under the condi- tioning event.	Difference in asset VaR of the system under the two conditioning events.	Additional tail loss in- duced into the system due to the firm.
<b>Scope</b>	All listed firms globally.	from Financial firms CRSP / COMPUSTAT.	All listed firms globally.
	Implementation For each firm, a bivari- ate GARCH-DCC com- prising of the equity in- dex and the firm.	Quantile regressions based off a set of state variables.	All firms modelled jointly using a dynamic factor model with lag effects. Factors guide identification.
Transmission of Risk	Co-movement of firm eq- uity with the equity in- dex.	Not explicit.	Co-movement of credit quality channelled through 3 factors and sparse inter-firm inter- actions.

Table 1: Key Features of SRISK, ∆CoVaR and This Paper

### 2 Factor Models and Simultaneity

In this section, we show how a structural representation of the system can be obtained by assuming a factor model and addressing the embedded endogeneity due to simultaneity. We begin with the purely contemporaneous case, and then introduce autoregressive structure.

Assumption 1. Firm dynamics can be described by a factor model:

$$
R_t \gamma = F_t \beta + e_t D \tag{1}
$$

where  $R_t$  is a  $1 \times p$  vector of credit risk shocks or changes to probabilities of default (PD)<sup>3</sup> of p firms at time t,  $F_t$  is a  $1 \times r$  vector of changes of r factors at time t,  $\beta$  is a  $r \times p$  matrix which maps the factors changes into firm PD changes,  $e_t$  is a  $1 \times p$  vector of i.i.d. firm structural shocks, and D is a  $p \times p$  diagonal matrix of standard deviations.  $\gamma$  is an invertible matrix with diagonal entries equal to 1, but may have off-diagonal sparse entries.

In other words, PD changes of the firms are driven firstly by factors, and secondly by inter-firm interactions over and above the factors. Note that by inverting  $\gamma$ , we have  $R_t = F_t \beta \gamma^{-1} + e_t D \gamma^{-1}$ which bears the familiar form of a typical factor model, except that it is more general here in that  $D\gamma^{-1}$  potentially has off-diagonal entries.

Assumption 2. Factors can be represented as an aggregation of firms:

$$
F_t = R_t W \tag{2}
$$

where W is a  $p \times r$  matrix of weights which maps the firm PD changes into factors.

The above two assumptions are commonly made in finance and economics; we are simply following them to the logical conclusion. For example, in asset pricing the market index is often

<sup>&</sup>lt;sup>3</sup>In this paper, we are primarily interested in credit risk shocks, but the technique laid out here should be understood to be more generally applicable to say, stock returns.

used as a factor, at the same time it is constructed from the firms. In intuitive terms, we are choosing factors to represent the system, which is reflected in W, the constituent weights. This belief provides the restrictions and drives the identification.

With the two assumptions, we can substitute  $(1)$  into  $(2)$  to obtain:

$$
F_t = e_t D \gamma^{-1} W (I - \beta \gamma^{-1} W)^{-1} =: e_t A
$$
\n(3)

which identifies the factor changes  $F_t$  in terms of structural shocks  $e_t$ . Next, substitute (3) into (1) to obtain the firm PD changes  $R_t$  in terms of structural shocks:

$$
R_t = e_t (A\beta + D)\gamma^{-1} =: e_t G \tag{4}
$$

The factors essentially provide the channels through which the structural shocks of firms are transmitted to all other firms, and it is in this manner that identification is provided. W coalesces the structural shocks of individual firms into factors, and A is the precise way in which it is done. The factors transmit the shocks to all other firms by way of  $\beta$ . Equations (3) and (4) express these externalities or *contemporaneous feedback effects* between the firms in relation to their structural shocks.

This result also brings us to an interesting observation. When factor models are employed, say in asset pricing,  $e_t$  is typically treated as the "idiosyncratic risk" of the firms, implying that they are orthogonal to the factors and each of them affects only its corresponding firm. Given that the state space is actually represented by  $e_t$ , it may be more appropriate to term them as "firm" structural shocks".

Equation (3) makes clear another reason why a factor model is useful, besides being representative of the state. Since the number of factors is typically much smaller than the number of firms, or  $r \ll p$ , we are able to obtain the  $r \times r$  inverse  $(I - \beta \gamma^{-1} W)^{-1}$  with better numerical accuracy and computation time, thereby obtaining an efficient (low rank) representation of the dynamics.

While high-dimension,  $\gamma^{-1}$  can reasonably to be expected to be sparse and diagonally dominant, hence does not pose problems with inversion. Without a factor model, identification would have to be resolved with a full dimension  $p \times p$  matrix (depending on assumptions).

Note that the observable data in the above setup is  $R_t$ , the firm PD changes. The weights W are chosen by design and lead directly to  $F_t$ . The firms' factor loadings  $\beta$ , dependency on other firms  $\gamma$ , and standard deviations of their structural shocks D must be estimated from the data. For firm i, let  $R_{t,i}$  be its PD changes and  $R_{t,-i}$  be the PD changes of other firms which it depends on. One might be tempted to run a standard OLS of  $R_{t,i}$  on  $F_t$  and  $R_{t,-i}$ . However, Equations (3) and (4) clearly indicate that  $F_t$  and  $R_{t,-i}$  are correlated with  $e_t$ , and therefore, any estimation is subject to endogeneity due to simultaneity.

Indeed, if direct estimation such as OLS is conducted on (1), a contradiction is imposed on the system by assuming that  $F_t$  and  $R_{t,-i}$  are uncorrelated with  $e_t$ , which is clearly violated here. Furthermore, even if we were to accept the OLS  $\beta$ , identification is no longer possible as  $I - \beta \gamma^{-1} W = 0$ . In other words, the identification of the system is entwined with the issue of simultaneity.

Fortunately, the simultaneity can be resolved by recognising that the simultaneity for firm i results specifically from its own structural shock  $e_{t,i}$ . Suppose that for now we know the structural representations of the factors and firms,  $A$  and  $G$  respectively. For firm  $i$ , the simultaneity corrections to apply are:

$$
F_{t,i}^{adj} = F_t - R_{t,i} \frac{A_i}{G_{ii}} \tag{5}
$$

$$
R_{t,-i}^{adj} = R_{t,-i} - R_{t,i} \frac{G_{i,-i}}{G_{ii}} \tag{6}
$$

where  $A_i$  is the *i*-th row of A corresponding to factors' loading on firm *i*'s structural shock  $e_{t,i}$ ,  $G_{i,-i}$  contains the elements in the i-th row of G corresponding to the relevant firms' loading on firm i's structural shock  $e_{t,i}$ , and  $G_{ii}$  is the entry of G corresponding to firm i's loading on its own structural shock.

 $F_{t,i}^{adj}$  and  $R_{t,-}^{adj}$  $_{t,-i}^{aq}$  no longer contain any component of  $e_{t,i}$  but only the other shocks  $e_{t,-i}$ . To see this, simply substitute equations (3) and (4) into the above to eliminate the entries relating to  $e_{t,i}$ . Given that  $e_t$  is cross-sectionally independent, it follows that  $cov(F_{t,i}^{adj}, e_{t,i}) = 0$  and  $cov(R_{t,-i}^{adj}, e_{t,i})$  $_{t,-i}^{aaj},e_{t,i}) =$ 0. Then, for each firm, we can run the following regression without violating the endogeneity (rather, exogeneity) assumption:

$$
R_{t,i} = F_{t,i}^{adj} \tilde{\beta}_{(i)} + R_{t,-i}^{adj} \tilde{\gamma}_{(i)} + e_t \tilde{D}_i
$$
\n
$$
\tag{7}
$$

Next, substitute (5) and (6) into (7) and rearrange the terms. This leads to the required coefficients, corrected for simultaneity:

$$
\beta_{(i)} = \frac{\tilde{\beta}_{(i)}}{1 + \frac{A_i}{G_{ii}} \tilde{\beta}_{(i)} + \frac{G_{i,-i}}{G_{ii}} \tilde{\gamma}_{(i)}} \tag{8}
$$

$$
\gamma_{(i)} = \frac{\tilde{\gamma}_{(i)}}{1 + \frac{A_i}{G_{ii}} \tilde{\beta}_{(i)} + \frac{G_{i, -i}}{G_{ii}} \tilde{\gamma}_{(i)}}
$$
\n(9)

The remaining problem is that  $\beta$ ,  $\gamma$ ,  $D$ ,  $A$  and  $G$  are all unknown initially.  $\beta$ ,  $\gamma$  and  $D$  must be estimated with knowledge of  $A$  and  $G$  to apply the simultaneity correction, yet  $A$  and  $G$  are defined in terms of  $\beta$ ,  $\gamma$  and D. This problem can be resolved numerically via fixed point iteration which is described in the appendix. The algorithm is efficient owing to the factor model which addresses the curse of dimensionality  $(r \ll p)$  and multicollinearity / rank deficiency.

#### 2.1 Autoregressive Structure

The above formulation can be easily extended to include an autoregressive structure. The full  $p$ -dimension system with  $k$  lags can be written as:

$$
R_t \gamma = \sum_{j=0}^k R_{t-j} \psi_j + e_t D \tag{10}
$$

where  $\{\psi_j\}$  are  $p\times p$  coefficient matrices corresponding to the  $j\text{-th}$  lags.

Assumption of a factor model essentially applies a low rank structure to the coefficients, or  $\psi_j = W \beta_j$ , and we have the low rank representation:

$$
R_t \gamma = \sum_{j=0}^k R_{t-j} W \beta_j + e_t D
$$
  
= 
$$
\sum_{j=0}^k F_{t-j} \beta_j + e_t D
$$
 (11)

$$
F_t = R_t W \tag{12}
$$

where  $\{\beta_j\}$  are  $r \times p$  coefficient matrices corresponding to the *j*-th lags. As before,  $r \ll p$ .

The workings above follow through as before, with as many lags as desired. In the case of lag-1 structure used in our application:

$$
F_t = F_{t-1}\beta_1\gamma^{-1}W(I - \beta_0\gamma^{-1}W)^{-1} + e_tD\gamma^{-1}W(I - \beta_0\gamma^{-1}W)^{-1}
$$
  
\n
$$
=: F_{t-1}A_1 + e_tA_0
$$
  
\n
$$
R_t = F_{t-1}(A_1\beta_0 + \beta_1)\gamma^{-1} + e_t(A_0\beta_0 + D)\gamma^{-1}
$$
  
\n
$$
=: F_{t-1}G_1 + e_tG_0
$$
\n(14)

The simultaneity adjustments follow the same logic of eliminating the contemporaneous firm structural shock:

$$
F_{t,i}^{adj} = F_t - R_{t,i} \frac{A_{0,i}}{G_{0,ii}} \tag{15}
$$

$$
R_{t,-i}^{adj} = R_{t,-i} - R_{t,i} \frac{G_{0,i,-i}}{G_{0,ii}} \tag{16}
$$

Likewise, we run the following regression:

$$
R_{t,i} = F_{t,i}^{adj} \tilde{\beta}_{0,(i)} + F_{t-1} \tilde{\beta}_{1,(i)} + R_{t,-i}^{adj} \tilde{\gamma}_{(i)} + e_t \tilde{D}_i
$$
\n(17)

And by substituting  $(15)$  and  $(16)$  into  $(17)$ , we can recover:

$$
\beta_{0,(i)} = \frac{\tilde{\beta}_{0,(i)}}{1 + \frac{A_{0,i}}{G_{0,ii}} \tilde{\beta}_{0,(i)} + \frac{G_{0,i,-i}}{G_{0,ii}} \tilde{\gamma}_{(i)}} \tag{18}
$$

$$
\beta_{1,(i)} = \frac{\tilde{\beta}_{1,(i)}}{1 + \frac{A_{0,i}}{G_{0,ii}} \tilde{\beta}_{0,(i)} + \frac{G_{0,i,-i}}{G_{0,ii}} \tilde{\gamma}_{(i)}} \tag{19}
$$

$$
\gamma_{(i)} = \frac{\tilde{\gamma}_{(i)}}{1 + \frac{A_{0,i}}{G_{0,ii}} \tilde{\beta}_{0,(i)} + \frac{G_{0,i,-i}}{G_{0,ii}} \tilde{\gamma}_{(i)}} \tag{20}
$$

As before, the fixed point algorithm can be used to compute  $A_0$ ,  $A_1$ ,  $G_0$ ,  $G_1$ ,  $\beta_0$ ,  $\beta_1$ ,  $\gamma$  and D. The structural representations (13) and (14) now include autoregressive terms of the factors, which is exactly what we are after. An equivalent representation for  $k \geq 1$  is:

$$
F_{t+k} = F_t A_1^k + \sum_{j=1}^k e_{t+j} A_0 A_1^{k-j}
$$
\n(21)

Using the earlier terminology, (21) describes the contemporaneous and *lagged feedback effects*, and is useful for assessing the firm's impact on the system over a forward-looking horizon. For example, to measure the systemic importance of firm i, one may prescribe shock profiles to  $e_{t+1,i}$ ,  $e_{t+2,i}$ , ...,  $e_{t+T,i}$  such that the firm is shocked to distress, and assess the resulting impact to the system.

What do the firm structural shocks  $e_t$  represent? Conceptually, they are shocks that pertain specifically to the firm, and keeping it generic allows the model to accommodate a range of causes in the conditioning event, i.e. the firm may deteriorate due to a number of reasons and there is no need to restrict the interpretation here. That said, as part of the robustness checks later, we will analyse them in the context of bailouts and supply chain effects.

We conclude this section with a practical note. While constructed factors are used in this paper, Assumption 2 does not necessarily require this. One may take a stronger position that the results also apply to non-constructed or prescribed factors. For example, although our analysis is conducted on credit risk shocks, we could just as well treat the stock market as a factor. The essence

of the argument is that defining a particular series as a factor already assigns the identification, as long as the factor can be represented as a weighted sum of firm PD changes<sup>4</sup>. However, if all we want is to assess the impact of the system on the stock market, there is no need to include it as a factor. We can project the stock market returns on the set of constructed factors and infer its dynamics from the factors.

### 3 Application to Systemic Importance

In this section, we discuss the considerations to be made in implementing the factor model identification framework for identifying global systemically important financial institutions. The focus here is on banks, which the Basel framework is concerned with, although the study can be extended to include, say, non-bank insurers<sup>5</sup>. For each of the 866 listed banks, the impact through 3 global factors on 21,544 firms is assessed.

#### 3.1 Credit Quality as the Denominator of Systemic Importance

A distinguishing feature of our work is that we use physical default probabilities to analyse contagion effects. Co-movement of credit quality is the appropriate denominator of systemic importance as it speaks directly to the health of all firms which make up the economy. As noted earlier, several other systemic risk measures infer systemic importance from the co-movement of equity (e.g. SRISK) or equity-based asset returns (e.g. ∆CoVaR, Hautsch et al 2014 and Billio et al 2012). Typically, such measures are aggregated into a loss value, such as capital shortfall or value-at-risk. In the broader literature, authors such as Dungey et al (2012) and Diebold and Yilmaz (2011) analyse volatility spillovers in a network setting.

The trouble for systemic importance is that equity prices and volatilities rise and fall jointly for numerous reasons, not all of which stem from systemic concerns. Take for example, Google which

<sup>&</sup>lt;sup>4</sup>This may require additional assumptions to implement. For example, one may project the stock market onto the firms via OLS to obtain the implied aggregation. The residual will be orthogonal to the system.

<sup>&</sup>lt;sup>5</sup>Banks typically have insurance operations as well.

holds USD 60 bn in cash, far in excess of its USD 1 bn in debt, or Apple which holds USD 140 bn in cash and USD 35 bn in debt. Neither company is plausibly expected to default on debt. However, their equity prices co-move every day, and such movements are obviously not driven by systemic concerns. Consequently, inferring systemic importance from stock returns bears the inherent risk of possibly misinterpreting a host of irrelevant factors as systemic importance. This point has also been noted in Chan-Lau et al (2016).

We opine that co-movements in credit risk is a sharper and more direct means of addressing systemic importance. The viability of the broad community of firms is the primary concern of regulators. If an individual firm can cause the joint credit deterioration of a large number of firms simultaneously, it sparks immediate worry. Analysing credit risk speaks directly to the health of firms making up the economy.

The use of credit quality is troublesome for most papers for two reasons. Firstly, CDS spreads contain a premium and do not convey purely credit quality information, due to risk aversion of the issuer, counterparty risk of the guarantor, trading illiquidity, and other concerns. An example would be Alter and Beyes (2014) which measures CDS spillovers<sup>6</sup>. Huang et al (2012) also use CDS and face the same issues with regards to CDS premium. Secondly, the availability of traded single-name CDS is extremely limited, and prices are usually illiquid.

We have the benefit of using physical PD computed by the Credit Research Initiative of the Risk Management Institute, National University of Singapore (RMI-CRI<sup>7</sup>). The RMI-CRI PD are estimates of physical default probabilities using a reduced-form econometric approach factoring in

 $6$ Additionally, Alter and Beyes (2014) do not address the issue of systemic importance in detail and their analysis is specific to Europe.

<sup>7</sup>The RMI-CRI system has implemented the forward intensity model of Duan et al (2012). It currently makes available and allows free access by all legitimate users to daily updated PD ranging from one month to five years for 60,400 exchange-listed firms in 106 economies around the world. The forward intensity functions used to generate the RMI-CRI PD are exponential linear functions of some input variables (2 macroeconomic factors and 10 firm-specific attributes) where the coefficients depend on the forward starting time, and are subject to the Nelson-Siegel type of smooth term structure restriction. Estimation of the parameters and statistical inference for the constrained system rely on the pseudo-Bayesian sequential Monte Carlo technique and self-normalized statistics devised in Duan and Fulop (2013). For details on the specific RMI-CRI implementation, please refer to RMI-CRI Technical Report (2013). The RMI-CRI model's parameters are re-calibrated monthly and the inputs to the functions are updated daily.

many commonly known drivers of default and bankruptcy. Therefore, in contrast to papers which rely solely on traded CDS, we have less of a credit premium issue. Furthermore, the dataset of traded single-name CDS is very limited in comparison to RMI-CRI's PD.

The quality of PD is critical, as we are building an entire model upon it. To allay possible concerns on reliability, RMI-CRI's PD have undergone backtesting and have been shown to be more accurate than competing models. For 1-year PD which we use, accuracy ratios are in excess of 80% for most countries, even for out-of-sample predictions. To our knowledge, the accuracy is superior to any other PD datasets which have comparable coverage, and far more accessible. See Duan et al (2012) and the NUS-RMI CRI Technical Report for details.

#### 3.2 Global Scope for Global SIFIs

It has been noted that several systemic risk measures are developed as some incarnation of correlation analysis, and rely on the conditioning event to infer the individual firm's systemic importance. Here, the use of structural analysis bears the advantage of isolating the channels of impact to the firm-specific level.

Even so, we are not immune to potential misreporting of systemic importance if our dataset is too small. Consider an extreme example to make the point. If we decided to analyse the 2008 Global Financial Crisis but omitted the key US and European financial institutions, the major drivers of the systemic risk would have been ignored. A less drastic example, some papers focus solely on US financial firms without accounting for their impact to and from non-financials, and discount influences to and from Europe, Asia and the rest of the world. In some sense, we face the same challenge as Billio et al (2012), which as mentioned uses aggregates of pairwise Granger Causality of returns to determine systemic risk. A general comment about such implementations is that they depend entirely on the appropriate selection of variables. Obviously, causal factors that are not incorporated into the regression model cannot be represented in the output.

National regulators might be predominantly interested in the impact on firms in their own

jurisdictions. Even so, without taking a global perspective, it is impossible to assess, manage or otherwise attribute away the external influences which may be affecting domestic firms in a globalised world.

Our problem is mitigated by using a global dataset, comprising roughly 60,400 firms in 106 economies. When restricted to the scope of data with at least 50% of observations in the period from Jan 2004 to Dec 2013, and existing balance sheet data as of 31 Dec 2013, there are 21,544 firms in 96 economies. In terms of raw scope, few efforts have come close to the scale we are contemplating. Huang et al (2012) analysed 19 bank holding companies covered by the US Supervisory Capital Assessment Program, while Hautsch et al (2014) focused on publicly traded US financial firms. Adrian and Brunnermeier (2011) used CRSP data in conjunction with COMPUSTAT, with the scope restricted to financial firms. Billio et al (2012) used CRSP data together with the TASS hedge fund database. The SRISK measure has a global scope but assesses each firm's systemic risk in a bivariate setting (each firm individually with the market index), in essence assuming that externalities between all firms are captured through the single reference market index. CriSIFI, which uses the same dataset as us, has over 1,200 banks, but excludes non-financials. One drawback with this data is that private banks are not covered, and hence cannot be measured. However, most of the large banks are listed, which are the ones we expect to be systemically important.

#### 3.3 Factor Selection

Our analysis of the global dataset is made feasible by the factor model. For reasons which are now clear given section 2, the choice of factors inherently represents our beliefs with regards to how the system evolves – by acting as conduits for individual firms to express their impact on all other firms and itself, and all firms induce their own effects through the factors. To capture these interactions adqeuately, such externalities of the firm must be expressed through multiple factors, rather than relying on a single one. Typically, factors are chosen to be representative of the overall state of the firms, and the factors themselves are often what we are concerned with or consider to be important.

Setting up the factors with these considerations can be coherent with their importance as a conduit.

Technically, selecting factors amounts to choosing the weights  $W$ . Several popular possibilities include value-weighting, equal-weighting, or even the use of principal components. Consider two firms, Citibank and National Penn Bancshares, which have total liabilities of USD 1,696,772 mil and USD 7,293 mil (several magnitudes smaller) respectively. Without knowing more about their interactions, one may expect the firm with higher liabilities to drive the credit environment to a greater extent, and this corresponds to the value-weighting case. Likewise, an equal-weighting case expresses the belief that both the large and small firms drive the factors to a similar extent, apart from the differing volatility of shocks D. Using the first few Principal Component Analysis (PCA) factors amounts to believing that firms with higher variation are more important, although one must note that smaller firms tend to be more volatile<sup>8</sup>.

It is not true, however, that there is no leeway in determining  $W$ . The firm's loading on the factors,  $\beta$ , will respond to the choice of W in order to best fit the data, taking into account simultaneity. If the bad choice of an unrepresentative factor is made,  $\beta$  will instead load on more relevant factors supposing they are available. Regardless,  $\beta$  is assigned to a factor and does not make the distinction between firms which are within the same factor. Suppose that in our example of Citibank and National Penn Bancshares, their default probabilities have a high correlation<sup>9</sup> of 0.9, implying that they move closely with one another. Either of them can fulfil the role of creating variation in the factor. If by design both firms are assigned equal weights and included into the same factor, they will have similar potential to drive the factor.

The results presented in this paper are based on constructing factors by aggregating individual firm default probabilities weighted by total liabilities. Since liabilities represent monies owed to other parties, credit deterioration of a firm with large liabilities is much more likely to devalue the assets of other firms, and thus cause credit co-movement. Also, liabilities are usually representative of the size of operations of the firm. In some sense, the value-weighting scheme starts with the

<sup>8</sup>One alternative around this is to apply value-weighting before factor extraction.

 $^{9}$ The actual is closer to 0.4.

belief of "too-big-to-fail"  $(W)$  and the data reveals "too-interconnected-to-fail"  $(\beta)$ .

In the formation of factors, we form each country as a pre-factor via value weighting of its component firms, then apply hierarchical clustering using Ward's method to identify correlation clusters. In essence, countries that co-move more closely will be grouped together. 3 country groups are obtained via this method, and correspond to Europe, Asia, and the Americas. A number of developing countries are grouped together with the Americas, reflecting its importance. Even though this is a purely statistical method, it is interesting to note that countries are grouped up according to geographical closeness, and supports our intuition that countries located close together are more likely to co-move. Figure 1 shows the cluster dendrogram leading to the 3 groupings<sup>10</sup>. It is possible to cut the dendrogram lower but we do not find it beneficial as there would be many factors comprised of tiny economies, and unnecessarily adds multicollinearity. For example, if we cut the dendrogram lower at 1.25, we can have two additional factors "Eastern Europe" branching out from "Europe", and "Rest of the World" branching out from "Americas". However, they have correlations in the order of 0.6 with other factors<sup>11</sup>. In addition, both of them are economically too small or fragmented to be meaningful, with sizes  $1/10$  or less of the original blocs.

Table 2 summarizes the composition of the 3 factors, while Table 3 provides an analysis of the  $R<sup>2</sup>$  captured by the factor model, cross-sectional correlations, and autocorrelations.

These factors are able to capture a fair amount of variation with  $R^2$  up to 37.8% (at the 99th percentile of firms<sup>12</sup>), which results in lowering of cross-sectional correlations across the spectrum of firms. There is also marginal improvement in autocorrelation, although they are not high enough in the first place to warrant explicit modelling of individual terms<sup>13</sup>.

 $10$ Previous versions of this paper further split the country groups into financial and non-financial factors. We found that this was not necessary due to the high correlation between them.

<sup>&</sup>lt;sup>11</sup>Standard variance inflation factors have been avoided as the formal statistics are not readily available for the coefficients estimated under our model. Nonetheless, if we compute them, adding the two new factors would raise VIFs from the 1.18-1.54 range to the 1.29-2.21 range.

<sup>&</sup>lt;sup>12</sup>More precisely, the factors are fitted to each of the 21,544 firms, resulting in  $R^2$  for each of them. The percentiles refer to the distribution of these.

<sup>&</sup>lt;sup>13</sup>If desired, we could model an AR(1) process for each structural shock, but given Equation (21) they would decay rather quickly. Instead, we have leveraged on the factor model and introduced lagged structural shocks via the lagged factors (which themselves are represented as structural shocks). This has the advantage of incorporating

Figure 1: Hierarchical Cluster to Obtain Country Groupings Figure 1: Hierarchical Cluster to Obtain Country Groupings 3 country groups, obtained from 93 country pre-factors via hierarchical clustering using Ward's method, correspond 3 country groups, obtained from 93 country pre-factors via hierarchical clustering using Ward's method, correspond loosely to Europe, Asia and the Americas. loosely to Europe, Asia and the Americas.





Apart from the baseline factor setup described below, an alternative factor specification of randomly assigning countries to factors has been explored and the results are robust. Details will be covered later.

The exclusion of pure exogenous macroeconomic factors is intentional. The purpose of our analysis alludes to macro-level observations underpinned by microfoundations based on individual firm data. In fact, the low frequency components of our default probability panel extracted by filtering have a high correlation with business cycle indicators and GDP. As discussed earlier, one may wish to infer the impact on GDP by projecting it on the system. It would be inconsistent, however, to adopt the view that macroeconomic factors drive the world's credit cycles exogenously. That said, pure exogenous factors can be easily added via (11) if so desired. Furthermore, if one wishes to introduce an aggregate economic shock into this model, say monetary or technology shock, it can be effected by shocking the broad set of relevant firms.

Table 2: List of Country-Sector Factors

Summary statistics of the number and total liabilities of the firms used to construct each factor.

Country	$#$ Firms	Liabilities (USDmil)
Americas	8263	44,961,898
Asia	9948	42,750,642
Europe	3333	50,610,997
21,544 firms represented by 3 factors		

#### 3.4 Conditioning Event and Measurement

We define systemic importance as:



lagged cross-sectional shocks.

#### Table 3: Analysis of Residuals

For each firm, the residuals from the factor model are compared with a model assuming no factors, and the  $R<sup>2</sup>$  and lag-1 autocorrelations are reported. Cross-sectional correlations reported here are drawn from the correlation matrix ignoring the diagonals which by definition is 1. Figures are reported in %.

		Cross-sectional Correlation		Lag-1 Autocorrelation	
Percentile	$\boldsymbol{R}^2$	No Factors	<b>Factor Model</b>	No Factors	<b>Factor Model</b>
99	37.8	25.7	16.1	13.9	13.6
95	24.3	15.4	7.7	8.9	8.5
90	17.9	11.1	5.4	6.9	6.4
75	9.6	6.5	3.0	3.4	2.6
50	3.9	3.4	1.0	$-0.7$	$-1.5$
25	1.3	1.0	$-0.8$	$-5.6$	$-6.1$
10	0.4	$-0.8$	$-2.3$	$-11.1$	$-11.3$
5	0.2	$-1.8$	$-3.3$	$-14.7$	$-15.1$
	$-1.4$	$-3.7$	$-5.2$	$-22.5$	$-22.9$

This definition is very natural and stems directly from Basel that "The Committee is of the view that global systemic importance should be measured in terms of the impact that a bank's failure can have on the global financial system and wider economy". The impact to the system stems from the financial institution rather than the opposite direction or a confoundment of it. In other words, we place the firm in the (hypothetical) position of the perpetuator instead of the victim, and ask what impact it would cause to the global economy of firms.

One must be careful that the impact is measured with respect to *other firms* in the economy and not the financial institution which is being conditioned. As noted earlier, several papers interpret systemic importance of a financial institution as its *own* potential loss contribution, and thereby imposes the strong assumption that its *own* loss somehow translates neatly into its impact *on all* other firms in the economy.

We also point out that systemic importance as defined above is a result of a compound event. First, the financial institution falls into distress due to its own structural shocks, and second, under such a situation, we are concerned when the entire system is aversely affected. If the distressed

firm does not cause system-wide problems, the situation is benign and not problematic.

#### 3.4.1 Mathematical Definition

We now proceed to derive our definition in mathematical terms. Standing at time  $t$ , PD changes for the single period  $t + k$  of the system of firms can be found using (14) and (21):

$$
R_{t+k} = F_{t+k-1}G_1 + e_{t+k}G_0
$$
  
=  $F_t A_1^{k-1} G_1 + \sum_{j=1}^{k-1} e_{t+j} A_0 A_1^{k-1-j} G_1 + e_{t+k} G_0$  (22)

where the second line applies to  $k \geq 2$ . The cumulative PD changes over T periods is:

$$
\sum_{k=1}^{T} R_{t+k} = F_t(\sum_{j=0}^{T-1} A_1^j)G_1 + \sum_{v=1}^{T-1} e_{t+v}(G_0 + A_0(\sum_{j=0}^{T-v-1} A_1^j)G_1) + e_{t+T}G_0
$$
  

$$
=: F_t(\sum_{j=0}^{T-1} A_1^j)G_1 + \sum_{v=1}^{T-1} e_{t+v}H_v + e_{t+T}G_0
$$
 (23)

As can be seen from the summations over powers of  $A_1$ , the interactions in the system are expressed in low rank through the factors, and implied back to the firms by  $G_1^{14}$ .

We need to derive the conditional properties of  $(23)$  when a particular firm i is shocked to a distressed state over the same period. First, at time  $t$ , the expectation unconditional on any event is:

$$
\mu_t = E_t \left( \sum_{k=1}^T R_{t+k} \right) = F_t \left( \sum_{j=0}^{T-1} A_1^j \right) G_1 \tag{24}
$$

Now, when focusing only on the structural shocks  $e_{t+k,i}$  due to i, we are essentially shutting down

<sup>&</sup>lt;sup>14</sup>Recall that  $G_1 = A_1 \beta_0 + \beta_1$ .

the other structural shocks (set to zero), so the covariance matrix unconditional on any event is:

$$
\Sigma^{i} = Cov^{i} \left( \sum_{k=1}^{T} R_{t+k} \right) = \sum_{v=1}^{T-1} H_{v,i}^{T} H_{v,i} + G_{0,i}^{T} G_{0,i}
$$
\n(25)

where  $H_{v,i}$  is the *i*-th row of  $H_v$  and likewise  $G_{0,i}$  is the *i*-th row of  $G_0$ .

Furthermore, taking the conditional linear expectation given a shock to  $i$  only requires the  $i$ -th row of  $\Sigma^i$  which we denote as  $\Sigma_i^i$ . Also let  $\mu_{t,i}$  and  $\Sigma_{ii}^i$  be the mean and variance respectively of firm i itself. The expected PD changes of the system conditional on i is<sup>15</sup>:

$$
\bar{R}|i = E_t \left( \sum_{k=1}^T R_{t+k} \left| \sum_{k=1}^T R_{i,t+k} = R_{i,distress} \right| \right) = \mu_t + \Sigma_i^i (\Sigma_{ii}^i)^{-1} (R_{i,distress} - \mu_{t,i}) \tag{26}
$$

where  $R_{i,t+k}$  refers to i's PD change at time  $t + k$  and  $R_{i,distress}$  is the cumulative PD change required to shock firm  $i$  to a distressed state.

We now have the expected cumulative PD changes of the system conditional on firm  $i$  being shocked to distress, but this is not the end as we want to compute additional tail loss induced into the system conditional on this event. The use of tail loss makes sense because we are predominantly concerned when extreme events occur. In other words, even if a firm induces higher PD systemwide, it is not meaningful to assess the impact under situations where actual defaults are minimal.

Let  $\mathcal{L}(Y_t)$  represent the random variable which describes the global total uncovered loss given credit profile  $Y_t$ . We have:

$$
\mathcal{L}(Y_t) = \sum_{j=1}^p Z_j LGD_j \tag{27}
$$

$$
Z_j \sim Bernoulli(Y_{j,t}) \tag{28}
$$

where  $LGD_j$  refers to the loss given default of firm j, and  $Z_j = 1$  if firm j defaults with probability

<sup>&</sup>lt;sup>15</sup>To save time when computing  $\Sigma_i^i$ , simply pick the *i*-th entry of the left vectors  $H_{v,i}^T$  and  $G_{0,i}^T$  instead of multiplying the entire matrices. Also, the computation of  $A_1^j$ s in (23) can be done recursively.

 $Y_{j,t}$ , and  $Z_j = 0$  otherwise. It is clear from the equation that  $\mathcal{L}(Y_t)$  is a summation of the losses given default of all defaulting firms.

Following Duan and Miao (2016), default correlations are incorporated via correlated future short-term PD. Conditional on correlated future short-term PD paths, actual default events  $\{Z_i\}$ are independent. This approach bears the advantage of enabling default correlations without having to model joint actual realisations of defaults which is much more challenging.

Finally, writing the definition of systemic importance in mathematical terms, we have:

$$
S_i = \mathcal{L}^q \left( Y_t + \bar{R} | i \right) - \mathcal{L}^q (Y_t) \tag{29}
$$

where  $\mathcal{L}^q(Y_t)$  represents  $\mathcal{L}(Y_t)$  at the q-th percentile.

 $\mathcal{S}_i$  is basically the additional tail loss induced into the system conditional on firm i being shocked to distress. There is some loose connection here with ∆CoVaR which prescribes systemic risk as the difference between the system-wide loss conditional on a firm being in distress and under the firm's median state. The important distinction is that our conditioning event bears the interpretation of the firm's own structural shock rather than an unidentified shock.

In our implementation, we use 1-year forward PD, which is the most commonly referenced term and apt given that FSB reviews its list of global systemically important banks on an annual basis. Calibration is done using data with daily frequency. We define the distressed state to be a credit deterioration shock of 3 standard deviations<sup>16</sup> to the 1-year PD over the time horizon of 1 month or  $T = 22$  days. The tail level for systemic importance is set to  $q = 99\%$ . Loss given default assumes a recovery rate of  $40\%^{17}$  on liabilities. Since only the negative impacts to the system are of interest, firms receiving a decrease in PD (i.e. credit improvement) are zeroed. Firm i's own impact is also zeroed since we are interested in the loss to other firms rather than self-impact. All banks are assessed for systemic importance with the base date t as of 31 Dec 2013.

<sup>&</sup>lt;sup>16</sup>The use of a quantile is similar to a number of other measures including  $\Delta$ CoVaR.

<sup>&</sup>lt;sup>17</sup>An alternative is to apply a firesales factor to the assets and assess the uncovered liabilities.

A direct simulation of  $\mathcal{L}^q(Y_t)$  is extremely taxing and computationally inefficient. Several alternatives are possible. Gordy (2002) advocates the use of saddlepoint approximation to accurately estimate the tail of credit risk profiles, while Duan and Miao (2016) provide an alternative algorithm for a similar problem. We opt to use saddlepoint approximation, and adapt Gordy (2002) to our setup, the details of which are provided in the appendix.

#### 3.5 Sparse Interactions

In practice, factors will remove most of the commonality but it is not reasonable to expect 3 factors to completely capture the variation in 21,544 firms (see Table 3). In particular, the largest firms are likely to drive parts of the economy due to the sheer size of their operations and market importance, and may even operate as their own factors. For each of these extremely large firms, its corresponding row in  $\gamma$  is allowed to contain sparse entries, meaning that when the said firm receives a structural shock, it not only impact other firms through the factors, but also directly through the non-zero entries of  $\gamma$ .

To give context, some of these influential corporate names which may drive their own supply chains include Walmart, Exxon Mobil, General Electric, and of course, the largest banks which include ICBC, HSBC, and BNP Paribas to name a few. The top 1% of firms by size are listed in the appendix.

As there are over 200 large firms, a few assumptions need to be made to make the model tractable. First, large firms are allowed to drive other firms, but not vice versa. In practice, one may think of a Walmart distress as causing stress to its suppliers, but a disruption of one of its many suppliers will not cause a huge problem for Walmart. Second, we assume that large firms do not directly feed their structural shocks into each other so that it does not introduce an additional identification subproblem.

Note that  $R_t = F_t \beta_0 \gamma^{-1} + F_{t-1} \beta_1 \gamma^{-1} + e_t D \gamma^{-1}$ . Off-diagonal entries in  $\gamma^{-1}$  should correspond to the off-diagonal entries in  $\gamma$ . To see this, the submatrices of  $\gamma$  corresponding to large firms and to small firms are both diagonal given our assumptions above. The submatrix corresponding to large firms' impact to small firms is non-zero, while the submatrix corresponding to small firms' impact to large firms is zero. Hence, given the block matrix inversion identity, the locations of the non-zero entries in  $\gamma$  and  $\gamma^{-1}$  are the same.

To identify the relevant sparse entries, a few alternatives are available. For a small scale problem, one may wish to run a variable selection algorithm such as the SCAD (Smoothly Clipped Absolute Deviation) algorithm provided by Fan and Li (2001) or the MC+ algorithm provided by Zhang (2010). There is also some relationship with spatial autoregressive models, in that the spatial weights matrix also introduces cross-dependency between dependent variables. However, we have 21,544 firms with potentially over 200 "explanatory" firms which makes variable selection for all firms collectively extremely time consuming. We can instead estimate the factor model and then apply a thresholding estimator to the residual correlation matrix, with the residuals being obtained via the plug-in method. The thresholding procedure in relation to factor models has been explored in Fan et al (2011) for observable factors, and extended in Fan et al (2013) for unobservable factors.

Recall that our implementation involves fixed-point iteration, so potentially, some of the nonzero entries in  $\gamma$  could change at every iteration and cause instability in our fixed-point algorithm. For practical purposes, the locations of the non-zero entries in  $\gamma$  are pre-identified by running a standard OLS of firm PD changes against the factors, and then thresholding the residual correlation matrix. However, the actual values of the non-zero entries in  $\gamma$  are estimated jointly with  $\beta$  with the necessary simultaneity corrections at every iteration.

A variety of thresholding techniques exist, including the traditionally used hard or soft thresholding. We elect to adopt the SCAD thresholding method, which has been shown to bear the oracle property, meaning that when the true parameters have some zero components, they are estimated as 0 with probability tending to 1, and the non-zero components are estimated as well as when the correct submodel is known. The procedure for SCAD thresholding of the residual correlation matrix is described in the appendix, and has been used, for example, in Duan and Miao (2016)

which we largely follow with some adaptation. For theoretical properties pertaining to thresholding of covariance matrices, we refer readers to Bickel and Levina (2008) and Rothman (2009).

#### 3.6 Data Treatment

As mentioned, we use series with at least 50% of observations in the period from Jan 2004 to Dec 2013, and existing balance sheet data as of 31 Dec 2013. It is necessary to fill missing PD data as factors need to be formed using all firms. We do so by linear interpolation if the missing segment is between two points with available data. If data does not exist at the start or end of a particular time series, it is forward-filled or back-filled respectively. An alternative method to fill data with fitted values from an underlying model (for example a PCA factor model). In our case, a model has already been assumed and needs to be estimated with simultaneity adjustments, which does not lend itself easily to the model-filling method. Another alternative would be to drop even more firms with missing data. However, this would exclude many of the smaller firms with shorter history. Even larger firms have missing data from time to time due to, for example, non-trading days.

Since Box-Muller transformations suggest that the 1-year PD of the majority of the firms require log-transformation, it is applied to all firms. We include a constant in addition to the 3 factors for completeness<sup>18</sup>. To reduce the impact of possibly spurious outliers, the  $log-PD$  changes are winsorized at the 1% level.

### 4 Results

In this section, we present the banks which have been identified as global systemically important<sup>19</sup>. The systemic importance measure for the top 30 global systemically important banks are reported in Table 4. Note that the country column indicates the main factor which the bank feeds its

<sup>&</sup>lt;sup>18</sup>That said, the mean of daily log-PD changes is negligible. The average daily mean has a magnitude of  $10^{-4}$ compared to a much higher standard deviation of  $5 \times 10^{-2}$ .

<sup>&</sup>lt;sup>19</sup>The computation has benefited from the PROPACK package in Matlab, and the stargazer and matchit packages in R.

structural shocks into by definition of W. However, through our identification strategy and taking into account simultaneity, the reality is that each bank feeds its structural shock into all factors via  $A_0$ .

All the top banks are convincing candidates by nature of their size, or "too-big-to-fail", and this is natural given our identification strategy of prescribing  $W$  as value-weighting by total liabilities. The nature of the restrictions means that small banks do not have the capacity to drive the system as much as large banks, and would tend to have lower systemic importance. This is evident from the scatter plot in Figure 2 where there is a general trend that larger banks tend to be more systemically important. However, the same plot clearly shows that *size is not everything*, since systemic importance does not line up perfectly with the size variable. On the contrary, there exists a number of huge banks with low systemic importance, and mid-sized banks which punch above their weight. Incorporating the full dynamics of our system, "too-interconnected-to-fail" plays a very important role. In a later section, we shall relate the systemic importance measure to firm-specific characteristics.

Overall, the results suggest that two important considerations – "too-big-to-fail" and "toointerconnected-to-fail" – are incorporated into the model. Size is a precursor which, together with interconnectedness, determines systemic importance.

### 5 Validation

This section presents a number of exercises which were conducted to validate the measurement of systemic importance, as the identification approach appears to be new and its applicability to systemic importance measurement is untested. First, bailout events from the US correspond with the model-implied firm structural shocks, more so than with actual PD changes. This suggests that the model has at least some success in removing the systematic components of PD changes and identifying the firm structural shocks. Second, we relate sparse entries in the residual correlation matrix to actual supply chains driven by two different large US firms, and show that SCAD thresh-

This table lists the 30 most systemically important banks globally according to our methodology. The systemic importance measure is the additional tail loss induced into the global system of firms conditional on the bank being shocked to distress due to its own structural shock. Figures in USD mil.



#### Figure 2: Size Is Not Everything

A scatter plot of systemic importance against total liabilities for each bank. Size, or "too-bigto-fail" is a precursor to being systemically important by nature of the restrictions and, together with "too-interconnected-to-fail" revealed by the data, determines systemic importance.



olding succeeds in discriminating related and unrelated firms. Third, we show that the results are robust to the choice of factors by investigating an alternative set of factors, formed by randomly assigning countries to factors. Forth, we show why the selection of "too-big-to-fail" restrictions (W) is sensible by comparing it to an unrestricted model which has reverse causality issues. Fifth, for cross-validation, calibration to the pre-crisis period produces results which are close to the full sample calibration. Sixth, the predictive power of our model using simultaneity adjusted  $\beta$  is comparable to the model using OLS, or at most marginally weaker. Lastly, we make a short comment on the numerical accuracy of our scheme.

#### 5.1 Relationship between Bailout Events and Firm Structural Shocks

Recall that the firm structural shocks are extracted by removing systematic components, while accounting for the inherent simultaneity. In this section, we assess if the model-implied firm structural shocks are meaningful by comparing them against bailout events from the US in 2008-2009. It will be shown that they have a significant relationship, and the relationship is better than the actual PD changes, suggesting that the model has some success in removing the systematic components of PD changes and identifying the firm structural shocks.

We focus on the Capital Purchase Program (CPP) and the Targeted Investment Program (TIP) where the US Treasury's Office of Financial Stability purchased preferred stock and equity warrants of individual named banks as part of the Troubled Asset Relief Program (TARP). The total commitments under CPP and TIP were USD 205 bn and USD 40 bn respectively, which together accounted for approximately 3/4 of TARP funds directed towards financial institutions. The remaining USD 70 bn was directed to AIG which is outside the scope of analysis since, as mentioned, the focus here is on banks which the Basel framework is concerned with. Other TARP funds which are not considered here include those directed towards the automotive industry (USD 81 bn) and homeowner foreclosure assistance (USD 39 bn). Also not considered are events of general liquidity provision and lowering of interest rates as these pertain more to industry-wide systematic effects rather than firm-specific events.

The consolidated list of bailouts was sourced from ProPublica, which contains 709 named banks which received aid under CPP, and 2 named banks under TIP. For each bank, the amount of capital committed and date entered are available. Of the 278 listed US banks in our system, we were able to identify 181 bailed out banks. With Bank of America and Citigroup appearing in both CPP and TIP, we have 183 observations.

The strategy here involves matching each bank which received aid to similar banks elsewhere which did not receive aid, and checking if the bailout produced a significant negative effect on PD level. A negative relation is only natural since more capital becomes available to meet liabilities, thereby directly improving the credit quality of the bank. In technical terms, distance to default increases and this is imputed as a negative effect into the PD.

The control group was formed from banks worldwide, but it was necessary to exclude<sup>20</sup> EU countries (including UK), US and Canada as there was a wave of bailouts in Oct 2008 led by the UK and followed by many European countries and the US. There were also reports of Canadian banks receiving secret bailouts<sup>21</sup>. These events might have confounded our results. In practice, this means that Japanese, BRICS, Middle-Eastern and other Asian banks were matched to the US banks. Although not ideal, the expected bias would be against our results as the banks in these countries were less affected by the sub-prime crisis and presumably had less of an upward PD trajectory, and we are looking for a negative coefficient on the bailout banks relative to these banks. The pool of daily observations from 28 Oct 2008 to 23 Oct 2009 corresponding to the first and last bailout dates was used.

For the purpose of matching, we apply nearest neighbour matching based on Mahalanobis distance along the dimensions of total liabilities (size), PD (likelihood of default) and 3-month firm

<sup>&</sup>lt;sup>20</sup>The alternative would have been to include all bailouts in the world, including EU and Canada, but an open and comprehensive list was not readily available. The major European bank bailouts can be found easily, but there is a lack of detailed data on smaller individual bank bailouts.

<sup>&</sup>lt;sup>21</sup>For example, see "The Big Banks' Big Secret: Estimating government support for Canadian banks during the financial crisis" published by Canadian Centre for Policy Alternatives.

PD trend (severity of short-term deterioration), reflecting what we think are motivations for the government to bailout a bank. Mahalanobis distance produced better balance of the covariates compared to logit propensity score. That said, the conclusions survive in both cases. We pick a matching ratio of 5 control units to 1 treatment unit to increase the power of the test. Figure 3 shows good balance between the treatment and control samples in the form of QQ plots of the covariates.

The results provided in Table 5 show that bailout events do not have a significant relationship with raw changes in firm PD. However, when our model removes the systematic components and extracts the firm-specific component, the relationship becomes significant, and the coefficient is negative as expected. The conclusion here supports the use of firm structural shocks,  $e_t$ , produced by our model. The results reported are based on [−1, 1] changes but are also valid for day-0 changes or [-2,2]. Both the firm structural shocks and the PD changes have been standardised according to their volatilities.

#### 5.2 Analysis of Sparsity in  $\gamma$

We had allowed for sparse entries in  $\gamma$  on the pretext that large pivotal firms may drive parts of the economy, over and above the factors channel. Using two separate examples of large US firms, Walmart and Ford Motor, we show that this is not merely a theoretical exercise but could reflect the actual supply chain relationships which each of these large firms may have with the wider community.

For this exercise, we use COMPUSTAT Segments Data which provides customer data for over 70% of the companies in the US database. We use Walmart because it has the most extensive supply chain in this dataset. After matching with our PD database, we have 101 related firms which report Walmart as an important customer. The intuition underlying our approach is that if Walmart's credit quality deteriorates, it could affect its ability to make payments to its creditors, in this case those who have reported Walmart as an important customer, and consequently, spillover

Figure 3: Balance Analysis of Matched Sample

QQ plots of treated and control groups, pre- and post- matching, using nearest neighbour matching based on Mahalanobis distance along the dimensions of total liabilities (size), PD (likelihood of default) and 3-month firm PD trend (severity of short-term deterioration).



#### Table 5: Relation between Bailouts and Firm Structural Shocks

The objective of this table is to show the relationship between bailouts and our model-implied firm structural shocks  $e_t$ . Regressions were based on a sample of 183 US bank bailout events matched with 915 control banks.  $\Delta ln(PD)_t$  refers to the actual change in firm PD, while  $e_t$  refers to the model-implied firm structural shock extracted from  $\Delta ln(PD)_t$  by removing the systematic components using our algorithm which accounts for simultaneity.

	Dependent variable			
	$\Delta ln(PD)_t$	$e_t$	$\Delta ln(PD)_t$	$e_t$
	(1)	$\left( 2\right)$	(3)	(4)
Bailout=1	0.082 (0.230)	$-0.439**$ (0.224)	0.074 (0.227)	$-0.445^{\ast\ast}$ (0.223)
ln(Liabilities)			$-0.222***$	$-0.083**$
ln(PD)			(0.043) $0.195**$	(0.042) $0.167**$
			(0.079)	(0.077)
Short-Term			$-0.010$	0.041
Deterioration			(0.129)	(0.126)
Constant	$0.418***$	$0.660***$	$3.601***$	$2.478***$
	(0.094)	(0.091)	(0.641)	(0.630)
Observations	1,098	1,098	1,098	1,098
$\mathbf{R}^2$	0.0001	0.004	0.032	0.012

<sup>∗</sup>p<0.1; ∗∗p<0.05; ∗∗∗p<0.01

the credit impact to them. Likewise, we also analyse Ford Motor which is the second most reported customer with 32 related firms after matching. There is a limit as to how many large firms we can study because customers are reported in text format without unique identifiers. Furthermore, many of the top firms in terms of size are domiciled outside of the US.

For each of the said related firms, we match it with control firms which do not report Walmart (or Ford Motor as the case may be) as an important customer, are also domiciled in the US, in the same industrial sector, and close in size. We use a match ratio of up to 5 firms (this can be relaxed), provided that the respective subsets have the number. Then, we check if the firms identified using the SCAD thresholding procedure is indicative of having an actual relationship.

As presented in Table 6, the SCAD thresholding procedure demonstrated effectiveness in discriminating between firms reporting Walmart (or Ford Motor) as a key customer and unrelated firms, even though it was implemented in a purely statistical sense without knowledge of the actual supply chain. We note that the COMPUSTAT data is based on companies' self-reporting and are truncated below a certain level of activity. However, under-reporting in this case would bias against our results, but we still find significant relationships.



Table 6: SCAD thresholding and firms reporting Walmart/Ford Motor as an important customer

#### 5.3 Alternative Selection of Factors

One might think that our results are largely driven by the selection of factors. As it turns out, the results are generally similar if we change the factor specification, provided that the notion of "too-big-to-fail" is preserved. In other words, as long as we recognise that larger firms have more capacity to drive the system, there is some flexibility as to which countries we group together, and how we compose the factors from firms. We speculate that there are two reasons for this. First, we allow interconnectedness to not only be expressed through the factors, but also through the sparse entries in  $\gamma$ , and this affords some flexibility. Second, there is inherent positive correlation between countries, so assigning a particular country to another factor does not completely alter the setup, compared to a scenario where all countries are uncorrelated.

In the alternative specification, instead of using correlation clusters and obtaining the 3 country groups, we randomly assign each country to 1 of 3 similarly sized groups, each representing a factor. As before, each firm is value-weighted by the size of its liabilities when composing the factors.

The computed systemic importance numbers are extremely close to the baseline case, and the correlation between the alternative and the baseline is 86.4%, supporting the conclusion that our results are robust to the choice of factors selected. The scatter plot is provided in Figure 4.

#### 5.4 Comparison with Unrestricted Model

Our specification of W prescribes the channels through which firms are allowed to express their impact on the system, and it was mentioned that they are essentially "too-big-to-fail" restrictions, where bigger banks have more capacity to affect the system. As shown, the data reveals "toointerconnected-to-fail" and both considerations together determine the systemic importance of the financial firm, and there instances of huge but not systemically important banks.

Without these restrictions, the direction of structural causality would be unclear and there would be numerous instances of reverse causality. To illustrate their importance, consider the case of a pure correlation model. Figure 5 shows that many small banks would be wrongly assigned a



This scatter plot compares the systemic importance computed using factors obtained via random assignment to 3 similarly sized country factors as an alternative specification, to the baseline case.



high systemic importance by a pure correlation model, implying that they have the same capacity as banks which are several magnitudes larger in size to perpetuate a crisis. The more likely situation is that these small banks are highly dependent of the rest of the firms, and the high "systemic importance" number simply indicates that they are severely affected when the economy is doing poorly, rather than them perpetuating a crisis. To push the point, small banks default from time to time, and we do not see systemic events. Clearly, a pure correlation model is inappropriate for imposing regulatory risk charges.

Figure 5: Unrestricted / Pure Correlation Model - Systemic Importance and Size

The unrestricted model fails to rule out reverse causality and wrongly assigns high systemic importance to small banks, implying that they have the same capacity as banks which are several magnitudes larger in size to perpetuate a crisis.



#### 5.5 Cross-Validation

Our model is able to learn from the relationships between the factors and firms even when the calibration is restricted to the pre-crisis period. Figure 6 shows that systemic importance measured using the reduced pre-crisis sample correlates closely with that using the full sample. The full period, as stated before, is from Jan 2004 to Dec 2013, while the reduced sample uses the period Jan 2004 to Feb 2008, right before the collapse of Bear Stearns and prior to the spikes in default probabilities. The correlation between the systemic importance values of the reduced and full samples is 82.0%, suggesting that the parameters are relatively stable.

#### Figure 6: Cross-Validation

This scatter plot compares the systemic importance using the parameters calibrated to the pre-crisis period from Jan 2004 to Feb 2008, to that of the full sample from Jan 2004 to Dec 2013.



#### 5.6 Predictive Power of Model using Simultaneity Adjusted  $\beta$

Given a sensible  $W$ , our model must be able to predict PD changes of the system well, and this forms another prong of the validation. By prediction, we mean that given the right-hand side actual changes in factors, the left-hand-side expected firm PD changes are a good indication of the actual firm PD changes. Essentially, we are validating (11).

1-year PD changes of the 3 factors are used to predict the PD changes of 21,544 firms over the same period, and checked against the actual PD changes. Both Bear Stearns and Lehman collapsed in 2008, which makes the 1-year PD changes of 2008 a suitable candidate for the check. Using the simultaneity adjusted  $\beta$  provides a cross-sectional  $R^2$  of 20.4%. This is only moderate degradation from using OLS which gives 25.6%.

#### 5.7 Numerical Accuracy

We make a short comment on taking the inverse  $(I - \beta_0 \gamma^{-1} W)^{-1}$  in Equation (13). Recall that in the OLS case,  $\beta_0 \gamma^{-1} W = I$  which prevents the identification. Although the simultaneity adjusted β is quite close to the OLS β, it does not appear that numerical accuracy is compromised. The condition number of  $(I - \beta_0 \gamma^{-1} W)$  at the final iteration is 2.27 = 10<sup>0.356</sup>, indicating minimal loss of precision. Also, instead of evaluating the standalone inverse, we solve for  $W(I - \beta_0 \gamma^{-1} W)^{-1}$ .

#### 6 Other Studies

#### 6.1 Relating Systemic Importance to Firm Characteristics

One may ask, other than size, what is systemic importance related to in the cross-section? For example, in Adrian and Brunnermeier (2011), the authors related ∆CoVaR to firm characteristics in order to model forward-looking ∆CoVaR. We reference the variables used in their study but augment it with additional variables which may be more relevant or insightful.

The first characteristic which they use is leverage based on book values. For our study, we use

book leverage but also include market leverage which is computed from implied market value of  $assets<sup>22</sup>$  and market value of equity. The second characteristic is maturity mismatch which they define as the difference between short-term debt and cash, divided by total liabilities. There is a problem with this definition for financial firms, because the largest source of liabilities for a bank is usually deposits which do not fall under short-term debt. We instead use the ratio of market value of assets to adjusted liabilities, which we term as asset adequacy. Adjusted liabilities essentially refer to the combination of short-term liabilities, half of long-term liabilities, and an appropriate fraction of other liabilities (deposits), and commonly referred to as the default point in credit analysis. This ratio better reflects the adequacy of a bank's assets to meet its obligations. Since it is readily available, we also include the volatility of assets. The third variable is market-to-book ratio which we also use. The fourth is size, measured as the book value of equity. In our context, size restrictions have been imposed using total liabilities, since they represent obligations to other parties and would impact them in the event of a default occur. The fifth and sixth are equity return volatility and equity market beta. We have already laid out the advantages of working with credit risk rather than equity risk. In the context of our model, we use the sum of non-zero coefficients in  $G_0$ , which captures the joint effects of factor sensitivity and volatility. As many of these variables seem to be alluding to the ability of the bank to meet its obligations, we also use distance-to-default (DTD) in one of the specifications.

As established before and shown in the first regression of Table 7, systemic importance is closely related to size. If we remove the size effect, the remaining variation is related to the ability of the bank to meet its obligations, as shown in the second regression. More assets relative to liabilities, and lower asset volatility reduce systemic importance. Market leverage, rather than book leverage, is mildly significant. In the third regression, DTD stands out as the most relevant firm characteristic. One possible interpretation is that banks closer to default are more likely to affect other firms and hence more systemically important. In other words, firms respond less to the

 $^{22}$ This is computed as part of the PD estimation procedure and accounts for the deposit-taking activities of banks. For details on market value of assets and adjusted liabilities, please refer to Duan and Wang (2012).

deterioration of a bank which is far from default. The weaker effects in the 2nd stage regression may be due to size being correlated with other firm characteristics listed. For example, size in some sense also captures the number of counterparties and extent of operations.

#### 6.2 Mimicking FSB's list of G-SIBs

One question that can be asked is how well our results line up with FSB's list of G-SIBs which uses Basel's methodology. Banks considered to be of global systemic importance are classified into tiers ranging from 1 to 5. Depending on the tier, higher loss absorbency requirements of between 1.0% and 3.5% will apply, with the higher tiers attracting more stringent requirements. Briefly, Basel prescribes an indicator-based approach which combines 5 indicators – cross-jurisdictional activity, size, interconnectedness, substitutability, and complexity. A summary extracted from the Basel document is presented in Figure 7.

However, upon closer inspection, it turns out that almost all of the indicators are essentially size measures, be it assets or liabilities. Even the interconnectedness indicator is in fact a size measure, granted it is a subset. It is not surprising therefore, that the list of G-SIBs by FSB is the list of biggest banks by size, other than a few exceptions. It certainly incorporates the notion of "too-big-to-fail". The exception is substitutability which seems to be less related to size. Our model does not incorporate this indicator, but nothing stops us from adjusting our final measure with this consideration.

What might improve the measure is to actually assess how the bank would impact its external environment, i.e. others firms, instead of relying solely on the bank's own size to proxy all indicators. Here, our measure has the advantage of incorporating "too-interconnected-to-fail" by measuring  $\beta_0$ ,  $G_0$  and  $G_1$ .

In any case, to illustrate, we try to mimic FSB's list in our framework, making only a few simple adjustments in the process. The intention is to show that FSB's list is first and foremost a size measure, and a matter of amplifying the impact of certain countries while downplaying other



### Table 7: Systemic Importance and Firm Characteristics

### Figure 7: Basel's Indicator-Based Approach



## Indicator-based measurement approach

countries. The adjustments are as follows. First, instead of having 3 country factors, we have a single world factor, through which all interactions take place. This limits interconnectedness as there is now only one channel. As before, we use value-weighting but now set to zero the weights of countries whose banks are not in the G-SIB list. For example, Russian, Indian, and even Canadian banks are excluded from forming the factor. Next, we emphasize certain countries over others by artificially inflating or deflating the systemic importance measure. US banks are adjusted upwards to 120%, while the European countries, UK, France, Germany, and Switzerland are adjusted upwards to 200%, 150%, 180%, and 200% respectively. This means that the Asian countries, such as Japan and China, are left unadjusted. Some of these adjustments might simply reflect the varying degrees to which Basel has been implemented in these countries, with some taking leadership and others playing it slow, or perhaps the difference in political impetus to embrace post-crisis developments.

Figure 8 shows the relationship between our adjusted systemic importance rankings and the tier assigned by FSB. Each point in the figure represents a bank. A more systemically important bank should presumably be assigned to a higher FSB tier, and a higher rank (lower number, i.e. rank 1 is highest). The figure demonstrates this relationship correctly, meaning that our method with the above adjustments to limit interconnectedness and emphasize certain countries over others can reproduce FSB's list approximately. Without the said adjustments, the spread is wider and it becomes harder to distinguish between the FSB tiers.

The exercise is just to demonstrate the point; with enough fine tuning, it should be possible to obtain a closer and more exact list.

### 7 Conclusion

In this paper, we measured systemic importance of a financial institution as the additional tail loss induced into the system when the financial institution falls into distress due to its *own structural* shocks. Using only public data, we modelled over 21,000 firms listed globally jointly as a system,

Figure 8: Systemic Importance Ranking vs FSB Tier

Comparison of systemic importance ranking, using our methodology with adjustments to limit interconnectedness, amplify the effects of certain countries, and downplay others, and the actual tier which FSB assigns these banks to. A higher tier is associated with a higher rank (lower number).



while relying on "too-big-to-fail" restrictions to provide structural identification. Together with "too-interconnected-to-fail", the two key considerations of systemic importance are incorporated.

Using data from the CPP and TIP under TARP, we found that bailout events were more closely related to the firm structural shocks which we extracted, than to actual raw PD changes, suggesting that our model has some success in removing the systematic components of PD changes and identifying firm structural shocks. We also analysed firms that reported Walmart as a major customer, and found that the SCAD thresholding which we applied was effective in discriminating between firms that rely on Walmart and firms that do not, supporting our use of the sparse matrix. A similar finding was obtained for Ford Motor. Additional validation checks, including random selection of factors, and cross-validation using the pre-crisis sample, provided favourable results. In addition, we showed that Basel's list of global systemically important banks can be interpreted in our framework as limiting interconnectedness and inflating the impact of US and European banks, while reducing the impact of Asian banks.

Overall, our approach to measuring systemic importance bears the advantages of exploiting structural analysis to attribute systemic importance to individual banks, and while using only public PD data, can be corroborated with actual firm level events.

### References

- [1] Acharya, V., Engle, R., & Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulating systemic risks. The American Economic Review, 102(3), 59-64.
- [2] Acharya, V. V., Pedersen, L. H., Philippon, T., & Richardson, M. P. (2010). Measuring systemic risk.
- [3] Adrian, T., & Brunnermeier, M. K. (2011). CoVaR (No. w17454). National Bureau of Economic Research.
- [4] Alter, A., & Beyer, A. (2014). The dynamics of spillover effects during the European sovereign debt turmoil. Journal of Banking & Finance, 42, 134-153.
- [5] Anand, K., Gai, P., Kapadia, S., Brennan, S., & Willison, M. (2013). A network model of financial system resilience. Journal of Economic Behavior & Organization, 85, 219-235.
- [6] Bai, J., & Wang, P. (2016). Econometric analysis of large factor models. Annual Review of Economics, 8, 53-80.
- [7] Bickel, P. J., & Levina, E. (2008). Covariance regularization by thresholding. The Annals of Statistics, 2577-2604.
- [8] Billio, M., Getmansky, M., Lo, A. W., & Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics, 104(3), 535-559.
- [9] Breheny, P., & Huang, J. (2011). Coordinate descent algorithms for nonconvex penalized regression, with applications to biological feature selection. The annals of applied statistics, 5(1), 232.
- [10] Brownlees, C. T., & Engle, R. F. (2012). Volatility, correlation and tails for systemic risk measurement. Available at SSRN 1611229.
- [11] Canedo, J. M. D., & Jaramillo, S. M. (2009). A network model of systemic risk: stress testing the banking system. Intelligent systems in accounting, finance and management, 16(12), 87- 110.
- [12] Chan-Lau, J. A., Chuang, C., Duan, J. C., & Sun, W. (2016). Banking Network and Systemic Risk via Forward-Looking Partial Default Correlations. working paper, International Monetary Fund and National University of Singapore.
- [13] Chen, N., Liu, X., & Yao, D. D. (2016). An optimization view of financial systemic risk modeling: Network effect and market liquidity effect. Operations research, 64(5), 1089-1108.
- [14] Diebold, F. X., & Ylmaz, K. (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms. Journal of Econometrics, 182(1), 119-134.
- [15] Duan, J. C., & Fulop, A. (2013). Multiperiod Corporate Default Prediction with the Partially-Conditioned Forward Intensity. Available at SSRN 2151174.
- [16] Duan, J. C., & Miao, W. (2016). Default Correlations and Large-Portfolio Credit Analysis. Journal of Business & Economic Statistics, 34(4), 536-546.
- [17] Duan, J. C., Sun, J., & Wang, T. (2012). Multiperiod corporate default predictionA forward intensity approach. Journal of Econometrics, 170(1), 191-209.
- [18] Duan, J. C., & Wang, T. (2012). Measuring distance-to-default for financial and non-financial firms. Global Credit Review, 2(01), 95-108.
- [19] Duan, J. C., & Zhang, C. (2013). Cascading Defaults and Systemic Risk of a Banking Network. Available at SSRN 2278168.
- [20] Dungey, M., Luciani, M., & Veredas, D. (2012). Ranking systemically important financial institutions (No. 12-115/IV/DSF44). Tinbergen Institute.
- [21] Eisenberg, L., & Noe, T. H. (2001). Systemic risk in financial systems. Management Science, 47(2), 236-249.
- [22] Fan, J., & Li, R. (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. Journal of the American statistical Association, 96(456), 1348-1360.
- [23] Fan, J., Liao, Y., & Mincheva, M. (2011). High dimensional covariance matrix estimation in approximate factor models. Annals of statistics, 39(6), 3320.
- [24] Fan, J., Liao, Y., & Mincheva, M. (2013). Large covariance estimation by thresholding principal orthogonal complements. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 75(4), 603-680.
- [25] Geweke, J. (1977). The dynamic factor analysis of economic time series. Latent variables in socio-economic models, 1.
- [26] Giudici, P., & Parisi, L. (2016). CoRisk: measuring systemic risk through default probability contagion (No. 116). University of Pavia, Department of Economics and Management.
- [27] Gordy, M. B. (2002). Saddlepoint approximation of CreditRisk+. Journal of banking & finance, 26(7), 1335-1353.
- [28] Gourieroux, C., Ham, J. C., & Monfort, A. (2012). Bilateral exposures and systemic solvency risk. Canadian Journal of Economics/Revue canadienne d'conomique, 45(4), 1273-1309.
- [29] Han, H., Linton, O., Oka, T., & Whang, Y. J. (2016). The cross-quantilogram: Measuring quantile dependence and testing directional predictability between time series. Journal of Econometrics, 193(1), 251-270.
- [30] Hautsch, N., Schaumburg, J., & Schienle, M. (2014). Financial network systemic risk contributions. Review of Finance, rfu010.
- [31] Huang, X., Zhou, H., & Zhu, H. (2012). Systemic risk contributions. Journal of financial services research, 42(1-2), 55-83.
- [32] Nier, E., Yang, J., Yorulmazer, T., & Alentorn, A. (2007). Network models and financial stability. Journal of Economic Dynamics and Control, 31(6), 2033-2060.
- [33] Okhrin, O., Ristig, A., Sheen, J. R., & Trck, S. (2015). Conditional Systemic Risk with Penalized Copula (No. 2015-038). SFB 649 Discussion Paper.
- [34] Pesaran, H. H., & Shin, Y. (1998). Generalized impulse response analysis in linear multivariate models. Economics letters, 58(1), 17-29.
- [35] ProPublica. The Bailout: Initiatives and Programs. https://projects.propublica.org/bailout [Accessed 18 Apr 2016].
- [36] Rothman, A. J., Levina, E., & Zhu, J. (2009). Generalized thresholding of large covariance matrices. Journal of the American Statistical Association, 104(485), 177-186.
- [37] Sargent, T. J., & Sims, C. A. (1977). Business cycle modeling without pretending to have too much a priori economic theory. New methods in business cycle research, 1, 145-168.
- [38] Stock, J. H., & Watson, M. W. (2005). Implications of dynamic factor models for VAR analysis (No. w11467). National Bureau of Economic Research.
- [39] Stock, J. H., & Watson, M. W. (2011). Dynamic factor models. Oxford Handbook of Economic Forecasting, 1, 35-59.
- [40] RMI staff. (2014). NUS-RMI Credit Research Initiative Technical ReportVersion: 2014 Update 1. Global Credit Review, 4(01), 117-202.
- [41] White, H., Kim, T. H., & Manganelli, S. (2015). VAR for VaR: Measuring tail dependence using multivariate regression quantiles. Journal of Econometrics, 187(1), 169-188.

[42] Zhang, C. H. (2010). Nearly unbiased variable selection under minimax concave penalty. The Annals of statistics, 38(2), 894-942.

### Appendix A: Fixed Point Iteration to Resolve Simultaneity Bias

- 1. Begin with arbitrary starting values for the simultaneity adjustments  $\left[\frac{A_{0,i}}{G_{0,ii}}\right]^0$  and  $\left[\frac{G_{0,i,-i}}{G_{0,ii}}\right]^0$ . For our implementation, we choose the initial values to correspond to the case where  $\beta =$  $0.9\beta_{OLS}$  and  $\gamma = I$ .
- 2. For iteration  $m = 1, 2, 3, \dots$  until convergence:
	- (a) Compute  $A_0^m$  and  $G_0^m$  according to (13) and (14).
	- (b) For each firm:
		- i. Compute the adjusted factors  $F_{t,i}^{adj,m}$  and  $R_{t,-i}^{adj,m}$  $_{t,-i}^{aq,m}$  according to (15) and (16).
		- ii. Run the regression (17) for each firm and obtain the estimates for  $\beta_{(i)}^{new}$  and  $\gamma_{(i)}^{new}$ via (18) to (20).

(c) Set the 
$$
\beta_{(i)}^m = d\beta_{(i)}^{new} + (1-d)\beta_{(i)}^{m-1}
$$
 and  $\gamma_{(i)}^m = d\gamma_{(i)}^{new} + (1-d)\gamma_{(i)}^{m-1}$ .

(d) Perform SCAD thresholding on  $\gamma^m$ .

d is a weight to reduce oscillation of the solution, which is set to 0.5 for our implementation. For regularity,  $\beta_0$  and  $\beta_1$  are constrained to be within  $\pm 1$  of the OLS regression, and off-diagonal entries of  $\gamma$  are not allowed to be collectively greater than 0.5 in magnitude. Other than  $F_{t,i}^{adj,m}$  and  $R_{t,-i}^{adj,m}$  $_{t,-i}^{a a j,m},$ the RHS variables of the regression consist of lagged factors  $F_{t-1}$  which are not simultaneous with the contemporaneous shock and the inverse  $(F_{t-1}^T F_{t-1})^{-1}$  can be computed once and stored for use at every iteration. Although we are unable to show theoretically that convergence is guaranteed, it has been possible to obtain convergence empirically for different factors and subsamples.

### Appendix B: Computation of Tail Loss

Our methodology calls for the computation of tail percentile for the global total uncovered loss. To recall, we need to compute  $\mathcal{L}^q(Y_t)$ , the q-th percentile for the loss of the system in (27). We apply a

saddlepoint approximation very similar to Gordy (2002). First, note that the cumulant generating function for  $\mathcal{L}(Y_t)$  and its first two derivatives are<sup>23</sup>:

$$
K(v) = \sum_{i} PD_i(e^{vLGD_i} - 1) \tag{30}
$$

$$
K'(v) = \sum_{i} PD_i LGD_i e^{vLGD_i}
$$
\n(31)

$$
K''(v) = \sum_{i} PD_i LGD_i^2 e^{vLGD_i}
$$
\n(32)

Let  $\hat{v}$  denote the solution to  $\mathcal{L}(Y_t) = K'(\hat{v})$ . Using the Lugannani-Rice approximation, the formula for the tail of  $\mathcal{L}(Y_t)$  is:

$$
q = \Phi(\omega) - \phi(\omega)\left(\frac{1}{u} - \frac{1}{\omega}\right)
$$
\n(33)

where

$$
\omega = \sqrt{2(\hat{v}\mathcal{L}(Y_t) - K(\hat{v}))} \tag{34}
$$

$$
u = \hat{v}\sqrt{K''(\hat{v})} \tag{35}
$$

and  $\Phi$  and  $\phi$  denote the cdf and pdf of the standard normal distribution respectively.

For the purpose of this paper, we search for  $\mathcal{L}(Y_t)$  corresponding to  $q = 0.99$ . The lower limit of the domain which we need to search in is  $\hat{v} = 0$  corresponding to  $E(\mathcal{L}(Y_t))$ . To determine the upper limit, note that the maximum value for  $K'(\hat{v})$  is the case where every firm defaults, or  $\sum_i LGD_i$ . At the same time, due to the convexity of the exponential function,

$$
\sum_{i} LGD_i \geq \sum_{i} PD_i LGD_i e^{\hat{v} LGD_i} \geq \sum_{i} e^{PD_i LGD_i^2 \hat{v}}
$$
\n(36)

$$
\implies \hat{v} \leq \frac{ln(\sum_{i} LGD_{i})}{PD_{i}LGD_{i}^{2}} \tag{37}
$$

 $^{23}$ As per Gordy (2002), this is a close approximation to the CGF rather than the exact form.

# Appendix C: Top 1% of Firms by Total Liabilities



Table 8: Top  $1\%$  of Firms by Total Liabilities

#	<b>Firm Name</b>	Total Liabilities (USD bn)
23	Credit Suisse Group AG	935
24	Allianz SE	883
25	Bank of Communications Co Ltd	875
26	Goldman Sachs Group Inc/The	845
27	Nordea Bank AB	807
28	Intesa Sanpaolo SpA	797
29	Toronto-Dominion Bank/The	777
30	Royal Bank of Canada	777
31	Commerzbank AG	766
$32\,$	Morgan Stanley	764
33	MetLife Inc	761
34	Banco Bilbao Vizcaya Argentaria SA	757
35	National Australia Bank Ltd	710
36	Natixis	696
37	Prudential Financial Inc	688
38	Bank of Nova Scotia	668
39	Commonwealth Bank of Australia	649
40	Australia & New Zealand Banking Group Ltd	613
41	Westpac Banking Corp	605
42	Standard Chartered PLC	605
43	China Merchants Bank Co Ltd	593
44	Assicurazioni Generali SpA	580
45	Danske Bank $A/S$	567

Table 8: Top  $1\%$  of Firms by Total Liabilities

#	<b>Firm Name</b>	Total Liabilities (USD bn)
46	Industrial Bank Co Ltd	562
47	Shanghai Pudong Development Bank	555
48	Banco do Brasil SA	537
49	General Electric Co	532
50	Schweizerische Nationalbank	524
51	China Citic Bank Corp Ltd	519
52	China Minsheng Banking Corp Ltd	509
53	Ping An Insurance Group Co of China Ltd	492
54	Bank of Montreal	485
55	Prudential PLC	481
56	Sberbank of Russia	460
57	Manulife Financial Corp	457
58	<b>CNP</b> Assurances	448
59	Aegon NV	445
60	American International Group Inc.	441
61	CaixaBank SA	430
62	Resona Holdings Inc	418
63	Itau Unibanco Holding SA	409
64	Nomura Holdings Inc.	403
65	Sumitomo Mitsui Trust Holdings Inc	389
66	DnB ASA	382
67	Skandinaviska Enskilda Banken AB	382
68	Banco Bradesco SA	378

Table 8: Top  $1\%$  of Firms by Total Liabilities

#	<b>Firm Name</b>	Total Liabilities (USD bn)
69	Zurich Insurance Group AG	377
70	Svenska Handelsbanken AB	375
71	Canadian Imperial Bank of Commerce/Canada	364
72	State Bank of India	338
73	Deutsche Boerse AG	337
74	Bank of New York Mellon Corp/The	334
75	US Bancorp/MN	319
76	KBC Groep NV	319
77	Volkswagen AG	318
78	Dexia SA	317
79	Shinkin Central Bank	314
80	Muenchener Rueckversicherungs AG	311
81	Power Corp of Canada	293
82	Power Financial Corp	292
83	<b>CIC</b>	291
84	DBS Group Holdings Ltd	290
85	Ping An Bank Co Ltd	288
86	Woori Finance Holdings Co Ltd	288
87	Great-West Lifeco Inc	284
88	China Life Insurance Co Ltd	281
89	Electricite de France	280
90	Banca Monte dei Paschi di Siena SpA	271
91	Swedbank AB	269

Table 8: Top  $1\%$  of Firms by Total Liabilities

#	<b>Firm Name</b>	Total Liabilities (USD bn)
92	Shinhan Financial Group Co Ltd	268
93	PNC Financial Services Group Inc/The	266
94	Hartford Financial Services Group Inc/The	265
95	Erste Group Bank AG	260
96	Hana Financial Group Inc	257
97	Capital One Financial Corp	248
98	Toyota Motor Corp	247
99	Berkshire Hathaway Inc	247
100	Huaxia Bank Co Ltd	240
101	VTB Bank OJSC	234
102	Oversea-Chinese Banking Corp Ltd	234
103	London Stock Exchange Group PLC	224
104	Lincoln National Corp	217
105	Deutsche Postbank AG	216
106	BOC Hong Kong Holdings Ltd	216
107	Banco de Sabadell SA	215
108	Old Mutual PLC	207
109	United Overseas Bank Ltd	197
110	State Street Corp	197
111	Banco Popular Espanol SA	194
112	Principal Financial Group Inc	192
113	Cathay Financial Holding Co Ltd	191
114	AT&T Inc	188

Table 8: Top  $1\%$  of Firms by Total Liabilities

#	<b>Firm Name</b>	Total Liabilities (USD bn)
115	Petroleo Brasileiro SA	187
116	BlackRock Inc	187
117	Verizon Communications Inc	186
118	PetroChina Co Ltd	186
119	Industrial Bank of Korea	185
120	Swiss Re AG	185
121	Bank of Beijing Co Ltd	185
122	Ford Motor Co	182
123	BP PLC	180
124	Daiwa Securities Group Inc	178
125	Royal Dutch Shell PLC	174
126	Exxon Mobil Corp	172
127	National Bank of Canada	172
128	Sun Life Financial Inc	171
129	Daimler AG	169
130	<b>GDF</b> Suez	167
131	Swiss Life Holding AG	166
132	Tokio Marine Holdings Inc	165
133	Bank of Ireland	165
134	Banco Popolare SC	163
135	Raiffeisen Bank International AG	163
136	Banco Santander Brasil SA/Brazil	161
137	Enel SpA	159

Table 8: Top  $1\%$  of Firms by Total Liabilities

#	<b>Firm Name</b>	Total Liabilities (USD bn)
138	BB&T Corp	159
139	Generali Deutschland Holding AG	159
140	Standard Bank Group Ltd	157
141	<b>SLM</b> Corp	156
142	Unione di Banche Italiane SCPA	154
143	Malayan Banking Bhd	152
144	SunTrust Banks Inc	151
145	MS&AD Insurance Group Holdings	144
146	Allied Irish Banks PLC	143
147	National Bank of Greece SA	141
148	Bayerische Motoren Werke AG	139
149	Softbank Corp	133
150	Macquarie Group Ltd	132
151	Wal-Mart Stores Inc	131
152	Ameriprise Financial Inc	131
153	American Express Co	131
154	T&D Holdings Inc	131
155	Tokyo Electric Power Co Inc	130
156	Total SA	130
157	Charles Schwab Corp/The	130
158	Hang Seng Bank Ltd	130
159	Fukuoka Financial Group Inc	129
160	Telefonica SA	127

Table 8: Top  $1\%$  of Firms by Total Liabilities

#	<b>Firm Name</b>	Total Liabilities (USD bn)
161	E.ON SE	126
162	Sony Corp	125
163	Bank of Yokohama Ltd/The	124
164	Rosneft OAO	124
165	China Petroleum & Chemical Corp	118
166	Ageas	116
167	Piraeus Bank SA	113
168	Banco Espanol de Credito SA	113
169	Deutsche Telekom AG	113
170	Airbus Group NV	111
171	Fifth Third Bancorp	111
172	Gazprom OAO	109
173	Chiba Bank Ltd/The	108
174	Banco Comercial Portugues SA	108
175	Hokuhoku Financial Group Inc	107
176	AMP Ltd	107
177	Comcast Corp	106
178	Aflac Inc	105
179	ENI SpA	105
180	China Pacific Insurance Group Co Ltd	104
181	CIMB Group Holdings Bhd	104
182	Espirito Santo Financial Group SA	103
183	Turkiye Is Bankasi	103

Table 8: Top  $1\%$  of Firms by Total Liabilities

#	Firm Name	Total Liabilities (USD bn)
184	Fiat SpA	103
185	Unipol Gruppo Finanziario SpA	102
186	Chevron Corp	102
187	Allstate Corp/The	102
188	Shizuoka Bank Ltd/The	101
189	Regions Financial Corp	101
190	Eurobank Ergasias SA	101
191	Mitsubishi Corp	100
192	Vodafone Group PLC	99
193	Siemens AG	99
194	Banco Espirito Santo SA	98
195	Bank Hapoalim BM	98
196	International Business Machines Corp	98
197	Wuestenrot & Wuerttembergische AG	98
198	Bank of Baroda	97
199	Bank Leumi Le-Israel BM	96
200	RWE AG	96
201	Korea Electric Power Corp	95
202	Mediobanca SpA	95
203	Hanwha Corp	94
204	Turkiye Garanti Bankasi AS	93
205	Genworth Financial Inc	92
206	Nissan Motor Co Ltd	91

Table 8: Top  $1\%$  of Firms by Total Liabilities

#	Firm Name	Total Liabilities (USD bn)
207	Nippon Telegraph $&$ Telephone Corp	91
208	Honda Motor Co Ltd	90
209	Yamaguchi Financial Group Inc	90
210	Taishin Financial Holding Co Ltd	89
211	Alpha Bank AE	89
212	Mega Financial Holding Co Ltd	88
213	Northern Trust Corp	88
214	American Capital Agency Corp	87
215	China Railway Group Ltd	87

Table 8: Top 1% of Firms by Total Liabilities

### Appendix D: Identifying Sparse Relationships

As discussed, to identify the non-zero entries in  $\gamma$ , we first run a standard OLS and inspect the residual correlation matrix  $\rho$ .

- 1. On row-by-row basis:
	- (a) If the firm corresponding to the row is not in the top 1%, off-diagonal entries in  $\gamma$  are zero.
	- (b) If the firm corresponding to the row is in the top 1%, off-diagonal entries in  $\gamma$  will be those that are non-zero after the following:
		- i. Perform significance filtering on each pairwise entry in the row of  $\rho$  and set insignificant entries to zero at the two-tailed 5% level.
		- ii. Apply SCAD thresholding to the remaining off-diagonal entries.

The SCAD thresholding operator, as defined in Fan et al (2001), is:

$$
\mathcal{T}(\rho_{ij}) = \begin{cases}\nsgn(\rho_{ij})(|\rho_{ij}| - \lambda)_+, & \text{if } |\rho_{ij}| \le 2\lambda \\
\{(a-1)\rho_{ij} - sgn(\rho_{ij})a\lambda\}/(a-2), & \text{if } 2\lambda < |\rho_{ij}| \le a\lambda \\
\rho_{ij}, & \text{if } |\rho_{ij}| > a\lambda\n\end{cases}
$$

The parameters used for SCAD thresholding in our implementation are based on cross-validation. Given the large size of the system, the  $\lambda$  parameter is cross-validated using a sub-sample of 5,000 random firms and equals 0.09 for the baseline case.  $a = 3.7$  as per standard practice. As per Fan et al (2013),  $\lambda$  is selected so as to ensure that the resulting correlation matrix is positive semidefinite. Assumptions applied for SCAD thresholding can be found in Fan et al (2011), and include (1) sparsity of the covariance matrix of  $\{u_t\} := e_t D_{\gamma}^{-1}$ , (2)  $\{u_t\}$  stationary and ergodic, (3) deviations of estimated errors from actual errors are bounded.